Emnemodul: Advanced Process Control

16. Nov. 2021. Time: 1300 – 1600.

Answer as carefully as possible, preferably using the available space. There are in total 6 questions. Answer all questions in English.

No printed or hand-written support material is allowed. A specific basic calculator is allowed.

Problem 1 – General questions (20%)

a) What is the goal of Skogestad's top-down procedure for control design? Briefly describe its main steps.

b) What is self-optimizing control?

c) What is the importance of active constraints in terms of optimal operation?

d) In the case that such constraints cannot be violated at any time, what additional precautions should be taken in the control design and operation?

e) What are the ideal controlled variables for the unconstrained degrees of freedom? Why is that?

f) What are the differences between the exact local method and the nullspace method? What are the main assumptions behind these methods?

g) How can a control structure be prepared for changes in active constraints during operation? What control elements can we use for that end?

h) Why is it usually a bad idea to directly control the cost with simple controllers? How can we use cost measurements for optimal operation instead?

a) The goal is to achieve near-optimal operation. The main necessary steps are defining the optimization problem (cost, constraints, and degrees of freedom), solving it for the expected disturbances, defining the primary controlled variables, and setting the throughput manipulator (TPM).

b) Self-optimizing control is a procedure of selection of controlled variables which allow for minimal economic loss.

c) Active constraints are the most important controlled variables for optimal operation, as the economic loss grows linearly with the implementation error with relation to the constraint value.

d) As the constraints need to be tightly controlled, one must carefully evaluate the necessary backoff to be implemented, which must be as small as possible given the controllers' tuning.

e) The ideal unconstrained CVs are the cost gradient, J_u, as $J_u^* = 0$ at the optimum.

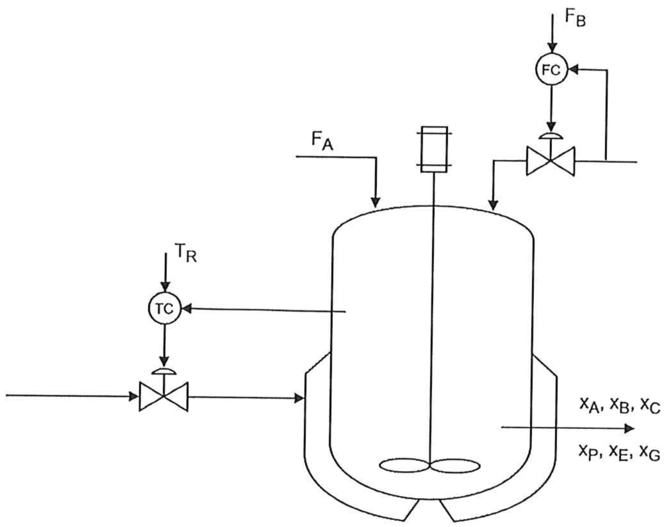
f) The exact local method considers measurement noise ($y^m = y + n^y$), and is not limited with relation to the number of measurements. On the other hand, the nullspace method is valid for perfect measurements ($n^y = 0$), and requires $n_y \geq n_u + n_d$. Both method assume steady-state, quadratic cost function, and linear process model.

g) Changes in active constraints during operation require changing controlled or manipulated variables. One can use selectors for CV-CV switching, or split-range control for MV-MV switching. MV-CV switching can be handled by pairing the MV that may saturate with the CV that may be given up.

h) One cannot know the optimal cost value, due to disturbances. Trying to operate with $J < J_{min}$ is infeasible, while trying to operate with $J > J_{min}$ leads to multiple operating points, which may in turn result in oscillations.

Problem 2 – Optimal process operation (20%)

Consider the Williams-Otto reactor with a constant volume, where A and B are converted to P and E, along with undesired byproducts C and G, according to the reactions shown below.



$$A + B \rightarrow C$$

$$B + C \rightarrow P + E$$

$$C + P \rightarrow G$$

The feed stream F_A, with pure A component, is a given disturbance to the process. The process has two degrees of freedom, namely F_B and the reactor temperature T_R, shown in red in the figure above. The objective is to maximize the production of the valuable products P and E, subject to purity constraints on G and A.

$$\min_{F_B, T_R} J = -1043.38\, x_P\, (F_A + F_B) - 20.92\, x_E\, (F_A + F_B) + 79.23\, F_A + 118.34\, F_B$$

$$\text{s.t.:} \quad \begin{aligned} x_G &\le 0.08 \\ x_A &\le 0.125 \end{aligned}$$

The nominal state of the plant is given by $F_A = 1.65\ kg/s$. For that condition, and for a disturbed condition $F_A = 1.75\ kg/s$, the optimal operating conditions are given by the table below:

Variable	Optimal value	
F_A	1.65	1.75
F_B	3.9008	4.0951
T_R	351.83	352.33
x_A	0.1214	0.1242
x_B	0.3901	0.3894
x_C	0.0248	0.0251
x_P	0.1101	0.1093
x_E	0.2735	0.2719
x_G	0.08	0.08

The nonlinear constraints may be approximated through a linear model around the operating conditions, given by:

$$\begin{bmatrix} \Delta x_G \\ \Delta x_A \end{bmatrix} = \begin{bmatrix} -0.0303 & 0.0044 \\ -0.0261 & -0.0030 \end{bmatrix} \begin{bmatrix} \Delta F_B \\ \Delta T_R \end{bmatrix} + G_d \, \Delta F_A$$

a) Identify the MVs and DVs for this process. How many constrained and unconstrained degrees of freedom do you have, and what are the CVs that you suggest controlling?

b) For the unconstrained degrees of freedom, you want to control a linear combination of measurements $c = Hy$. You have two available measurements for this, namely x_P and x_E. Based on the operating conditions given before, compute the optimal sensitivity matrix F, and use the nullspace method to find the optimal selection matrix H.

c) Alternatively, assume that you can directly estimate the plant cost gradients with relation to the inputs, J_u, and now you want to control a linear combination of the gradient, $c = N^T J_u$. How do you choose N? Compute N for this problem.

Now consider that the range of disturbances has increased, and F_A now may reach higher values. The optimal conditions for some selected disturbances are shown in the table below:

Variable	Optimal value			
F_A	1.65	1.75	1.85	1.95
F_B	3.9008	4.0951	4.3294	4.5824
T_R	351.83	352.33	353.09	353.92
x_A	0.1214	0.1242	0.125	0.125
x_B	0.3901	0.3894	0.3910	0.3935
x_C	0.0248	0.0251	0.0250	0.0247
x_P	0.1101	0.1093	0.1085	0.1078
x_E	0.2735	0.2719	0.2704	0.2690
x_G	0.08	0.08	0.08	0.08

d) What can you observe in terms of the system's optimal behavior? What does that imply in terms of optimal operation?

e) Propose a control strategy that can deal with values of F_A in its full considered range.

a) MVs: $u = [F_B,\ T_R]^T$ DVs: $d = F_A$

Active constraint: $x_G = 0.08$ $\Rightarrow$ 1 constrained DOF
1 unconstrained DOF

One should select $x_G = 0.08$ as a primary CV; must find one self-optimizing variable.

b) Nullspace method: $HF = 0$

$$F = \frac{\Delta y^{opt}}{\Delta d} = \begin{bmatrix} \frac{\Delta x_P^{opt}}{\Delta F_A} \\ \frac{\Delta x_E^{opt}}{\Delta F_A} \end{bmatrix} = \begin{bmatrix} \frac{0.1093 - 0.1101}{1.75 - 1.65} \\ \frac{0.2719 - 0.2735}{1.75 - 1.65} \end{bmatrix} = \begin{bmatrix} -8 \cdot 10^{-3} \\ -16 \cdot 10^{-3} \end{bmatrix}$$

$$[h_1 \quad h_2] \begin{bmatrix} -8 \\ -16 \end{bmatrix} \cdot 10^{-3} = 0 \Rightarrow -8h_1 - 16h_2 = 0 \Rightarrow h_2 = -\frac{1}{2} h_1$$

$$\Rightarrow H = [1 \quad -1/2]$$

c) $c = N^T \nabla_u J$, N is the nullspace of the gradient of the active constraints, $\nabla_u g_1$. From the provided model, $\nabla_u g_1 = [-0.0303 \quad 0.0044]$.

$$[-0.0303 \quad 0.0044] \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = 0 \Rightarrow n_2 = \frac{0.0303}{0.0044} n_1 = 6.8864 n_1$$

$$\Rightarrow N = \begin{bmatrix} 1 \\ 6.8864 \end{bmatrix}$$

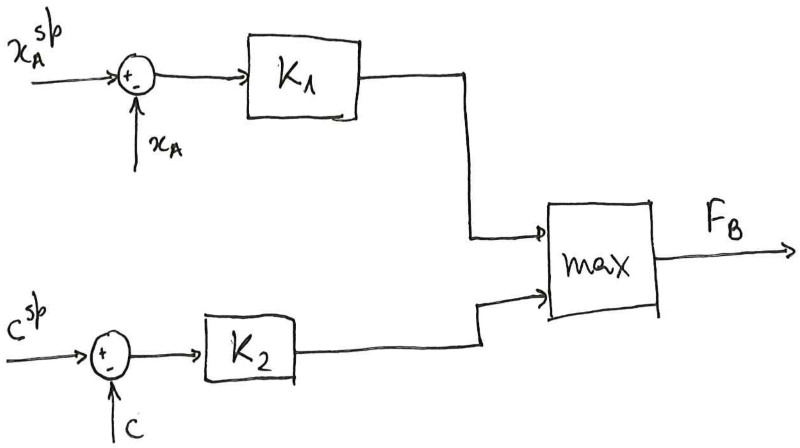
d) $x_A \leq 0.125$ becomes active for $F_A \geq 1.85$. This implies that we must give up on the self-optimizing variable to control this constraint, and we must therefore detect the change in the active constraint.

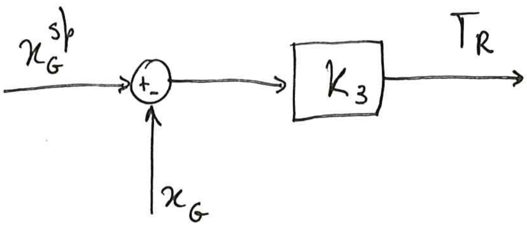
e) RGA calculation :
$$\lambda = \frac{(-0.0303)\times(-0.0030)}{(-0.0303)\times(-0.0030) - (0.0044)\times(-0.0261)} = 0.4418$$

⤳ strong interaction! Suggested pairing:

MV	–	CV
T_R		x_G
F_B		x_A

Control of x_A may be given up ⇒ use of selector!



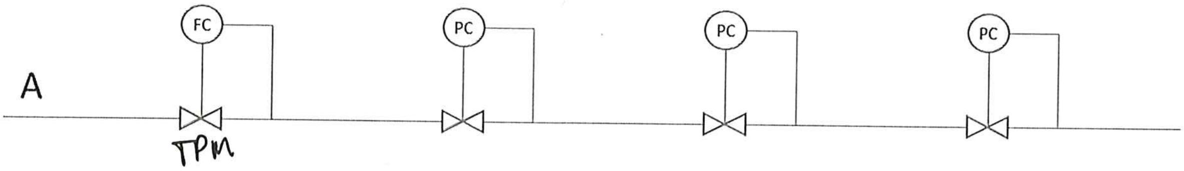


Problem 3 – TPM and inventory control (10%)

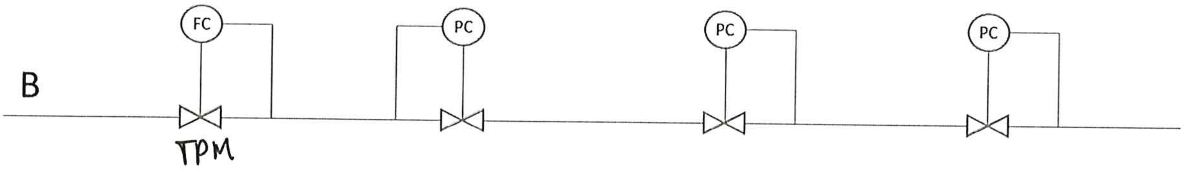
a) What do you understand by inventory control? How is it related to the throughput manipulator (TPM) of the process?

b) Indicate whether these control structures are consistent or not, and the location of the TPM in each structure. In addition, indicate if they are local or global consistent. Justify your answers.

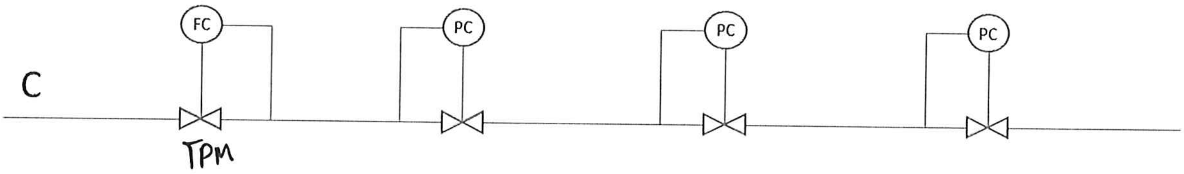
A



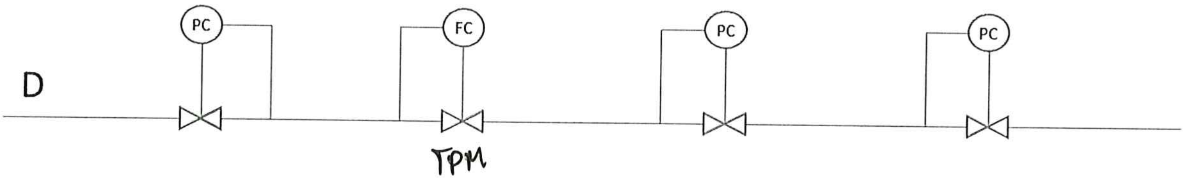
B



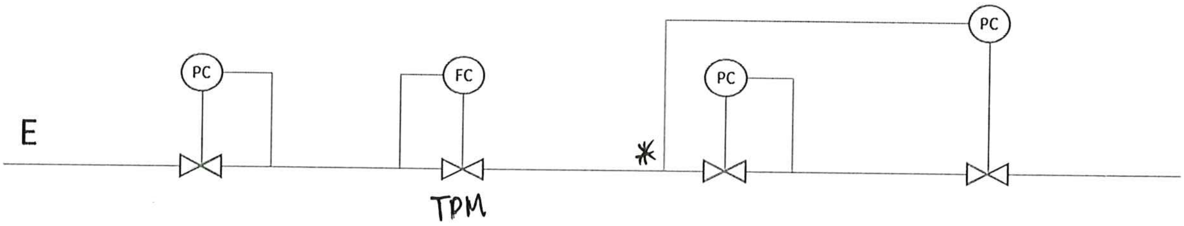
C



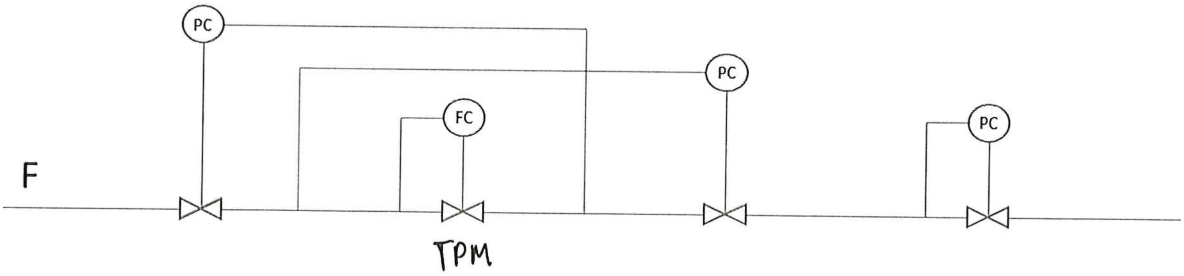
D



E



F



a) Inventory control is a part of the regulatory layer of the process. It aims for satisfaction of the mass balances during operation. The radiating rule states that inventory control loops must "radiate" from the TPM of the process.

b) A : inconsistent (inventory left uncontrolled)

B: inconsistent

C: local consistent (follows radiating rule)

D: local consistent (")

E: global, but not local consistent (inventory * not regulated by its direct outlet)

F: inconsistent (inventory control crossing TPM)

Problem 4 – PID tuning (20%)

Consider the following model:

$$\begin{cases} y_1 = \dfrac{6\,(-0.8s+1)}{(5s+1)(20s+1)}e^{-0.7s}\,(y_2+d_1)+d_2 \\ y_2 = \dfrac{3}{2s+1}e^{-0.3s}\,(u+d_3) \end{cases}$$

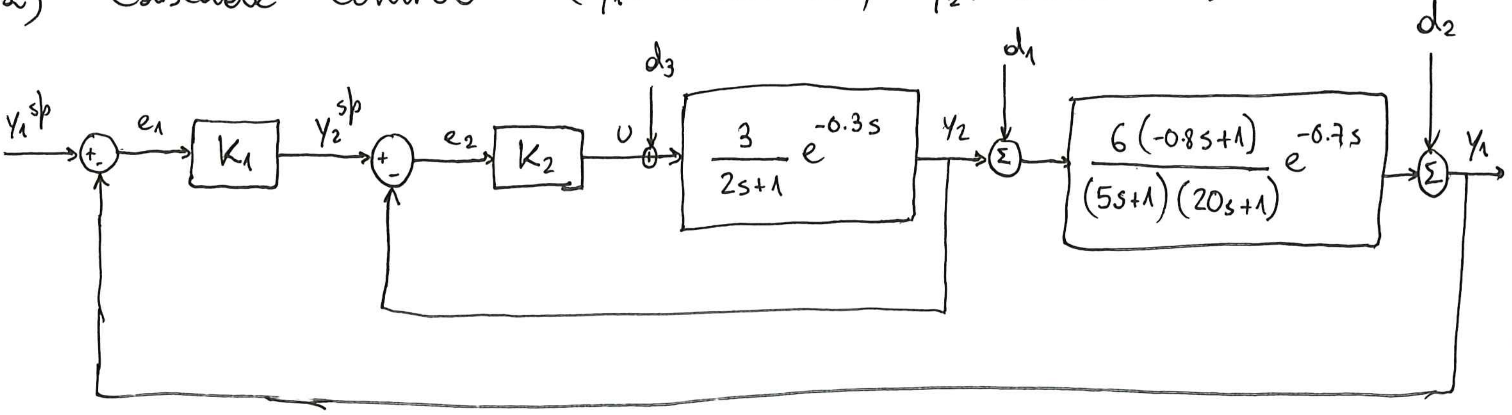
We wish to design a control scheme that drives y_1 to a desired setpoint. The extra measurement y_2 may be used to help in the control of the main output.

a) What is the control structure that should be used in this case? Draw the block diagram, along with all process transfer functions, of the proposed control structure.

b) Tune the controllers from the structure you proposed using the SIMC rules. Use the half-rule when necessary to reduce the models to a first order plus time delay form and choose the respective tuning parameters such that tight control is achieved.

c) Similarly, tune a single PI controller using the SIMC rules for the direct control of y_1 using u. Compare the obtained controllers. For which situations do you believe your proposed control structure performs better than simple feedback?

a) Cascade control (y_1: master CV, y_2: slave CV)



b) SIMC tuning, K_2: $K_{c,2} = \dfrac{1}{k}\cdot\dfrac{\tau}{\tau_c+\theta} = \dfrac{1}{3}\cdot\dfrac{2}{0.3+0.3} = \dfrac{10}{9}$

(tight control: $\tau_c = \theta = 0.3$)

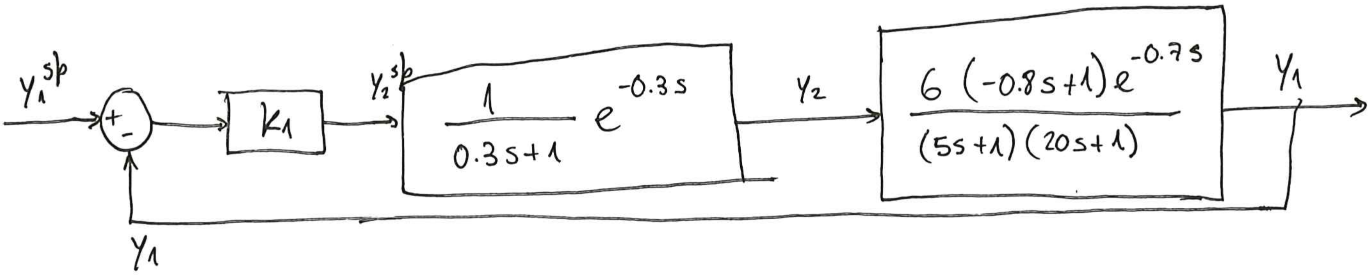
$\tau_{I,2} = \min\left(\tau_1,\ 4(\tau_c+\theta)\right) = \min(2,\ 4\times 0.6)$
$= \min(2,\ 2.4) = 2$

$K_2(s) = K_{c,2}\left(\dfrac{\tau_{I,2}s+1}{\tau_{I,2}s}\right)$

Closed-loop TF ($y_2^{sp} \to y_2$) : $T_2(s) = \dfrac{K_2 G_2}{1 + K_2 G_2}$

$$T_2(s) = \frac{\frac{10}{9}\frac{\cancel{(2s+1)}}{2s} \cdot \frac{3}{\cancel{2s+1}} e^{-0.3s}}{1 + \frac{10}{9}\frac{\cancel{(2s+1)}}{2s} \cdot \frac{3}{\cancel{2s+1}} e^{-0.3s}} = \frac{\frac{10}{3} e^{-0.3s}}{2s + \frac{10}{3} e^{-0.3s}}$$

$$\Rightarrow T_2(s) = \frac{1\ e^{-0.3s}}{\frac{6}{10}s + \underbrace{e^{-0.3s}}_{\approx 1 - 0.3s}} \approx \frac{1}{0.3s+1} e^{-0.3s}$$



$$G(s) = \frac{6(-0.8s+1)\, e^{-1.0s}}{(20s+1)(5s+1)(0.3s+1)} \Rightarrow$$

Half-rule: $\theta = 1.0 + 0.8 + 0.3 + \frac{5}{2} = 4.6$

$\tau_1 = 20 + \frac{5}{2} = 22.5$

$$G_{HR}(s) = \frac{6\, e^{-4.6s}}{22.5s + 1} \Rightarrow$$

SIMC tuning, K_1: $K_{c,1} = \frac{1}{6} \times \frac{22.5}{4.6 \times 2} = 0.4076$

$\tau_c = 4.6$

$\tau_{I,1} = \min(22.5, \underbrace{4 \times 2 \times 4.6}_{36.8})$
$= 22.5$

c) $G(s) = \dfrac{18\,(-0.8s+1)\,e^{-1.0s}}{(20s+1)(5s+1)(2s+1)}$ Half-rule $\Rightarrow$ $\theta = 1.0 + 0.8 + 2 + \frac{5}{2} = 6.3$

$\tau_1 = 20 + \frac{5}{2} = 22.5$

$G_{HR}(s) = \dfrac{18\,e^{-6.3s}}{22.5s+1}$

SIMC tuning $(\tau_c = 6.3)$: $K_c = \dfrac{1}{18} \times \dfrac{22.5}{2 \times 6.3} = 0.0992$

$\tau_I = \min\left(22.5,\ \underbrace{4 \times 2 \times 6.3}_{50.4}\right) = 22.5$

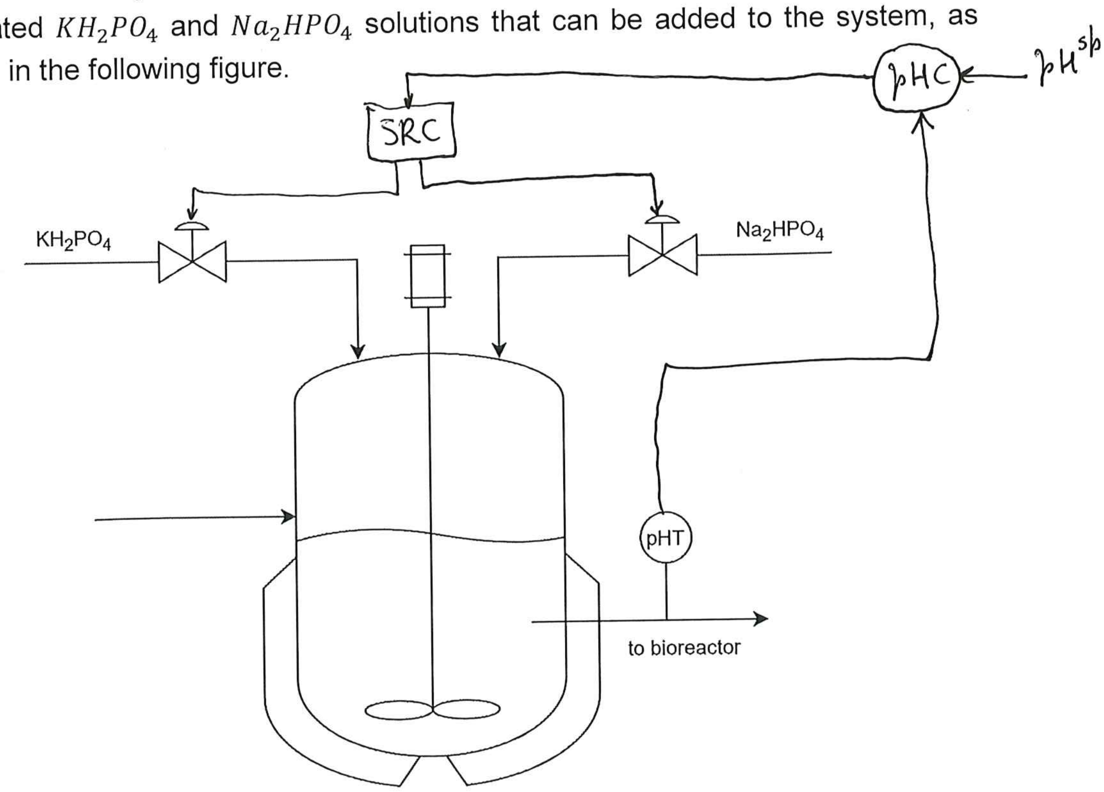
The use of cascade control accelerates the internal loop, which may improve setpoint tracking slightly. The main advantage of the cascade structure would be rejecting disturbance d_3.

Problem 5 – Advanced control elements (15%)

a) Assume a system with measured disturbance $y = Gu + G_d d$. Draw the block diagram of the system with a simple feedforward controller and write the ideal expression for the controller transfer function. How would you combine this with feedback control?

b) In the context of multivariable control, what are the advantages of using decentralized control? When should one consider more advanced approaches? Give examples of such approaches.

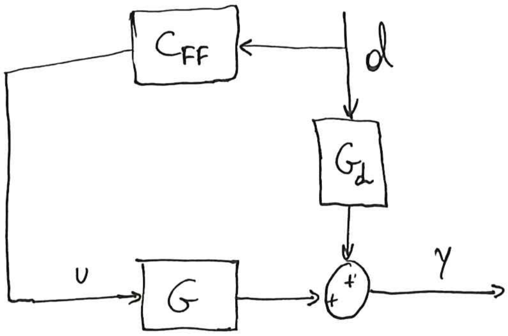
c) Phosphate buffered saline (PBS) is a widely used salt solution for cell growth applications. The pair $H_2PO_4^-/HPO_4^{-2}$ has a pK_a of 7.20, and is therefore ideal for keeping the pH at mild conditions. Consider you are in charge of a pretreatment unit that fine-tunes the pH of the feed to a bioreactor. For that, you have streams of concentrated KH_2PO_4 and Na_2HPO_4 solutions that can be added to the system, as illustrated in the following figure.



As the pH from the upstream units may vary considerably, you must propose a control structure that deals with this disturbance efficiently. What control structure would you propose, and why?

c) As the MV to be used changes during operation, one may use split-range control to automatically select the best MV to reject the disturbance, adding either the acid or the respective conjugated base.

a)

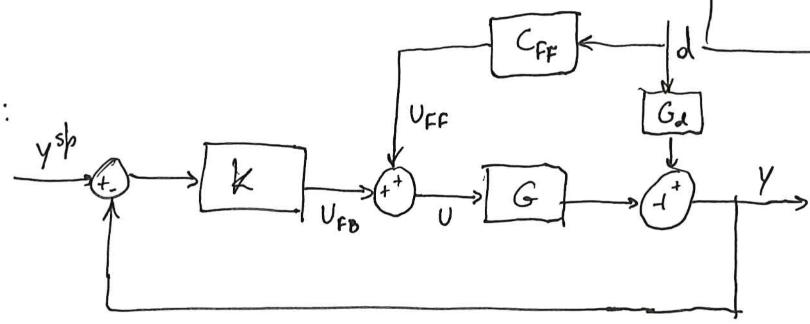


$$y = Gu + G_d d \quad , \quad u = C_{FF}\, d$$

$$\Rightarrow y = \left(G\, C_{FF} + G_d \right) d$$

Ideally: $G\, C_{FF} + G_d = 0 \Rightarrow \boxed{C_{FF} = \frac{-G_d}{G}}$

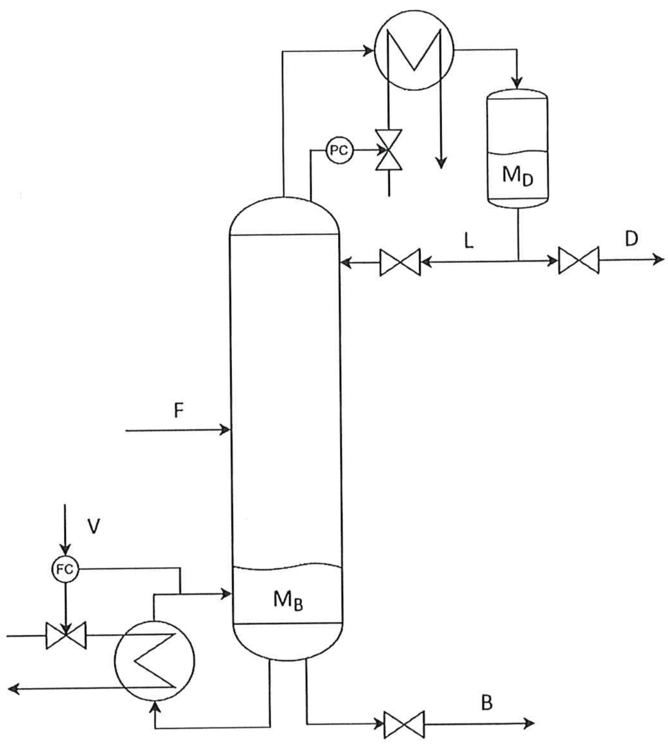
Combination with feedback:



b) Decentralized control strategies are simpler to design and maintain, as the loops operate mostly independently. One should consider more advanced, centralized, strategies when interactions are strong in the off-diagonal elements. The most used strategy for that is MPC.

Problem 6 – Optimal process operation (15%)

Consider a single distillation column that separates B and D as shown in the figure below.



The feed and its composition are given by the upstream processes and outside of our analysis. The pressure in the column is controlled by the coolant flowrate and is deemed constant.

Purity constraints are imposed on the two products as follows:

$$x_B \geq 97\%\, B$$
$$x_D \geq 97\%\, D$$

To prevent column flooding, there is a maximum allowed vapor flowrate given by:

$$V \leq 5\, mol/s$$

The objective is to minimize the cost given by:

$$\min p_V V - p_B B - p_D D$$

The product prices are given by $p_B = 1\, \$/mol$ and $p_D = 3\, \$/mol$. The energy price p_V is determined by the market and is considered a disturbance.

Based on the information above, answer the following questions.

a) How many dynamic and steady-state degrees of freedom does this system have? Propose a control structure to control the dynamic degrees of freedom.

b) How many active constraint regions are possible? List all the possible regions.

c) Based on your engineering intuition, can you eliminate some of the active constraint combinations that you listed above? Justify your answer.

d) For a given feed rate of $F = 1.4\, mol/s$, when the energy price is very low ($p_V = 0.001\, \$/mol$), what are the expected active constraints? Suggest suitable pairings for this case and draw the corresponding full control structure in the flowsheet provided in the next page.

a) Dynamic DOFs (excluding PC): 4

Number of levels: 2 (M_D, M_B)

Steady-state DOFs: $4-2=2$

Pairing:

M_B — B

M_D — D

b) Theoretically, $2^3 = 8$ regions:

	I	II	III	IV	V	VI	VII	VIII
x_D	●	●	●	●	○	○	○	○
x_B	●	●	○	○	●	●	○	○
V	●	○	●	○	●	○	●	○

(●: active ; ○: inactive)

I is mathematically impossible (infeasible), as $n_g > n_u$

⇒ 7 possible regions (II – VIII)

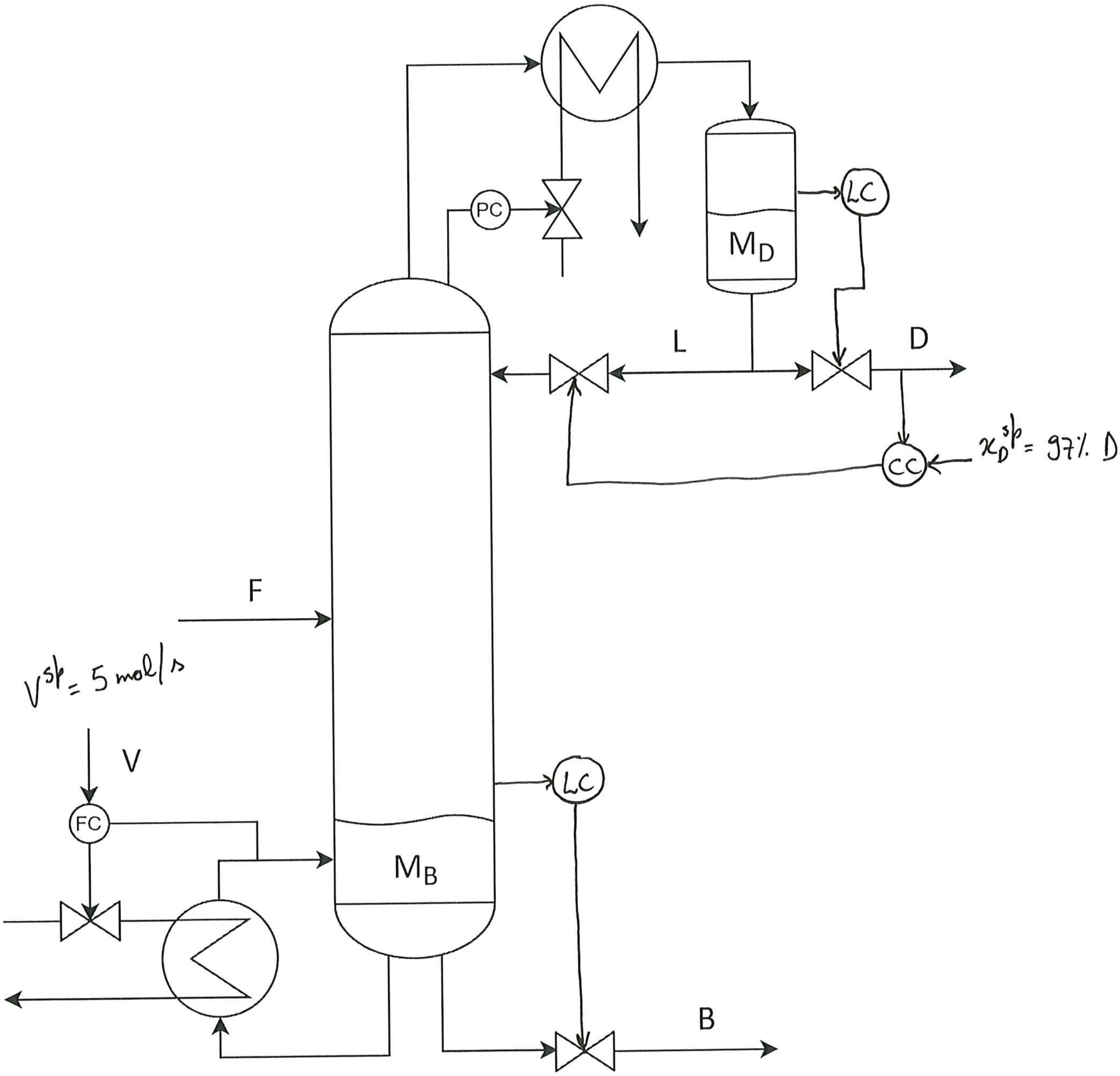
c) Based on p_D being relatively high, we may expect $x_D \geq 97\%$ D being always active. We may therefore exclude regions V to VIII.

d) For a very low energy price, one would expect $V \leq 5$ mol/s to be active. We would therefore operate in region III.

Pairing:

L — x_D

V — V^{max}



PC
LC
M_D
L
D
$x_D^{sp} = 97\%$ D
CC
F
$V^{sp} = 5$ mol/s
V
FC
LC
M_B
B