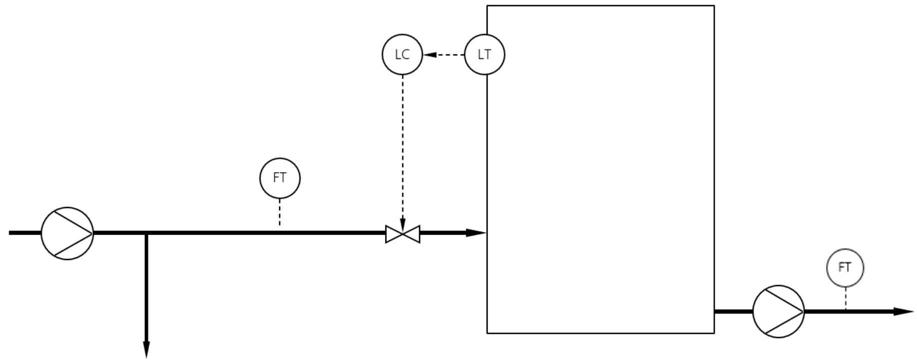
Exam Advanced Process control. 18 Nov. 2025 (3 hours)

Note: The sum of percentages is 80% since the homework counts 20%

Problem 1. Advanced level control (15%)

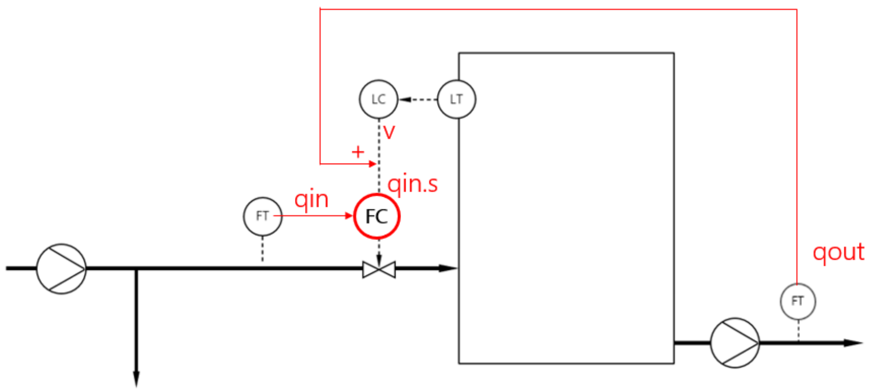


(a) The level in the tank varies too much, because there are pressure variations in the line for the incoming flow. We can't tune the PI level controller (LC) more aggressively because then it may become unstable. Propose a possible control structure to improve performance for this case.

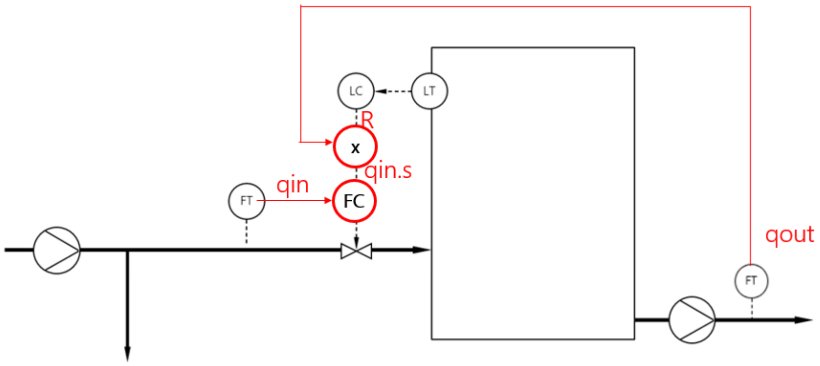
(b) Another reason for the level variations is large variations in the tank outflow. Propose an additional control modification to improve performance in this case.

Solution: (a) Slave flow controller; the level controller sets qins.

(b) Feedforward control based on the model dV/dt = qin – qout [m3/s]. We let the controller compute v = qin-qout and then feedforward is just an addition: qin,s = qout + v



An alternative solution for b) using ratio control is shown below. The level controller computes the ratio setpoint R. If the flow measurements are correct and in the same units then R=1 at steady state. The ratio control solution is better if the two flow measurements are in different units.



Problem 2. PID control (5%)

In the slides as help for Exercise 5, the slide below is presented for "clarification. Can you explain what kind of PID controller is given by (1) and (2), and point out possible errors in what is presented.

General Clarification

- SIMC tuning give results in series form:

$$u(s) = K_c\left(1+\frac{1}{\tau_I s}\right)\left(y_s(s) - \frac{\tau_D s + 1}{(\tau_D / \ N)s+1}y(s)\right) \qquad (1)$$

- Simulink uses however:

$$u(s) = \left(P + I\frac{1}{s} + D\frac{N}{1+N\frac{1}{s}}\right)y_s(s) \qquad (2)$$

- So $P = K_c$ and $I = \frac{K_c}{\tau_I}$
- The model is designed in **min**, not s!

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Solution: (1) is a cascade/series-form PID without D-action on SP.

(2) for Simulink is an ideal/parallel form PID with D-action on SP.

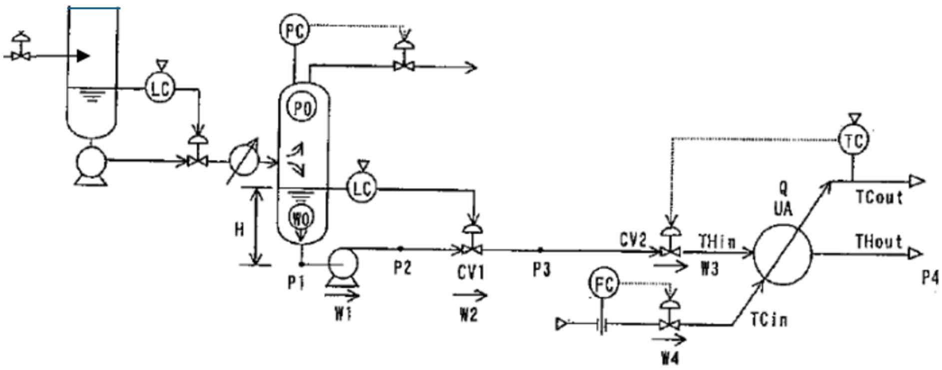
Error 1 (misprint): In (2), ys should be ys-y.

"Error" 2: The formula for P and I is correct for PI only. However, for PID it should also depend on the ratio taud/taul since we go from cascade to ideal PID.

"Error" 3. N is not the same in (1) and (2). In (1) N is dimensionless, whereas in (2) N has units 1/time. To make them more similar, we would need to replace (τ_D /N) in (1) by (1/N). Personally, I (Sigurd) think it is clearer to use τ_F = 1/N (filter time constant) in(2), and also to put the filter 1/(τ_F s+1) on the whole PID-controller and not only the D-term.

Problem 3. TPM and consistency (10%)

(a) Is the control structure below consistent? Where is the TPM?
(b) If it is not consistent, then explain how it can be changed to make it consistent.



Solution: (a) No it's not consistent as there are two TPMs: Feed and W4. (b)Rearrange the two inventory loops (LC) so that they go to the feeds of the tanks. (CV1 may be left fully open to reduce pressure drop).

Problem 4. Optimal operation (15%)

(a) In the unconstrained case the "ideal" controlled variable is c=J_u. Explain with a figure why.

Show figure of J as a function of u. The optimal point is at the "bottom" where Ju=0.

(b) In the heat exchanger case below, where the cold stream (T_0) goes to two heaters, we want to adjust the split u=α in order to maximize the combined outlet temperature T. (i)What is the cost function J in this case? (ii)What are the disturbances? (iii)How derive analytical expression for Ju?

Solution. (i) J=-T. (ii). Disturbances are 3 flowrates (0,1,2), 3 inlet temperatures (0,1,2). In addition, there are the 2 UA-values (iii). We need a model for J, which in this case is the energy balance for the two heat exchangers and the expression for heat transfer Q which depends on UA and the temperature difference. In general, this gives and expression for J=_T which depends on u and the 8 disturbances. To find, Ju, we differentiate this expression for J with respect to u (with constant disturbances). In general, Ju will depend on u and all 8 disturbances.

Comment: In this case, we could with algebraic manipulations eliminate many of the disturbances (UA-values and flowrates), see Exercise 1, but in general this is complex and cannot be done. So the linear exact local method described next is simpler and more general.

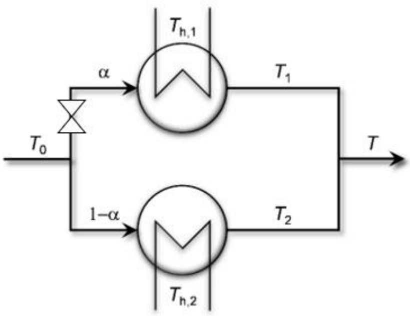
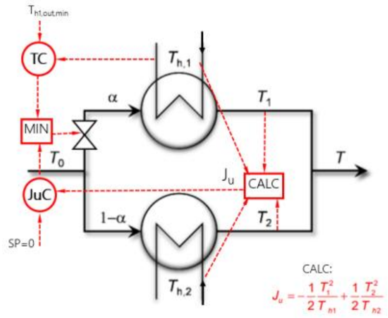


Figure 1: 2 heat exchanger network used in problem 1

$$J_u = -\frac{1}{2}\frac{T_1^2}{T_{h1}} + \frac{1}{2}\frac{T_2^2}{T_{h2}}$$

(c) An alternative approach is to derive c as a linear combination of the available measurements. (i)What two methods do you know for this? (ii)Explain the main steps in applying this approach to the case in Figure 1. (iii)Which of the two linear methods do you recommend?
Solution: (i) Nullspace method and exact local method (iii) First, we need to find the optimal sensitivity F = dyopt/dd from the model for J, where y are the measurements (which could be temperatures, flowrates, etc.). In our case, there are 8 disturbances! Then we plug F into formulas to find H, (and use c=Hy, or use H to find Ju). (iii) In general, I recommend the exact local method, because it is more general than the nullspace method and requires little extra information (weights Wd and Wn; for magnitude of disturbances and measurement error). In this case, there is no doubt that the exact local method is preferred, because the nullspace method would require 9 measurements (y). So if we only want to use the 4 temperatures as measurements, we need to use the exact local method.

(d) Given that we can "measure" Ju, show a control structure to achieve optimal operation. Show how you can handle the case where there is minimum constraint on the exit temperature for Th1 (Th1,out > Th1,min).



Solution: We have one gradient controller (with setpoint 0) and one temperature controller (with setpoint Th1,min). Both could be PI-controllers with anti windup. The constraint on Th1 is satisfied by a small α, so the two controller outputs go to a MIN-selector. If we reach the temperature constraint, we have to give up controlling the gradient (it is "overridden").

Problem 4. Model for tuning PID (5%)

You probably have tried the following method for getting a model (this is from Exercise 5 slides):

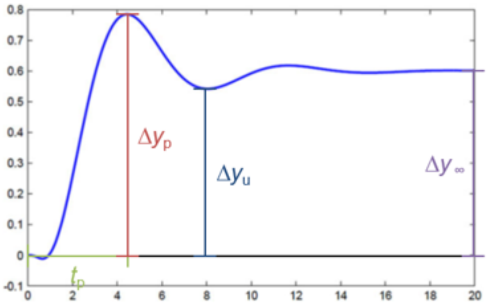
- Identify Δy_p, Δy_u (or Δy_∞), and t_p.
- Calculate:

$$\Delta y_\infty = 0.45(\Delta y_p + \Delta y_u)$$

$$D = \frac{\Delta y_p - \Delta y_\infty}{\Delta y_\infty} \quad B = \left|\frac{\Delta y_s - \Delta y_\infty}{\Delta y_\infty}\right|$$

$$A = 1.152D^2 - 1.607D + 1$$

$$r = 2A / B$$



- Calculate 1st order + time delay parameters

$$k = 1/(K_{c0}B)$$

$$\theta = t_p\left(0.309 + 0.209e^{-0.61r}\right)$$

$$t_1 = r\theta$$

(a) What is this method called? Is it based on analytical derivations?
(b) How do you use this method in practice? Maybe you can tell about the experience you had with the method on Exercise 5?
(c) When should it be used (rather than some alternative method)?
(d) Describe an alternative method.

Solution

(a) This is Shams closed-loop method to find the open-loop model G (first-order with delay). It is purely empirical.
(b) Use a proportional controller and increase gain until the overshoot is about 30%.
Comment: This is similar to ZN closed-loop method but we use a smaller controller gain - and we get more information (ZN identification gives only 2 pieces of information, Ku and Pu, which is not enough to obtain 3 parameters needed for a first-order with delay model. For this reason ZN can only estimate the combined ratio k/t1, so it doesn't work for fast processes with a small t1 where we need to estimate k to get good tunings.)
(c) Use when you cannot put the loop in manual mode and do open-loop responses.
(d) The simplest alternative approach is an open-loop step response experiment.

Problem 6. Happy cows. (15%)

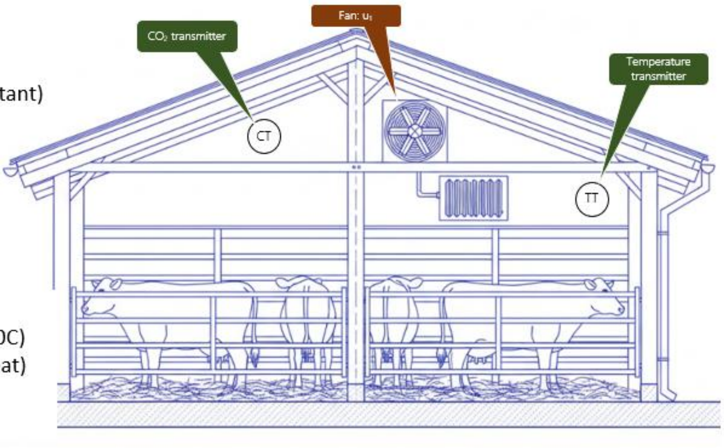
Propose a control structure based on PI controllers for the cow case study (below).

Cow case study (Norway, winter)

Happy cows
- y1=C (CO2) less than 2000 ppm (most important)
- y2=T above 5C and below 20C
- Not too much draft or noise from fan
  - 50% fan speed is good

MV
- u=fan (reduces y1 and y2)

Disturbances:
1. Outdoor temperature (between 15C and -30C)
2. Number of cows (they generate CO2 and heat)
   - Because of heat from cows it's always colder outside

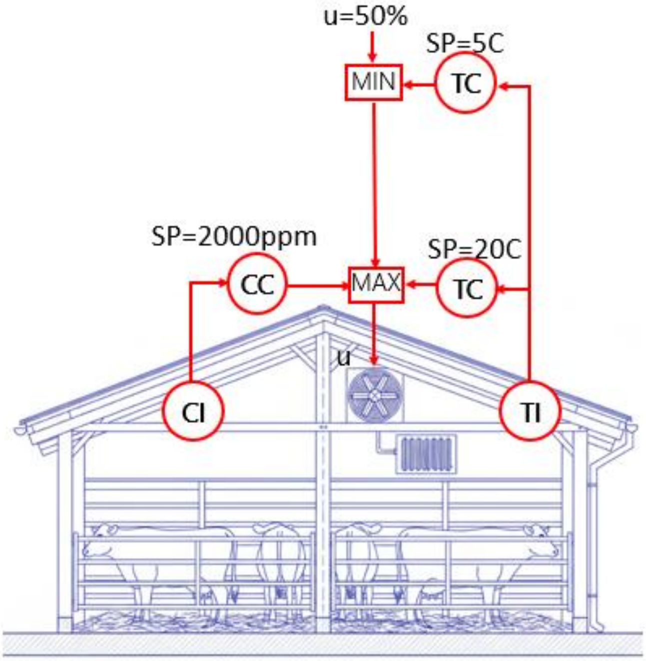


Nominally (and hopefully most of the time), the fan speed will be fixed at 50%. However, if there are too many cows, the air quality may get bad (with CO2 above 2000 ppm) or the temperature may get too high (above 20C) and we need to increase the fan speed above 50%. On the other hand, on a cold winter day, the temperature may get too low (below 5C) and we need to reduce the fan speed below 50%. Note that the air quality constraint (y1<2000 ppm) is more important than the temperature constraints.

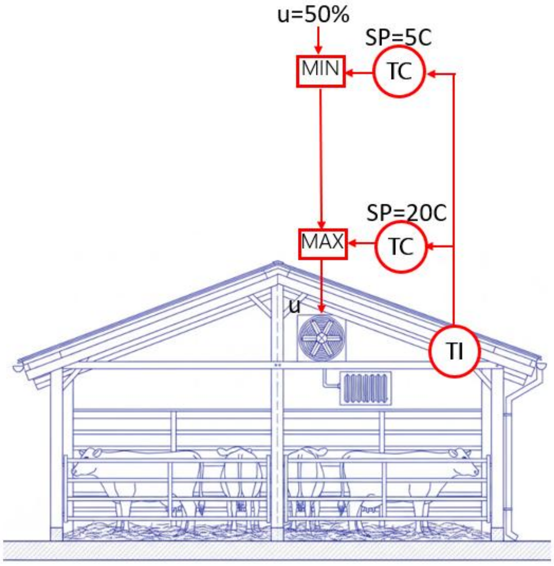
Solution:

Solution:
Constraint y1<2000 ppm : Satisfied by large fan speed -> MAX selector
Constraint y2>5C: Satisfied by small fan speed -> MIN selector
Constraint y2<20C: Satisfied by large fan speed -> MAX selector

The MIN- and MAX-selectors for y2=T will never conflict because T cannot be both 5C and 20 C at the same time. However, when it's cold outside and many cows, the MAX-selector for y1 (max 2000 ppm) (requiring a large fan speed), may conflict with the MIN-selector for y2 (min 5C) (requiring a small fan speed). However, the constraint on y1 is the most important, so the MAX-selector should be "at the end" (closest to the input u). That is, we may need to accept that y1=T drops below 5C when it's cold outside. The final control structure is then as shown below.



Comment: Below is shown the temperature control only, for the case when we don't care about the CO2. Here, the order of MAX- and MIN-selector may be interchanged because the two constraints (5C and 20C) cannot be active at the same time.



Problem 7. Transformed inputs (15%)

(a) Consider a steady-state model y=f(u,d). Define the transformed input v and explain with a block diagram how it can be used for control purposes.

(b) In a paper you find a model (see below) for a bioreactor where the reaction

substrate (concentration S) → cells (concentration X) takes place. Define output y=X, input u=D (feed rate), disturbance d=S_f (feed concentration). What is the transformed input v in this case? Fins also the inverse (to compute u from a given v).

to the concentration of cells (X), i.e., $r = \mu(S)X$. We assumed that the specific growth rate (μ (h^{-1})) is given by the Monod model

$$\mu(S) = \frac{\mu_m S}{K_s + S} \qquad (8)$$

where μ_m (h^{-1}) and K_s (g/L) represent the maximum specific growth rate and the saturation constant respectively. The dynamical behavior of the *simple microbial growth process* is most often described by the following "unstructured"[2] model which is the result of the material balances on the cell mass and the substrate in a constant-volume continuous bioreactor (e.g., Bastin and Dochain, 1990)

$$\frac{dX}{dt} = \mu(S)X - DX \qquad (9)$$

$$\frac{dS}{dt} = D(S_f - S) - \frac{\mu}{Y_{X/S}}X \qquad (10)$$

where $Y_{X/S}$ is the yield coefficient [g cells/g substrate] which is assumed constant in this paper.

Equation (9) gives at steady state that **μ(S)=D** (because X=0 is not a desired solution). The steady-state model (which is important for finding the transformed input v) then becomes

$$\bar{S} = \frac{K_s \bar{D}}{\mu_m - \bar{D}} \qquad (11)$$

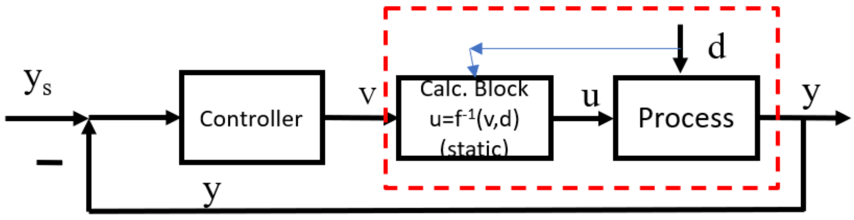
$$\bar{X} = Y_{X/S}(\overline{S_f} - \bar{S}) \qquad (12)$$

where the overbars represent steady-state values. (Comment: (11) follows from (8) and (12) is derived from (10) by setting dS/dt=0 and using μ=D.)

Note that μ_m=0.5,K_s=0.1 and $Y_{X/S}$=0.4 are assumed to be constant parameters.

Solution

(a) For a model y=f(u,d), the transformed input is v=(u,d). The feedback controller (which controls y at ys) computes v and then we have an "inverse" calculation to obtain u=f^{-1}(v,d).



Comment: If v=y=ys (which is will be at steady state with a perfect model) then this is essentially feedforward control (based on the measured d and ys), but the "trick" of using input transformations is that it incorporates feedback control in a nice manner (through manipulating v) and it also makes the feedback problem linear and decoupled (at steady state). The feedback controller is needed to correct for model error and unmeasured disturbances, and also to shape the dynamics.

(b) The static model gives by putting (11) into (12)

$$y = X = Y_{X/S}(S_f - S) = Y_{X/S}(S_f - K_s D/(\mu_M - D)) = f(u,d)$$

Here, $\mu_m=0.5$, $K_s=0.1$ and $Y_{X/S}=Y=0.4$ are assumed to be constant parameters.

We define v=f(u,d) where u=D and d=S_f.

$$v = Y_{X/S}(S_f - K_s D/(\mu_M - D)) = 0.4(d-0.1\,u)/(0.5-u)$$

The inverse $u=f^{-1}(v,d)$ then becomes (writing $Y = Y_{X/S}$)

$$D = \frac{\mu_M(v - YS_f)}{v - YS_f + YK_s}$$

Or

$$u = \frac{0.5\,(v - 0.4\,d)}{v - 0.4d + 0.04}$$