2. Self-optimizing control Theory

Sigurd Skogestad

Outline

Skogestad procedure for control structure design:

I. Top Down
   - Step S1: Define operational objective (cost) and constraints
   - Step S2: Identify degrees of freedom and optimize operation for disturbances
   - Step S3: Implementation of optimal operation
     - Control active constraints
     - Control self-optimizing variables for unconstrained, c=Hy
   - Step S4: Where set the production rate? (Inventory control)

II. Bottom Up
   - Step S5: Regulatory control: What more to control (secondary CV's)?
   - Step S6: Supervisory control
   - Step S7: Real-time optimization

Step S3: Implementation of optimal operation

- Optimal operation for given d*:

$$\min_{u} J(u,x,d)$$

subject to:

Model equations: $f(u,x,d) = 0$

Operational constraints: $g(u,x,d) < 0$

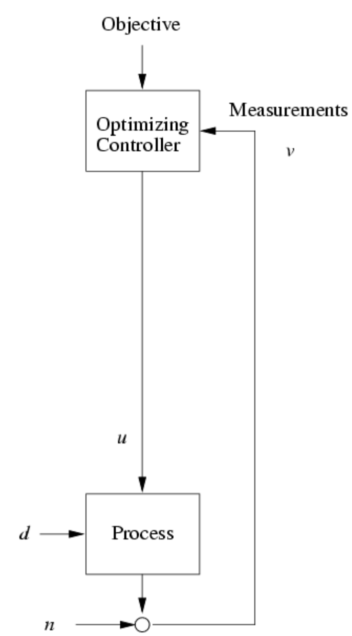
$$\rightarrow u_{opt}(d)$$

Problem: Usally cannot keep u_{opt} constant because disturbances d change

How should we adjust the degrees of freedom (u)?

What should we control?

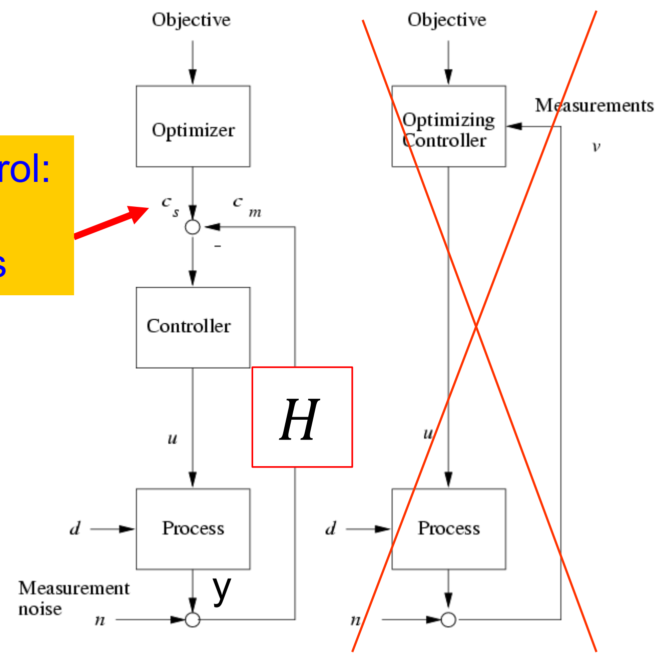
"Optimizing Control"



“Self-Optimizing Control”

Self-optimizing control:
Constant setpoints
give acceptable loss

What should we control?
(What is c? What is H?)



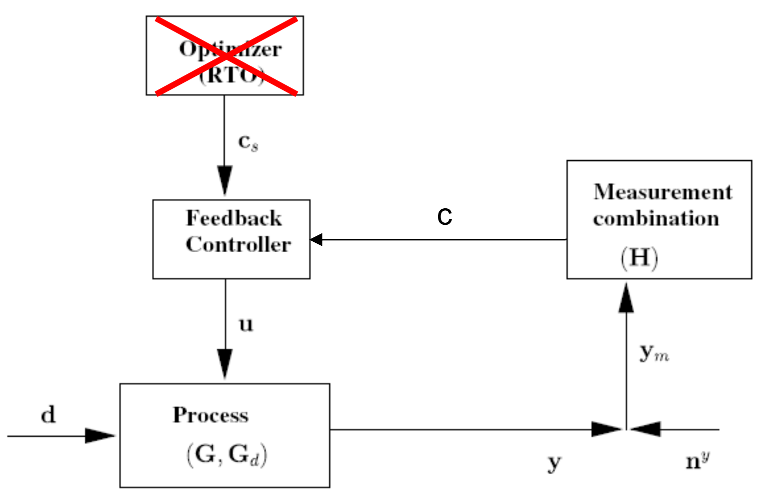
$c = Hy$

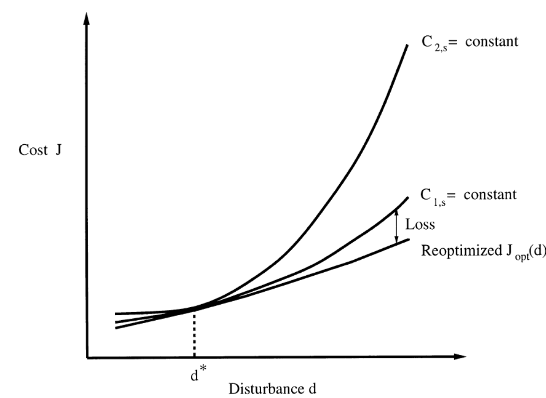
H: Nonsquare matrix

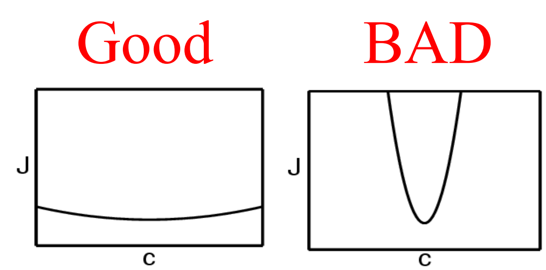
- Usually selection matrix of 0’s and some 1’s (measurement selection)
- Can also be full matrix (measurement combinations)

Self-optimizing control

Self-optimizing control is when we can achieve an acceptable loss with constant setpoint values for the controlled variables



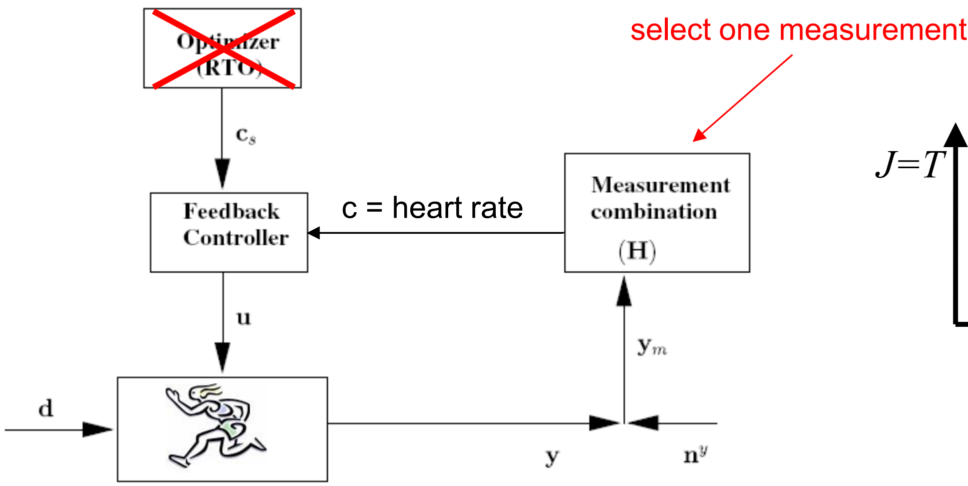


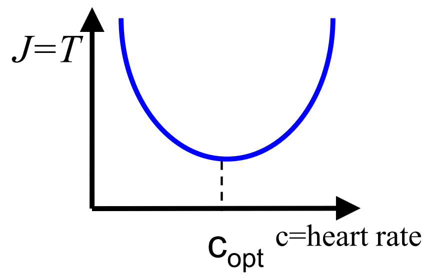


(b) Flat optimum: Implementation easy

(c) Sharp optimum: Sensitive to implementation erros

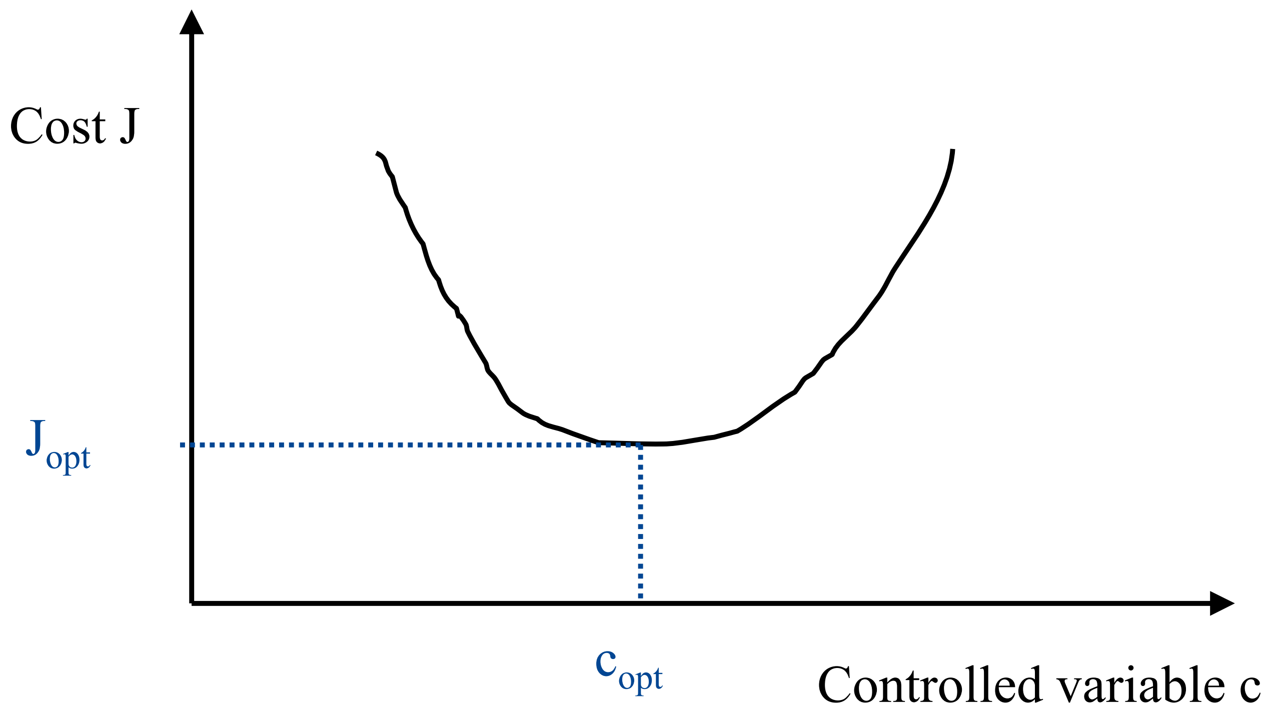
Recap: marathon runner





- CV = heart rate is good “self-optimizing” variable
- Simple and robust implementation
- Disturbances are indirectly handled by keeping a constant heart rate
- <u>May</u> have infrequent adjustment of setpoint (c_s)

Optimal operation

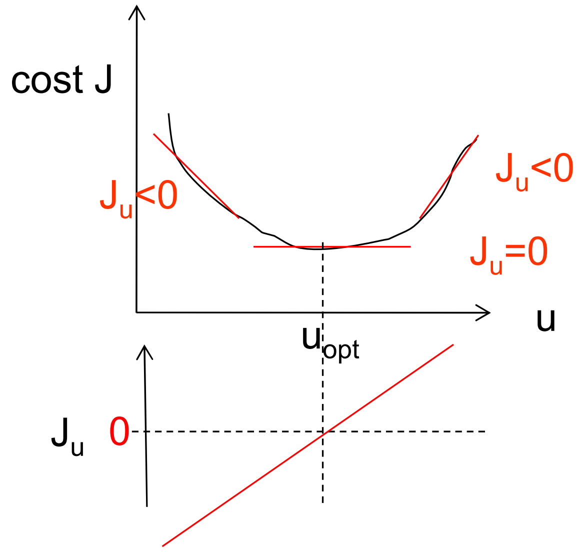


Unconstrained degrees of freedom

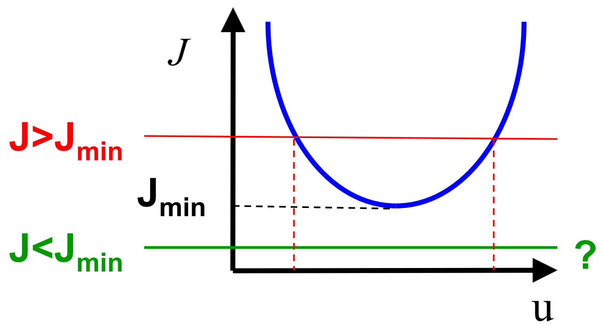
The ideal "self-optimizing" variable is the gradient, J_u

$$c = \Delta J/\Delta u = Ju$$

– Keep gradient at zero for all disturbances ($c = J_u = 0$)
– Problem: Usually no measurement of gradient



*I.J. Halvorsen, S. Skogestad, Indirect on-line optimization through setpoint control, in: AIChE 1997 Annual Meeting, Los Angeles; paper 194h.
*I.J. Halvorsen, S. Skogestad, J.C. Morud, V. Alstad, Optimal selection of controlled variables, Industrial & Engineering Chemistry Research 42 (14) (2003) 3273–3284



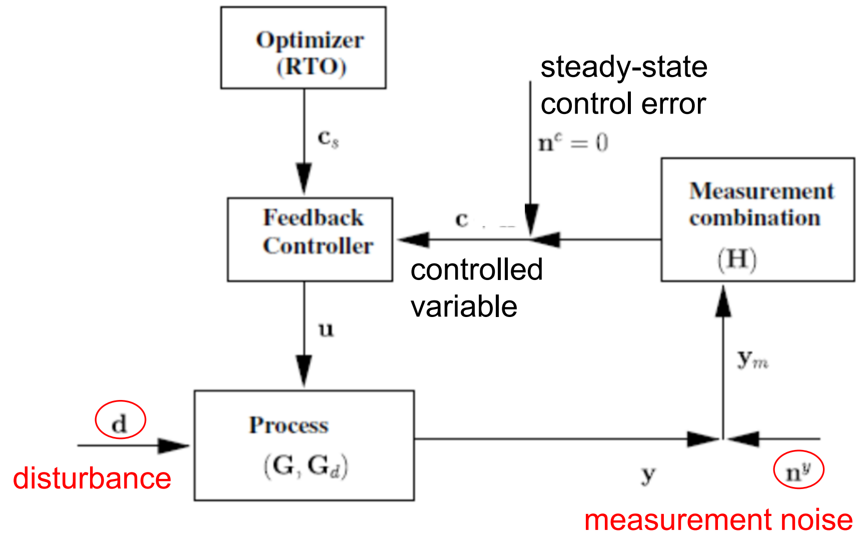
Unconstrained optimum: NEVER try to control a variable that reaches max or min at the optimum

- In particular, never try to control directly the cost J
- Assume we want to minimize J (e.g., J = V = energy) - and we make the stupid choice os selecting CV = V = J
  - **Then setting $J < J_{min}$: Gives infeasible operation (cannot meet constraints)**
  - **and setting $J > J_{min}$: Forces us to be nonoptimal (two steady states: may require strange operation)**

Measurements or mesurement combinations

Ideally: c = J_u

In practice: c = Hy



- Single measurements:

$$\mathbf{c} = \mathbf{H}\mathbf{y} \qquad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

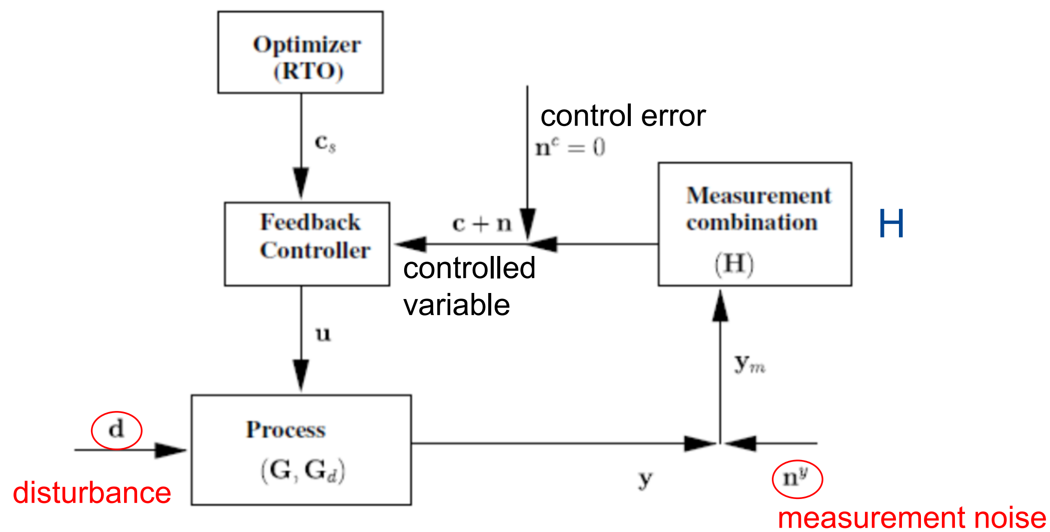
- Combinations of measurements:

$$\mathbf{c} = \mathbf{H}\mathbf{y} \qquad \mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{bmatrix}$$

Optimal measurement combination

$$\Delta c = h_1 \Delta y_1 + h_2 \Delta y_2 + \cdots = H \Delta y$$

- Candidate measurements (y): Include also inputs u



No measurement noise ($n^y=0$)

Nullspace method

Theorem

Given a sufficient number of measurements ($n_y \geq n_u + n_d$) and no measurement noise, select **H** such that

$$\mathbf{HF} = 0$$

where

$$\mathbf{F} = \frac{\partial \mathbf{y}^{opt}}{\partial \mathbf{d}}$$

Controlling $\mathbf{c} = \mathbf{Hy}$ to zero yields locally zero loss from optimal operation.

Proof: Given $\partial y^{opt} = F\,\partial d$, and $c = Hy$:

$\partial c^{opt} = H\,\partial y^{opt} = HF\,\partial d$

To make $\partial c^{opt} = 0$ for any ∂d, we must have $HF = 0$.

V. Alstad, S. Skogestad, Null space method for selecting optimal measurement combinations as controlled variables, Industrial & Engineering Chemistry Research 46 (2007) 846–853.
Jäschke, J., Cao, Y., & Kariwala, V. (2017). Self-optimizing control–A survey. Annual Reviews in Control, 43, 199–223.

Nullspace method gives J_u=0

Proof:

$$J_u = J_{uu}\Delta u + J_{ud}\Delta d = [J_{uu}\ J_{ud}]\begin{bmatrix}\Delta u\\ \Delta d\end{bmatrix}$$

$$\Delta y = \begin{bmatrix}G^y\ G_d^y\end{bmatrix}\begin{bmatrix}\Delta u\\ \Delta d\end{bmatrix} = \tilde{G}_y\begin{bmatrix}\Delta u\\ \Delta d\end{bmatrix} \rightarrow \begin{bmatrix}\Delta u\\ \Delta d\end{bmatrix} = \tilde{G}_y^{+}\Delta y$$

Formula for F:

$$J_u^{opt} = J_{uu}\Delta u^{opt} + J_{ud}\Delta d = 0 \rightarrow \Delta u^{opt} = -J_{uu}^{-1}J_{ud}\Delta d$$

$$\Delta y^{opt} = \tilde{G}_y\begin{bmatrix}\Delta u^{opt}\\ \Delta d\end{bmatrix} = \tilde{G}_y\begin{bmatrix}-J_{uu}^{-1}J_{ud}\\ I\end{bmatrix}\Delta d$$

$$\rightarrow F = \tilde{G}_y\begin{bmatrix}-J_{uu}^{-1}J_{ud}\\ I\end{bmatrix}$$

Let $H = [J_{uu}\ J_{ud}]\tilde{G}_y^{+}$. We can verify that $HF = 0$. Therefore, $J_u = [J_{uu}\ J_{ud}]\tilde{G}_y^{+}\Delta y = H\Delta y = \Delta c$, and thus controlling c ($\Delta c = 0$) leads to $J_u = 0$.

- Proof. Appendix B in: Jäschke and Skogestad, "NCO tracking and self-optimizing control in the context of real-time optimization", *Journal of Process Control*, 1407-1416 (2011)

Nullspace method (HF=0) gives J_u=0

Proof (constant d):

$$J_u(u,d) = \underbrace{J_u(u_{opt}(d),d)}_{=0} + J_{uu} \cdot (u - u_{opt})$$

$u - u_{opt} = (HG^y)^{-1}(c - c_{opt})$

Here: $c - c_{opt} = \Delta c - \Delta c_{opt}$

where we have introduced deviation variables around a nominal optimal point (c^*, d^*) (where $c^* = c_{opt}(d^*)$)

Assume perfect control of c (no noise): $\Delta c = 0$

Optimal change: $\Delta c_{opt} = H \Delta y_{opt} = HF \Delta d$

Gives: $J_u = -J_{uu}(HG^y)^{-1} HF \Delta d$

$\Rightarrow HF = 0$ gives $J_u = 0$ for any disturbance Δd

Proof. Appendix B in: Jäschke and Skogestad, ”NCO tracking and self-optimizing control in the context of real-time optimization”, *Journal of Process Control*, 1407-1416 (2011)

Example. Nullspace Method for Marathon runner

u = power, d = slope [degrees]

y_1 = hr [beat/min], y_2 = v [m/s]

F = dy_{opt}/dd = $\begin{bmatrix} 0.25 \\ -0.2 \end{bmatrix}$

H = [h_1 h_2]

HF = 0 → $h_1 f_1 + h_2 f_2 = 0.25 h_1 - 0.2 h_2 = 0$

Choose $h_1 = 1$ → $h_2 = 0.25/0.2 = 1.25$

Conclusion: **c = hr + 1.25 v**

Control **c = constant** → hr increases when v decreases (OK uphill!)

Marathon runner: Exact local method

$$F = \begin{bmatrix} 0.25 \\ -0.2 \end{bmatrix}, W_d = 1, W_{ny} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, G^y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Y = [FW_d \quad W_{ny}] = \begin{bmatrix} 0.25 & 1 & 0 \\ -0.2 & 0 & 1 \end{bmatrix}$$

$$H = G^{y^T}(Y\,Y^T)^{-1} \rightarrow H = [0.989 \quad 1.009]$$

Normalized H1 = D*H = [1 1.02]

Conclusion: **c = hr + 1.02 v**

- Before (nullspace method): **c = hr + 1.25 v**
- Note: Gives same as nullspace when W_{ny} is small

Extension: "Exact local method" (with measurement noise)

$$\min_H \| J_{uu}^{1/2} (HG^y)^{-1} H \underbrace{[FW_d \quad W_{n^y}]}_{Y} \|_F$$

- General analytical solution ("full" H):

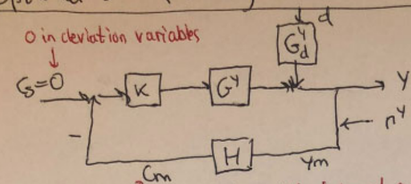
$$H = G^{yT} (YY^T)^{-1}$$

- No disturbances (W_d= [ ]) + same noise for all measurements (W_{ny}= Y = I):
  Optimal is H=G^{yT} ("control sensitive measurements")
  - Proof: Use analytic expression
- No noise (W_{ny}=0): Cannot use analytic expression, but optimal is clearly
  HF = 0 (Nullspace method)
  - Assumes enough measurements: #y ≥ #u + #d
  - If "extra" measurements (>) then solution is <u>not unique</u>

V. Alstad, S. Skogestad, E.S. Hori, Optimal measurement combinations as controlled variables, Journal of Process Control 19 (1) (2009) 138–148.
Jäschke, J., Cao, Y., & Kariwala, V. (2017). Self-optimizing control–A survey. Annual Reviews in Control, 43, 199–223.

Optimal self-optimizing measurement combination, H

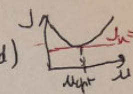
28/8-23
SiS



Expected:
$d = W_d d'$
$n^y = W_n n^{y'}$
Expected magnitude = 1
$\left(\left\| \begin{matrix} d' \\ n^{y'} \end{matrix} \right\|_2 \le 1 \right)$ ← 2-norm

$c_m = 0$ at steady with integral action in K.

Question: What to control, $c = Hy$. What is best H?



Steady-state: Economic loss L for given d is caused by $u \ne u_{opt}(d)$

Taylor expansion of loss around optimum:

$$L = J(u,d) - J(u_{opt}(d), d) = J_u^T (u - u_{opt}) + \frac{1}{2} (u - u_{opt})^T J_{uu} (u - u_{opt}) + \text{higher-order (neglected)}$$

$J_u = 0$: Gradient $J_u = \frac{dJ}{du} = 0$ at optimum.

J_{uu}: Hessian of J, $J_{uu} = \frac{d^2 J}{du^2}$

Here: $c - c_{opt} = H(y - y_{opt}) = HG^y (u - u_{opt})$

$$\Rightarrow (u - u_{opt}) = (HG^y)^{-1} (c - c_{opt})$$

So get $L = \frac{1}{2} z^T z$ where $z = J_{uu}^{1/2} (u - u_{opt}) = \boxed{J_{uu}^{1/2} (HG^y)^{-1} (c - c_{opt})}$

L depends on d and n^y (it's = 0 otherwise)

1. Disturbances change u_{opt} (and y_{opt} and c_{opt}):
Optimal sensitivity $F = \frac{dy_{opt}}{dd} \Rightarrow y_{opt} = Fd \Rightarrow c_{opt} = HFd = HFW_d d'$
(deviation variables)

Note: HF=0 -> copt=0 (Nullspace method)

2. Noise makes $c \ne 0$:
$c_m = 0$ (integral action in K) $\Rightarrow c_m = Hy_m = H(y + n^y) = 0 \Rightarrow c = Hy = -Hn^y = +HW_n^y n^{y'}$
(change from – to + because sign of $n^{y'}$ does not matter.)

So get:

$$z = \underbrace{J_{uu}^{1/2} (HG^y)^{-1} H \overbrace{[FW_d \;\; W_{n^y}]}^{Y}}_{M(H)} \begin{bmatrix} d' \\ n^{y'} \end{bmatrix} \quad (*)$$

Average loss for $N\begin{pmatrix} d' \\ n^{y'} \end{pmatrix} = \{0, 1\}$ (average, variance) is $L_{avg} = \frac{1}{2} \|M\|_F^2$, F = Frobenius norm $= \sqrt{\sum M_{ij}^2}$

Worst-case loss for $\left\| \begin{matrix} d' \\ n^{y'} \end{matrix} \right\|_2 \le 1$ (2-norm of vector) is $L_{wc} = \frac{1}{2} \bar{\sigma}(M)^2$, $\bar{\sigma}$ = singular value

→ Both cases: Analytical formula for optimal H is $H^T = (YY^T)^{-1} G^y$ (exact local method)

Special case with no noise, $W_{n^y} = 0$ get $L = 0$ with $HF = 0$ (nullspace method)

Obtaining F

F is defined as the gain matrix from the disturbances to the optimal measurements → $\Delta y^{opt} = F\,\Delta d$

Brute force method:

- For every disturbance $d_i,\ i = 1, \dots, n_d$:
  - Perturb the system with $\hat{d}_i = d_i + \Delta d_i$, Δd_i small
  - Reoptimize the system → obtain change in measurements $\Delta y^{opt,i}$
  - Obtain i-th column of F: $F_i = \Delta y^{opt,i}/\Delta d_i$
- Return F

Linearization method for F

F can also be obtained through a linearized state-space model:

$$\Delta y = G^y \Delta u + G_d^y \Delta d$$

$$J_u(u^* + \Delta u, d^* + \Delta d) \approx J_u^* + J_{uu}\Delta u + J_{ud}\Delta d = 0$$
$$\Rightarrow \Delta u^{opt} = -J_{uu}^{-1} J_{ud} \Delta d$$

0

$$\Delta y^{opt} = G^y \Delta u^{opt} + G_d^y \Delta d = \left(-G^y J_{uu}^{-1} J_{ud} + G_d^y\right) \Delta d$$

$$F = -G^y J_{uu}^{-1} J_{ud} + G_d^y$$

Toy Example.

$J = (u - d)^2$

$n_u = 1$ unconstrained degrees of freedom

$u_{\text{opt}} = d$

Alternative measurements:

$y_1 = 0.1(u - d)$

$y_2 = 20u$

$y_3 = 10u - 5d$

$y_4 = u$

Scaled such that:

$|d| \leq 1$, $|n_i| \leq 1|$, i.e. all y_i's are ± 1

Nominal operating point:

$d = 0 \Rightarrow u_{\text{opt}} = 0, y_{\text{opt}} = 0$

What variable c should we control?

Single measurements

$$L_{wc} = \frac{1}{2}\ \bar{\sigma}(M)^2$$

$$M = J_{uu}^{\frac{1}{2}}(HG^y)^{-1}H\,Y,$$

$$Y = \left[FW_d\ W_{ny}\right], F = -G^y J_{uu}^{-1} J_{ud} + G_d^y$$

. Exact evaluation of loss:

$L_{wc,1} = 100$

$L_{wc,2} = 1.0025$

$L_{wc,3} = 0.26$

$L_{wc,4} = 2$

Here: $W_d = 1, W_{ny} = 1,\ J_{uu} = 2, J_{ud} = -2,$

For y_1: $HG^y = 0.1, HG_d^y = -0.1, F = 0, HY = [0\ \ 1], M = \sqrt{2} \cdot 10 \cdot [0\ 1], L_{wc} = \frac{1}{2}\ \bar{\sigma}(M)^2$ =100

For y_2: $HG^y = 20, HG_d^y = 0, F = 20, HY = [20\ \ 1], M = \sqrt{2} \cdot \frac{1}{20} \cdot [20\ 1], L_{wc} = \frac{1}{2}\ \bar{\sigma}(M)^2 = 1.0025$

For y_3: $HG^y = 10, HG_d^y = -5, F = -15, HY = [5\ \ 1], M = \sqrt{2} \cdot \frac{1}{10} \cdot [5\ \ 1], L_{wc} = \frac{1}{2}\ \bar{\sigma}(M)^2 = 0.26$

Reference: I. J. Halvorsen, S. Skogestad, J. Morud and V. Alstad, "Optimal selection of controlled variables", Industrial & Engineering Chemistry Research, 42 (14), 3273-3284 (2003).

Toy Example. Exact local method. Combine all measurements

$J = (u-d)^2$
$n_u = 1$ unconstrained degrees of freedom
$u_{\text{opt}} = d$

Alternative measurements:

$y_1 = 0.1(u-d)$
$y_2 = 20u$
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Scaled such that:

$|d| \leq 1$, $|n_i| \leq 1|$, i.e. all y_i's are ± 1

Nominal operating point:

$d = 0 \Rightarrow u_{\text{opt}} = 0, y_{\text{opt}} = 0$

What variable c should we control?

$$Y = [F W_d \; W_{ny}],$$
$$F = -G^y J_{uu}^{-1} J_{ud} + G_d^y$$
$$H = (Y Y^T)^{-1} \; G^y$$

Here: $W_d = 1, W_{ny} = I \; (4x4),\; J_{uu} = 2, J_{ud} = -2,$
$G^y = [0.1 \;\; 20 \;\; 10 \;\; 1]',\; G_d^y = [-0.1 \;\; 0 \;\; -5 \;\; 0]',$
$F = [0 \;\; 20 \;\; 5 \;\; 1]',$

$$Y = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 20 & 0 & 1 & 0 & 0 \\ 5 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$(Y Y^T)^{-1} =$

1.0000	0	0	0
0	0.0632	-0.2342	-0.0468
0	-0.2342	0.9415	-0.0117
0	-0.0468	-0.0117	0.9977

$H = (Y Y^T)^{-1} \; G^y = [0.1000 \;\; -1.1241 \;\; 4.7190 \;\; -0.0562]$

Normalized to have 2-norm = 1.

$H = [\,0.0206 \;\; -0.2317 \;\; 0.9725 \;\; -0.0116]$

Reference: V. Alstad et al. , Journal of Process Control 19 (2009) 138–148

Toy Example: Nullspace method (not unique)

$$c = Hy = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = h_1 y_1 + h_2 y_2 + h_3 y_3 + h_4 y_4$$

B1. Nullspace method

Neglect measurement error ($n = 0$):

$$HF = 0$$

Sensitivity matrix

$$\Delta y_{\text{opt}} = F \Delta d; F = \begin{pmatrix} 0 & 20 & 5 & 1 \end{pmatrix}^T$$

To find H that satisfies $HF = 0$ must combine at least two measurements:

$$n_y \geq n_u + n_d = 1 + 1 = 2$$

Toy Example. Nullspace method with 2 measurements

C. Optimal combination

Need two measurements. Best combination is y_2 and y_3:

$$\begin{pmatrix} y_2 \\ y3 \end{pmatrix} = \begin{pmatrix} 20 & 0 \\ 10 & -5 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}; \; \underline{\sigma} = 4.45$$

Optimal sensitivity:

$$y_{\text{opt}} = Fd; F = \begin{pmatrix} 20 \\ 5 \end{pmatrix}$$

Optimal combination:

$$HF = 0 \Rightarrow \begin{pmatrix} h_1 & h_2 \end{pmatrix} \begin{pmatrix} 20 \\ 5 \end{pmatrix} = 0 \Rightarrow 20h_1 + 5h_2 = 0$$

Select $h_1 = 1$. Get $h_2 = -20h_1/5 = -4$, so

$$c_{\text{opt}} = y_2 - 4y_3$$

Check: $c = y_2 - 4y_3 = 20u - 40u + 20d = -20(u - d)$ *(OK!)*

Example where nullspace method «fails»

u= reflux
d=feed rate

J = $(u-d)^2$
y1 = 0.01(u-d)   % temperature product (very small gain!)
y2 = u-0.8d % tempereture inside column
uopt = d
y1opt = 0
y2opt = 0.2 d

Nullspace: H0=[1 0]   % Not good! Use only y1
Exact local method: H=[1 96] % Use y2 instead

```
F =[0 0.2]'
Wd=1*eye(1)
Wn=1*eye(2)
Gy = [0.01 1]'
H0=null(F'); H0=H0'/H0(1)  % nullspace method
Y = [F*Wd Wn],
H1 = Gy' * inv(Y * Y')
H = H1/H1(1)   % exact local method
```

Conclusion: GOOD "SELF-OPTIMIZING" CV = c

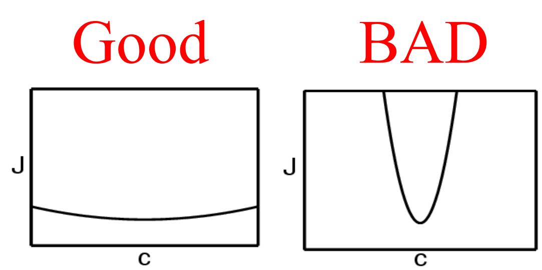
1. Optimal value c_{opt} is constant (independent of disturbance d):

→ Want small optimal sensitivity: $F_c = \frac{\Delta c_{opt}}{\Delta d} = \mathrm{HF}$

2. c is "sensitive" to input u (MV) (to reduce effect of measurement noise)

→ Want large gain $G = HG^y = \frac{\Delta c}{\Delta u}$

(Equivalently: Optimum should be flat!)



(b) Flat optimum: Implementation easy

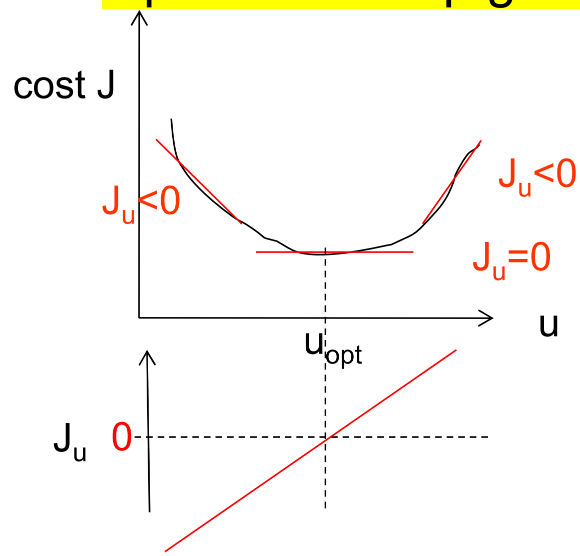
(c) Sharp optimum: Sensitive to implementation erros

New 2024: Optimal steady-state operation using gradient estimate

$\min_u J(u,d)$

s.t. $g(u,d) \geq 0$ (constraints)

- J = economic cost [$/s]
- Unconstrained case: Optimal to keep gradient $J_u = \partial J/\partial u = 0$



- Constrained case: KKT-conditions:

$$g_A = 0,$$
$$L_u = J_u + \lambda^T g_u = 0$$

Want tight control of active constraints for economic reasons

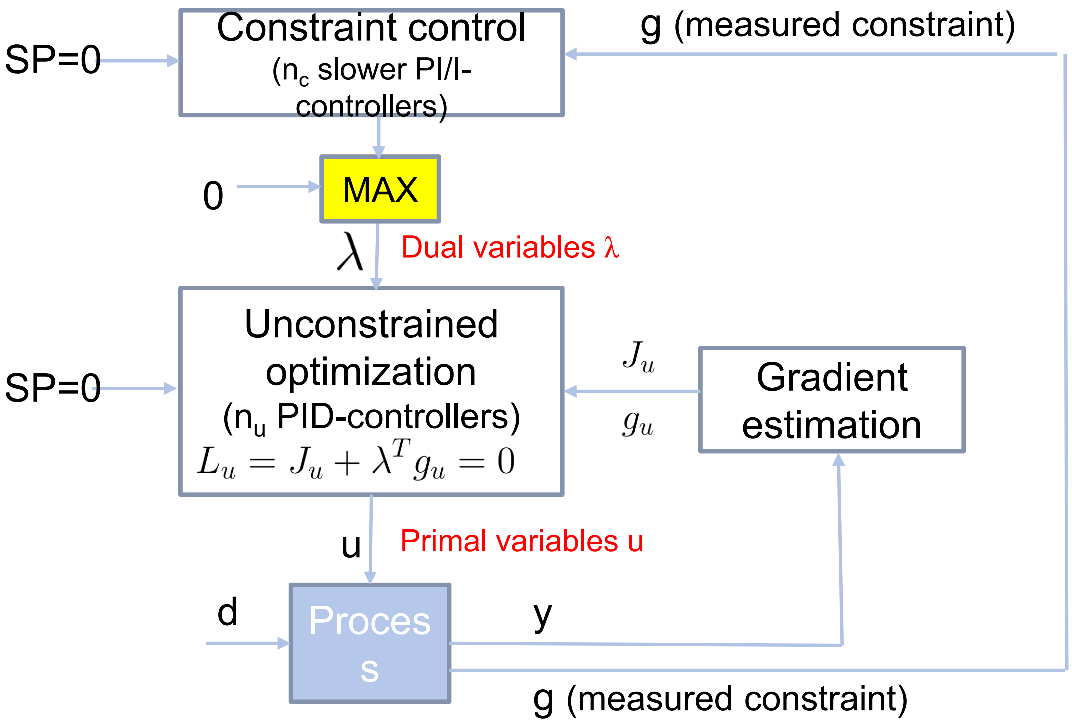
  - Active constraint: $g_A=0$
  - Tight control of g_A minimizes «back-off»

- How can we identify and control active constraints?
- How can we switch constraints?
- How do find the correct gradient when the constraints change?

I. Primal-dual control based on KKT conditions: Feedback solution that automatically tracks active constraints by adjusting Lagrange multipliers (= shadow prices = dual variables) λ

$$L_u = J_u + \lambda^T g_u = 0$$

Inequality constraints: $\lambda \geq 0$



Primal-dual feedback control.

- Makes use of «dual decomposition» of KKT conditions
- Selector on dual variables λ
- Problem: Constraint control using dual variables is on slow time scale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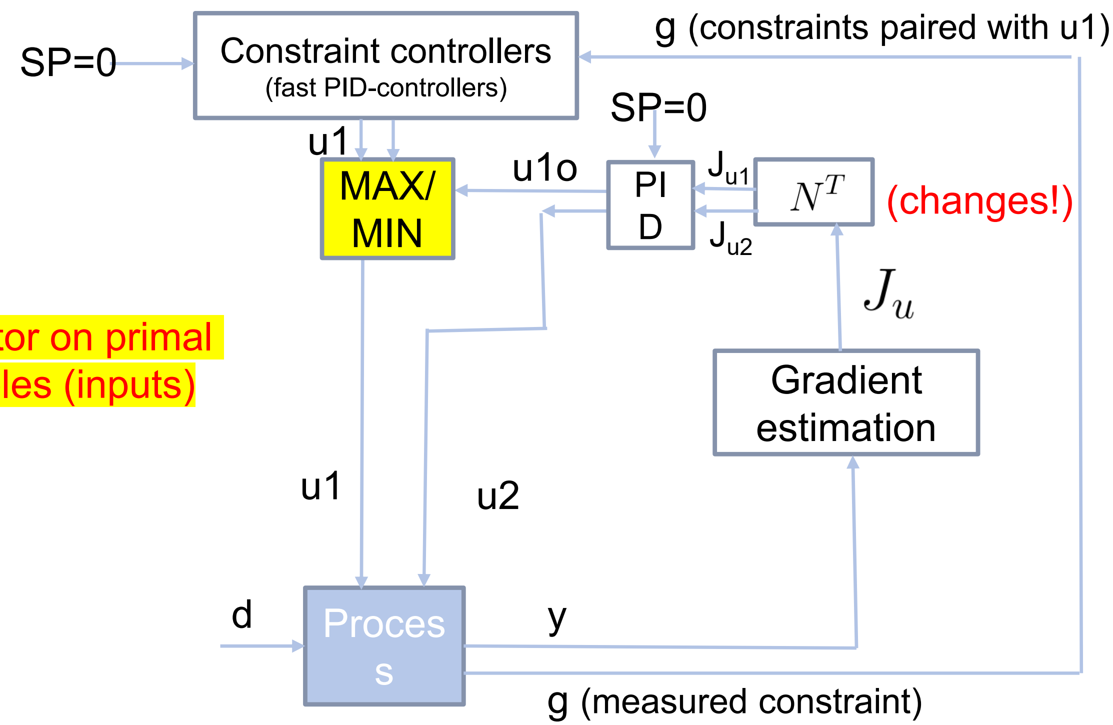
Primal–dual feedback-optimizing control with override for real-time optimization

Risvan Dirza, Sigurd Skogestad

- D. Krishnamoorthy, A distributed feedback-based online process optimization framework for optimal resource sharing, J. Process Control 97 (2021) 72–83,
- R. Dirza and S. Skogestad . Primal–dual feedback-optimizing control with override for real-time optimization. J. Process Control, Vol. 138 (2024), 103208.

II. Region-based feedback solution with «direct» constraint control (for case with more inputs than constraints)



- Selector on primal variables (inputs)

KKT: $L_u = J_u + \lambda^T g_u = 0$

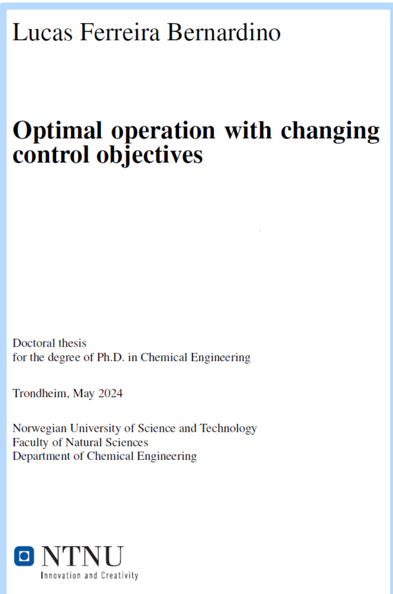
Introduce N: $N^T g_u = 0$

Control
1. Reduced gradient $N^T J_u = 0$
   - «self-optimizing variables»
2. Active constrints $g_A = 0$.

- Jaschke and Skogestad, «Optimal controlled variables for¨polynomial systems». S., J. Process Control, 2012
- D. Krishnamoorthy and S. Skogestad, «Online Process Optimization with Active Constraint Set Changes using Simple Control Structure», I&EC Res., 2019
- Bernardino and Skogestad, Decentralized control using selectors for optimal steady-state operation with changing active constraints, J. Process Control, Vol. 137, 2024


Journal of Process Control
Decentralized control using selectors for optimal steady-state operation with changing active constraints
Lucas Ferreira Bernardino, Sigurd Skogestad

Static gradient estimation:
Very simple and works well!


Lucas Ferreira Bernardino

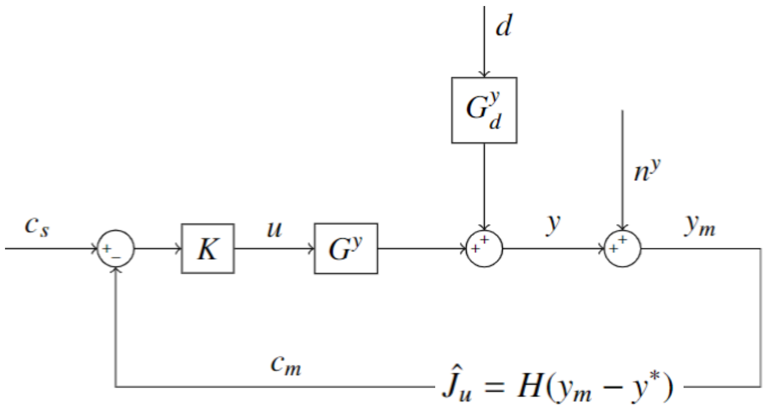
Optimal operation with changing control objectives

Doctoral thesis
for the degree of Ph.D. in Chemical Engineering

Trondheim, May 2024

Norwegian University of Science and Technology
Faculty of Natural Sciences
Department of Chemical Engineering

NTNU
Innovation and Creativity



From «exact local method» of self-optimizing control:

$$H^J = J_{uu}\left[G^{yT}\left(\tilde{F}\tilde{F}^T\right)^{-1}G^y\right]^{-1}G^{yT}\left(\tilde{F}\tilde{F}^T\right)^{-1}$$

where $\tilde{F} = [FW_d \quad W_{n^y}]$ and $F = \frac{dy^{opt}}{dd} = G_d^y - G^y J_{uu}^{-1} J_{ud}$.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Optimal measurement-based cost gradient estimate for feedback real-time optimization

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- Bernardino and Skogestad, Optimal measurement-based cost gradient estimate for real-time optimization, Comp. Chem. Engng., 2024

III. Region-based MPC with switching of cost function (for general case)

Standard MPC with fixed CVs: Not optimal

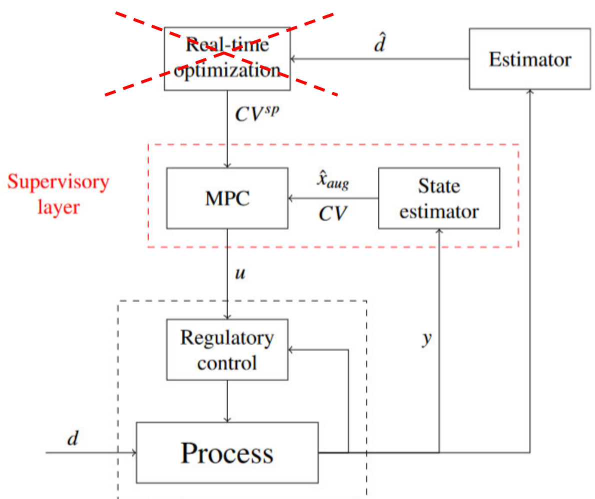

Figure 1: Typical hierarchical control structure with standard setpoint-tracking MPC in the supervisory layer. The cost function for the RTO layer is J^{ec} and the cost function for the MPC layer is J^{MPC}. With no RTO layer (and thus constant setpoints CV^{sp}), this structure is not economically optimal when there are changes in the active constraints. For smaller applications, the state estimator may be used also as the RTO estimator.

$$J^{MPC} = \sum_{k=1}^{N} \|CV_k - CV^{sp}\|_Q^2 + \|\Delta u_k\|_R^2$$

Proposed: With changing cost (switched CVs)

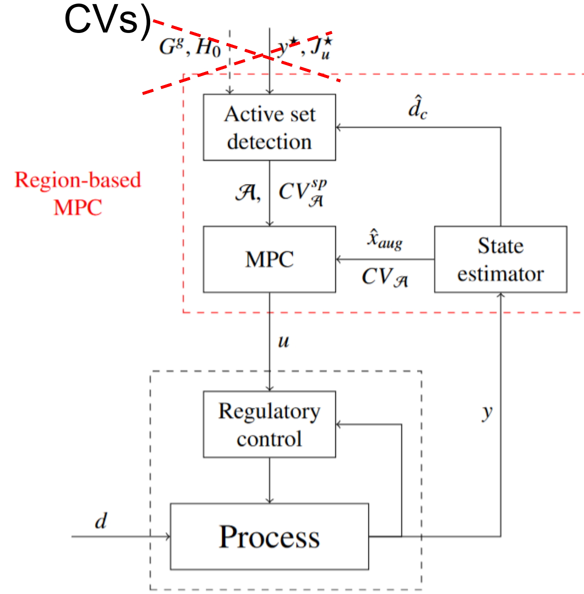

Figure 2: Proposed region-based MPC structure with active set detection and change in controlled variables. The possible updates from an upper RTO layer (y^*, J_u^* etc.) are not considered in the present work. Even with no RTO layer (and thus with constant setpoints $CV_{\mathcal{A}}^{sp}$, see (14) and (15), in each active constraint region), this structure is potentially economically optimal when there are changes in the active constraints.

$$J_{\mathcal{A}}^{MPC} = \sum_{k=1}^{N} \|CV_{\mathcal{A}} - CV_{\mathcal{A}}^{sp}\|_{Q_{\mathcal{A}}}^2 + \|\Delta u_k\|_{R_{\mathcal{A}}}^2$$

$$CV_{\mathcal{A}} = \begin{bmatrix} g_{\mathcal{A}} \\ c_{\mathcal{A}} \end{bmatrix} = \begin{bmatrix} g_{\mathcal{A}} \\ N_{\mathcal{A}}^T H_0 y \end{bmatrix} \quad (14)$$

$$H_0 = \begin{bmatrix} J_{uu} & J_{ud} \end{bmatrix} \begin{bmatrix} G^y & G_d^y \end{bmatrix}^\dagger$$

- Bernardino and Skogestad, Optimal switching of MPC cost function for changing active constraints. J. Proc. Control, 2024