KTH VETENSKAP OCH KONST

14/9-2016

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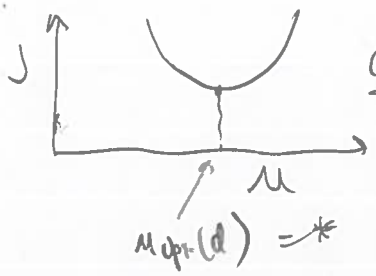
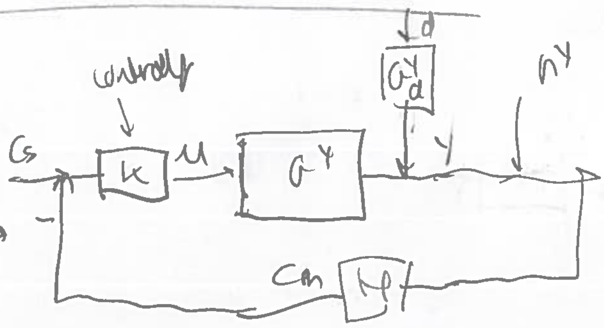
Lecture notes for Ch. 10.1-10.5

Derivation of "exact local method"

Given:





1. **Assumptions**:
   1. Steady state cost $J(u,d)$
   2. $J(u)$ is quadratic
   3. Linear measurement model
      $y = G^y u + G_d^y d$ (in deviation variables $\Delta u, \Delta y, \Delta d$)

Note: $c = Hy$

$$\frac{\Delta y_{opt}}{\Delta d} = F \quad \text{(see also below)}$$

$$\frac{\Delta c_{opt}}{\Delta d} = HF$$

2. **J(u) is quadratic**; For given d

$$J(u,d) = J(u_{opt},d) + \underbrace{J_u^T}_{=0 \text{ (around } u_{opt})} (u-u_{opt}) + \frac{1}{2}(u-u_{opt})^T J_{uu} (u-u_{opt})$$

1. **Comments**

4. ~~3~~ Analytic Expression for F

Optimal input: Keep gradient $J_u = 0$ for any d.

For small changes, Taylor expansion:

$$\underbrace{J_u}_{0} = \underbrace{J_u^*}_{0 \text{ (at nom.)}} + J_{uu}^* \Delta u_{opt}(d) + J_{ud}\,\Delta d$$

$$\Rightarrow \Delta u_{opt}(d) = -J_{uu}^{-1} J_{ud}\,\Delta d$$

$$\Rightarrow \Delta y_{opt} = G^y \Delta u_{opt} + G_d^y \Delta d = \underbrace{(-G^y J_{uu}^{-1} J_{ud} + G_d^y)}_{F}\,\Delta d$$

Comment: Often easier to find F by reoptimizing numerically: $F = \frac{\Delta y_{opt}}{\Delta d}$

3.4. Evaluation of loss

$$Loss = J(u,d) - J(u_{opt}(d),d) = \frac{1}{2} z^T z = \frac{1}{2}\|z\|_2^2$$

where $z = J_{uu}^{1/2}(u - u_{opt})$

Want to express z as a function of d and n^y

We have:

$$c = \overbrace{(HG^y)}^{G} u + \overbrace{(HG_d^y)}^{G_d} d$$

$$c_{opt} = (HG^y) u_{opt} + (HG_d^y) d$$

$$c - c_{opt} = HG^y (u - u_{opt})$$

$$\Rightarrow \boxed{u - u_{opt} = (HG^y)^{-1}(c - c_{opt})}$$

(a) Here c is controlled at setpoint $c_s = 0$ (assuming perfect control at steady-state)

so $c_m = c_s = 0$

Also $c_m = Hy_m = H(y + n^y) = \underbrace{Hy}_{= c} + Hn^y$

$$\Rightarrow \boxed{c = -Hn^y}$$

(b) And $\boxed{c_{opt} = Hy_{opt} = HFd}$

so $c - c_{opt} = -Hn^y - HFd$

and $z = J_{uu}^{1/2}(HG^y)^{-1}(-HFd - Hn^y)$

$$= \underbrace{J_{uu}^{1/2}(HG^y)^{-1} H \underbrace{[F W_d \;\; W_{n^y}]}_{\tilde{F}}}_{M} \begin{bmatrix} -d' \\ -n^{y'} \end{bmatrix}$$

W_d, W_{n^y}: weights giving magnitude of d and n^y

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(3)

Assume normalized disturbance and noise (normally ... is similar)

$$\left\|\begin{pmatrix} d' \\ n^y \end{pmatrix}\right\|_2 \leq 1$$

Both ± allowed so sign(·) does not matter)

1. Average loss (expected) for $\begin{pmatrix} d' \\ n^y \end{pmatrix} \sim N(0,1)$, Loss $= \frac{1}{2}\|M\|_F^2$

2. Worst-case loss for $\left\|\begin{pmatrix} d' \\ n^y \end{pmatrix}\right\|_2 \leq 1$

$$\text{Loss} = \frac{1}{2}\bar{\sigma}(M(H))^2$$

Both these have the same solution

$$\min_H \|M(H)\|_F$$

where $M = J_{uu}^{1/2}(HG^y)^{-1}HY$

Convex reformulation (Sigurd)

$$\min_H \|HY\|_F \quad \text{s.t.} \quad HG^y = J_{uu}^{1/2}$$

Analytic solution for "full" M (Vidar)

$$H^T = (YY^T)^{-1}G^y$$

← Exact local method

Notes: J_{uu} drops out for "full" M

4. Comment in here

$$F = (-G^y J_{uu}^{-1} J_{ud} + G^y_d)$$