APC8

Tranformed inputs

Online optimization

RTO

ESC

Briefly: Pros and cons of MPC

Sigurd Skogestad

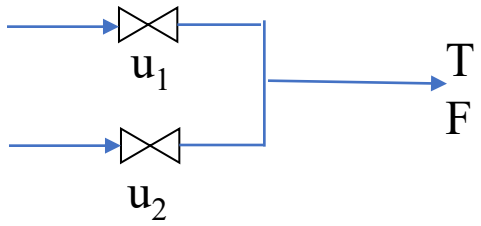
07 October 2025

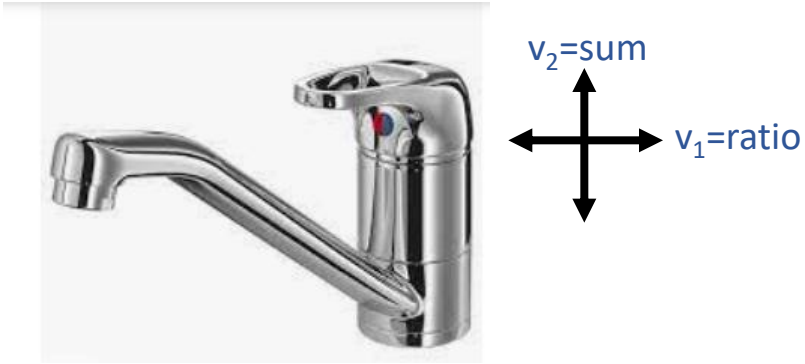
Feedforward (and decoupling) control

- Feedforward control relies on model
- as opposed to feedback which relies mostly on data

- Feedback control: Linear model is often OK
- Feedforward control: Much less likely that linear model is OK because of process changes and disturbances
- Here: Nonlinear feedforward control using Input transformations based on static process model

Motivating Example decoupling: Mixing of hot (u_1) and cold (u_2) water

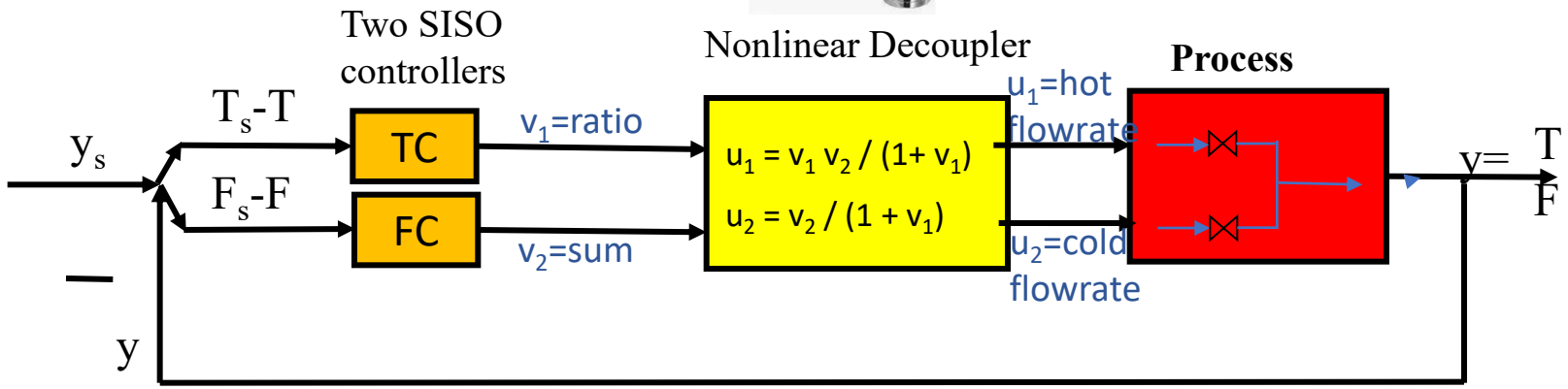




- Want to control (outputs, CVs)
  - y_1 = Temperature T
  - y_2 = total flow q
- Inputs, u_1,u_2=flowrates
- May use two SISO PI-controllers
  - TC
  - FC
- Insight: Get decoupled response with transformed inputs
  - TC sets flow ratio, $v_1 = u_1/u_2$
  - FC sets flow sum, $v_2 = u_1 + u_2$
- Decoupler: Need «static calculation block» to solve for inputs

$$u_1 = v_1 v_2 / (1+ v_1)$$

$$u_2 = v_2 / (1 + v_1)$$



Pairings:

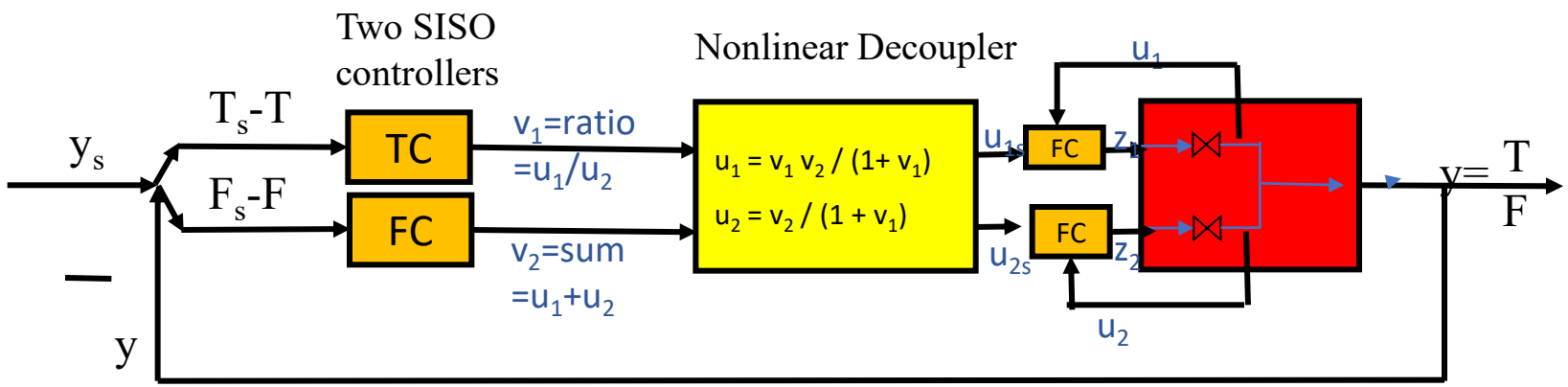
- $T - v_1$
- $F - v_2$

No interactions for setpoint change

Note:

- In practice, Physical inputs are valve positions (z)
- So must add two flow (q) controllers
  - These generate inverse $z=f^{-1}(q)$ by feedback

In practice must add two slave flow controllers



v = transformed inputs
u = flowrates
z = valve positions

Even better (see next):

$$v_1 = \frac{q_{hTh}+q_cT_c}{q_h+q_c} \quad (1)$$ Generalized ratio

$$v_2 = q_c + q_h \quad (2)$$

Decoupler with feedforward:

$$q_h = \frac{v_2(v_1 - T_c)}{T_h - T_c}$$

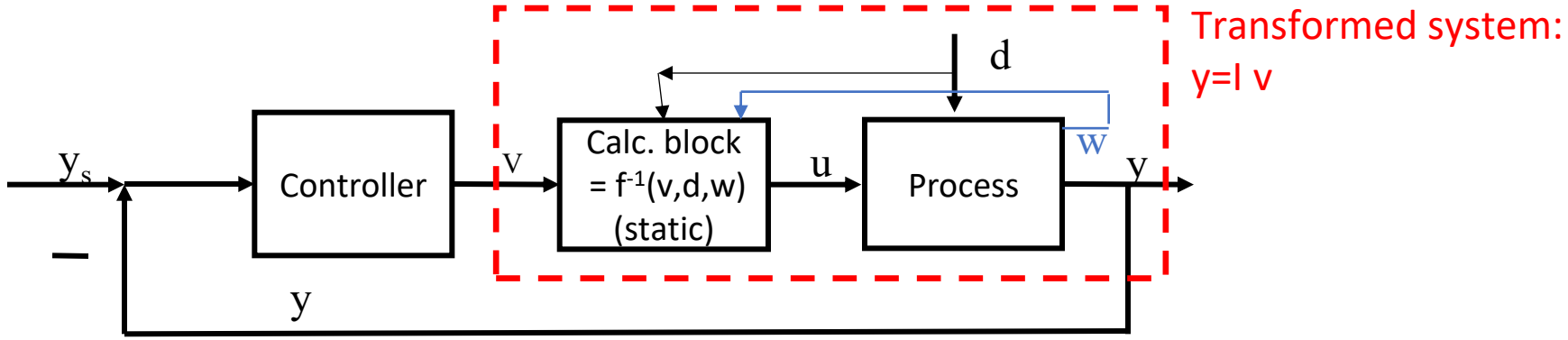
$$q_c = v_2 - q_h$$

Nonlinear feedforward, decoupling and linearization: Input transformations

- Transformed inputs: Extremely simple and effective way of achieving feedforward, decoupling and linearization

General approach: Combined Nonlinear decoupling, feedforward and linearization using Transformed Inputs

- Extremely simple: Introduce transformed input v and use Nonlinear calculation block



Genaral Method:

Steady-state model: $y = f(u,d,w)$

Select transformed input: $v = f(u,d,w)$ («right-hand side» of model)

Calculation block: Invert for given v: $u = f^{-1}(v,d,w)$ (may be replaced by slave v-controller)

w=dependent variable (flow, temperature), but treated as measured disturbance - to simplify model

Transformed system becomes: $y=I\ v$ («decoupled, linear, indepedent of d»)

It is so simple that many people (e.g., Seborg textbook) think it cannot work in all cases – but it does!

Comment: To simplify often may use only «parts» of f(u,d,w) as v (because of unknown parameters etc.)

Sigurd Skogestad, Cristina Zotica, Nicholas Alsop. "Transformed inputs for linearization, decoupling and feedforward control" Journal of Process Control 122 (2023) 113-133

Example: Tranformed input to get nonlinear decoupling and feedforward. Mixing of hot and cold water

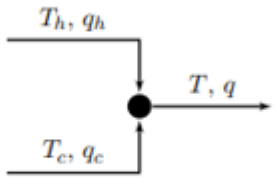
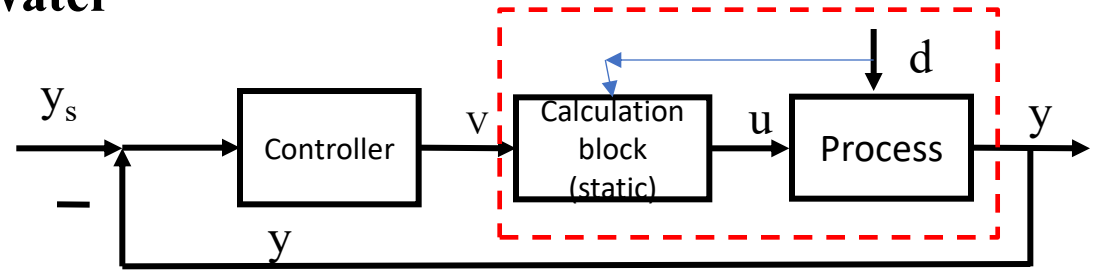


Figure 1: Mixer system



$$u = \begin{pmatrix} q_h \\ q_c \end{pmatrix}$$

$$d = \begin{pmatrix} T_h \\ T_c \end{pmatrix}$$

$$y = \begin{pmatrix} T \\ q \end{pmatrix}$$

Steady-state model written as y=f(u,d):

$$T = \frac{q_{hTh}+q_{cTc}}{q_h+q_c}$$

$$q = q_c + q_h$$

Select transformed inputs as right hand side, v = f

$$v_1 = \frac{q_{hTh}+q_{cTc}}{q_h+q_c} \quad (1)$$ Generalized ratio

$$v_2 = q_c + q_h \quad (2)$$

Model from v to y (red box) is then decoupled and with perfect disturbance rejection:

$$T = v_1$$

$$q = v_2$$

- Can then use two single-loop PI controllers for T and q!
  - These controllers are needed to correct for model errors and unmeasured disturbances
- Note that v_1 used to control T is a generalized ratio, but it includes also feedforward from Tc and Th.

Implementation (calculation block) : Solve (1) and (2) with respect to u=(qc qh):

Decoupler with feedforward:

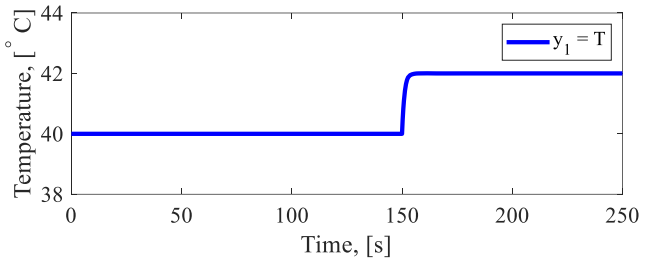
$$q_h = \frac{v_2(v_1 - T_c)}{T_h - T_c}$$

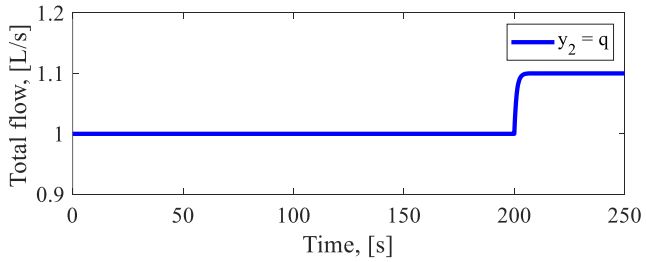
$$q_c = v_2 - q_h$$

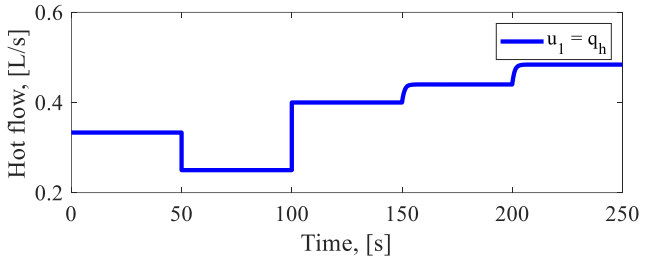
Transformed MVs for decupling, linearization and disturbance rejection
Mixing of hot and cold water (static process)
New system: T=v_1 and q=v_2

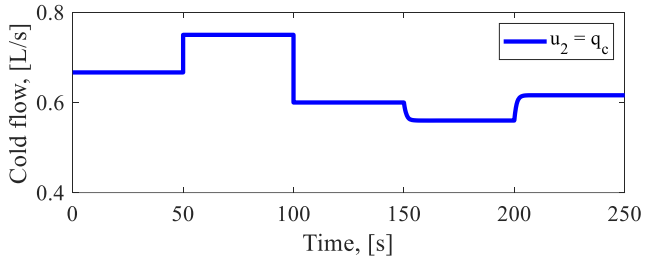
Outer loop: Two I-controllers with $\tau_C = 1$ s

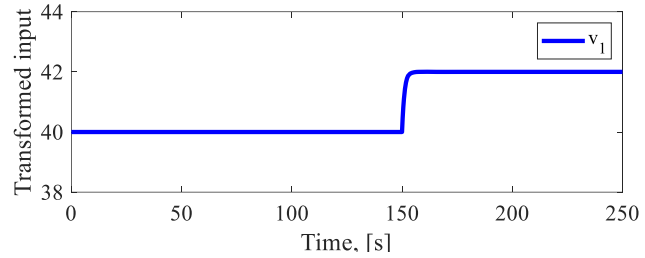
1. T_h: 60->70 °C at t = 50 s
2. T_c: 30->20 °C at t = 100 s
3. T_h^s: 40->42 °C at t = 150 s
4. q^s: 1->1.1 L/s at t = 200 s

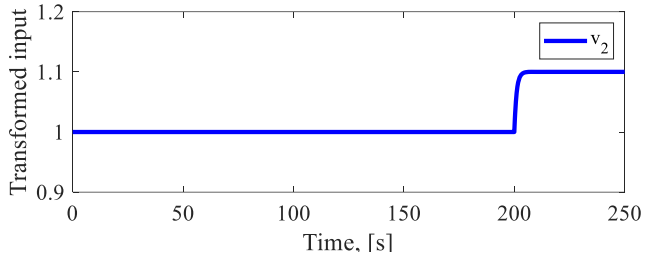




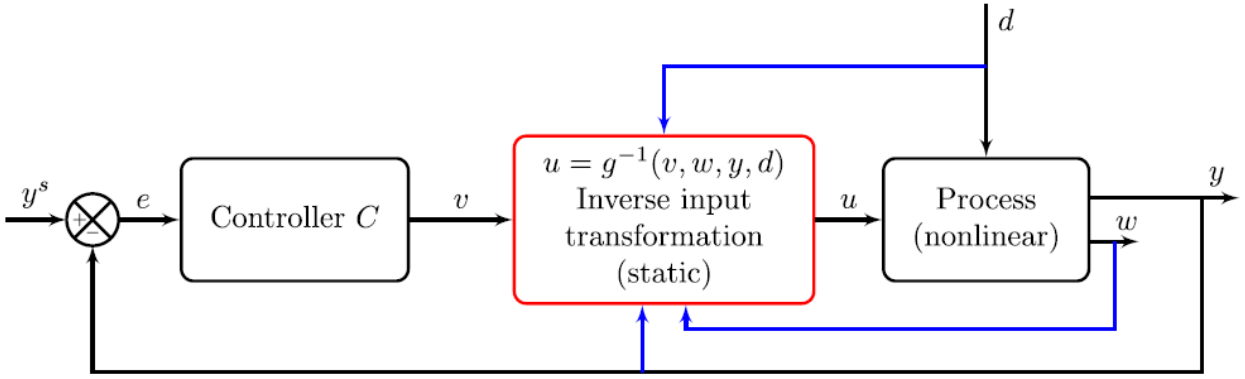




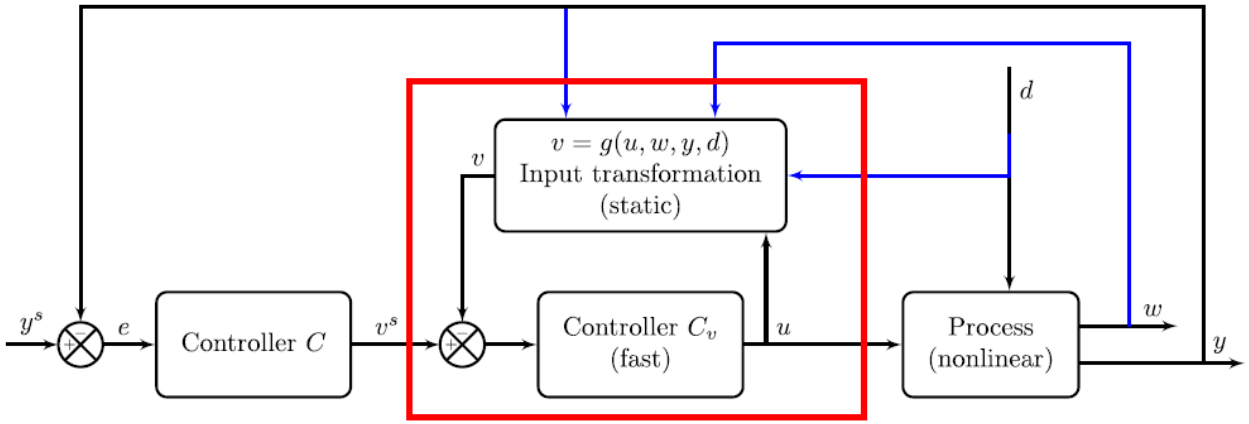




Alternative B: Calculation block solved by feedback (using fast slave controller C_v)



(a) Alternative A. Model-based implementation of transformed input $v = g(u, w, y, d)$. The physical input $u = g^{-1}(v, w, y, d)$ is generated by a static (algebraic) calculation block which inverts the transformed input model equations. The model-based implementation generates the exact inverse for the case with no model error.



(b) Alternative B. Feedback implementation of transformed input $v = g(u, w, y, d)$ using cascade control with a slave v-controller. The computed value of v is driven to its setpoint v_s by the inner (slave) feedback controller C_v which generates the physical input u. This implementation generates an approximate inverse.

Example: Power control

5.4.2. Transformed input $v_{0,w}$ based on parts of static model and measured state $w = T_2$

The second transformed variable, $v_{0,w}$, follows by using the measured state $w = T_2$ to replace the heat transfer Eq. (68c) for Q. We use (68a) to find

$$T_1 = T_1^0 + \frac{Q}{F_1 c_{p1}}$$

and then we substitute Q using (68b) to get

$$y = T_1 = \underbrace{T_1^0 + \frac{F_2 c_{p2}}{F_1 c_{p1}}(T_2^0 - T_2)}_{f_{0,w}(u,w,d)} \tag{71}$$

From (71) the corresponding ideal static transformed input becomes

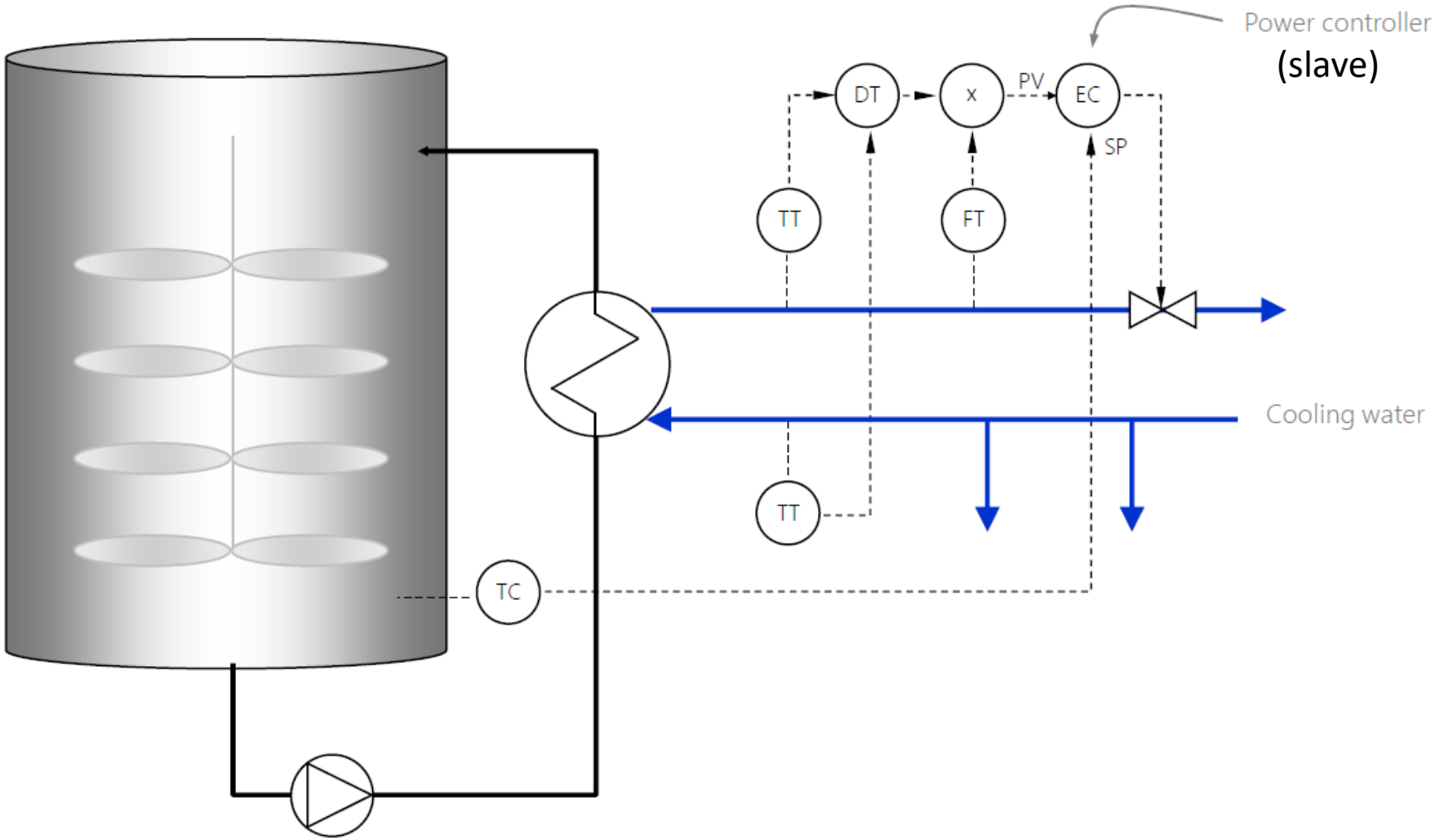
$$v_{0,w} = f_{0,w}(u, w, d) = T_1^0 + \frac{F_2 c_{p2}}{F_1 c_{p1}}(T_2^0 - T_2) \tag{72}$$

which depends on $w = T_2$ but not on the UA-value.

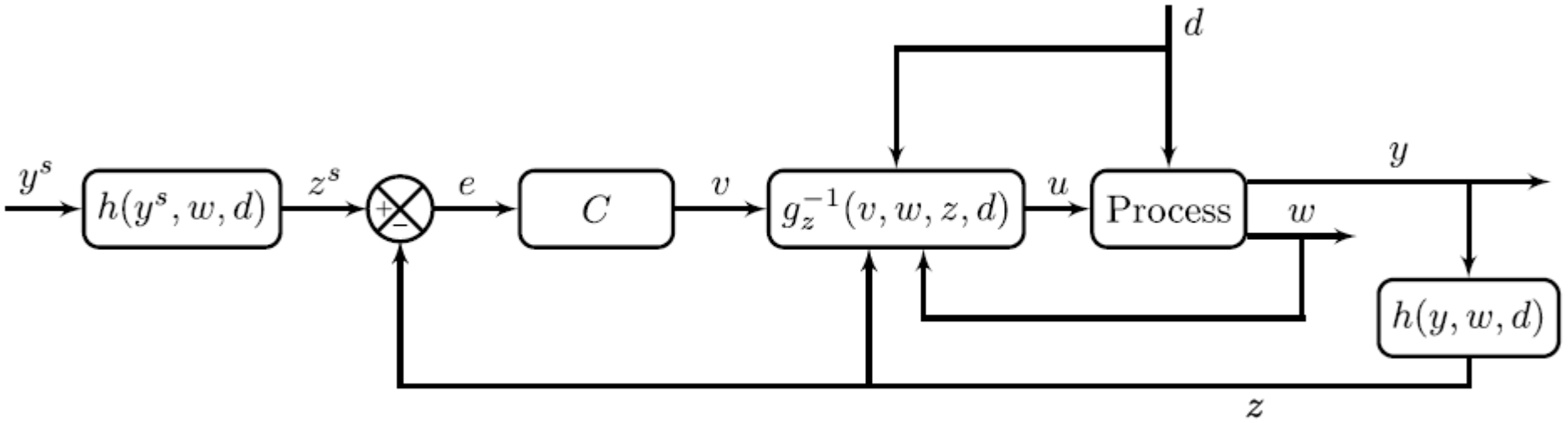
In practice (Perstorp) use only part of this:

$v=F_2(T_2^0 - T_2)$

New control structure: Power control



Also: Transformed outputs z



(a) General implementation of transformed output z

- No fundamental advantage, but can simplify input transformation
- For example, y=T, z=H (enthalpy)

More on transformed inputs

Journal of Process Control 122 (2023) 113–133

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Review

Transformed inputs for linearization, decoupling and feedforward control

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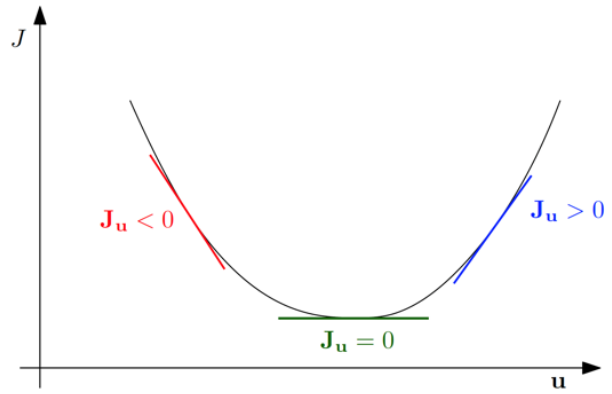
Economic optimization (RTO)

- Consider the task of minimizing a *scalar cost function J (*or equivalently, maximizing the profit –J).
- Typically, J represents an economic quantity with units such as [$/s]. We assume that we can influence J through input variables u, and that we may also have access to measurements y. Furthermore, we assume the system is static (i.e., without dynamics), so we can write J=J(u,d).



On-line steady-state Optimization

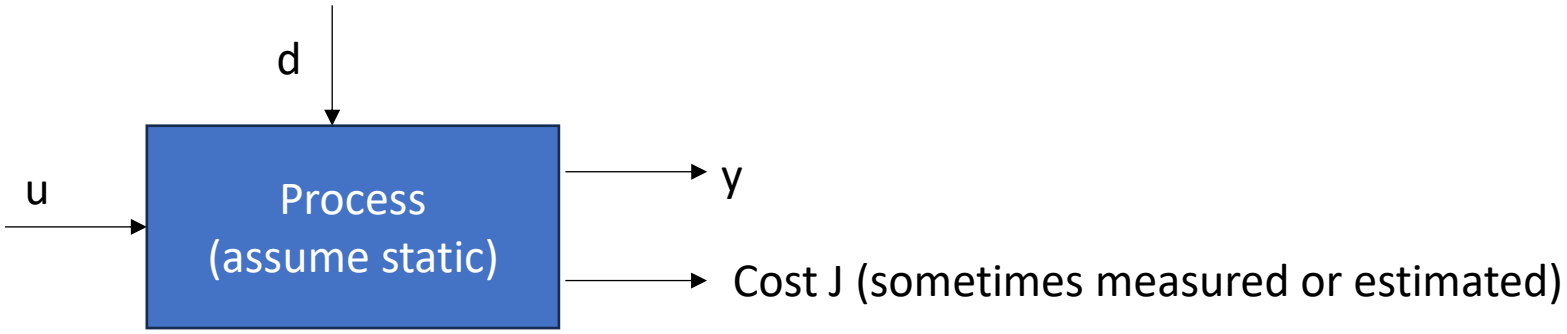
There are three main approaches for manipulating u to minimize the cost J:

I. **Real-time optimization (RTO): Model-based** - use nonlinear *online process model* with model updates from measurements y.

II. **Extremum seeking control (ESC):** Purely data-based using ***measured cost J.***

III. **Self-optimizing control (SOC):** c=Hy. *Combine data (y) and offline model (to get H)*.

In addition, there are hybrid methods and combinations. Actually, all three methods can be combined.



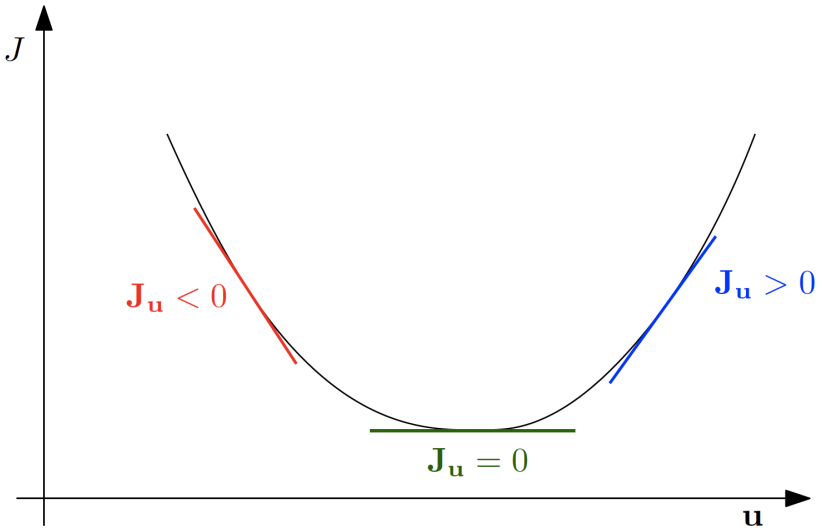
In addition: Feedback methods make use of gradient J_u = dJ/du (estimated)

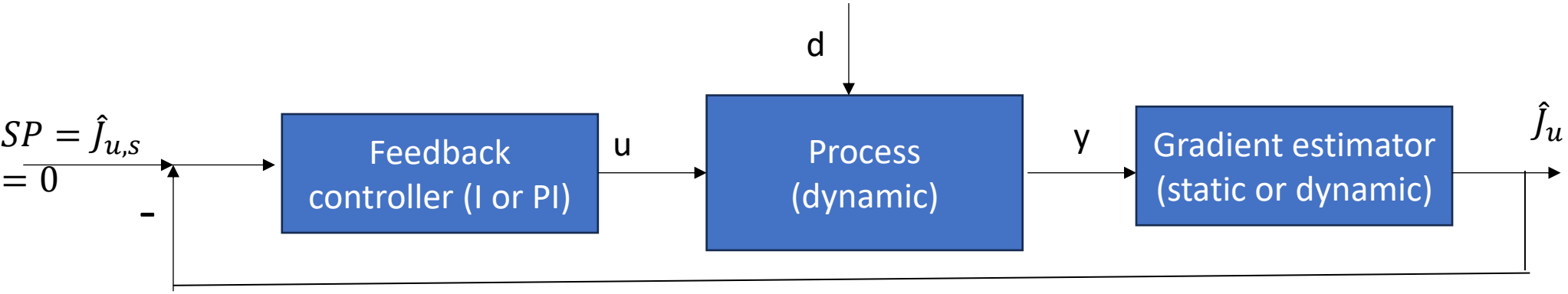
Feedback optimization (control gradient J_u to zero)

Unconstrained optimization.

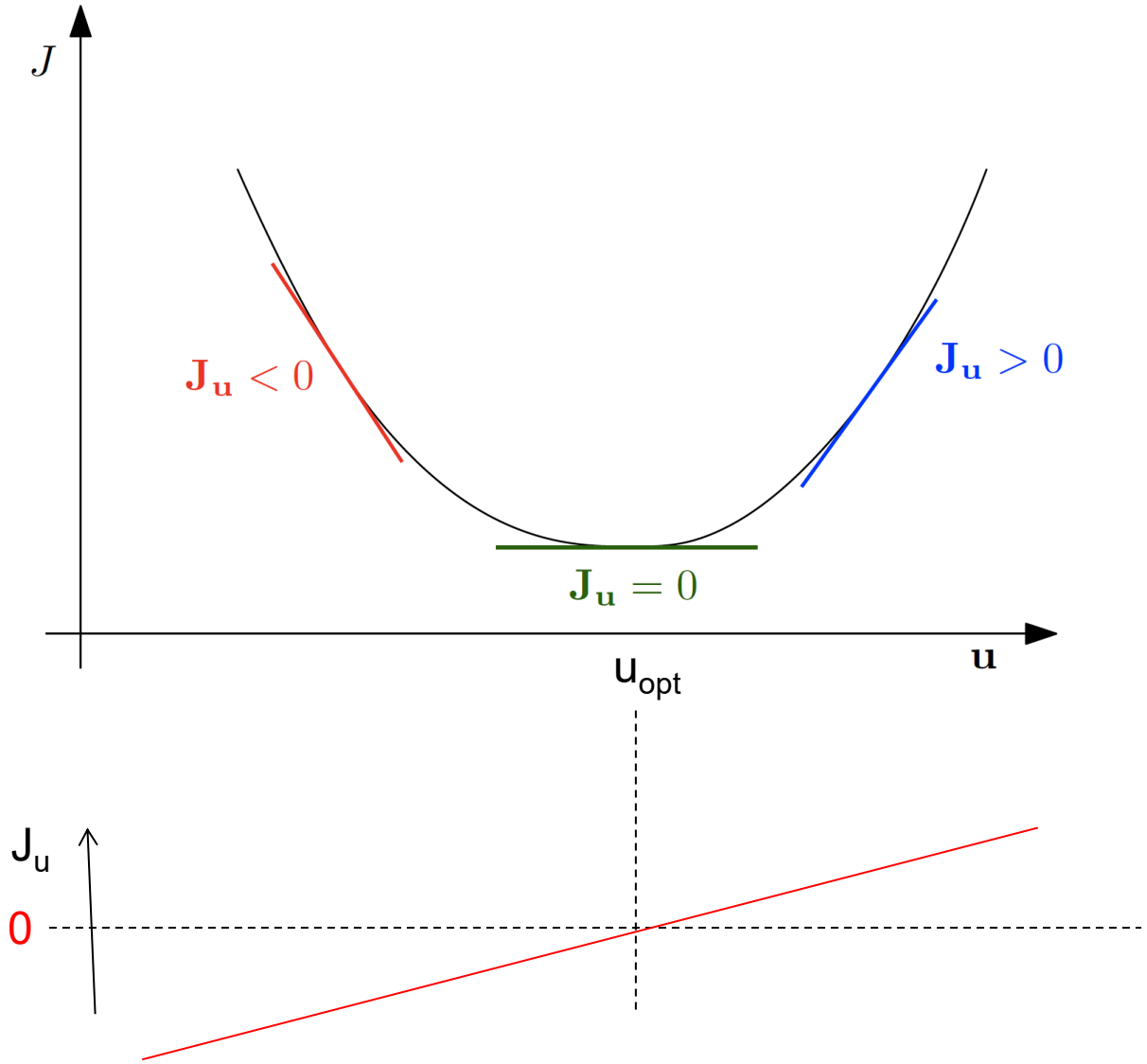
Necessary condition of optimality (NCO):

- Gradient of cost function = 0
- $\mathrm{J_u} \equiv \frac{\partial J}{\partial u} \equiv \nabla_u J = 0$





Often: Pure I-controller



- Optimal setpoint: J_u=0
- If Hessian J_{uu} is constant:
  - J_u as a function of u is a straight line with slope J_{uu}
- Nice properties for feedback control of J_u
- No dynamics: Pure I-controller optimal
  - SIMC-rule: $K_I = 1/(J_{uu}\,\tau_c)$

I. Standard Model-Based Optimization (RTO)

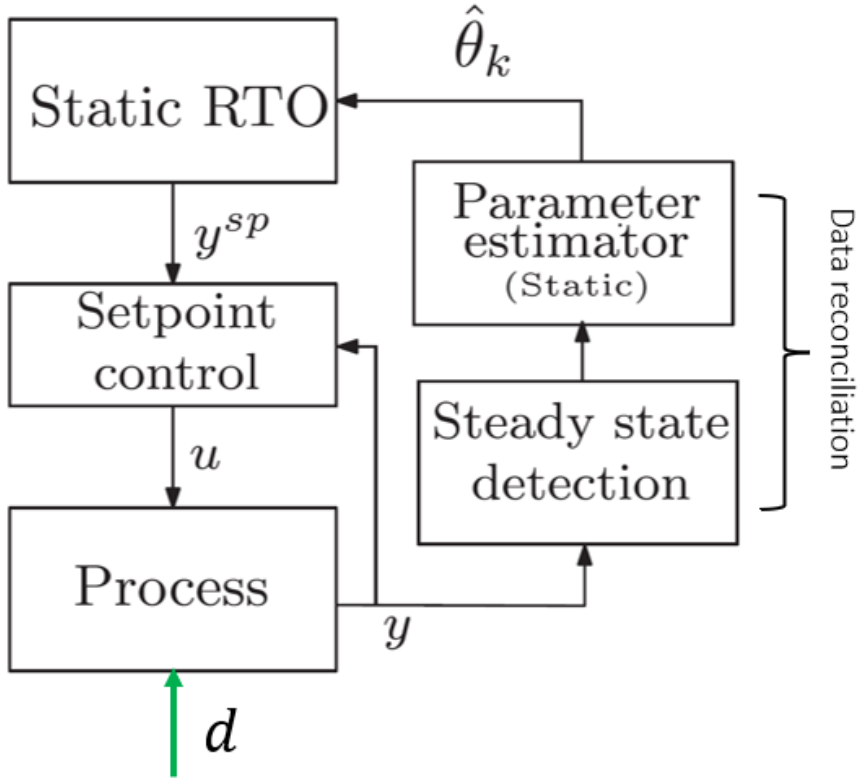
Real-Time Optimization (RTO) uses a detailed nonlinear model and does not require that the cost J is measured.

The system model is used at each time step k to compute the input u(k) that minimizes the cost J. Measurements y are used online to update selected model parameters (e.g., efficiencies). RTO minimizes the cost J while explicitly handling constraints.

Two main approaches

IA. Without gradient: Standard RTO (minimize cost J)

IB. With gradient: Feedback-based RTO (control gradient Ju to zero)



II. Purely Data-Based Feedback Optimization (ESC)

Also known as extremum seeking control (ESC), hill-climbing, greedy search, or perturb-and-observe.

This method relies entirely on **measurements of the cost J** (which may not always be available) and does not use a process model. It is simple, but generally slow, especially when gradient estimation is involved.

Also known as extremum seeking control (ESC), hill-climbing, greedy search, or perturb-and-observe.

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IIA. ESC without gradient: "Perturb and observe" (simplest)

This is the simplest and most intuitive version. It is used, for example, to fine-tune the position of solar mirrors.

IIB. ESC with gradient Ju (most common in academia)

In classical ESC, sinusoidal perturbations are used to estimate the gradient Ju, which is subsequently controllerd to zero - but sinusoids is not always an efficient method – so later others methods for estimating Ju have been developed

III. Self-Optimizing Control (SOC)

Here, the model is used offline to find good "self-optimizing" variables c. The online implementation uses a simple PID controller to keep c and its setpoint c_s.

The goal of SOC is to find a **"magic variable"** c to control—ideally one where a constant setpoint c_s leads to near-optimal performance despite disturbances. This allows the optimization to be embedded into the fast control layer.

In its simplest form, c=y is a single measurement. The variable c is chosen such that its setpoint c_s is **insensitive to disturbances**, yet c is **sensitive to input changes** (i.e., has a large gain from u to c).

More generally, we can use combinations of measurements: c=Hy where H is a matrix. Ideally, this corresponds to the **gradient**: $c=J_u$ =Hy. See the paper by Bernardino and Skogestad (2024) for methods to find H to estimate the gradient.

III-C. Self-Optimizing Control with Constraints

SOC can be extended to handle constraints by incorporating Lagrange multipliers into the control strategy. In this approach, the **multiplier acts as a manipulated variable in an upper slow control layer**. Constraint violation can be avoided using **override logic** (Dirza and Skogestad, 2024). In essence, this is a clever trick where a PI controller is used to iteratively solve a set of equations numerically.

Combinations

These approaches can be combined in all ways:

a) **I + II**: ESC (slow) sends bias for gradient J_u to RTO.

   This methods is sometimes called "modifier adaptation" but I don't like this name because it is not very descriptive.

   The bias means that the RTO-gradient will not be zero. This is to correct for model errors, unmeasured disturbances and measurement errors in the RTO-layer.

b) **II + III**: ESC (slow) updates setpoints to SOC (fast).

c) **I + III**: RTO (slow) updates setpoints to SOC (fast).

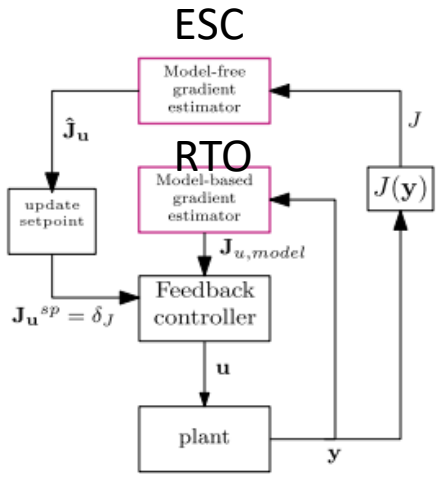
d) **I + II + III**: ESC (slow) sends bias for J_u to RTO (faster), which updates setpoints to the SOC layer (fastest).

There needs to be a time scale separation between the layers, typically about 10.

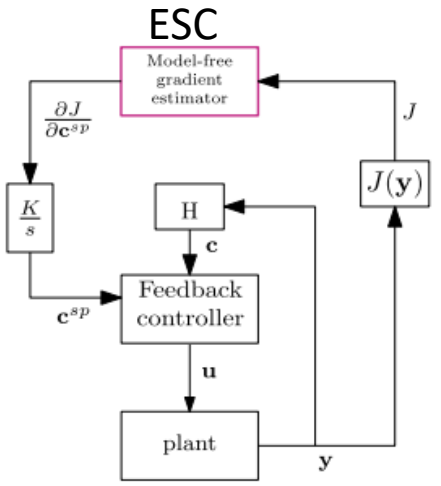
References

- D Krishnamoorthy, S Skogestad. Real-time optimization as a feedback control problem–A review. Computers & Chemical Engineering (2022
- R Dirza and S Skogestad .Primal-dual feedback-optimizing control with override for real-time optimization. Journal of Process Control 138, 103208 3 (2024)
- LF Bernardino and S Skogestad. Optimal measurement-based cost gradient estimate for feedback real-time optimization. Computers & Chemical Engineering, 108815 (2024)

(a) ESC + RTO



(b) ESC + SOC



More details on RTO and ESC now follow

I. RTO
- A. Without gradient (most common)
- B. With control of gradient (feedback-RTO)
  - Ju=Hy with H from SOC can used to get fast gradient estimation

II. ESC
- A. Without gradient (simplest)
- B. With control of gradient (most common)

IA. Conventional (commercial) steady-state RTO

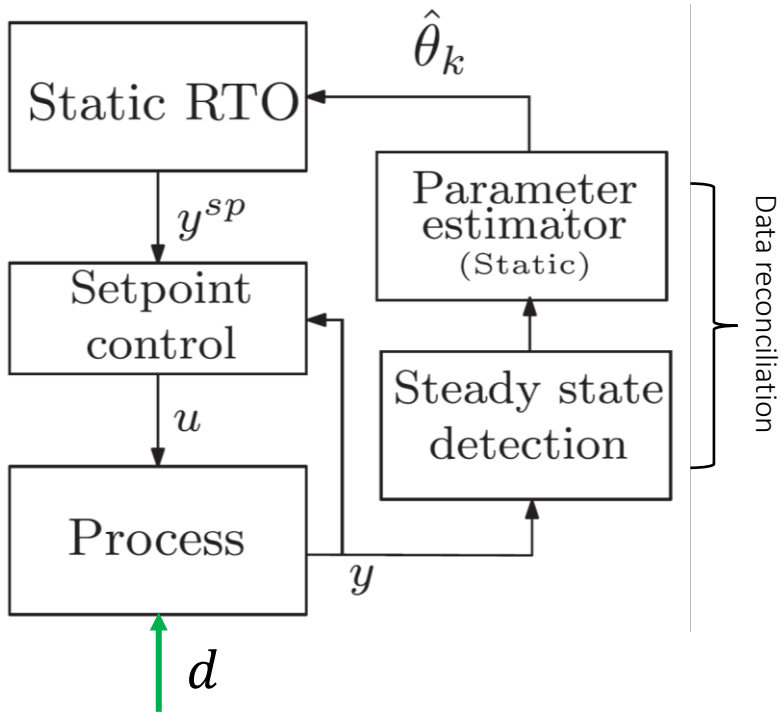
Fairly common in refining and petrochemical industy.

Two-step approach:

Step 1. “Data reconciliation”:

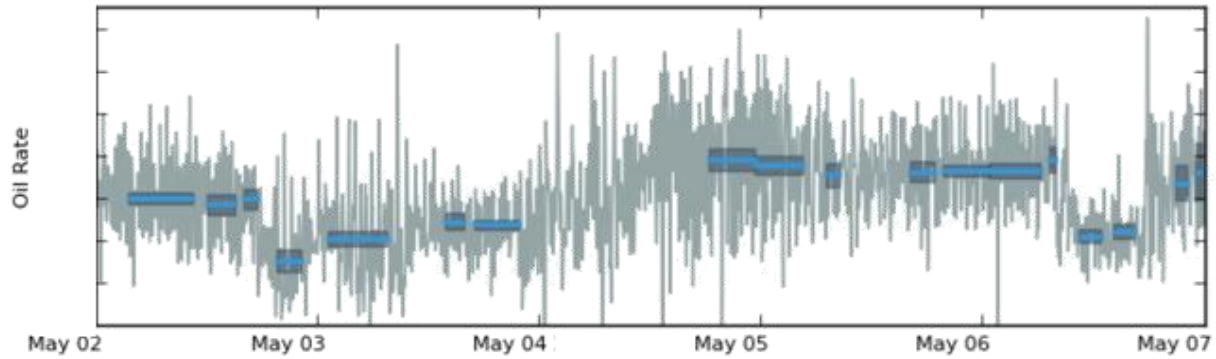
- Steady-state detection
- Update estimate of d: model parameters, disturbances (feed), constraints

Step 2. Re-optimize numerically to find new optimal steady state



Steady-state wait time

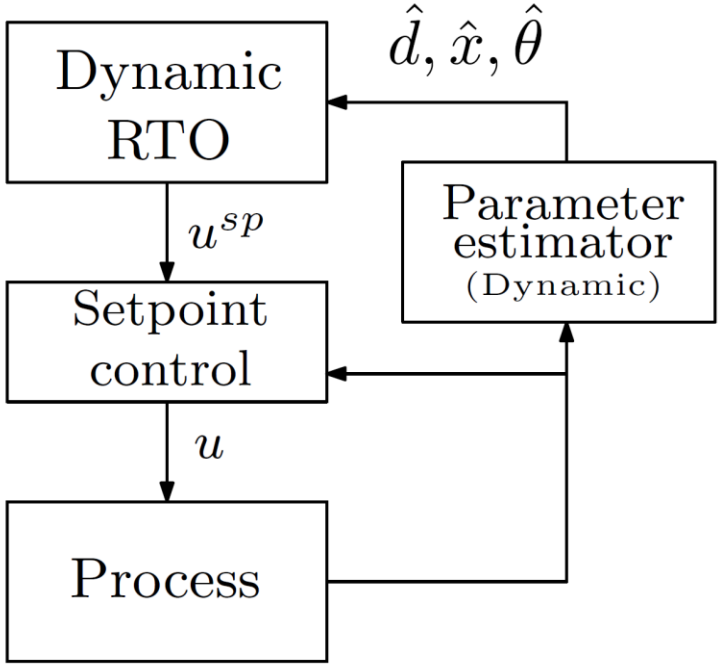
- Transient measurements cannot be used → system must “settle”
- Large chunks of data discarded
- Steady state detection issues
  - Erroneously accept transient data
  - Non-stationary drifts



Dinesh Krishnamoorthy, Bjarne Foss, Sigurd Skogestad, Steady-state real-time optimization using transient measurements, Computers & Chemical Engineering (2018)

How to avoid steady state wait time?

1. Dynamic RTO = EMPC



Could be nonlinear Kalman filter

Dinesh Krishnamoorthy, Bjarne Foss, Sigurd Skogestad, Steady-state real-time optimization using transient measurements, Computers & Chemical Engineering (2018)

D Krishnamoorthy, S Skogestad. Real-time optimization as a feedback control problem–A review. Computers & Chemical Engineering (2022)

RTO problem formulation

Steady-state RTO (used in Hybrid RTO):

$$\min_{x,u} J(x,d,u)$$

s.t.:

$$\begin{aligned} 0 &= F(x,d,u) \\ 0 &= h(x,d,u) \\ g(x,d,u) &\leq 0 \end{aligned}$$

Dynamic RTO ≡ (Economic) nonlinear MPC **:**

$$\min_{x(t),u(t)} \int_{t_0}^{t_f} J\big(x(t),d(t),u(t)\big)\,dt$$

s.t.:

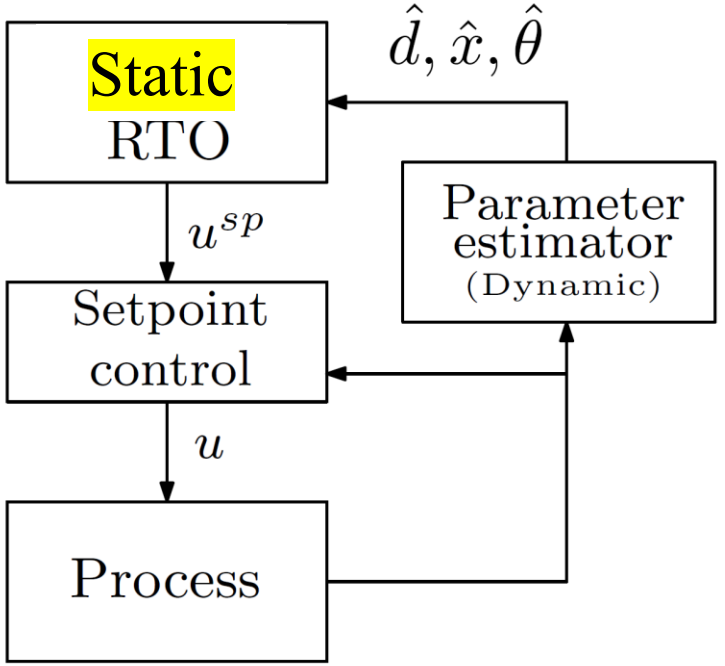
$$\begin{aligned} \dot{x}(t) &= F\big(x(t),d(t),u(t)\big) \\ 0 &= h\big(x(t),d(t),u(t)\big) \\ g\big(x(t),d(t),u(t)\big) &\leq 0 \\ x(t_0) &= \hat{x}_0 \end{aligned}$$

Now we calculate not only an optimal point, but an optimal trajectory!

BUT Much more complex that static RTO, and may not give much economic benefit

How to avoid steady state wait time?

2. Hybrid RTO

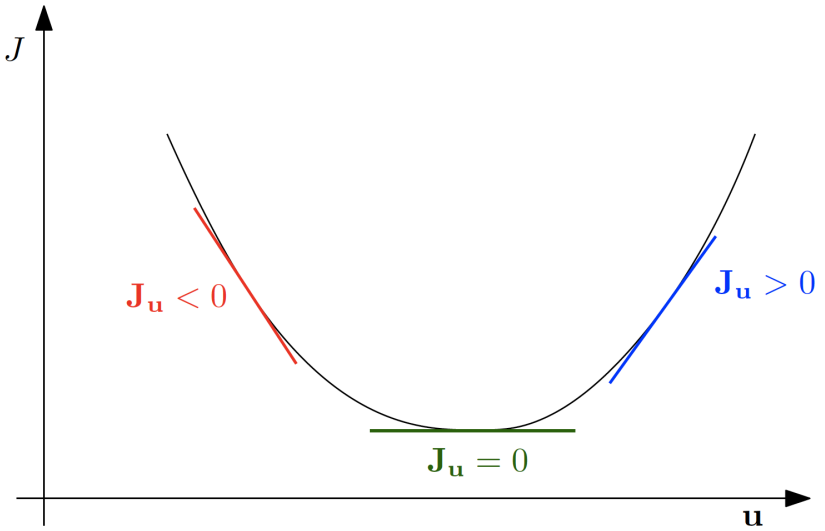


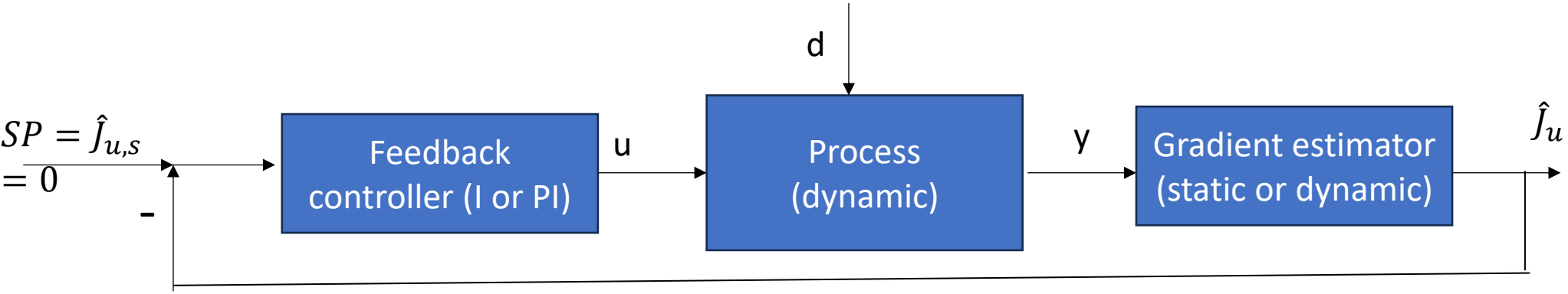
Could be nonlinear Kalman filter

Dinesh Krishnamoorthy, Bjarne Foss, Sigurd Skogestad, Steady-state real-time optimization using transient measurements, Computers & Chemical Engineering (2018)

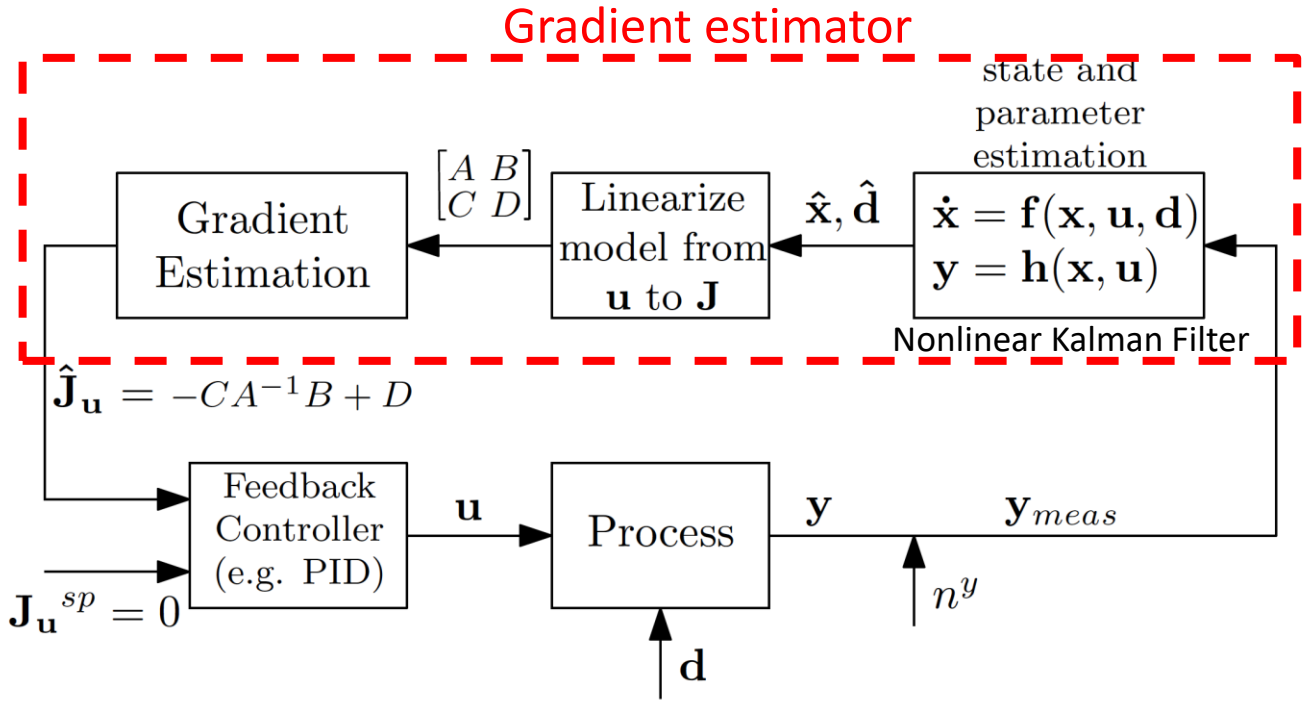
IB. Feedback RTO (control gradient J_u to 0) (unconstrained case)

«Solving RTO-problem using PI control»





IB. Feedback RTO (control gradient J_u to 0) (unconstrained case)



Linearize the dynamic model

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{d}) \\ J &= \mathbf{g}(\mathbf{x}, \mathbf{u}) \end{aligned} \quad \Rightarrow \quad \begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u} \\ J &= C\mathbf{x} + D\mathbf{u} \end{aligned}$$

$$A = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{x=\hat{x}} \quad B = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{x=\hat{x}}$$

$$C = \left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{x=\hat{x}} \quad D = \left.\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right|_{x=\hat{x}}$$

Trick, set $\dot{x} = 0$, *to get estimate of static gradient*:

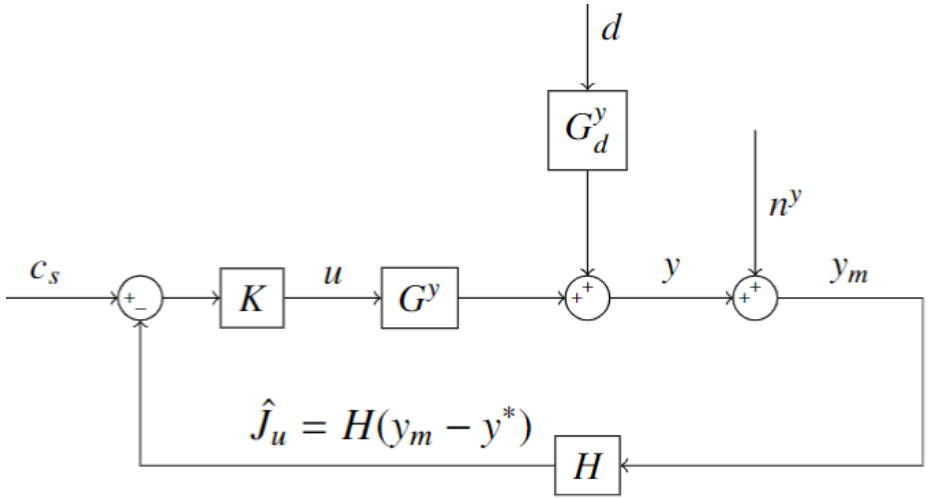
$$\Delta J = \underbrace{\left(-CA^{-1}B + D\right)}_{\hat{\mathbf{J}}_\mathbf{u}} \mathbf{u}$$

Note: This is one simple way of doing the gradient estimation, but need nonlinear dynamic model (e.g., Kalman Filter)

D Krishnamoorthy, E Jahanshahi, S Skogestad. Feedback Real-Time Optimization Strategy Using a Novel Steady-state Gradient Estimate and Transient Measurements. Industrial & Engineering Chemistry Research, 2019

Here is another simpler Static gradient estimation:

Based on self-optimizing control. Very simple and works well!



Computers and Chemical Engineering 189 (2024) 108815

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Optimal measurement-based cost gradient estimate for feedback real-time optimization

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ARTICLE INFO

Keywords:
Self-optimizing control
Optimal operation
Controlled variable design
Gradient estimation

ABSTRACT

This work presents a simple and efficient way of estimating the steady-state cost gradient J_u based on available uncertain measurements y. The main motivation is to control J_u to zero in order to minimize the economic cost J. For this purpose, it is shown that the optimal cost gradient estimate for unconstrained operation is simply $\hat{J}_u = H(y_m - y^*)$ where H is a constant matrix, y_m is the vector of measurements and y^* is their nominally unconstrained optimal value. The derivation of the optimal H-matrix is based on existing methods for self-optimizing control and therefore the result is exact for a convex quadratic economic cost J with linear constraints and measurements. The optimality holds locally in other cases. For the constrained case, the unconstrained gradient estimate $\hat{J}_u$ should be multiplied by the nullspace of the active constraints and the resulting "reduced gradient" controlled to zero.

From «exact local method» of self-optimizing control:

$$H^J = J_{uu}\left[G^{yT}\left(\tilde{F}\tilde{F}^T\right)^{-1}G^y\right]^{-1}G^{yT}\left(\tilde{F}\tilde{F}^T\right)^{-1}$$

where $\tilde{F} = [F W_d \quad W_{n^y}]$ and $F = \frac{dy^{opt}}{dd} = G_d^y - G^y J_{uu}^{-1} J_{ud}$.

- Bernardino and Skogestad, Optimal measurement-based cost gradient estimate for real-time optimization, Comp. Chem. Engng., 2024

With constraints

Constrained optimization problem

$$\min_{\mathbf{u}} \quad \mathrm{J}\ (\mathbf{u}, \mathbf{y}, \mathbf{d})$$
$$\text{s.t.} \quad \mathbf{g}(\mathbf{u}, \mathbf{y}, \mathbf{d}) \leq 0$$

Solution: Turn into unconstrained optimization problem using Lagrange multipliers

$$\mathcal{L}(\mathbf{u}, \mathbf{y}, \mathbf{d}, \boldsymbol{\lambda}) = \mathrm{J}\ (\mathbf{u}, \mathbf{y}, \mathbf{d}) + \boldsymbol{\lambda}^{\mathsf{T}} \mathbf{g}(\mathbf{u}, \mathbf{y})$$

$\min_{u,\lambda}$ L

u = primal variables = inputs

λ ≥ 0 = dual variables = Lagrange multipliers = shadow prices

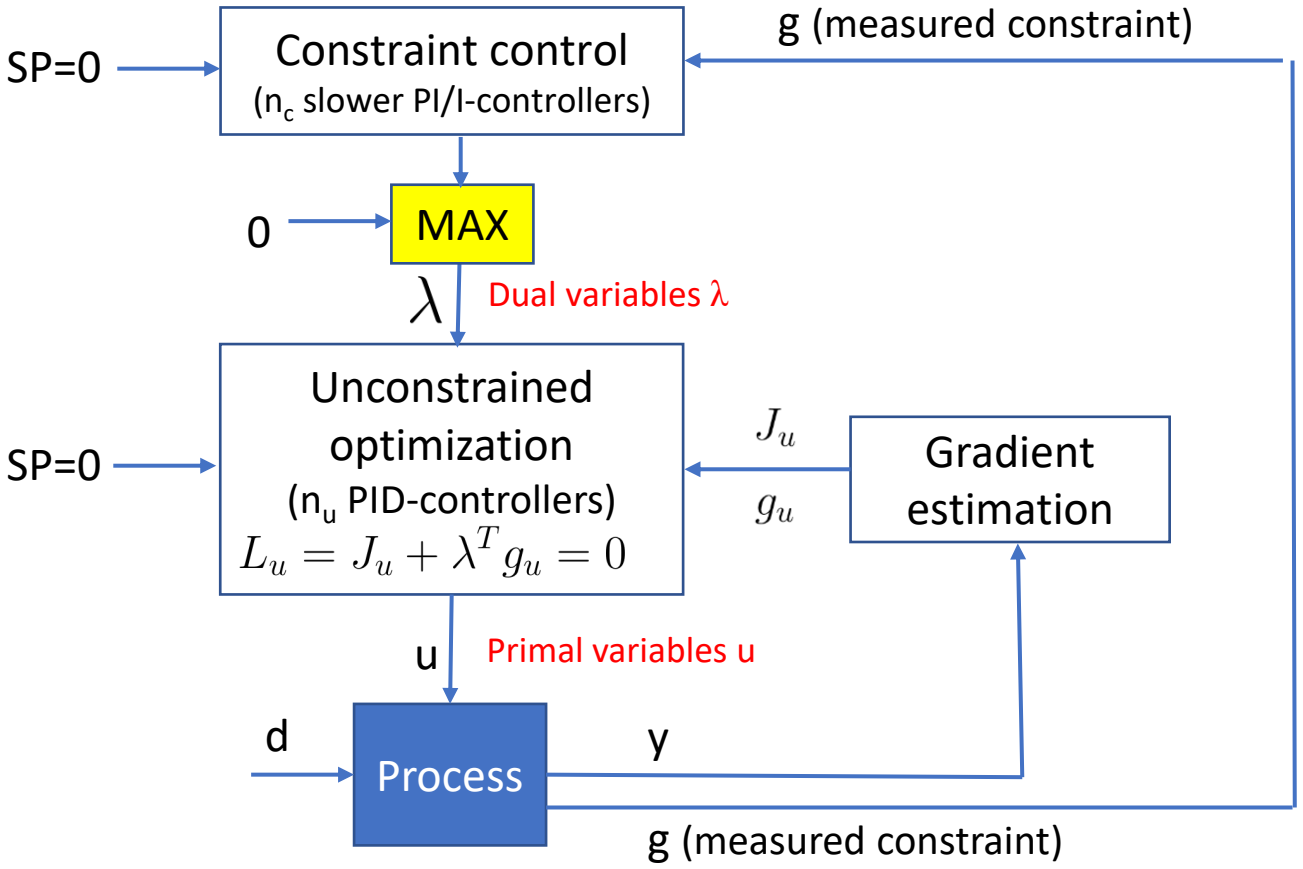
Necessary conditions of optimality (KKT-conditions)

$$\nabla_u \mathcal{L} = 0, \qquad \lambda \geq 0, \qquad g \cdot \lambda = 0$$

(complementary condition)

$$L_u = J_u + \lambda^T g_u = 0$$

IB-c. Primal-dual control based on KKT conditions: Feedback solution that automatically tracks active constraints by adjusting Lagrange multipliers (= shadow prices = dual variables) λ



KKT: $L_u = J_u + \lambda^T g_u = 0$

Inequality constraints: $\lambda \geq 0$

Primal-dual feedback control.

- Makes use of «dual decomposition» of KKT conditions
- Selector on dual variables λ
- Problem 1: Constraint control is indirect – it uses dual variables on slow time scale (upper layer)
  - Can be fixed using override at bottom of hiearchy (Dirza)
- Problem 2: Single-loop PID control in lower layer (L_u=0) may not be possible for coupled processes so may need to use Solver.

- D. Krishnamoorthy, A distributed feedback-based online process optimization framework for optimal resource sharing, J. Process Control 97 (2021) 72–83,
- R. Dirza and S. Skogestad . Primal–dual feedback-optimizing control with override for real-time optimization. J. Process Control, Vol. 138 (2024), 103208.

Alternative: Direct control of constraints

KKT: $L_u = J_u + \lambda^T g_u = 0$

Introduce N: $N^T g_u = 0$

Controlled variables (CVs) that gives optimal operation:

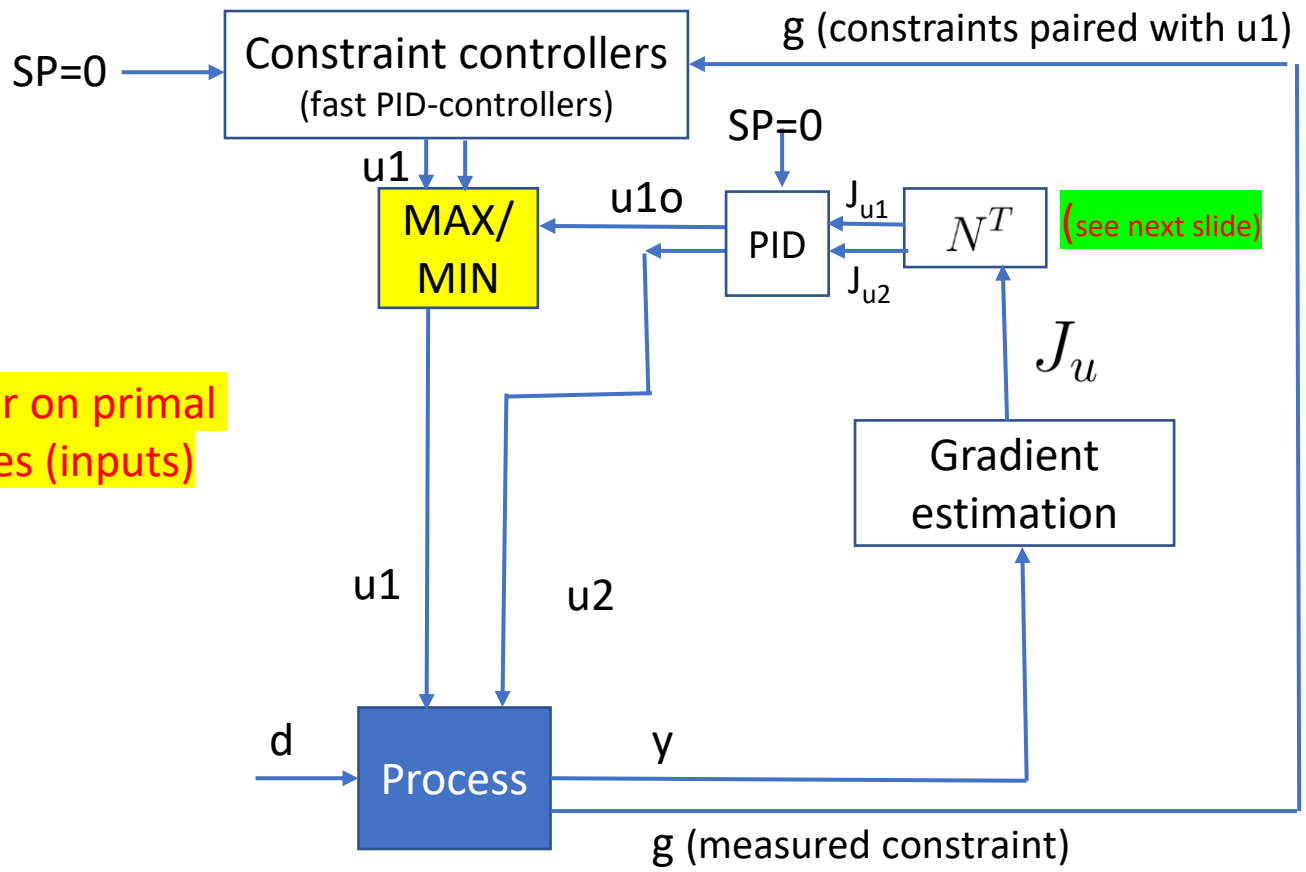
1. Active constraints $g_A = 0$.
2. Reduced gradient $N_A^T J_u = 0$
   - for the remaining inbconstrained degrees of freedom
   - «self-optimizing variables»

Seems easy. But how do we handle changes in constraints?

- Because g_A and N_A vary
- Originally, I thought we need a new control structure (with pairings) in each region

- Jaschke and Skogestad, «Optimal controlled variables for polynomial systems». S., J. Process Control, 2012
- D. Krishnamoorthy and S. Skogestad, «Online Process Optimization with Active Constraint Set Changes using Simple Control Structure», I&EC Res., 2019

IB-c. Region-based feedback solution with «direct» constraint control



- Selector on primal variables (inputs)

KKT: $L_u = J_u + \lambda^T g_u = 0$

Introduce N: $N^T g_u = 0$

- Selector on primal variables (inputs)
- Similar to selectors in ARC
- Limitation: need to pair each constraint with an input u, may not work if many constraints

- Jaschke and Skogestad, «Optimal controlled variables for polynomial systems». S., J. Process Control, 2012
- D. Krishnamoorthy and S. Skogestad, «Online Process Optimization with Active Constraint Set Changes using Simple Control Structure», I&EC Res., 2019
- L. Bernadino and S. Skogestad, Decentralized control using selectors for optimal steady-state operation with active constraints, J. Proc. Control, 2024

Feedback RTO with constraints

Assume: Have at least as many inputs as constraints

Can them have fixed pairings between constraints and unconstrained CVs! (with N is fixed)

Journal of Process Control 137 (2024) 103194

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Decentralized control using selectors for optimal steady-state operation with changing active constraints

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ARTICLE INFO

Keywords:
Optimal operation
Decentralized control
Selectors

ABSTRACT

We study the optimal steady-state operation of processes where the active constraints change. The aim of this work is to eliminate or reduce the need for a real-time optimization layer, moving the optimization into the control layer by switching between appropriately selected controlled variables (CVs) in a simple way. The challenge is that the best CVs, or more precisely the reduced cost gradients associated with the unconstrained degrees of freedom, change with the active constraints. This work proposes a framework based on decentralized control that operates optimally in all active constraint regions, with region switching mediated by selectors. A key point is that the nullspace associated with the unconstrained cost gradient needs to be selected in accordance with the constraint directions so that selectors can be used. A main benefit is that the number of SISO controllers that need to be designed is only equal to the number of process inputs plus constraints. The main assumptions are that the unconstrained cost gradient is available online and that the number of constraints does not exceed the number of process inputs. The optimality and ease of implementation are illustrated in a simulated toy example with linear constraints and a quadratic cost function. In addition, the proposed framework is successfully applied to the nonlinear Williams–Otto reactor case study.

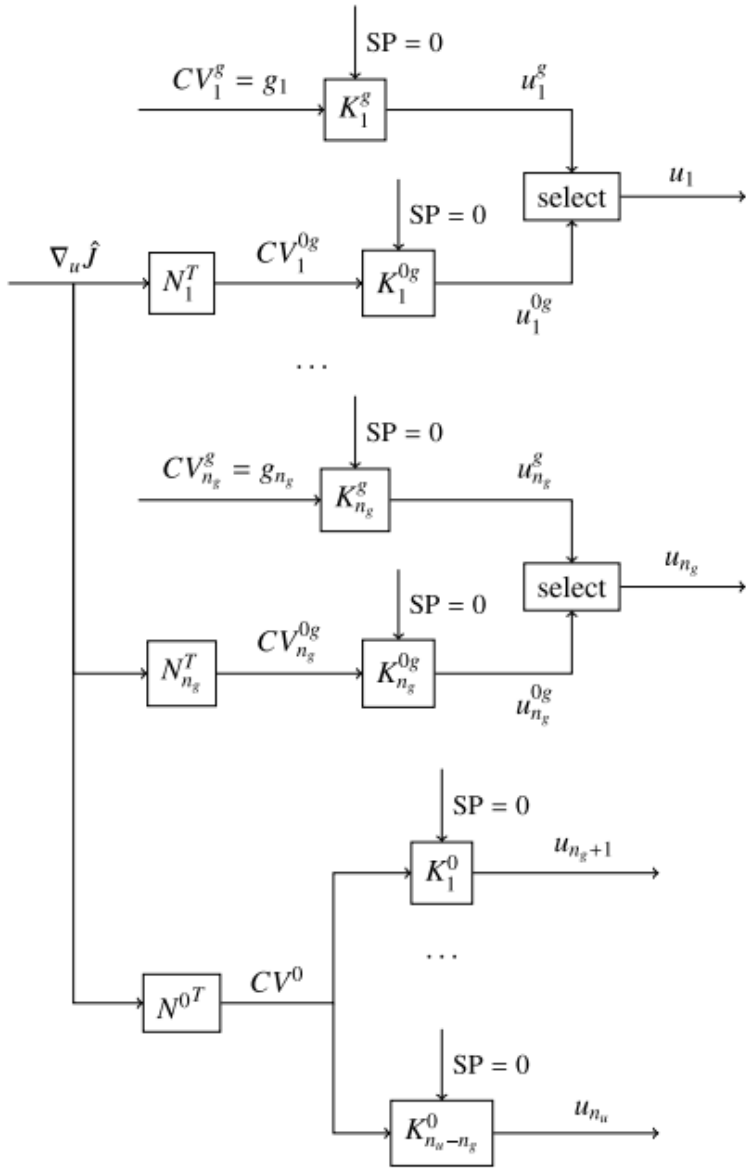


Fig. 3. Decentralized control structure for optimal operation according to Theorem 2. The "select" blocks are usually max or min selectors (see Theorem 3).

L. Bernadino and S. Skogestad, Decentralized control using selectors for optimal steady-state operation with active constraints, J. Proc. Control, 2024

IB-c. Region-based MPC with switching of cost function (for general case)

Standard MPC with fixed CVs: Not optimal

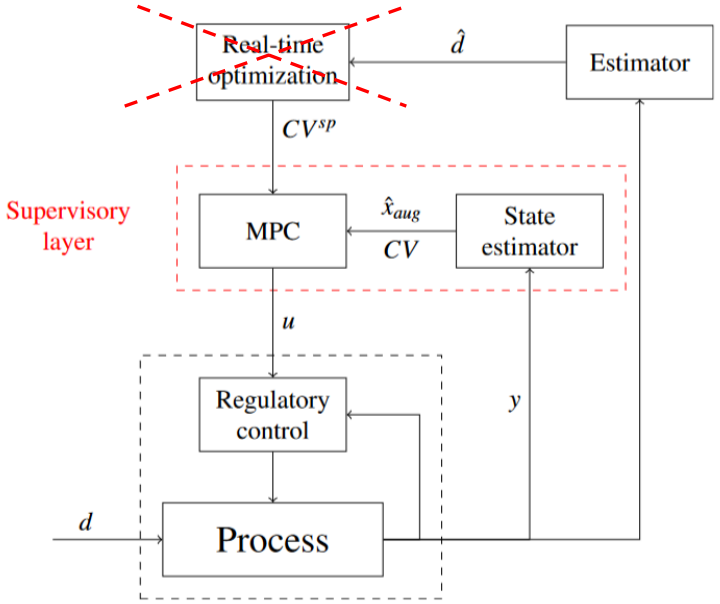


Figure 1: Typical hierarchical control structure with standard setpoint-tracking MPC in the supervisory layer. The cost function for the RTO layer is J^{ec} and the cost function for the MPC layer is J^{MPC}. With no RTO layer (and thus constant setpoints CV^{sp}), this structure is not economically optimal when there are changes in the active constraints. For smaller applications, the state estimator may be used also as the RTO estimator.

$$J^{MPC} = \sum_{k=1}^{N} \|CV_k - CV^{sp}\|_Q^2 + \|\Delta u_k\|_R^2$$

Proposed: With changing cost (switched CVs)

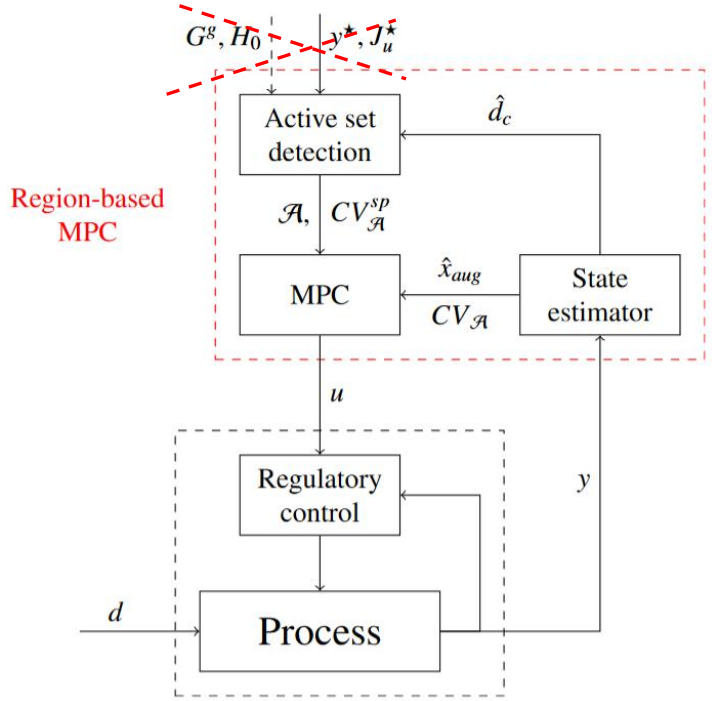


Figure 2: Proposed region-based MPC structure with active set detection and change in controlled variables. The possible updates from an upper RTO layer (y^*, J_u^* etc.) are not considered in the present work. Even with no RTO layer (and thus with constant setpoints $CV_{\mathcal{A}}^{sp}$, see (14) and (15), in each active constraint region), this structure is potentially economically optimal when there are changes in the active constraints.

$$J_{\mathcal{A}}^{MPC} = \sum_{k=1}^{N} \|CV_{\mathcal{A}} - CV_{\mathcal{A}}^{sp}\|_{Q_{\mathcal{A}}}^2 + \|\Delta u_k\|_{R_{\mathcal{A}}}^2$$

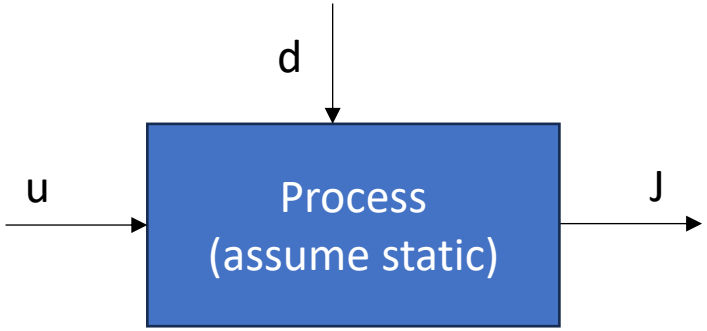
$$CV_{\mathcal{A}} = \begin{bmatrix} g_{\mathcal{A}} \\ c_{\mathcal{A}} \end{bmatrix} = \begin{bmatrix} g_{\mathcal{A}} \\ N_{\mathcal{A}}^T H_0 y \end{bmatrix} \tag{14}$$

$$H_0 = \begin{bmatrix} J_{uu} & J_{ud} \end{bmatrix} \begin{bmatrix} G^y & G_d^y \end{bmatrix}^\dagger$$

$$H^J = J_{uu} \left[G^{yT} \left(\tilde{F} \tilde{F}^T \right)^{-1} G^y \right]^{-1} G^{yT} \left(\tilde{F} \tilde{F}^T \right)^{-1}$$

- Bernardino and Skogestad, Optimal switching of MPC cost function for changing active constraints. J. Proc. Control, 2024

II. Model-free optimization:
Extremum Seeking Control (ESC) based on measuring cost J



Why ESC («hill-climbing»)?

- Simple if we can measure J
- Avoid expensive model (for J) needed for model-based RTO
- May also be used on top of a faster RTO-layer:
  - «Adapt» setpoint for J_u (to a nonzero bias value) to correct for model error
  - This hybrid approach is sometimes called «modifier adaptation» (bad name)

Two ESC alternatives

A. Without estimating gradient («perturb and observe»)
B. Gradient-based (classical ESC) (must perturb u to estimate gradient)

Main problems with ESC:

- Cost function J often not measured
  - For chemical process: $J=p_F F - p_P P - p_Q Q$
  - So need model (!) to estimate flows F, P and utility Q
- Slow. Typically 100 times slower than process dynamics (at least for gradient-based ESC)

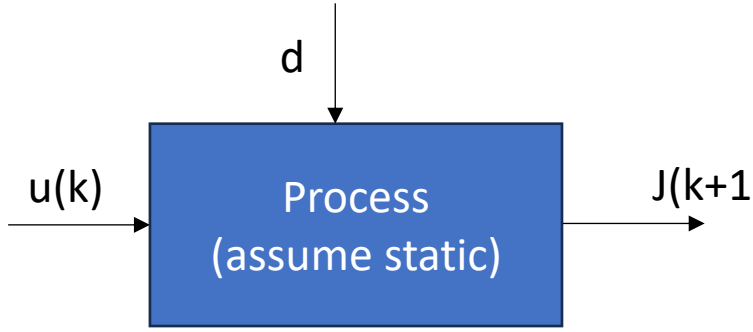
IIA. Extremum-seeking control (ESC) without gradient

A simple and intuitive version of ESC is the **perturb-and-observe** algorithm

- **Step 1**: Apply a perturbation Δu to the input:
  $u(k)=u(k-1)+\text{sign}(k)\cdot \Delta u$
  where k is the current time sample and sign(k) alternates between +1 and -1.
- **Step 2**: Observe (after time T_s) the cost J(k+1). If $J(k+1)<J(k)$, then the system is moving in the right direction so keep the same sign; otherwise, reverse the direction (change sign(k)). Repeat from Step 1.

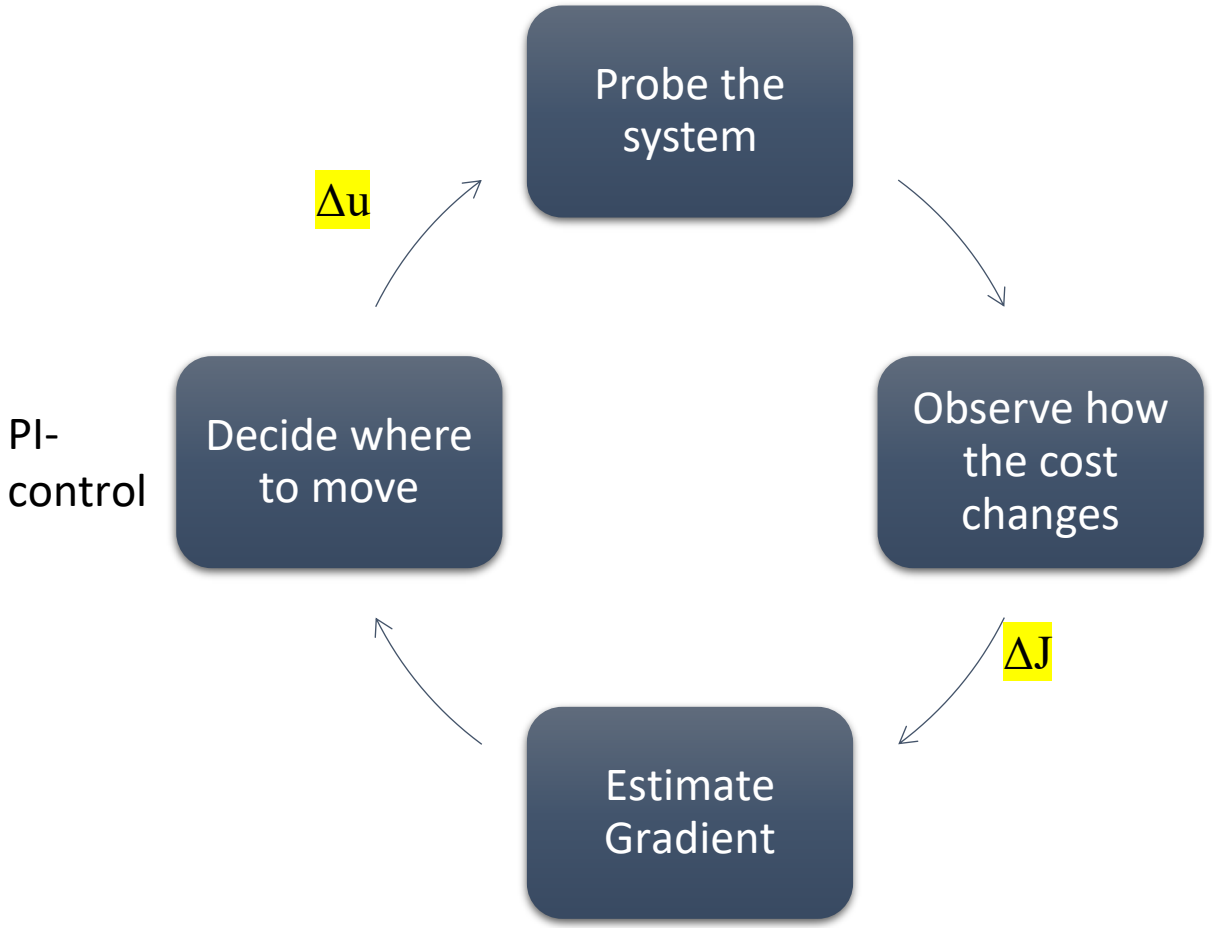
This method has two main tuning parameters:

- The **step size** Δu: Larger steps speed up convergence but cause larger oscillations around the optimum.
- The **sampling time** T_s: This must be long enough to let the system respond. A rule of thumb is $T_s \approx 3\tau$, where τ is the system's time constant (to settle to steady state).

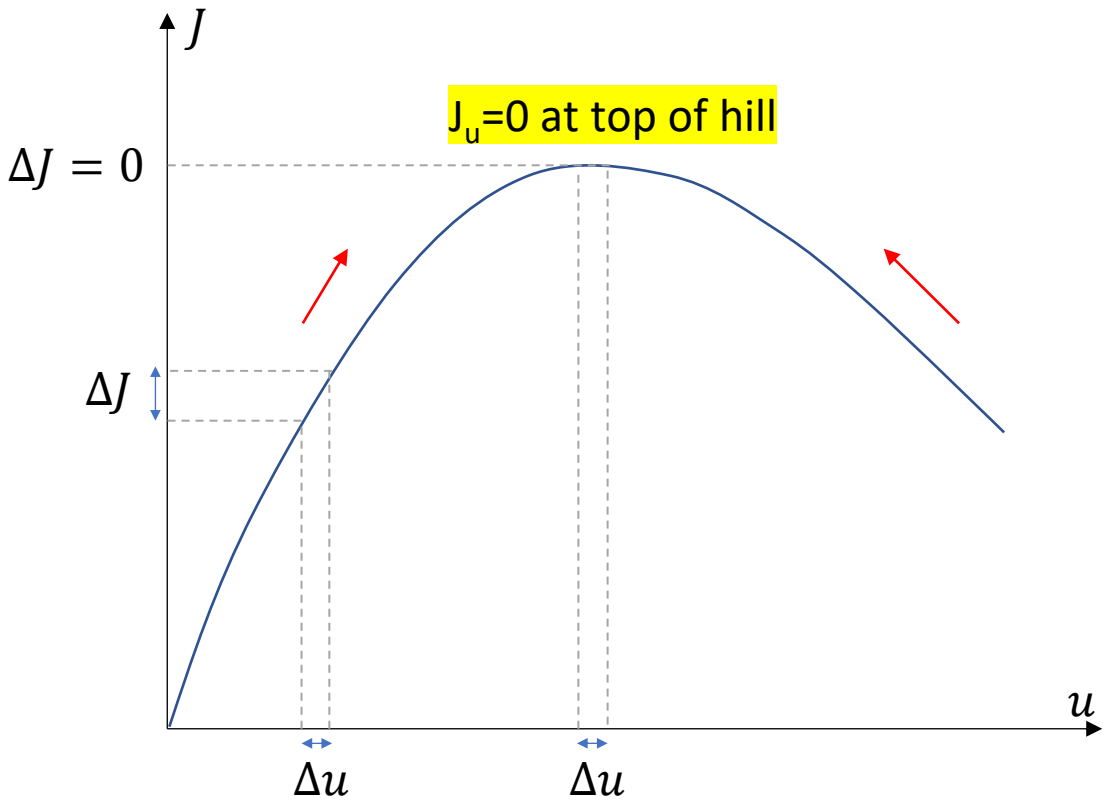


This approach is used, for example, to fine-tune the position of solar mirrors – and we use it currently to optimize a dividing-wall distillation column (Halvorsen et al., 2025)

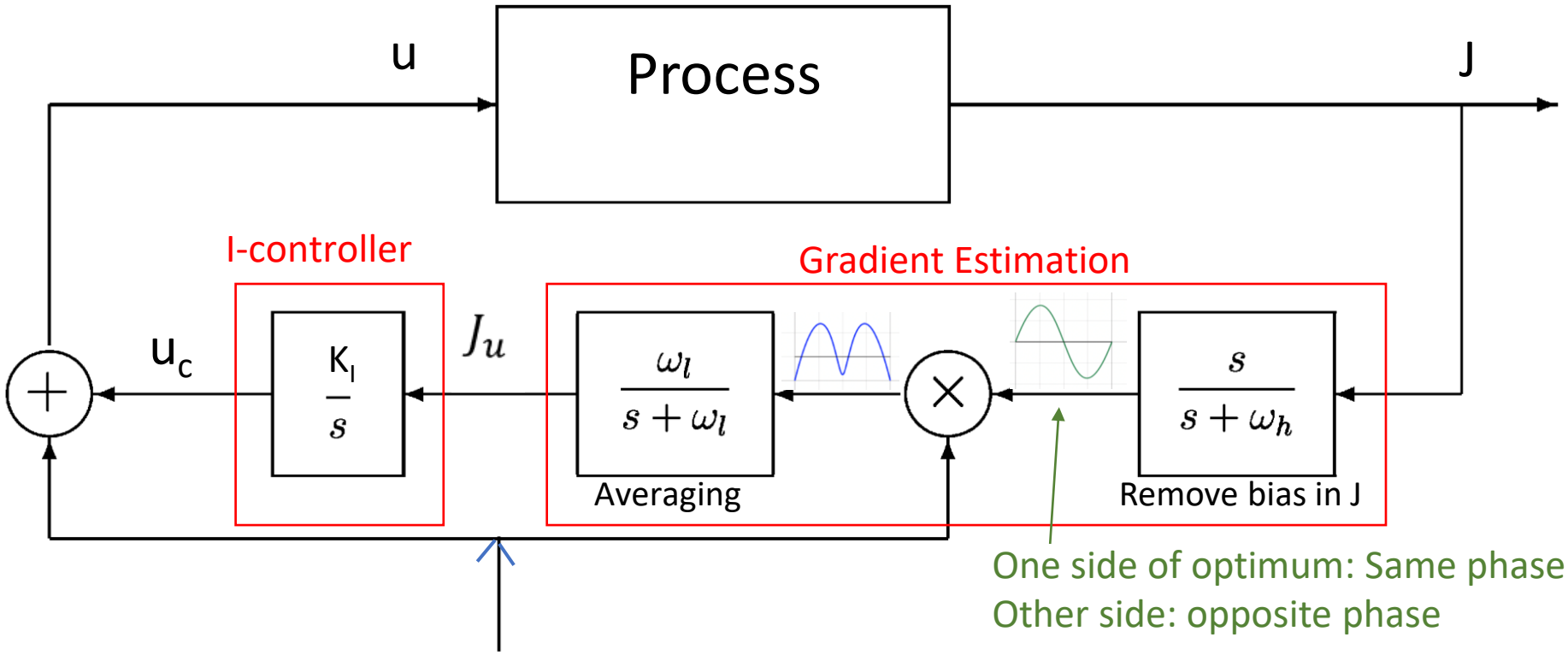
IIB. ESC with gradient: Drive gradient $J_u=dJ/du$ to zero.



Simplest: $J_u=\Delta J/\Delta u$



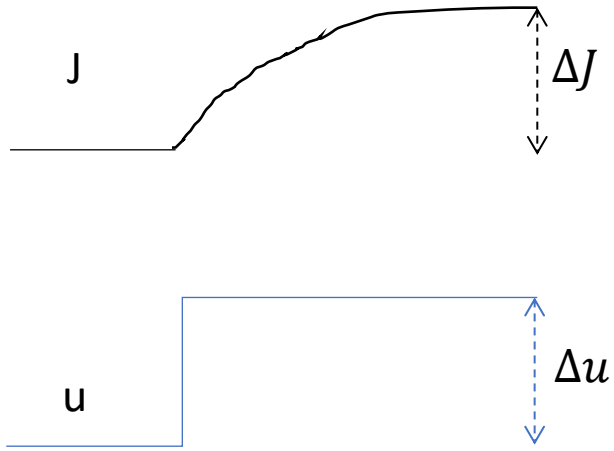
Classical Extremum seeking control using sinusoids



Multiplication trick: Draper & Li (1951)
Theory: Krstic & Wang (Automatica, 2000)

- **Simple to implement (don't need computer),** but
- Prohibitively slow convergence for systems with slow dynamics
- Typically 100 times slower than the system dynamics !

More common today: Estimate Steady-state gradient using discrete perturbations (steps)



$$J_u = \frac{\Delta J}{\Delta u}$$

Usually only one input. Simplest: step change in u:

- Hill climbing control (Shinskey, 1967)
- Evolutionary operation (EVOP) (1960's)
- NCO tracking (Francois & Bonvin, 2007)

More advanced variants which may also be applied to multivariable systems

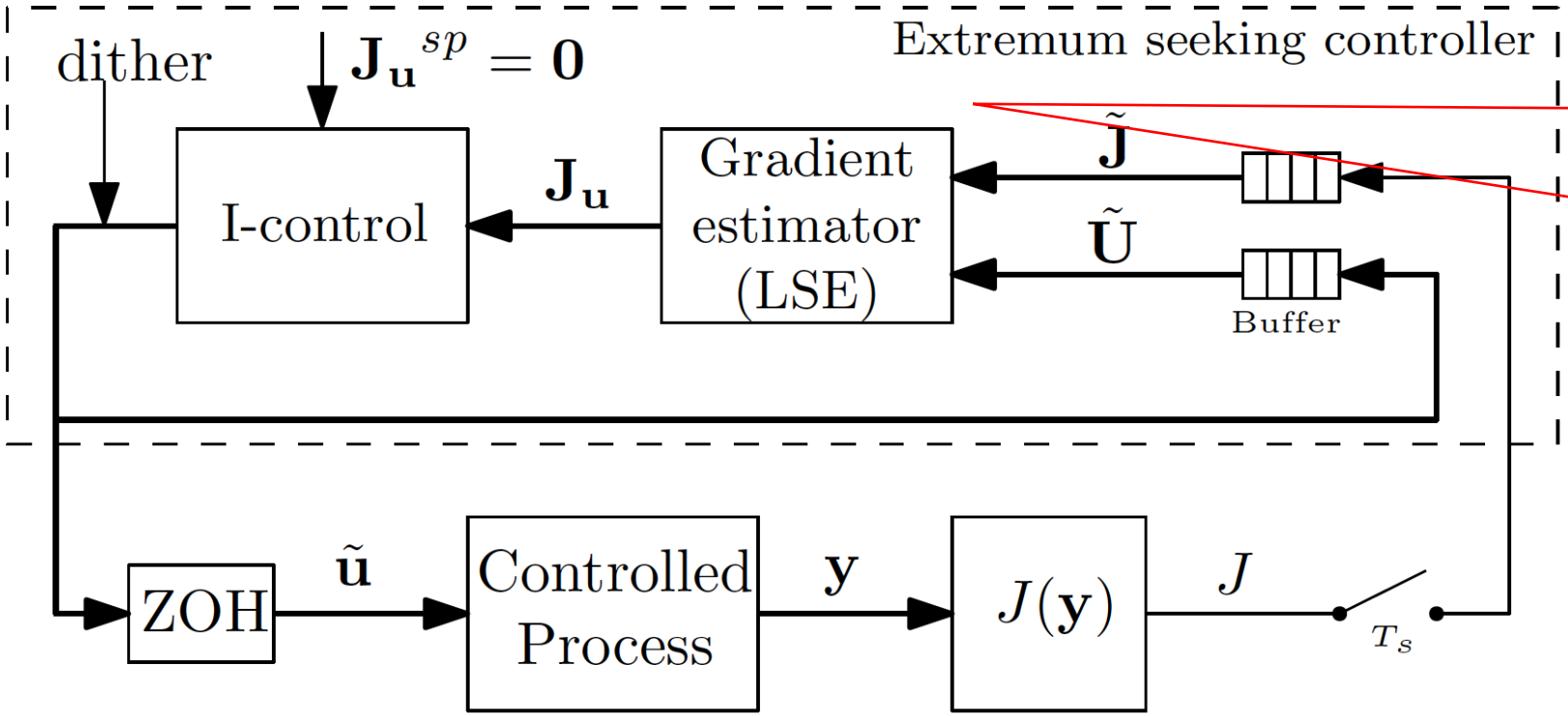
- **Least squares estimation**
- Fast Fourier transform (Dinesh Krishnamoorthy)

To avoid waiting for steady state

- Fitting of data to ARX model (difficult to make robust)

Note: Assumes steady state -> samling (step) time > 3-10 time process time constant

Least square gradient estimation for ESC



LSE: Fit a linear model

$$J = \mathbf{J}_{\tilde{\mathbf{u}}}{}^{\mathsf{T}}\tilde{\mathbf{u}} + m$$

Using least squares fit

$$\mathbf{Y} = [J_k, J_{k-1}, \ldots, J_{k-N+1}]^T$$
$$\mathbf{U} = [\mathbf{u}_k, \ldots, \mathbf{u}_{k-N+1}]^T$$
$$\theta = [\hat{\mathbf{J}}_{\mathbf{u}}^T, m]^T$$

$$\hat{\theta} = \arg\min_{\theta} \|\mathbf{Y} - \Phi^T\theta\|_2^2$$

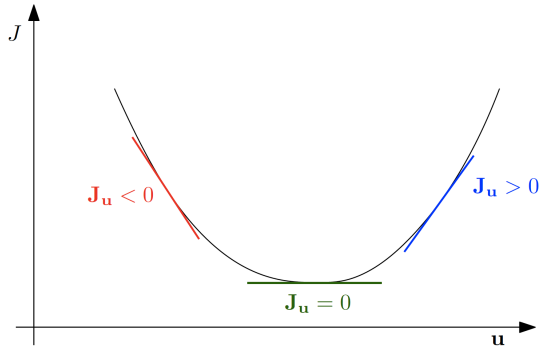
to which the analytical solution is given by

$$\hat{\theta} = [\Phi^T\Phi]^{-1}\Phi^T\mathbf{Y}$$

Note: Assumes no dynamics -> samling time > 3-10 time process constant

Hunnekens et al. (2011, 2014)

IIB. Summary extremum seeking control with gradient



Idea: Estimate the cost gradient J_u from data and drive it to zero

- Common to all methods:
  - Need measurement of cost J
  - Must wait for steady state (except ARX method which fails frequently)
  - Must assume no «fast» disturbances (while optimizing)

Algorithm needs two layers on top of process:

1. Optimization layer (slowest): Control J_u to zero (may use I-controller)
2. Lower estimation layer: Estimate the local gradient J_u using data
   - Must wait for the process to reach steady state

- Need time scale separation between layers.
  - At best this means that the optimization needs to be 10 times slower than the process.
  - Often it needs to be 100 times slower.
- **Useful for fast processes with settling time a few seconds**
- Not useful for many chemical processes where time constant typically are several minutes
  - 10 minutes * 100 = 1000 minutes = 16 hours
  - Unllikely with 16 hours without disturbance

Now we are finished…

ARC: Research tasks

8.1. A list of specific research tasks

Here is a list of some research topics, which are important but have received limited (or no) academic attention:

1. Vertical decomposition including time scale separation in hierarchically decomposed systems (considering performance and robustness)
2. Horizontal decomposition including decentralized control and input/output pairing
3. Selection of variables that link the different layers in the control hierarchy, for example, self-optimizing variables (CV1 in Fig. 4) and stabilizing variables (CV2).
4. Selection of intermediate controlled variables (w) in a cascade control system.[9]
5. Tuning of cascade control systems (Figs. 9 and 10)
6. Structure of selector logic
7. Tuning of anti-windup schemes (e.g., optimal choice of tracking time constant, τ_T) for input saturation, selectors, cascade control and decoupling.
8. How to make decomposed control systems based on simple elements easily understandable to operators and engineers
9. Default tuning of PID controllers (including scaling of variables) based on limited information
10. Comparison of selector on input or setpoint (cascade)

8.2. The harder problem: Control structure synthesis

The above list of research topics deals mainly with the individual elements. A much harder research issue is *the synthesis of an overall decomposed control structure, that is, the interconnection of the simple control elements for a particular application*. This area definitely needs some academic efforts.

One worthwhile approach is case studies. That is, to propose "good" (= effective and simple) control strategies for specific applications, for example, for a cooling cycle, a distillation column, or an integrated plant with recycle. It is here suggested to design also a centralized controller (e.g., MPC) and use this as a benchmark to quantify the performance loss (or maybe the benefit in some cases) of the decomposed ARC solution. A related issue, is to suggest new smart approaches to solve specific problems, as mentioned in item 11 in the list above.

A second approach is mathematical optimization: Given a process model, how to optimally combine the control elements E1–E18 to meet the design specifications. However, even for small systems, this is a very difficult combinatorial problem, which easily becomes prohibitive in terms of computing power. It requires both deciding on the control structure as well as tuning the individual PID controllers.

As a third approach, machine learning may prove to be useful. Machine learning has one of its main strength in pattern recognition, in a similar way to how the human brain works. I have observed over the years that some students, with only two weeks of example-based teaching, are able to suggest good process control solutions with feedback, cascade, and feedforward/ratio control for realistic problems, based on only a flowsheet and some fairly general statements about the control objectives. This is the basis for believing that machine learning (e.g., a tool similar to ChatGPT) may provide a good initial control structure, which may later be improved, either manually or by optimization. It is important that such a tool has a graphical interface, both for presenting the problem and for proposing and improving solutions.

What about MPC?

- First industrial use in the 1970s
- Became common in the refining and petrochemical industry in the 1980s
- In the 1990s a bright future was predicted for MPC in all process industries (chemical, thermal power, …)
- 30 years later: We know that this did not happen
- Why? First, the performance benefits of MPC compared to ARC are often minor (if any)
- In addition, MPC has some limitations
    1. Expensive to obtain model
    2. Does not easily handle integral action, cascade and ratio control
    3. Normally, cannot be used at startup (so need ARC anyway)
    4. Can be difficult to tune. Difficult to incorporate fast control tasks (because of centralized approach)
    5. Computations can be slow
    6. Robustness (e.g., gain margin) handled indirectly
- Advantages of MPC
    1. Very good for interactive multivariable dynamic processes
    2. Coordinates feedforward and feedback
    3. Coordinates use of many inputs
    4. Makes use of information about future disturbances, setpoints and prices (predictive capabilities of MPC)
    5. Can handle nonlinear dynamic processes (nonlinear MPC)
- What about constraints
    - Not really a major advantage with MPC; can be handled well also with ARC

MPC = model predictive control
ARC = advanced regulatory control

7.6.7. Summary of MPC shortcomings

Some shortcomings of MPC are listed below, in the expected order of importance as seen from the user's point of view:

1. MPC requires a "full" dynamic model involving all variables to be used by the controller. Obtaining and maintaining such a model is costly.
2. MPC can handle only indirectly and with significant effort from the control engineer (designer), the three main inventions of process control; namely integral control, ratio control and cascade control (see above).
3. Since a dynamic model is usually not available at the startup of a new process plant, we need initially a simpler control system, typically based on advanced regulatory control elements. MPC will then only be considered if the performance of this initial control system is not satisfactory.
4. It is often difficult to tune MPC (e.g., by choosing weights or sometimes adjusting the model) to give the engineer the desired response. In particular, since the control of all variables is optimized simultaneously, it may be difficult to obtain a solution that combines fast and slow control in the desired way. For example, it may be difficult to tune MPC to have fast feedforward control for disturbances because it may affect negatively the robustness of the feedback part (Pawlowski et al., 2012).
5. The solution of the online optimization problem is complex and time-consuming for large problems.
6. Robustness to model uncertainty is handled in an ad hoc manner, for example, through the use of the input weight R. On the other hand, with the SIMC PID rules, there is a direct relationship between the tuning parameter τ_c and robustness margins, such as the gain, phase and delay margin Grimholt and Skogestad (2012), e.g., see (C.13) for the gain margin.

7.6.8. Summary of MPC advantages

The above limitations of MPC, for example, with respect to integral action, cascade control and ratio control, do not imply that MPC will not be an effective solution in many cases. On the contrary, MPC should definitely be in the toolbox of the control engineer. First, standard ratio and cascade control elements can be put into the fast regulatory layer and the setpoints to these elements become the MVs for MPC. More importantly, MPC is usually better (both in terms of performance and simplicity) than advanced regulatory control (ARC) for:

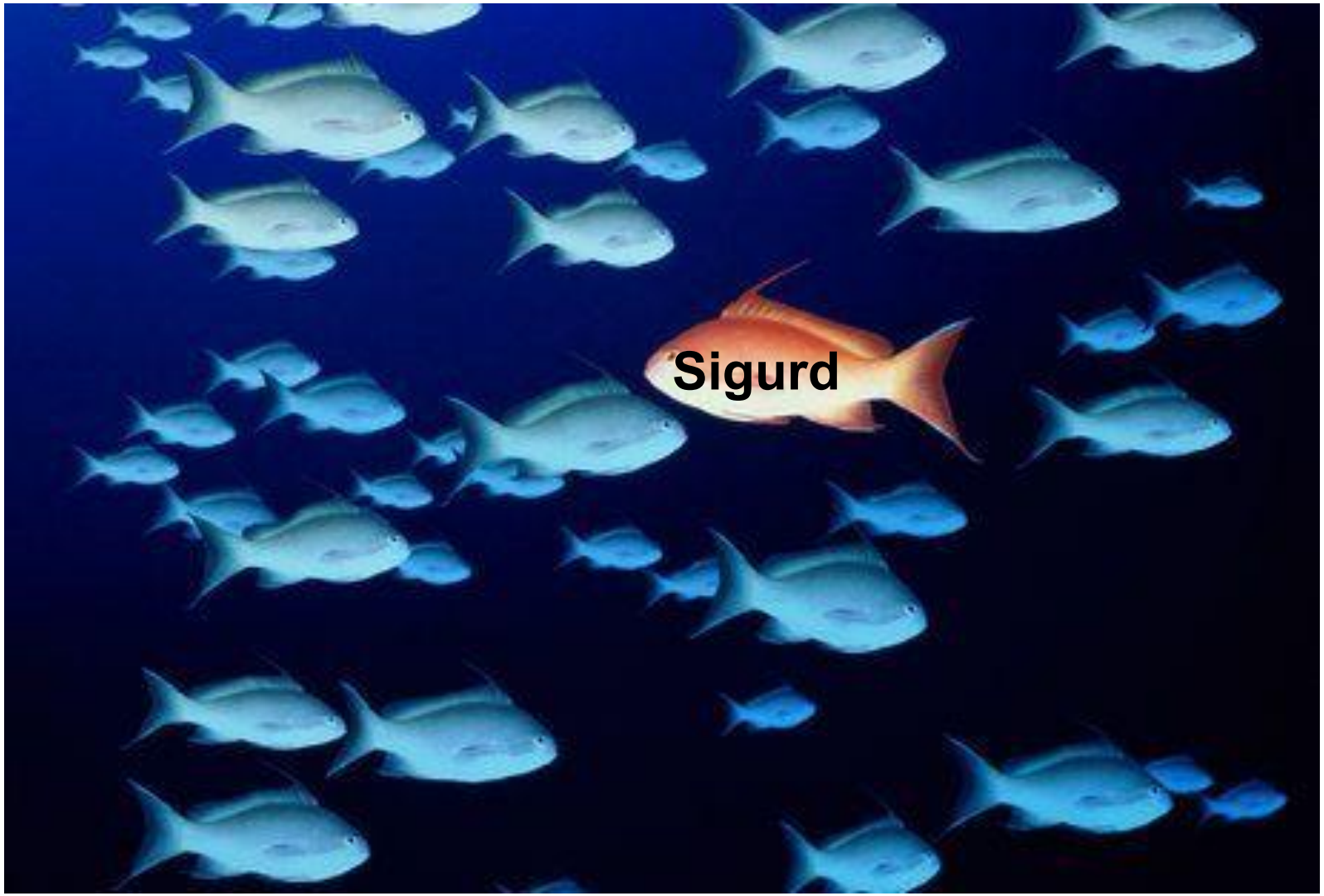
1. Multivariable processes with (strong) dynamic interactions.
2. Pure feedforward control and coordination of feedforward and feedback control.
3. Cases where we want to dynamically coordinate the use of many inputs (MVs) to control one CV.
4. Cases where future information is available, for example, about future disturbances, setpoint changes, constraints or prices.
5. Nonlinear dynamic processes (nonlinear MPC).

The handling of constraints is often claimed to be a special advantage of MPC, but it can it most cases also be handled well by ARC (using selectors, split-range control solutions, anti-windup, etc.). Actually, for the Tennessee Eastman Challenge Process, Ricker (1996) found that ARC (using decentralized PID control) was better than MPC. Ricker (1996) writes in the abstract: "There appears to be little, if any, advantage to the use of NMPC (nonlinear MPC) in this application. In particular, the decentralized strategy does a better job of handling constraints – an area in which NMPC is reputed to excel". In the discussion section he adds: "The reason is that the TE problem has too many competing goals and special cases to be dealt with in a conventional MPC formulation".

Present Academic control community fish pond

Complex optimal centralized Solution (**EMPC, FL**)

Simple solutions that work (ARC, PID)



FL = feedback linearization

Future Academic control community fish pond

Complex optimal centralized Solution (**EMPC, FL**)

Simple solutions that work (SRC,PID)