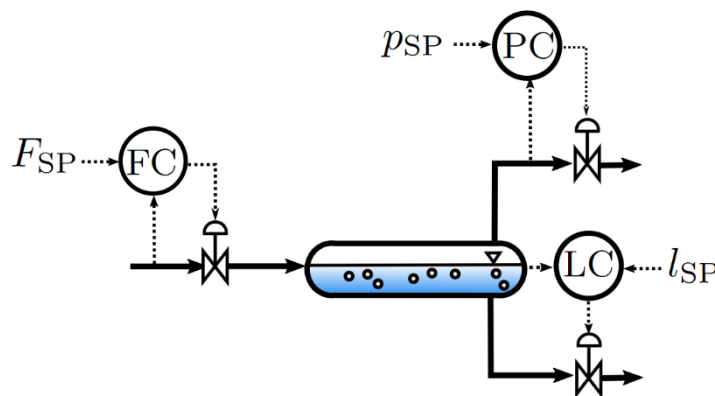


Exam APC, Autumn 2024. Solutions

Problem 1: General questions and hydrogen process separator (40 %)

(a) Terms: (7%)

- (3%) TPM: MV for controlling the throughput of the process.
- (2%) Global consistency: All steady-state mass balances of the process are satisfied through the inventory control system, including those of the individual process units, phases and components.
- (2%) Local consistency: All parts/units of the process can satisfy their respective mass balance using local inventory control loops. This means that the total inventory of any subprocess is locally regulated by its in- or outflows, which also hold for component inventories and phases.



(b) (4%) Applying the radiation rule around the TPM z_1 yields the above control structure, where the liquid and gas inventories in the separator are locally regulated by their outflows, which is a “classical” approach for the control of such equipment. Based on our above definitions we can determine that the system is indeed locally consistent, and thereby, also globally consistent.

(c) (5%) The input saturation rule states that we should pair a CV that may saturate only with MVs that can be given up (when the MV saturates). These will generally not be variables that are needed for the stabilization of the plant (3%). If the controller that may saturate has integral action, we must include anti-windup measures (2%).

(d) (max 5%) A cascade structure could help, e.g., to “linearize” the inner loop (2% for this), to counteract disturbances affecting the inner loop (1% for this), or to improve control performance if the outer loop and its measurements have slow dynamics (1% for extra this). Figure (2%): The inner loop can effectively handle input or output disturbances d_1 affecting G_2 , and the outer loop input or output disturbances d_2 affecting G_1 .

(e) (7%. 3% for model, 3% for tuning, 1% for TF) We use the SIMC-rule for an integrating process. According to the simulation data, the slope is as follows

$$k' = \frac{\Delta y}{\Delta t \Delta u} = \frac{(14.7 \text{ bar} - 20 \text{ bar})}{400 \text{ s} \cdot 0.05} = -0.27 \frac{\text{bar}}{\text{s}}.$$

Since we assume no delay θ and a closed loop time constant of $\tau_c = 3 \text{ s}$, the controller tunings are readily found as:

$$K_c = \frac{1}{k'\tau_c} = -1.25 \text{ 1/bar}, \quad \tau_I = 4\tau_c = 12 \text{ s.}$$

The transfer function from p_{SP} to p can be well approximated by noting that the SIMC rules are derived such that a first-order response is obtained for a setpoint change. We have integral action and no delay, so

$$\frac{p}{p_{sp}} \approx \frac{1}{3s + 1}.$$

(f) (6%. 3% model + 3% tuning) Again using the simulation data, we find the following values for θ, τ, k assuming a first-order like response:

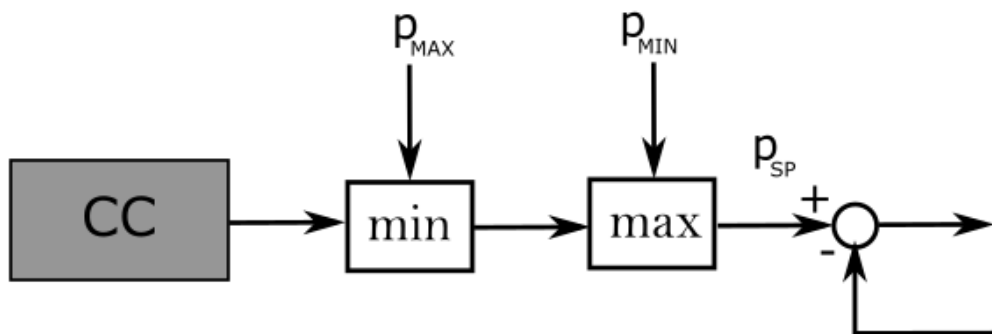
$$\theta = 30 \text{ s}, k = \frac{\Delta y(\infty)}{\Delta u} = \frac{(0.137 \% - 0.153 \%)}{-2 \text{ bar}} = 0.0077 \frac{\%}{\text{bar}}, \tau = 72 \text{ s.}$$

With the SIMC rules for a first order process with tight tuning ($\tau_c = \theta = 30\text{s}$) we then get:

$$K_c = \frac{1}{k} \frac{\tau}{2\theta} = 155.84 \frac{\text{bar}}{\%}, \tau_I = \min(\tau, 4(2\theta)) = 72 \text{ s.}$$

(g) (3%) The time-scale separation $\frac{\tau_{c,outer}}{\tau_{c,inner}} = \frac{30}{3} = 10$, which is desirably high for a cascade-control structure (it should be at least 4).

(h) (4%) The students should either include a combination of min/max selectors, or a simple saturation block with upper and lower limit directly after CC. For the combination of selectors, an upper bound would be enforced by using a min-selector that selects between the control signal and the upper bound, and vice versa a max-selector for the lower bound.



Problem 2: Self-optimizing and feedback-optimizing control (20 %)

From mathematics we have the necessary conditions of optimality (KKT-conditions)

- (1) $L_u = J_u + \lambda^T g_u = 0$
- (2) $\lambda \times g = 0$ (x=element-by-element multiplication)
- (3) $\lambda \geq 0$

(a) (5%) Explain what λ is and what these conditions represent

- Lambda is the Lagrange multiplier = dual variable = shadow price
- The first condition is the “unconstrained” optimality condition in terms of the lagrange function.
- (2) is the complementary condition.
- (3) is needed since this is an inequality constraint, so it has no effect when it's not active
- The last two conditions say that either $g=0$ with $\lambda > 0$ (g active constraint) or $g<0$ with $\lambda = 0$ (g not active).

(b) (5%) In self-optimizing control, we look for a variable for which we indirectly achieve optimal operation (minimize J) by controlling a variable at a constant setpoint. Explain this in a little more detail. Consider three cases

1. Use of a single measurement
2. Use of measurement combinations using the nullspace method
3. Use of measurement combinations using the exact local method

In all cases we can write $c = Hy$ where y are all available measurements (including u) .

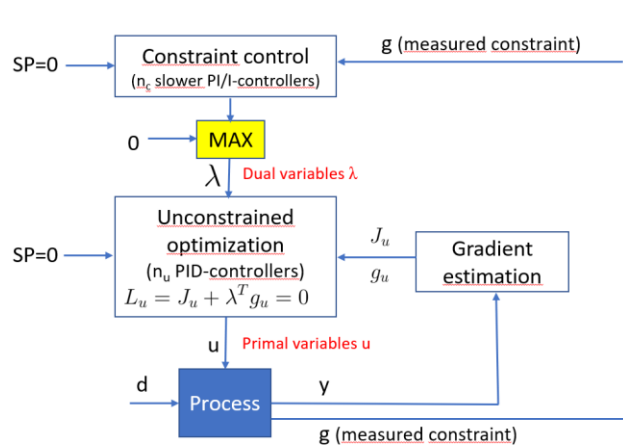
- In case 1, H_2 is a selection matrix, so with one 1 are the rest 0's in in each row. The optimal selected measurement is found by “brute-force” evaluation of the loss for the expected disturbances.
- In case 2, H_2 is found such that $H_2 F = 0$ where $F = dy_{opt}/dd$ is the optimal sensitivity. It assumes $n_y = n_u + n_d$ and is optimal for the case with no measurement noise.
- In case 3, we look for the truly optimal H_3 and we have a bit more complex formula for H which depends on F and “weights” that for expected magnitudes of disturbances d and measurement noise n.

(c) (5%) Returning to equation (1): How can the cost gradient J_u be estimated?

There are many ways.

- It could me from online-data (including sinusoids or steps in u) and estimating $J_u = dJ/dd$.
- Maybe better from model
 - Use a Kalman Filter and from this finding the state and linearizing the model du find $dx/dt = Ax + Bu$ and linearizing the cost expression to find $J = Cx + Du$ and then set $dx/dt = 0$ to eliminate x to find $J = J_u u$ where $J_u = -CA^{-1}B + D$.
 - Simplest: Use a variant of H from exact local method. That is, $J_u = H_3$ (with some adjustment on the directions in H_3 , so it starts with Hessian J_{uu} .) .

(d) (5%) Returning to equations (1) to (3): Show a cascaded feedback-optimizing control structure which implements these equations in practice (it involves a max-selector).



$$\text{KKT: } L_u = J_u + \lambda^T g_u = 0$$

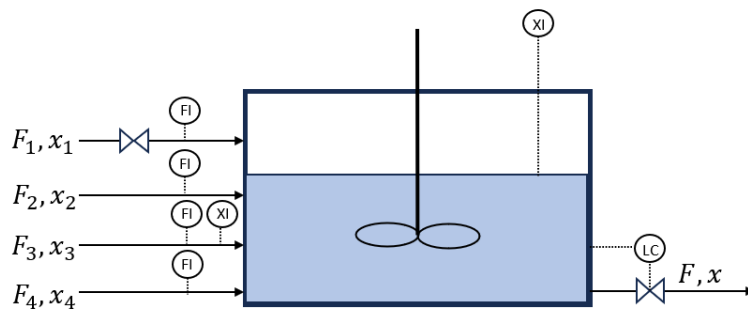
$$\text{Inequality constraints: } \lambda \geq 0$$

Primal-dual feedback control.

- Makes use of «dual decomposition» of KKT conditions
- Selector on dual variables λ
- Problem: Constraint control using dual variables is on slow time scale (upper layer)
 - Can be fixed using override at bottom of hierarchy (Dirza)
- Problem 2: Single-loop PID control in lower layer ($L_u=0$) may not be possible for coupled processes so may need to use Solver.

- D. Krishnamoorthy, A distributed feedback-based online process optimization framework for optimal resource sharing, *J. Process Control*, Vol. 138 (2024), 103208.
- R. Dirza and S. Skogestad, Primal-dual feedback-optimizing control with override for real-time optimization, *J. Process Control*, Vol. 138 (2024), 103208.

Problem 3: Feedforward control / transformed inputs (15 %)



Four streams are mixed to make a product F with a desired fraction $x_s \approx 0.3$ kg/kg (setpoint) of methanol.

F_1 [kg/s]: Manipulated. $x_1=0$ (pure water)

F_2 [kg/s]: Disturbance. $x_2 = 1$ kg/kg (pure methanol)

F_3 [kg/s]: Disturbance. $x_3 \approx 0.5$ kg/kg (x_3 it varies, but fortunately it's measured)

F_4 [kg/s]: Disturbance. $x_4=0.7$ kg/kg (fixed composition)

- Propose a feedforward control system with a feedback correction (write on the flowsheet), for example, a nonlinear model-based feedforward block based on the idea of transformed inputs.
- Why do we need the feedback correction?

Hint: Start by writing steady-state material balances to find an equation for x as a function of the other variables.

Solution :

(1) Component balance methanol: $F x = F_1 x_1 + F_2 x_2 + F_3 x_3 + F_4 x_4 = F_2 + F_3 x_3 + 0.7 F_4$

(2) Total balance $F = F_1 + F_2 + F_3 + F_4$

(get 2% for model only)

Combining gives: $x = (F_2 + F_3 x_3 + 0.7 F_4) / (F_1 + F_2 + F_3 + F_4)$

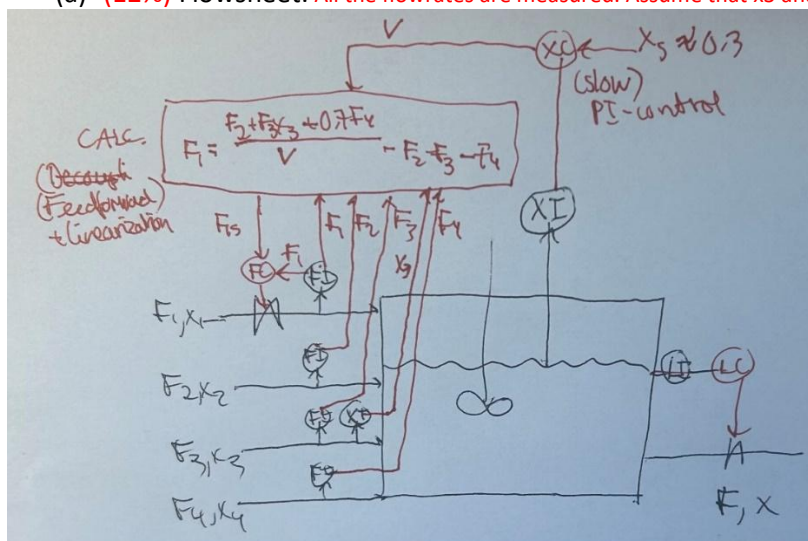
Proposed transformed input (output from feedback controller) is the right-hand side:

(3) $v = (F_2 + F_3 x_3 + 0.7 F_4) / (F_1 + F_2 + F_3 + F_4)$

Invert (3) to find flow F_1 that gives desired value for v (coming from the feedback controller).

(4) $F_1 = (F_2 + F_3 x_3 + 0.7 F_4) / v - F_2 - F_3 - F_4$

- (12%) Flowsheet: All the flowrates are measured. Assume that x_3 and x are measured

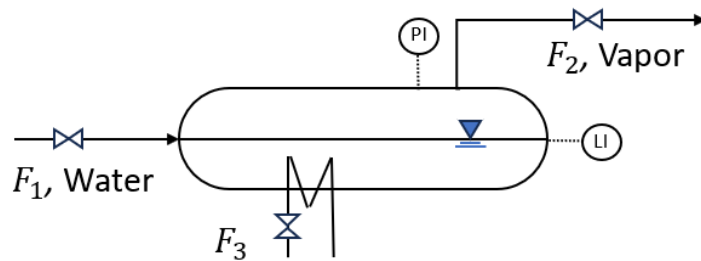


Note that the feedforward part is fast (actually, almost immediate because it's a static calculation).

Comment: A more "conventional" solution is to base the FF to F1 on the nominal value for x_s (by fixing $v=0.3$ in (4)) and adding the FF and FB signals together and send the sum as F1s to the FC for F1. It's not quite as good, for example, it does not give linearization for the FB controller. (FF=feedforward, FB=feedback)

- (b) (3%) The feedback correction is needed because of measurements errors (in flowrates and x_3) and model errors (maybe the methanol fraction in F4 is not always 0.7) and it could even be that someone decides to add another inflow.
 - Note that if the model is perfect then $v=x_s$ at steady state, but in general they are different because of model error and disturbances.
- (c) **Not asked for: Why do we need the feedforward calculation block?** The main reason must be that the measurement of x is delayed and/or unreliable, because otherwise feedback alone would be much simpler and good enough

Problem 4: Evaporator control (25%)



Consider the process diagram of an evaporator. Propose a control structure when:

- F_1 is the TPM
- F_2 is the TPM
- F_3 is the TPM
- A bidirectional structure that handles all cases

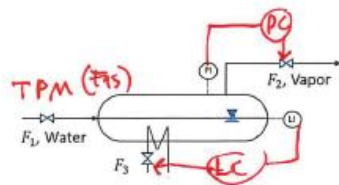
Note: Level can be a bit difficult to measure, so in the first three cases add also possible feedforward action from the TPM to the level control.

You can add flow sensors (FI) as needed. Note that F_1 affects only level (H), F_2 only pressure (p), whereas F_3 affects both.

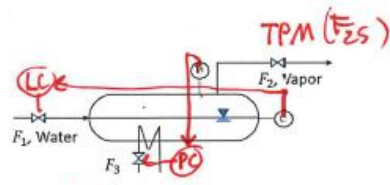
Solution: see Figures on the next page

- F_1 is the TPM (4% + 2% FF): Control level with F_3 and Pressure with F_2 .
 - Can add FF action from F_1 to F_3 . Conventional FF control involves adding together the contribution from FF and FB (FB = LC in this case). However, this FF requires a model for how F_1 affects the level.
 - Figure next page: FF action is implemented using ratio control, that is, the level controller sets the setpoint for F_3/F_1 . The advantage with ratio control compared to conventional FF is that we don't need a model, and the correct ratio is set by the feedback controller (LC in our case). However, note that with ratio control we also need a flow controller (FC) for F_3 .
- F_2 is the TPM (4% + 2% FF): Control level with F_1 and Pressure with F_3 .
 - FF action may be added using ratio control (F_1/F_2).
 - Alternative FF which does not require model: Let the output from level controller be F_1/F_2 and then adding F_2 to get F_1 .
- F_3 is the TPM (4% + 2% FF) . Control level with F_1 and pressure with F_2 .
 - FF action may be added by using ratio control (F_1/F_3).
- (7%) Bidirectional. We can start from case c and propose "overrides" for the situation when the MVs used for level and pressure saturate.
 - Why start from case c? Note that F_1 can be used only for level control , F_2 only for pressure control, whereas F_3 can be used both for pressure and level control. Because of this, bidirectional control for both LC and PC can go back to F_3 . Other schemes, for example, bidirectional control going back to F_1 , will depend on other loops being closed ("long loops")
 - It's also possible to add FF control (ratio control) for bidirectional, see last flowsheet (but this was not asked for)

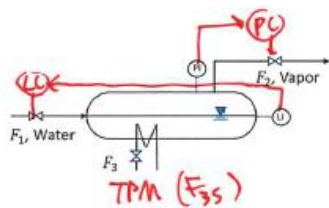
WITHOUT FEEDFORWARD :



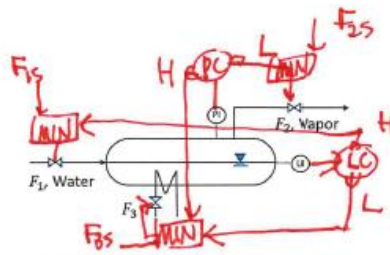
(a) $TPM = F_1$



(b) $TPM = F_2$

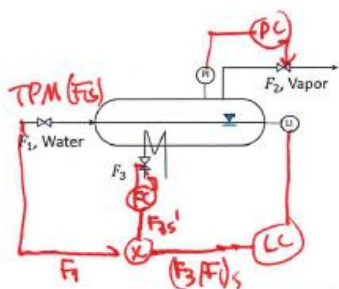


(c) $TPM = F_3$

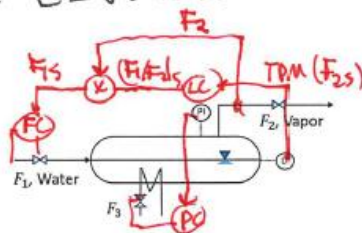


(d) Bidirectional

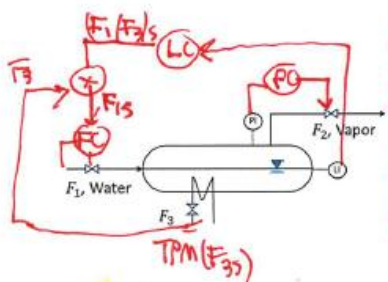
WITH FEEDFORWARD FOR LEVEL CONTROL :



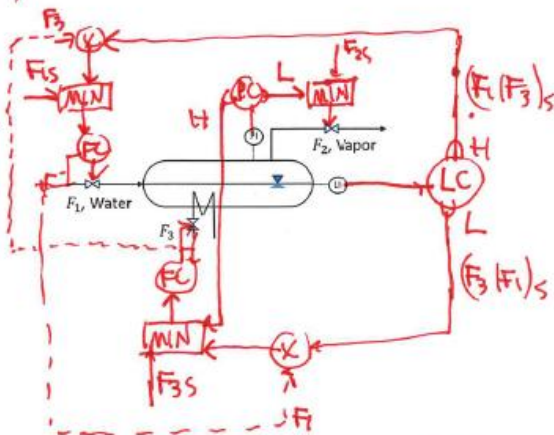
(a) with FF from $TPM(F_1)$



(b) with FF from $F_2 = TPM$



(c) with FF for level control



(d) Bidirectional with FF for level control.
(NOT ASKED FOR)

(The yellow ones are the ones asked for)