Negative feedback amplifier

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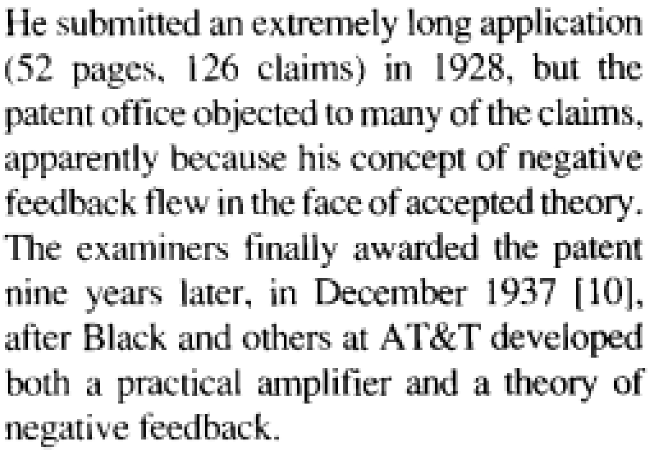
Linearizing effect of feedback

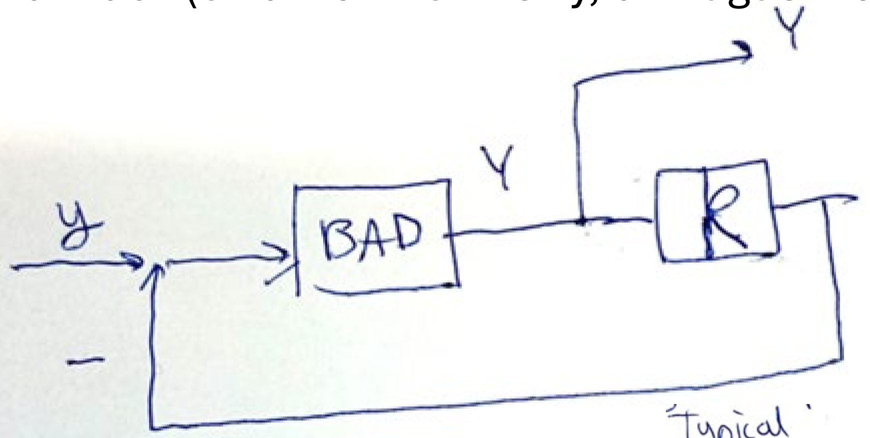
The negative feedback amplifier

- Amplification was required to send telephone signals across the US in the 1920s and it required 12 amplifications on the way, so they better be fairly accurate.
- The original idea of all engineers is to think feedforward (Bell Labs)
- Y = BAD y



He submitted an extremely long application (52 pages, 126 claims) in 1928, but the patent office objected to many of the claims, apparently because his concept of negative feedback flew in the face of accepted theory. The examiners finally awarded the patent nine years later, in December 1937 [10], after Black and others at AT&T developed both a practical amplifier and a theory of negative feedback.

Harold Black (on a New York ferry, 02 August 1927):



Typical:

$BAD \approx 10^4$

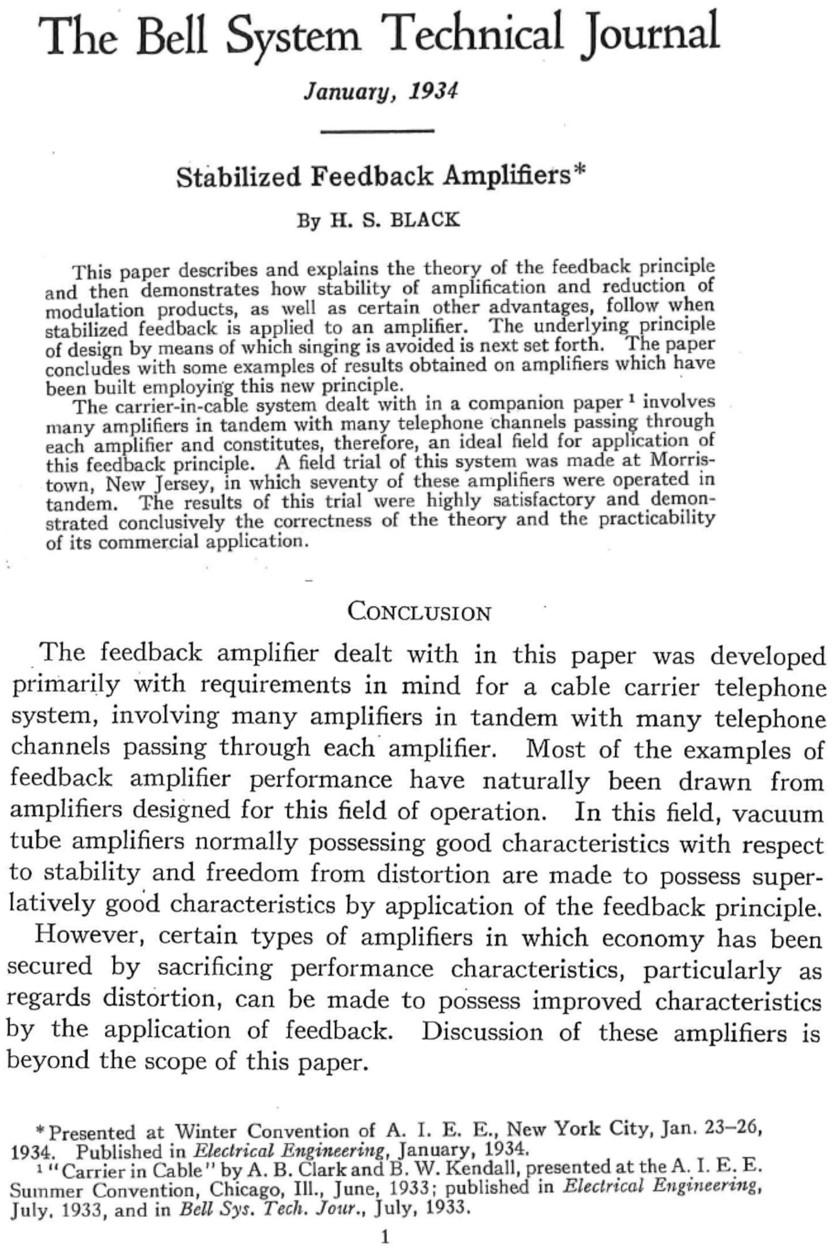
$R \approx 10^{-2}$

$$Y = BAD \cdot (y - R \cdot Y)$$

$$Y = \frac{BAD}{1 + BAD \cdot R} \cdot y$$

Assume $|BAD \cdot R| \gg 1$ then

$$y \approx \frac{1}{R} y$$



The Bell System Technical Journal

January, 1934

Stabilized Feedback Amplifiers*

By H. S. BLACK

This paper describes and explains the theory of the feedback principle and then demonstrates how stability of amplification and reduction of modulation products, as well as certain other advantages, follow when stabilized feedback is applied to an amplifier. The underlying principle of design by means of which singing is avoided is next set forth. The paper concludes with some examples of results obtained on amplifiers which have been built employing this new principle.

The carrier-in-cable system dealt with in a companion paper[1] involves many amplifiers in tandem with many telephone channels passing through each amplifier and constitutes, therefore, an ideal field for application of this feedback principle. A field trial of this system was made at Morristown, New Jersey, in which seventy of these amplifiers were operated in tandem. The results of this trial were highly satisfactory and demonstrated conclusively the correctness of the theory and the practicability of its commercial application.

CONCLUSION

The feedback amplifier dealt with in this paper was developed primarily with requirements in mind for a cable carrier telephone system, involving many amplifiers in tandem with many telephone channels passing through each amplifier. Most of the examples of feedback amplifier performance have naturally been drawn from amplifiers designed for this field of operation. In this field, vacuum tube amplifiers normally possessing good characteristics with respect to stability and freedom from distortion are made to possess superlatively good characteristics by application of the feedback principle.

However, certain types of amplifiers in which economy has been secured by sacrificing performance characteristics, particularly as regards distortion, can be made to possess improved characteristics by the application of feedback. Discussion of these amplifiers is beyond the scope of this paper.

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* Presented at Winter Convention of A. I. E. E., New York City, Jan. 23–26, 1934. Published in *Electrical Engineering*, January, 1934.

[1] "Carrier in Cable" by A. B. Clark and B. W. Kendall, presented at the A. I. E. E. Summer Convention, Chicago, Ill., June, 1933; published in *Electrical Engineering*, July, 1933, and in *Bell Sys. Tech. Jour.*, July, 1933.

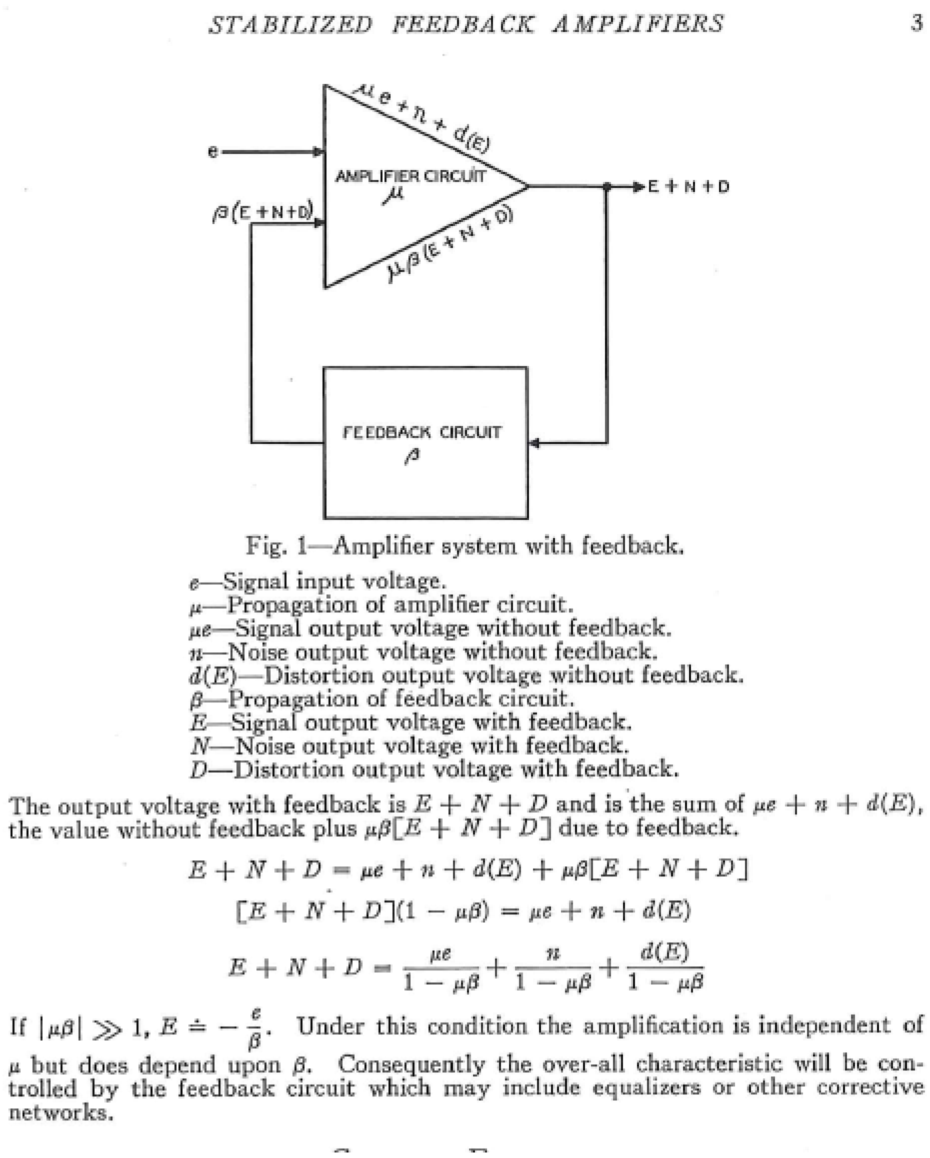


Fig. 1—Amplifier system with feedback.

e—Signal input voltage.
μ—Propagation of amplifier circuit.
μe—Signal output voltage without feedback.
n—Noise output voltage without feedback.
$d(E)$—Distortion output voltage without feedback.
β—Propagation of feedback circuit.
E—Signal output voltage with feedback.
N—Noise output voltage with feedback.
D—Distortion output voltage with feedback.

The output voltage with feedback is $E + N + D$ and is the sum of $\mu e + n + d(E)$, the value without feedback plus $\mu\beta[E + N + D]$ due to feedback.

$$E + N + D = \mu e + n + d(E) + \mu\beta[E + N + D]$$

$$[E + N + D](1 - \mu\beta) = \mu e + n + d(E)$$

$$E + N + D = \frac{\mu e}{1 - \mu\beta} + \frac{n}{1 - \mu\beta} + \frac{d(E)}{1 - \mu\beta}$$

If $|\mu\beta| \gg 1$, $E \doteq -\frac{e}{\beta}$. Under this condition the amplification is independent of μ but does depend upon β. Consequently the over-all characteristic will be controlled by the feedback circuit which may include equalizers or other corrective networks.

Experiment

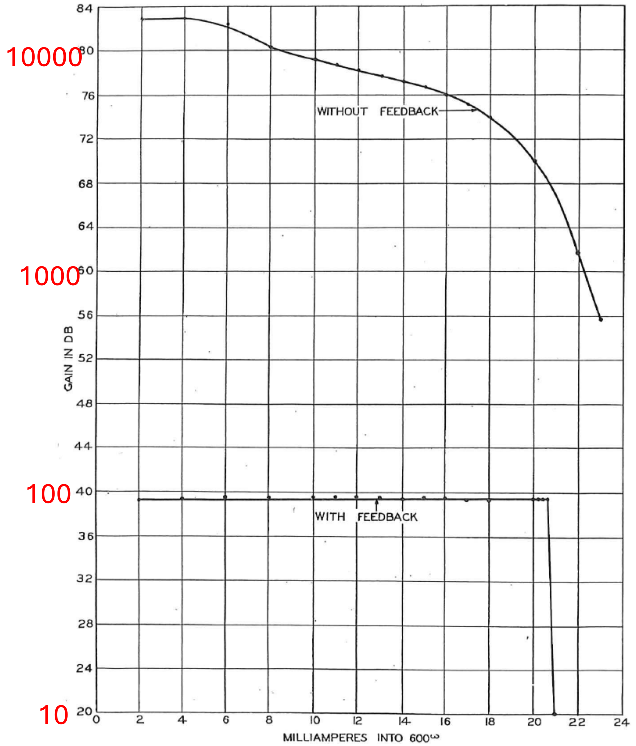


Fig. 10—Gain-load characteristic with and without feedback for a low level amplifier designed to amplify frequencies from 3.5 to 50 kc.

Bad amplifier:

$$\mu \approx 10000$$

Accurate resistance:

$$\beta \approx 0.01$$

Get

$$\mu\beta \approx 100 \gg 1$$

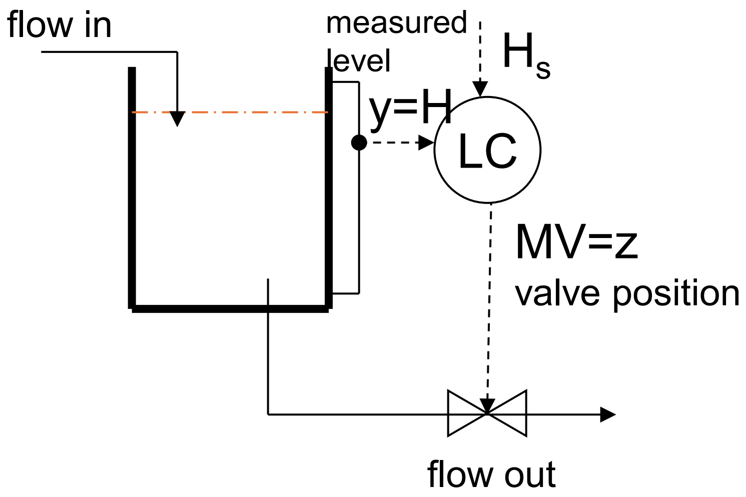
Resulting amplification (negative feedback)

$$\frac{\mu}{1 + \mu\beta} \approx \frac{1}{\beta} = 100$$

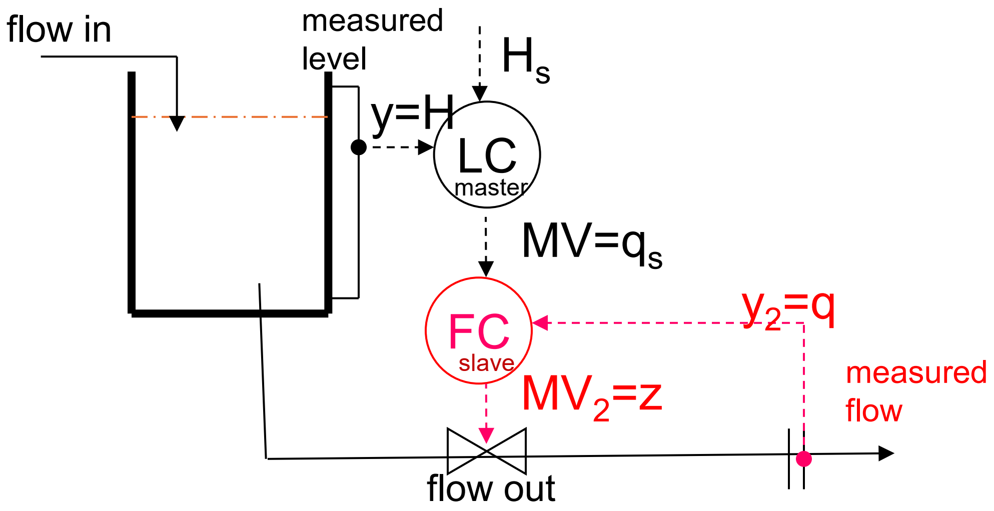
Linearization of valve using cascade control

- Benefits: 1. Local distrurbance rejection, 2. Linearization
- Does nonlinearity disappear?

WITHOUT CASCADE



WITH CASCADE (2 controllers)



No, it moves to the time constant for slave loop
– OK if we we have time scale separation between master and slave

Nonlinear valve with varying gain k_2: $G_2(s) = k_2(z) / (\tau_2 s+1)$

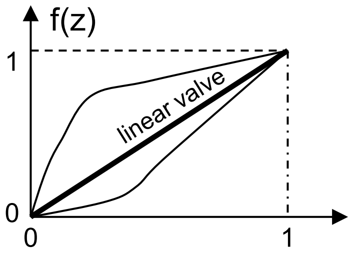
- Slave (flow) controller K_2: PI-controller with gain K_{c2} and integral time $\tau_I = \tau_2$ (SIMC-rule). Get

$$L_2 = K_2(s)G_2(s) = \frac{K_{c2}k_2}{\tau_2 s}$$

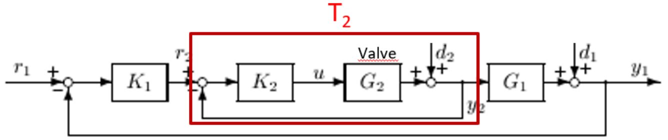
- With slave controller: Transfer function T_2 from y_{2s} to y_2 (as seen from master loop):

$$T_2 = L_2/(1+L_2) = 1/(\tau_{C2}\, s + 1), \text{ where } \tau_{C2} = \tau_2/(k_2\, K_{c2})$$

- Linearization: Gain for T_2 is always 1 (independent of k_2) because of intergal action in the inner (slave) loop
- But: Gain variation in k_2 (inner loop) translates into variation in closed-loop time constant τ_{C2}. This may effect the master loop



$k_2(z)$ = slope = df/dz



G_1T_2 = «Process» for tuning master controller K_1