Problem 1 – General questions (20%)

2p each

~~2p each fixed total 20~~

Multiply ~~...~~ by 1/2 (max 20)

a) What are the two main approaches for decomposing the control system into smaller elements? Make a figure (block diagram) to illustrate the decomposition.

b) What are the main objectives of the supervisory control layer?

c) What are the main objectives of the basic (stabilizing) control layer?

d) Explain the main ideas behind self-optimizing control, RTO and extremum seeking control. How can these three strategies be combined?

e) What is back-off and when is it used? When is it important to minimize back-off?

f) What is a typical cost function in the RTO layer?

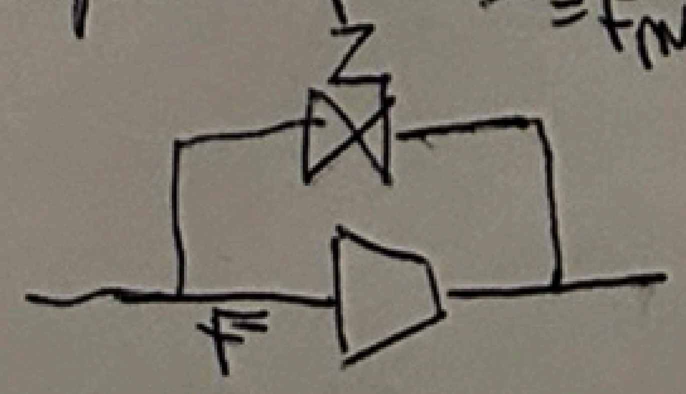
g) What is the difference between economic MPC and setpoint-based MPC? What is a typical cost function in the two cases?

h) The "pair-close" rule is the most important for choosing pairings for decentralized controller. Assume that you have $G(s) = k\,e^{-\theta s}/(\tau s+1)$. What values does this rule that imply that you would you like to have for each of the three parameters k, θ and τ (large, no effect or small)?

i) What is the "input saturation pairing rule"? When should it be used? If it is satisfied, what does it imply in terms of MV-CV switching. Give an example.

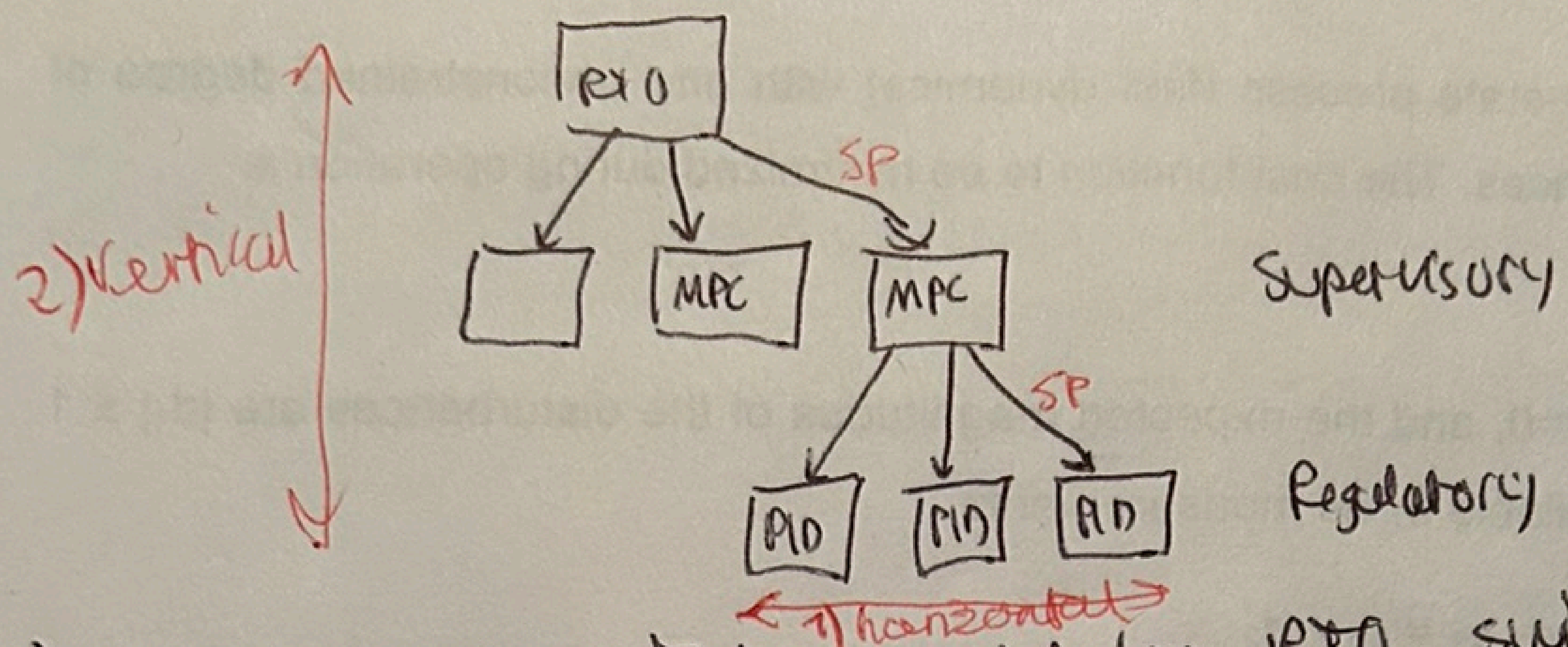
(h) Pair-close rule for $G = ke^{-\theta s}/\tau s+1$
want: k large
θ small
τ small

(i) Input saturation rule: Pair MV that can saturate with CV that can be given up. Implies that we can use "simple MV-CV switching". Example: Anti-surge control
(can give up $CV = F$ when $z=0$)
$= F_{max}$



Two decomposition approaches:

(a) 1) Horizontal (decentralized) and 2) vertical (hierarchical into layers)



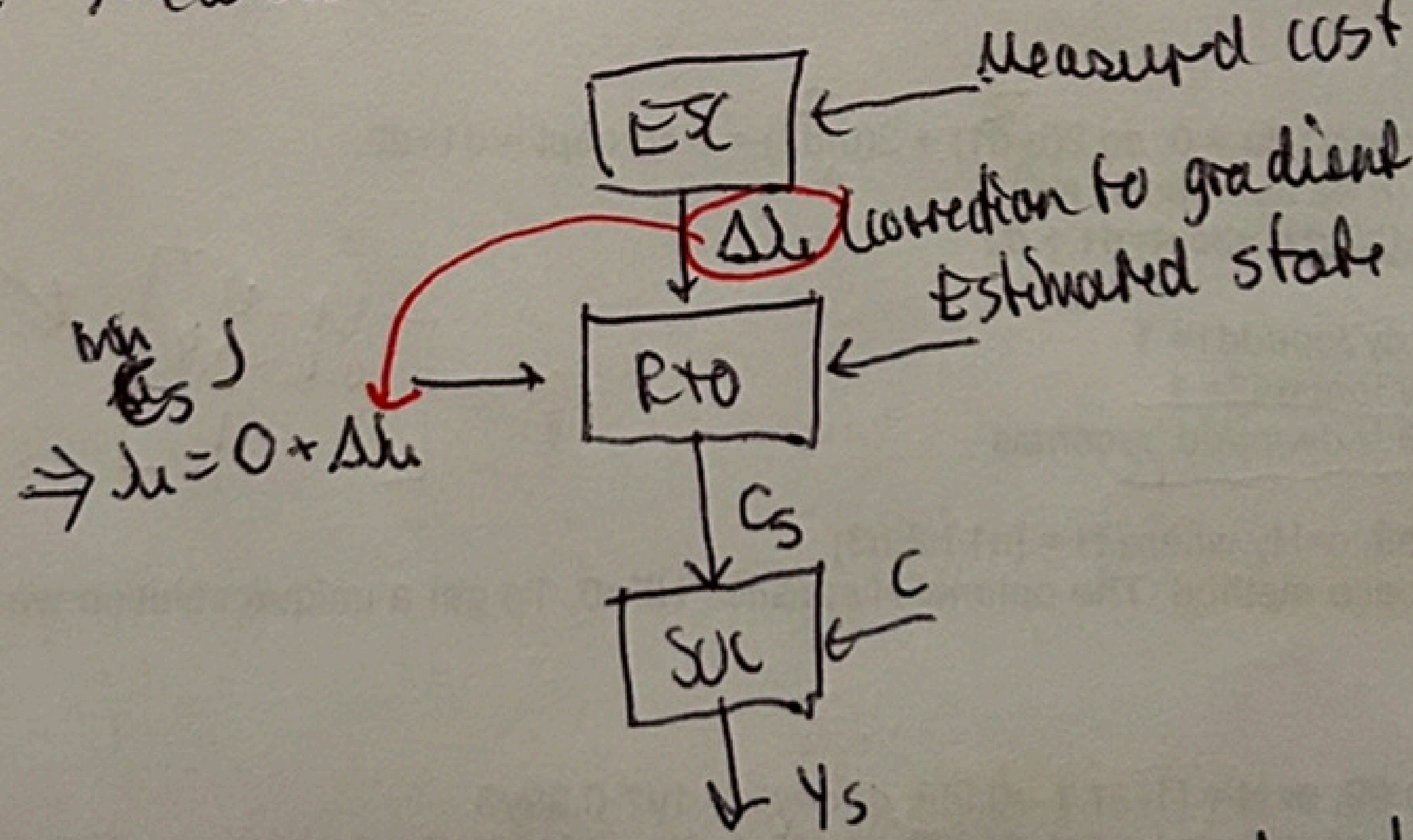
(b) Supervisory control: Follow setpoints from RTO, switch constraints, look after regulatory layer.

(c) Stabilizing control: stabilize process, Reject fast disturbances, Follow setpoints from supervisory, allow for manual control (i.e. without supervisory). Fast control tasks.

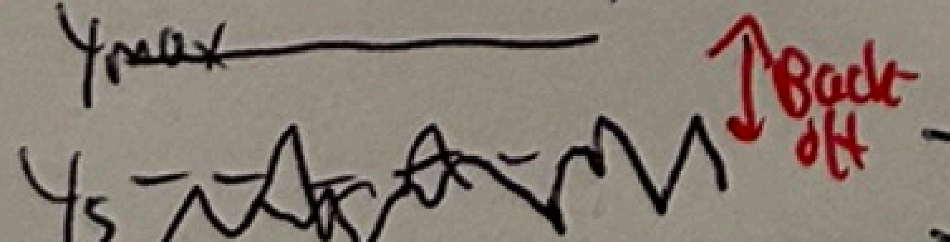
(d) self-optimizing: Keep c_1 at SP (related usually to unconstrained DOFs) to indirectly optimize operation

RTO: optimize economic criterion based on model (usually steady-state)

ESC: Measure ~~constr~~ cost J and keep $dJ/du = 0$ (unconstrained) ~~No~~ No model (steady-state)



(e) Back-off: Difference between y_s and actual constraint (e.g. y_{max}). hard const.



- It is used for constraints that cannot be violated
- Should be reduced because it gives economic loss.

(f) $J_{ec} = p_F F + p_Q Q - p_P P$ (RTO-cost) ← Need penalty for changing u in EMPC

(g) EMPC: $J = J_{ec} + W_u u^2$ — or $W_u|\Delta u|$ ← No setpoints, so combines RTO and MPC

MPC: $J = W_y (y - y_s)^2 + W_u u^2$

Problem 2 – Optimal process operation (20%)

Consider a simple steady-state process (fast dynamics) with one unconstrained degree of freedom and two disturbances. The cost function to be minimized during operation is

$J = (u-d_1)^2 + (u-d_2)^2$

Nominally we have d1=d2=0, and the expected magnitudes of the disturbances are $|d_1| \leq 1$ and $|d_2| \leq 1$. We have available three measurements.

$y_1 = u + 0.1\, d_1$, $y_2 = 0.1u - d_2$, $y_3 = d_1 + d_2$

10 (a) Use the nullspace method to find the self-optimizing c=Hy which when kept constant gives zero loss for the expected disturbances (at least locally and with perfect measurements). What is the optimal setpoint for c?

10 (b) Explain how to find the optimal H if we include measurements noise of magnitude $|n_{yi}| \leq 0.2$ for each of the three measurements. (Give the formula for H and give the matrices G^y and Y needed to compute H). What is this method called? Finding the final H is maybe a bit too much for hand calculations as it involves matrix inversion; how large is this matrix in this case?

Solution. (a) The optimal input u gives dJ/du = 0, so 2(u-d1) + 2(u-d2) = 0 so uopt = d1+d2..
The corresponding optimal outputs (measurements) are
y1opt=1.1 d1 + d2, y2opt= 0.1 d1 - 0.9 d2, y3opt=d1 + d2
So
dy1opt/dd1 = 1.1, dy2opt/dd1=0.1, dy3opt/dd1= 1
dy1opt/dd2 = 0, dy2opt/dd2=-0.9, dy3opt/dd2= 1
Thus, the optimal sensitivity matrix $\boxed{\text{F = dyopt/dd}}$ becomes
F = [1.1 0.1 1; 0 -0.9 1]'

$F = \begin{bmatrix} 1.1 & 0 \\ 0.1 & -0.9 \\ 1 & 1 \end{bmatrix}$

Note: $F = \frac{dy^{opt}}{dd}$ and not $\frac{dy}{dd}$ (many got this wrong)

We want to find the controlled variable c=Hy where H = [h1 h2 h3].
With no noise we can use the nullspace method. The optimal H satisfies HF=0. To get a unique solution we set h1=1. We then get
1.1 + 0.1 h2 + h3 = 0
0 -0.9 h2 +h3 = 0
Solving gives: h1=1 h2=-1.1, h3=-0.99, or H = [1 -1.1 -0.99], or c=y1-1.1y2-0.99y3.
Note that this gives (as expected): c1opt = H yopt = 1.1 d1+d2 - 1.1*0.1 d1 + 1.1*0.9 d2 - 0.99 d1 - 0.99 d2 = 0 d1 + 0 d2
Optimal setpoint is cs=0.
(b)With noise we use the exact local method . H = Gy' (Y Y')-1.
Gy' = dy'/du = [1 0.1 0]
Magnitude of disturbances and noise: Wd=1*I , case (2) Wny = 0.2*I. case (3) This gives
Y = [FWd Wny] = [1.1 0 0.2 0 0; 0.1 -0.9 0 0.2 0; 1 1 0 0 0.2]
Get H = Gy' (Y Y')-1 = [7.4858 -7.2355 -6.8739]
Normalized to make h1=1: H = [1 -0.9666 -0.9183]. So in this case the result is similar to nullspace method (the exact local method puts a little less weight on y1 and y2, but not much).
Note that in this case copt will depend on d1 and d2. Nevertheless, also in this case the optimal setpoint is cs=0.
Check also Wn=I.
Matlab: F = [1.1 0.1 1; 0 -0.9 1]'

See updated.

Solution.

(a) The optimal input u corresponds to dJ/du = 0, so 2(u-2d1) + 2(u-2d2) = 0 so uopt = d1+d2
The corresponding optimal outputs (measurements) are

y1opt=1.1 d1 + d2, y2opt= 0.1 d1 - 0.9 d2, y3opt=d1 + d2

So

dy1opt/dd1 = 1.1, dy2opt/dd1=0.1, dy3opt/dd1= 1
dy1opt/dd2 = 0, dy2opt/dd2=-0.9, dy3opt/dd2= 1

Thus, the optimal sensitivity matrix F = dyopt/dd becomes

F = [1.1 0.1 1; 0 -0.9 1]'

We want to find the controlled variable c=Hy where H = [h1 h2 h3].

With no noise we can use the nullspace method. The optimal H satisfies HF=0.
To get a unique solution we set h1=1. We then get
1.1 + 0.1 h2 + h3 = 0
0 -0.9 h2 +h3 = 0
Solving gives:

h1=1 h2=-1.1, h3=-0.99, or H = [1 -1.1 -0.99], or c=y1-1.1y2-0.99y3.

Note that this gives (as expected): c1opt = H yopt = 1.1 d1+d2 - 1.1*0.1 d1 + 1.1*0.9 d2 - 0.99 d1 - 0.99 d2 = 0 d1 + 0 d2
Optimal setpoint is cs=0.

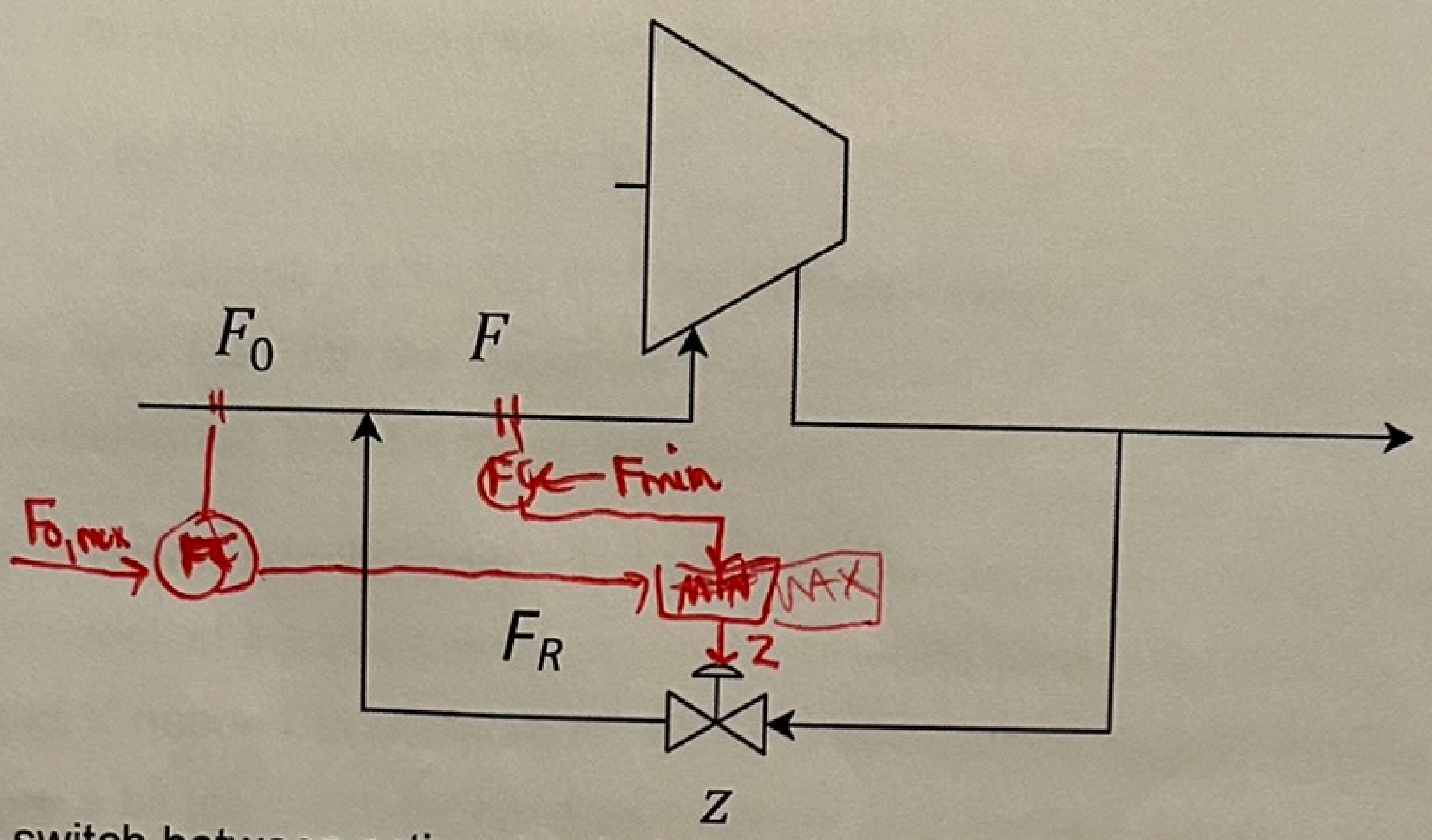
(b) With noise we use the exact local method . H = Gy' (Y Y')-1.
Gy = dy/du = [1 0.1 0]'
Magnitude of disturbances and noise: Wd=1*eye(2) , Wny = 0.2*eye(3). This gives
Y = [FWd Wny] = [1.1 0 0.2 0 0; 0.1 -0.9 0 0.2 0; 1 1 0 0 0.2]
Get H = Gy' (Y Y')-1 = [7.4858 -7.2355 -6.8739]
Normalized to make h1=1: H = [1 -0.9666 -0.9183]. So in this case the result is similar to nullspace method (the exac
method puts a little less weight on y1 and y2, but not much).
Note that in this case copt will depend on d1 and d2. Nevertheless, also in this case the optimal setpoint is cs=0.

```
%Matlab:
F = [1.1 0.1 1; 0 -0.9 1]',
H0=null(F'); H0=H0'/H0(1) % nullspace method
Gy= [1 0.1 0]', Wd=1*eye(2), Wn=0.2*eye(3),
Y = [F*Wd Wn],
H1 = Gy' * inv(Y * Y')
H = H1/H1(1) % exact local method
```

Comment: With noise increased to Wn=1*eye(3) we get more change: H= [1 -0.1422 -0.4046]
Comment: With noise increased to Wn=100*eye(3) we get: H= [1 0.1 0] (as expected, H = Gy')

Problem 3 – Compressor with constraints (15%)

Consider a constant-speed compressor (which means that it has no degrees of freedom for control) with an adjustable recycle F_R which should be minimized whenever possible. The following constraints need to be satisfied: $F_0 \leq F_{0,max}, F \geq F_{min}, z \geq 0, z \leq 1$. The first constraint is because the compressor is feeding a reactor and too reactor high flowrates are bad for the catalyst; the second constraint is to avoid compressor surge,



5 (a) To switch between active constraints we want to use one or more selectors. Give the three selector rules (Rule 1 is about min- or max- selector, Rule 2 is about order of selectors, Rule 3 is about valve constraints).

10 (b) Use the selector rules to propose a control system.
You can add flow sensors (FT) whenever needed.

$u = z$
$CV_1 = F \geq F_{min}$
$CV_2 = F_0 \leq F_{0,max}$

(a) Rule 1: Use max-selector for constraint which is satisfied by large u
" min " " small

Rule 2: If we need both max- and min-selector then there may be conflict. Put most important constraint at the end.

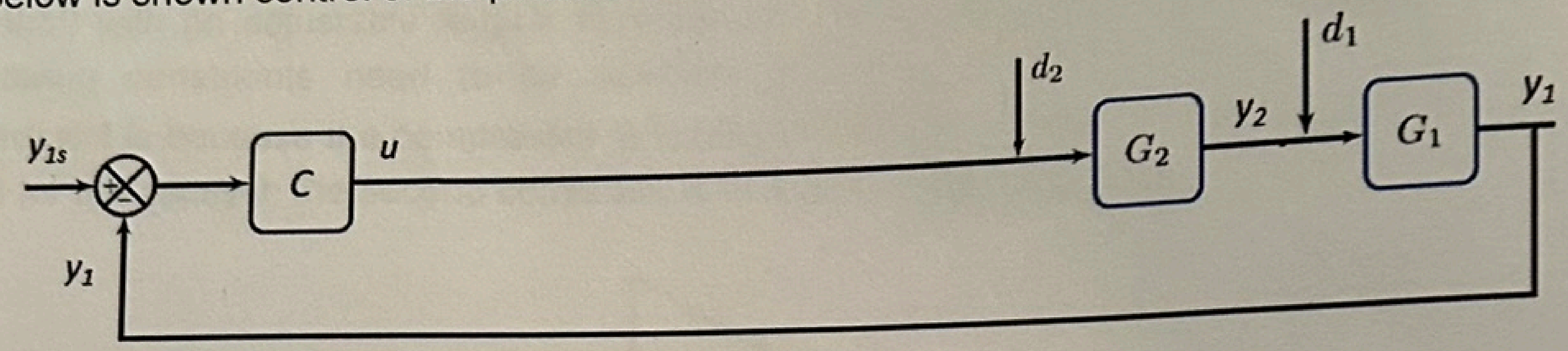
Rule 3: A valve at minimum ($z=0$) has build-in max-selector
" max ($z=1$) " min

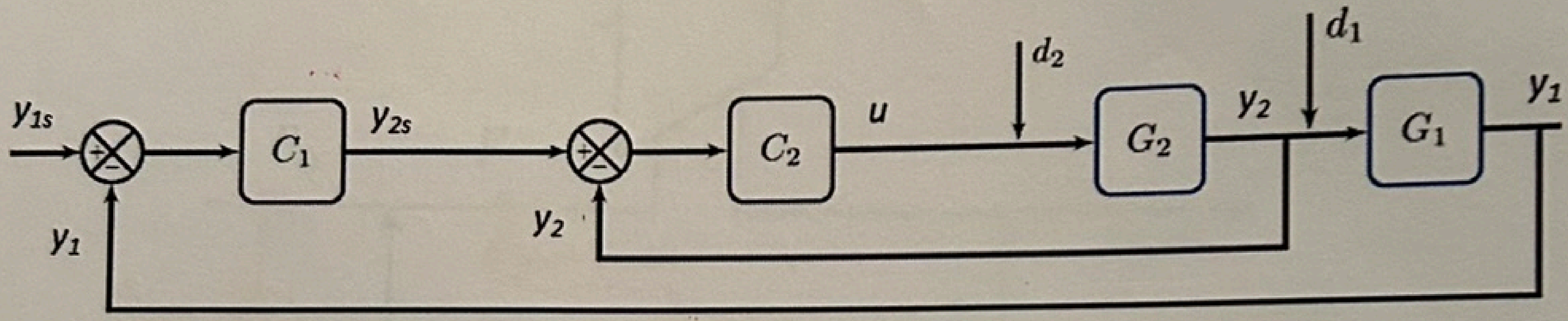
(b) $F \geq F_{min}$ is satisfied by large $u=z$ } max-selector
$F_0 \leq F_{0,max}$ — " —
$z \geq 0$ — " — } max-selector (build in)
$z \leq 1$ small } min-selector. Possible conflict so hopefully this is never encountered.

- see control structure in block diagram. Controllers need anti-windup.
- The valve will be open ($z=0$) (which is desired) when there are no constraints.

Problem 4 – Cascade control and tuning (25%)

Below is shown control of the process G=G1G2 for the case without and with cascade control





3 (a) What motivates the use of cascade control? Disturbance d_2, nonlinearity in G_2.

8 (b) Derive the transfer function from d_2 to to y_1 for the two cases. $\frac{y}{d} = \frac{g_1 g_2}{1+c g_1 g_2}$, $\frac{y}{d} = \frac{g_1 g_2}{1+c_2 g_2 + c_1 c_2 g_1 g_2}$

In the following let: $G_1 = 3/(18s+1)$, $G_2=e^{-0.4s}/(s+1)$

2 (c) Do you recommend cascade control in this case? ~~Yes, because~~ Maybe, no obvious benefit since delay in G_1 is small (confirmed by simulations)

3 (d) Suggest "tight" SIMC PI-tunings for c (for the case without cascade).

In the rest consider cascade control.

3 (e) Suggest "tight" SIMC PI-tunings for the slave controller c2.

3 (f) Suggest "tight" SIMC PI-tunings for the master controller c1.

~~5~~ (g) It is given that the time scale separation between the slave and master loops should be at least 5. What is the motivation for such a requirement? Is this satisfied for your case? If not suggest how to change the PI tunings.

3 or 5 (total was 25)

(d) $G = \frac{3 \cdot e^{-0.4s}}{(18s+1)(s+1)} \overset{\text{Half rule}}{\approx} \frac{3\, e^{-0.9s}}{(18.5s+1)}$, $K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = \frac{1}{3}\frac{18.5}{0.9+0.9} = 3.43$, $\tau_I = \min(18.5, \underbrace{4(0.9+0.9)}_{7.2}) = 7.2$ $\boxed{\tau_c=\theta=0.9}$

(e) Slave controller: $G_2 = \frac{e^{-0.4s}}{s+1}$ $K_c = \frac{1}{k}\frac{\tau}{\tau_{c2}+\theta} = \frac{1}{1}\frac{1}{0.4+0.4} = 1.25$, $\tau_I = \min(1, \underbrace{4(0.4+0.4)}_{3.2}) = 1$ $\boxed{\tau_{c2}=\theta=0.4}$

(f) Master (G): $G_p \approx G_1 T_2$. $T_2 \approx \frac{e^{-0.4s}}{0.4s+1}$, $G \approx \frac{3}{18s+1}\frac{e^{-0.4s}}{(0.4s+1)} = 3\frac{e^{-0.6s}}{18.2s+1}$

$K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = \frac{1}{3}\frac{18.2}{0.6+0.6} = 5.05$, $\tau_I = \min(18.2, 4(0.6+0.6)) = 4.8$ $\boxed{\tau_{c1}=0.6}$

(g) Motivation: Make the loops independent. Allow for linearization of G_2. This is not satisfied in this case, as time scale sep is 1.5. Would need $\tau_{c1} = 5 \cdot \tau_{c2} = 2$!

Need blank page!

New tuning of C_1: $K_c = \frac{1}{k}\frac{\tau}{\tau_c+\theta} = \frac{1}{3}\frac{18.2}{2+0.6} = 2.95$, $\tau_I = \min(18.2, \overbrace{4(2.6)}^{10.4}) = 10.4$ $\tau_{c1}=2$

b

(~~a~~) . $\frac{y_1}{d_2} = \frac{G_1 G_2}{1 + C G_1 G_2}$

$$\frac{y_1}{d_1} = \frac{G_1 G_2}{1 + G_2 C_2 + G_1 G_2 C_1 C_2}$$

(most easily derived from equations and eliminating.)

See also simulations in the "other" solution.

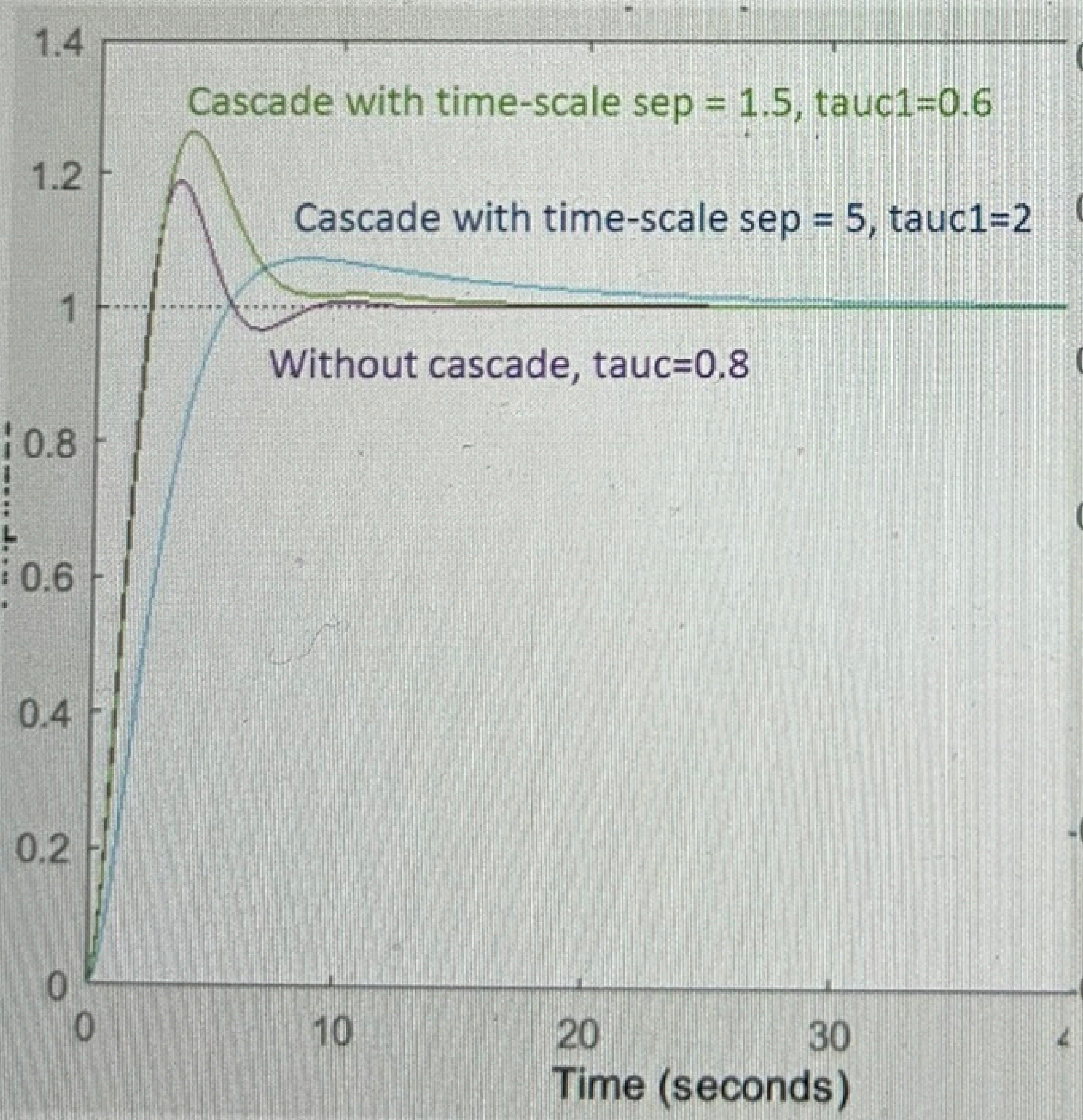
Exam 2023 solution - problems 2 and 4.docx No Label

```
s=tf('s')
k1=3; tau1=18; theta1=0; k2=1; tau2=1, theta2=0.4;  %
Process parameters
g1 = k1*exp(-theta1*s)/(tau1*s+1);
g2 = k2*exp(-theta1*s)/(tau2*s+1);

% (d) SIMC PI without slave.
%Effective delay in G=G1G2 is theta=tau2/2 + theta2= 0.9,
tau = tau1 + tau2/2 = 18.5
tau=tau1+tau2/2;theta=theta2 + tau2/2;
tauc=theta;
figure(2)
Kc = (1/(k1*k2))*tau/(tauc+theta),
taui=min(tau,4*(tauc+theta)), c = Kc*(1+1/(taui*s));
S0= minreal(inv(1+minreal(g1*g2*c))); Ms0=hinfnorm(S0)
T0d= g1*g2*S0, figure(2), step(Td,50)
T0=1-S0; figure(3), step(T0,50)

% (e) Slave Controller
tauc2=theta2;
Kc2 = (1/k2)*tau2/(tauc2+theta2),
taui2=min(tau2,4*(tauc2+theta2)),
c2 = Kc2*(1+1/(taui2*s));
S2 = minreal(inv(1 + minreal(g2*c2)))
T2= minreal(1/ (1 + ((minreal(inv(g2*c2))) ))) % Note. In this
case T2=g2=1/(s+1)
```

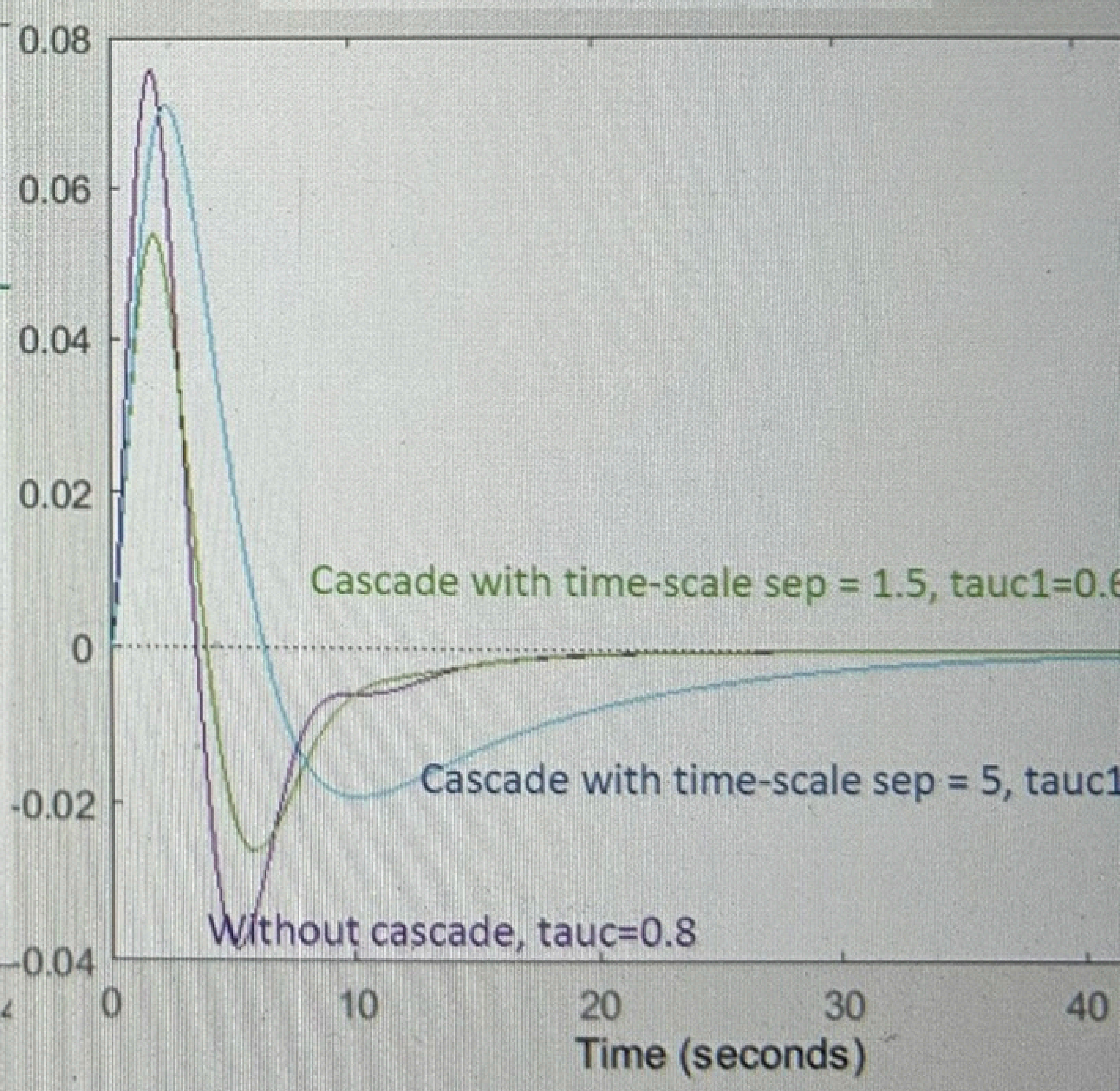
```
% (f) Master controller
tau1m=tau1+tauc2/2; theta1m=theta2+tauc2/2;
tauc1=theta1m %
Kc1 = (1/k1)*tau1m/(tauc1+theta1m),
taui1=min(tau1m,4*(tauc1+theta1m)), c1 =
Kc1*(1+1/(taui1*s));
% Total system with slave and master
S = minreal(inv(1 + minreal(g2*c2) + minreal(g1*c1*g2*c2)));
Td = g1*g2*S,; Ms=hinfnorm(S)
figure(2), hold on, step(Td,50)
T=minreal(g1*g2*c1*c2*S); figure(3), hold on, step(T,50)
% (c) Simulations show that there is not much improvement
with cascade (as expected since no delay in G1)

% (g) Try now with time scale separation of 5 in master
controller
tauc1=5*tauc2 %
Kc1 = (1/k1)*tau1m/(tauc1+theta1m),
taui1=min(tau1m,4*(tauc1+theta1m)), c1 =
Kc1*(1+1/(taui1*s));
S = minreal(inv(1 + minreal(g2*c2) + minreal(g1*c1*g2*c2)));
Td = g1*g2*S,; Ms=hinfnorm(S)
figure(2), hold on, step(Td,50)
T=minreal(g1*g2*c1*c2*S); figure(3), hold on, step(T,50)
```

From APC Exam, Nov. 2023

Setpoint response



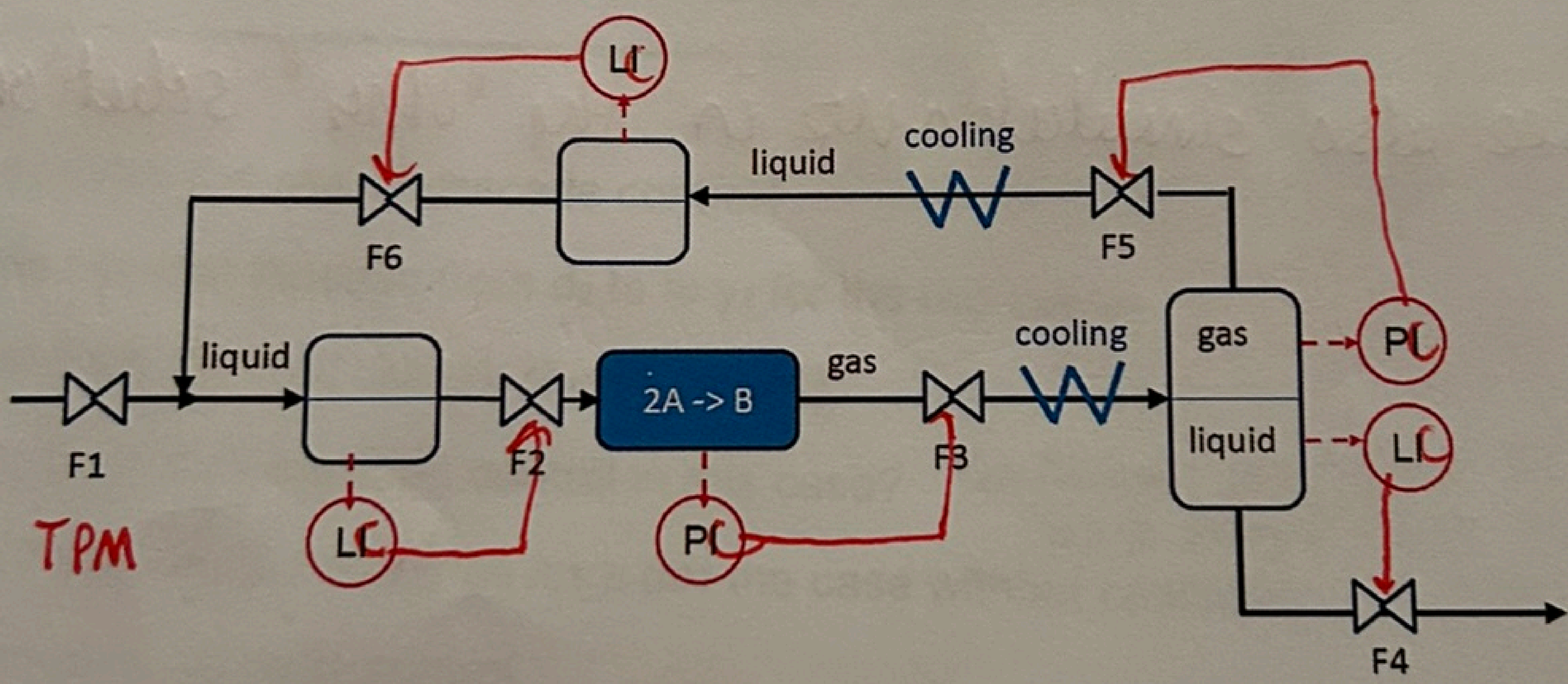
Input disturbance (d1)



Problem 5 – Inventory control for reactor process (20%) (7 for each, max 20)

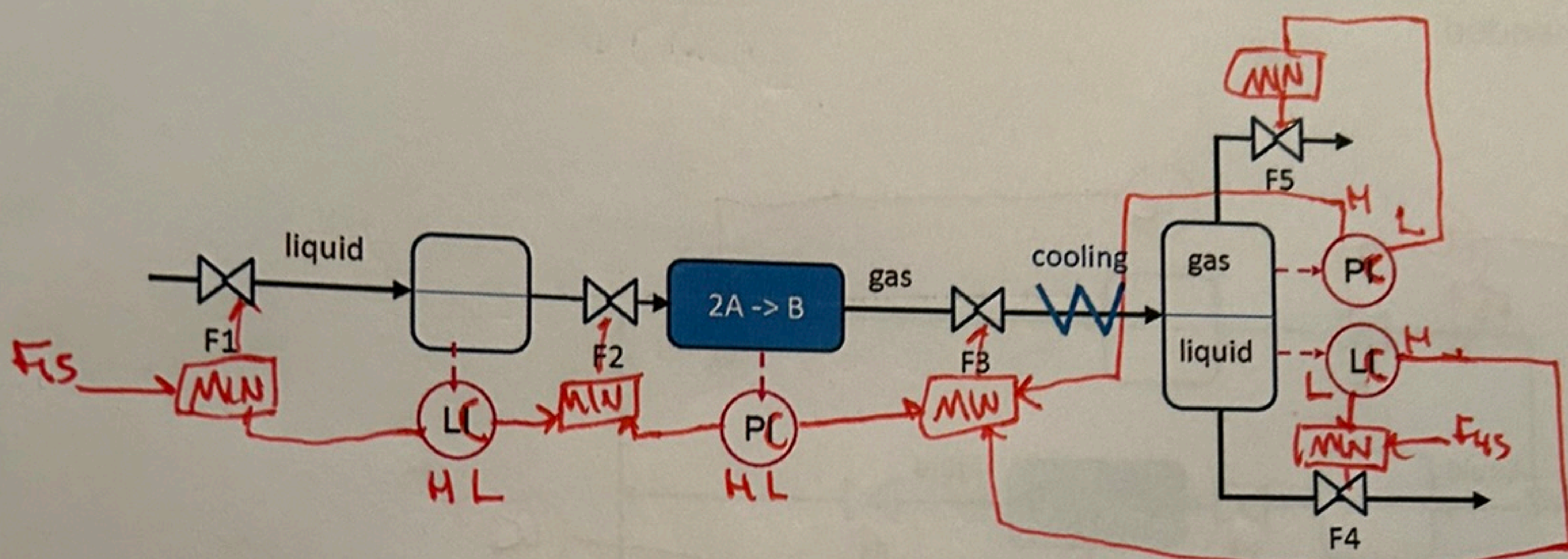
(a) Consider the flowsheets below. The reactor is exothermic with a liquid feed and gas product. The feed F1 contains mostly component A and the liquid product F4 contains mostly component B. The gas from the separator is recycled. The pressure (PI) and level (LI) sensors should be used for inventory control.

Suggest a consistent control scheme for the case when the federate is set (F1 is the TPM).



Follows radiation rule.

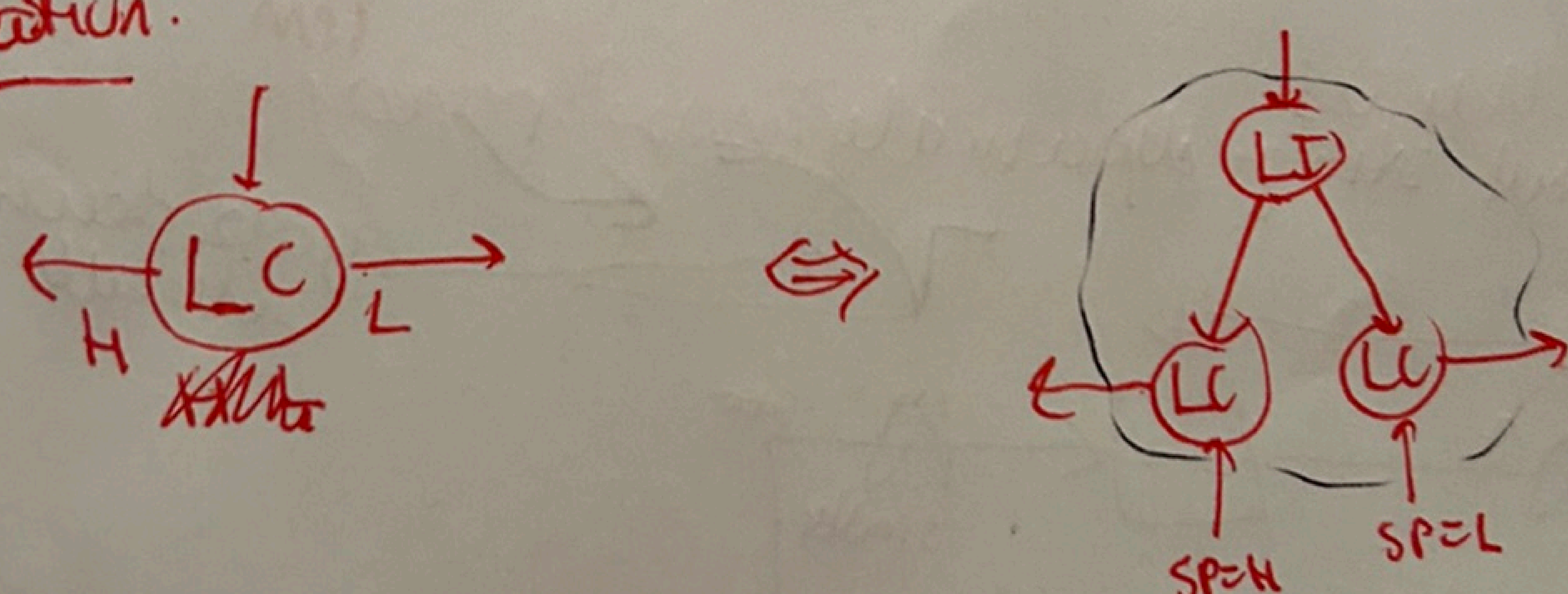
(b) Consider a similar process without recycle. Suggest a bidirectional inventory control scheme. What advantages does it has compared to a "fixed" control scheme (similar to the one in part a)?



Note: No fixed TPM for bidirectional control

Note: 3 controllers go to this min-selector. (+3 points if they see this!)

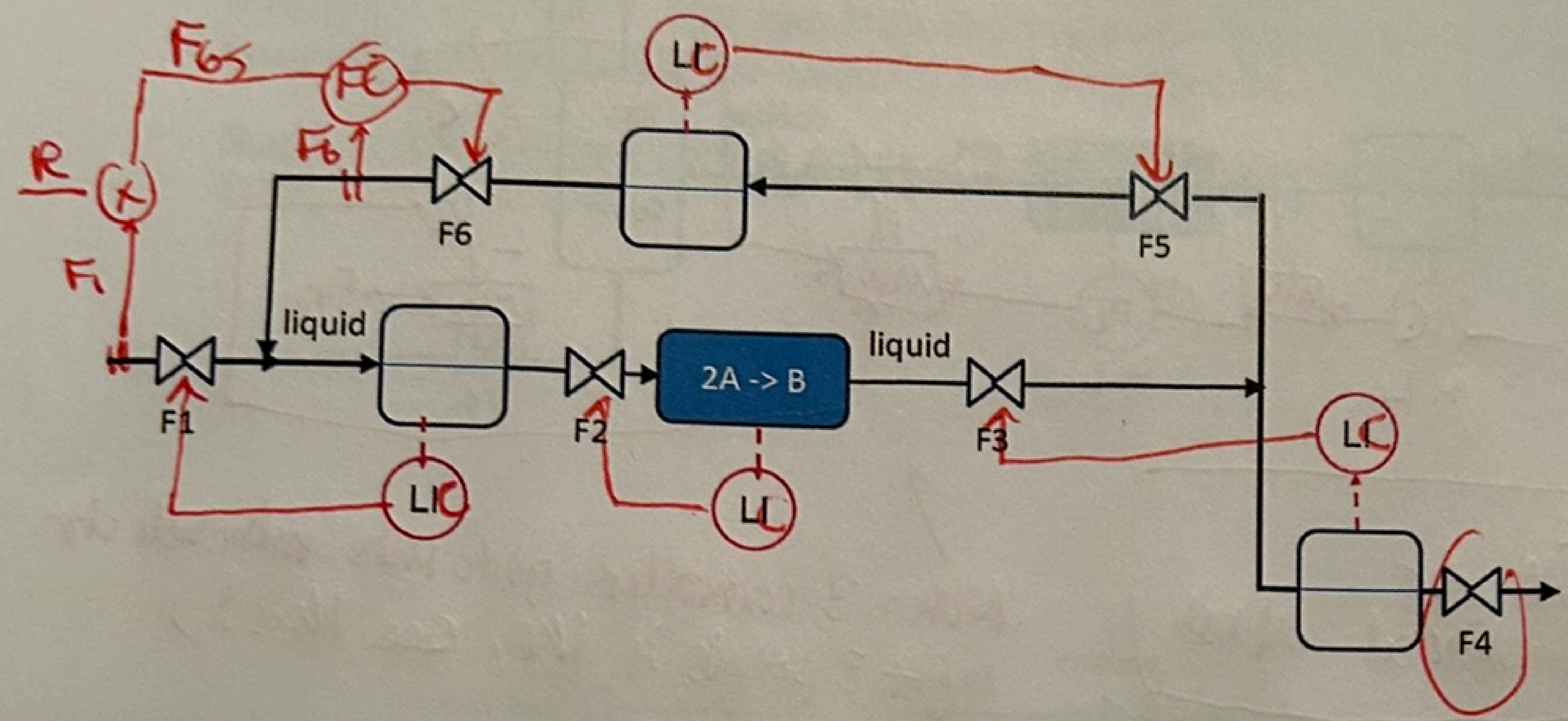
Notation:



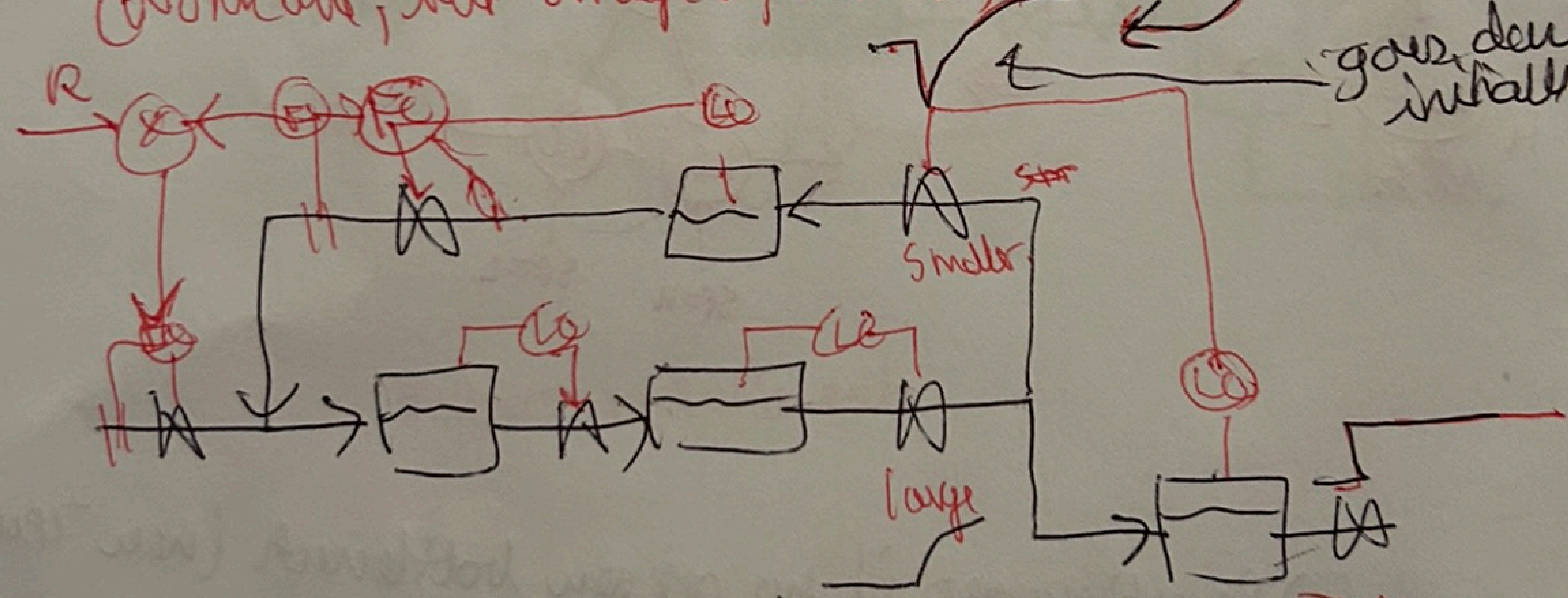
Advantages: (1) Reconfigures if we get new bottleneck (new TP

(2) Use of H and L setpoints maximizes product for temporary bottlenecks.

(c) Consider a case where the separator is replaced by a split. All flows are liquid in this case. It is given that we should have a given ratio R=F6/F1. Propose a control scheme for the case where F4 is given (F4 is the TPM). You can add extra flow sensors ~~(FI)~~ as needed.



Suggested by Martus:
(workable, but strange dynamically, "inverse" response)



FEASIBLE BUT NOT GOOD

Nice to mention and ask about problem!

Seems ~~a bit~~ slow
= Don't use largest flow for last loop
- Have to around whole circle, has like an inverse resp.