Self-optimizing control

From key performance indicators to control of biological systems

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Outline

- Optimal operation
- Implememtation of optimal operation: Self-optimizing control
- What should we control?
- Applications
  - Marathon runner
  - KPI's
  - Biology
  - ...
- Optimal measurement combination
- Optimal blending example

Focus: **Not** optimization (optimal decision making)
But rather: How to **implement** decision in an uncertain world

Optimal operation of systems

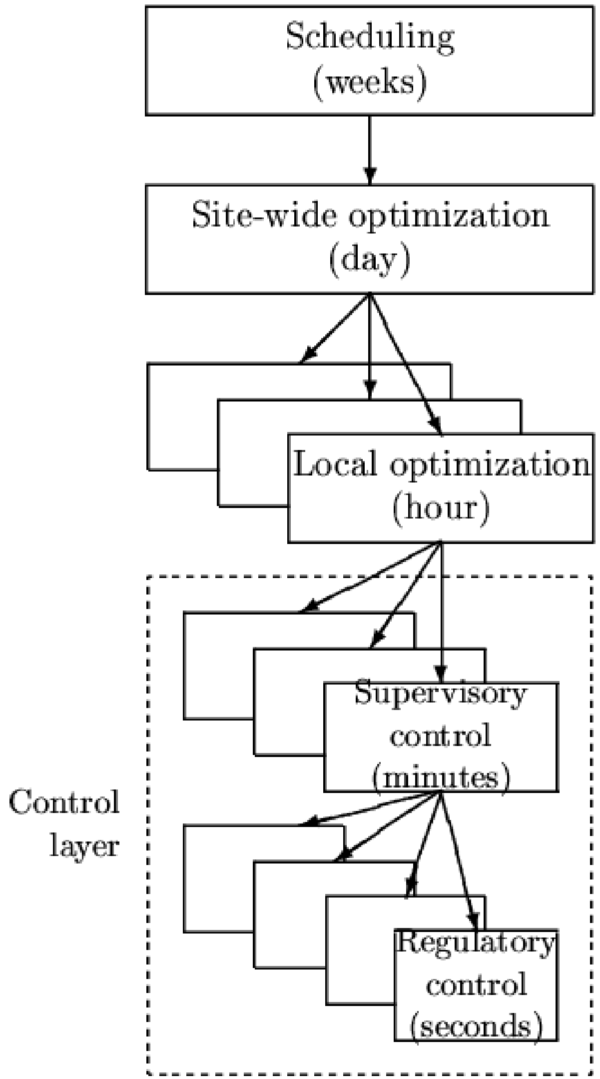
- Theory:
  - Model of overall system
  - Estimate present state
  - Optimize all degrees of freedom
- Problems:
  - Model not available and optimization complex
  - Not robust (difficult to handle uncertainty)
- Practice
  - Hierarchical system
  - Each level: Follow order (”setpoints”) given from level above
  - Goal: Self-optimizing

Process operation: Hierarchical structure

RTO

MPC

PID



Engineering systems

- Most (all?) large-scale engineering systems are controlled using hierarchies of quite simple single-loop controllers
  - Large-scale chemical plant (refinery)
  - Commercial aircraft
- 1000's of loops
- Simple components:

  on-off + P-control + PI-control + nonlinear fixes + some feedforward

Same in biological systems

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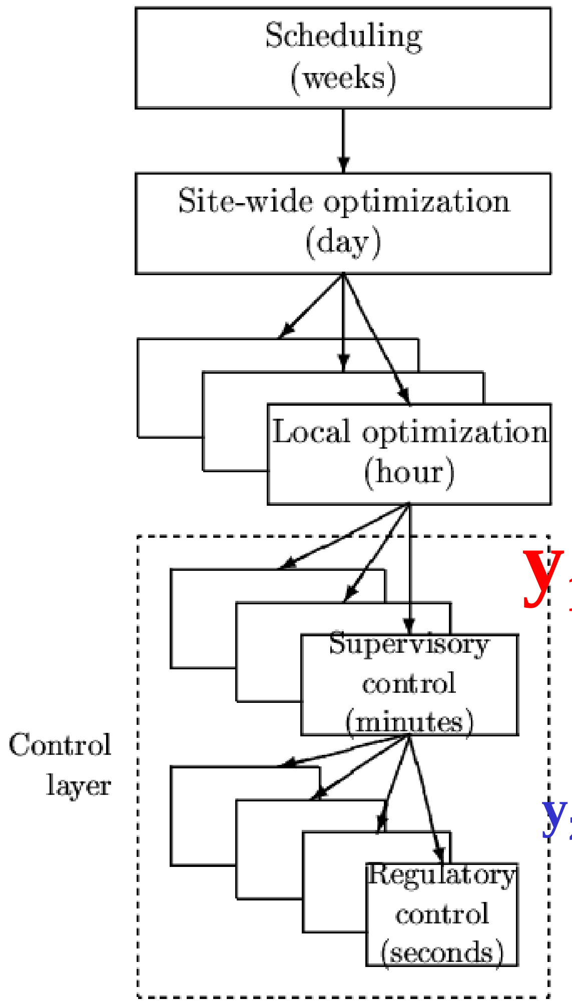
What should we control?

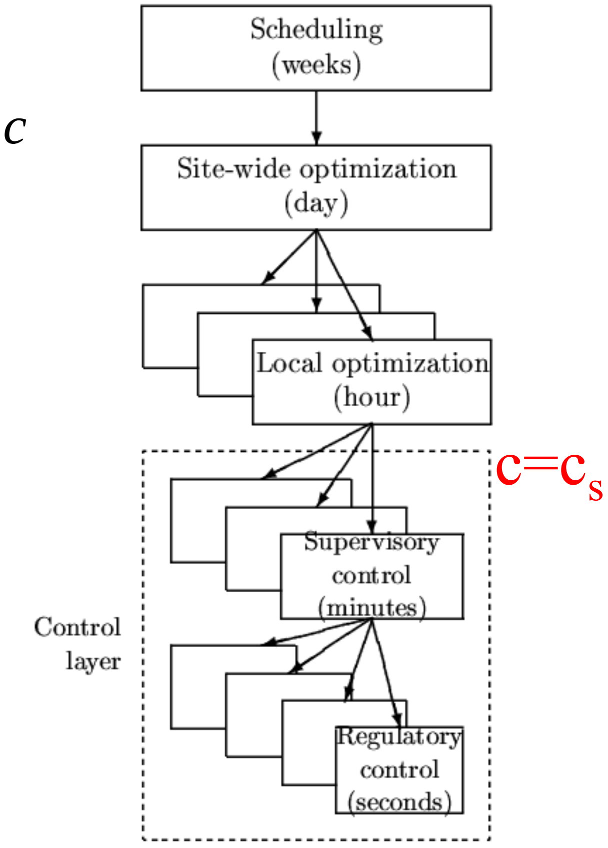


$y_1 = c$? (economics)

$y_2 = $? (stabilization)

Self-optimizing Control

Self-optimizing control is when acceptable operation can be achieved using constant set points (c_s) for the controlled variables c (without re-optimizing when disturbances occur).



Optimal operation (economics)

- Define scalar cost function $J(u_0,d)$
  - u_0: degrees of freedom
  - d: disturbances

- Optimal operation for given d:

$$\min_{u0} \; J(u_0,d)$$

subject to:

$f(u_0,d) = 0$

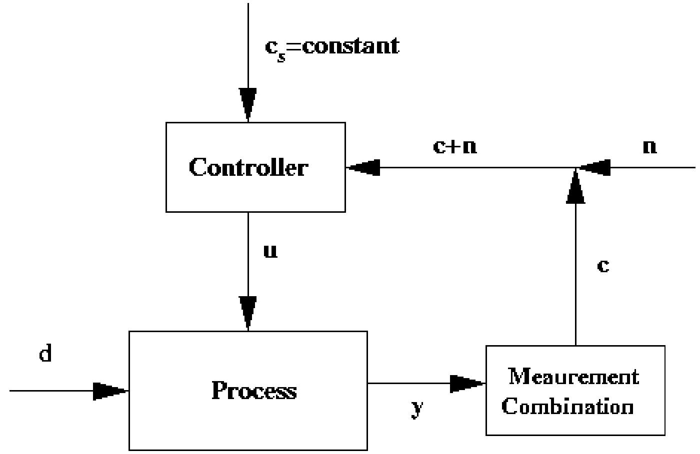
$g(u_0,d) < 0$

Implementation of optimal operation

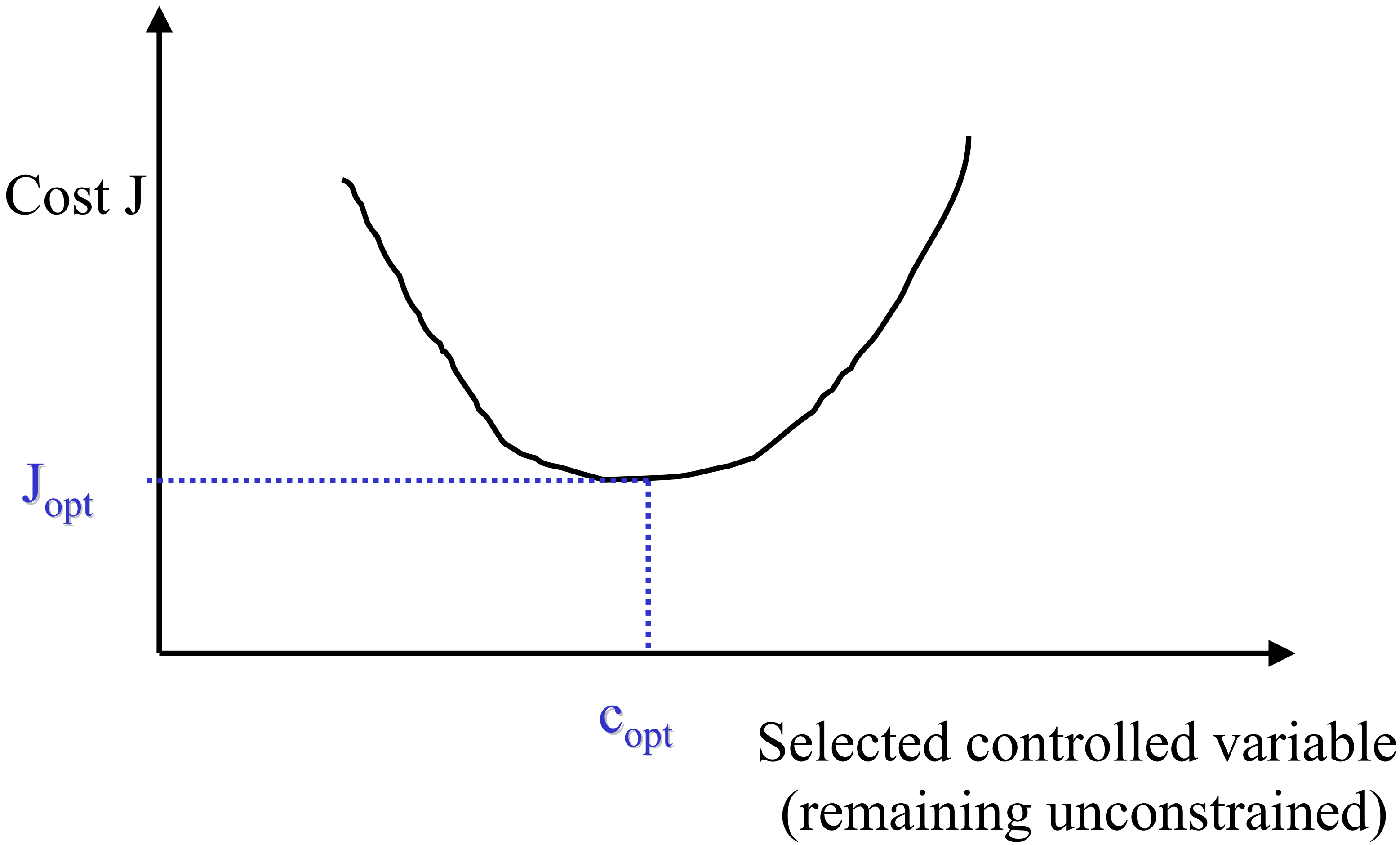
- *Idea*: Replace optimization by setpoint control

- Optimal solution is usually at constraints, that is, most of the degrees of freedom u_0 are used to satisfy "active constraints", $g(u_0,d) = 0$
- CONTROL ACTIVE CONSTRAINTS!
  - Implementation of active constraints is usually simple.

- WHAT MORE SHOULD WE CONTROL?
  - **Find variables c for remaining unconstrained degrees of freedom u.**



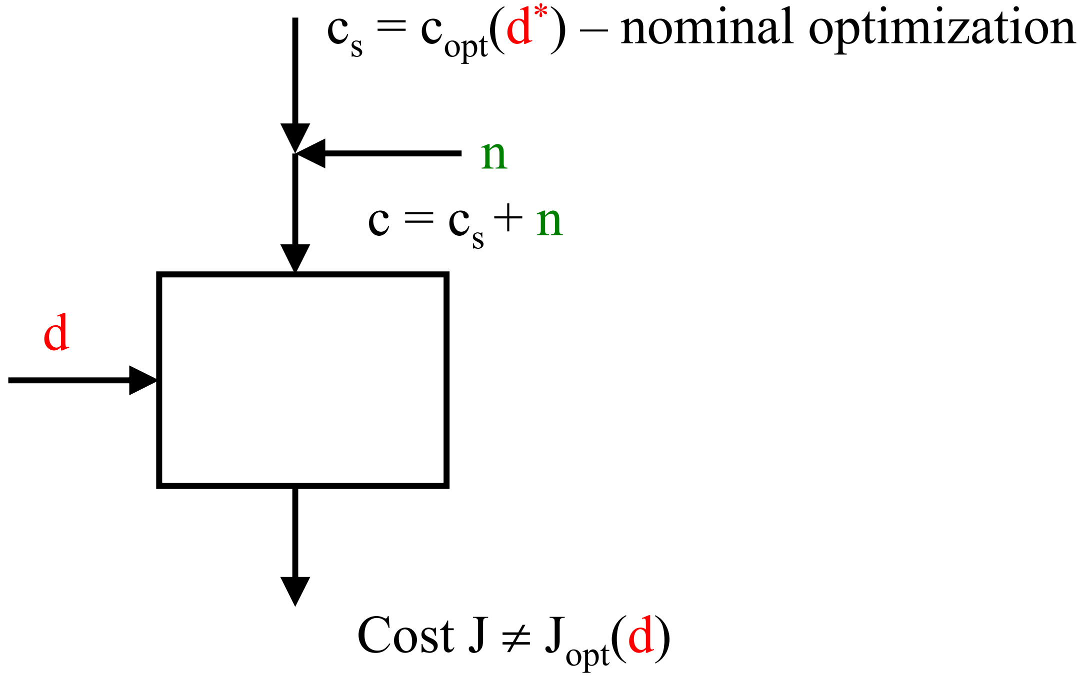
Unconstrained variables



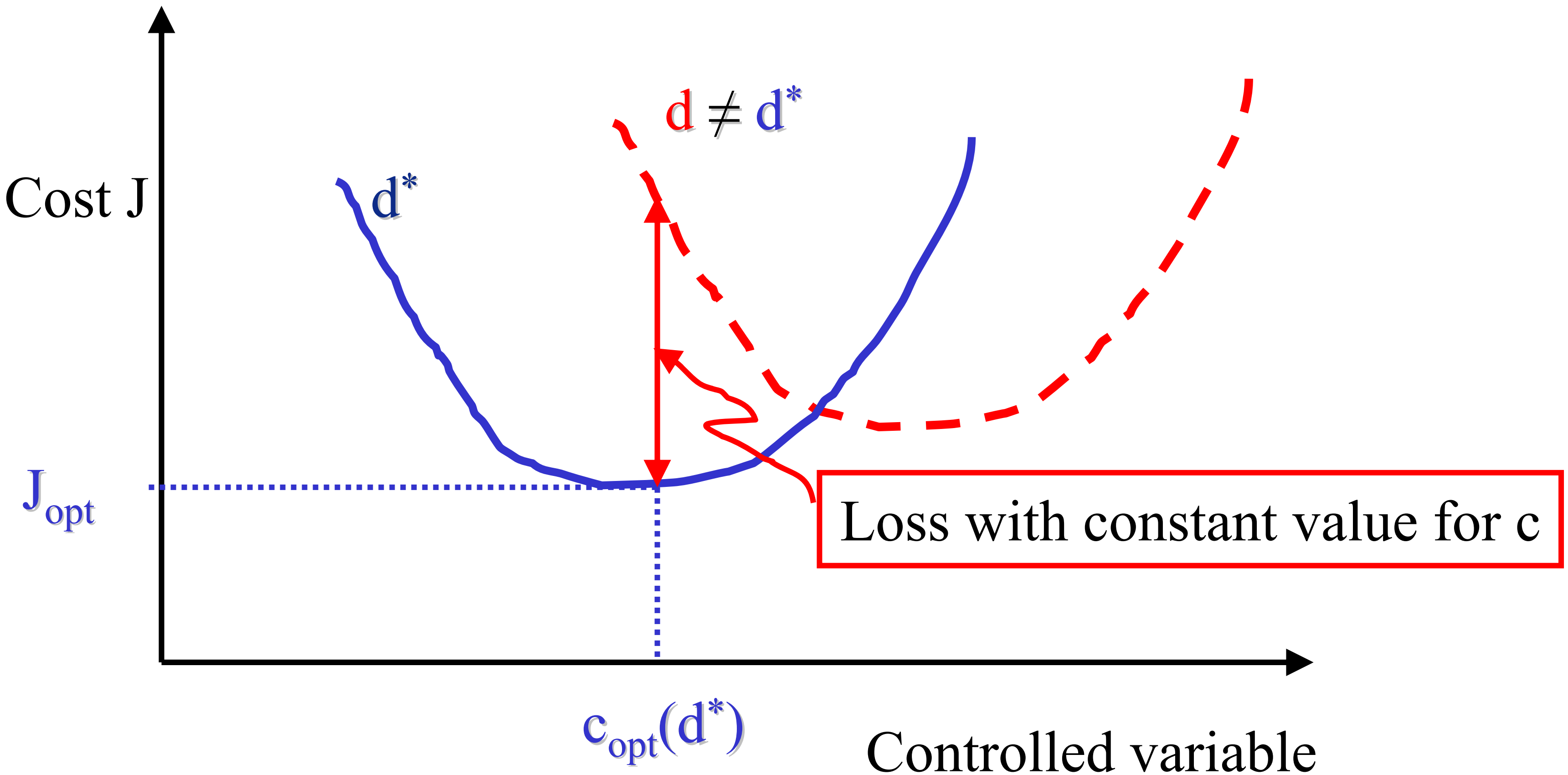
Implementation of unconstrained variables is not trivial:

How do we deal with uncertainty?

- 1. Disturbances d
- 2. Implementation error n

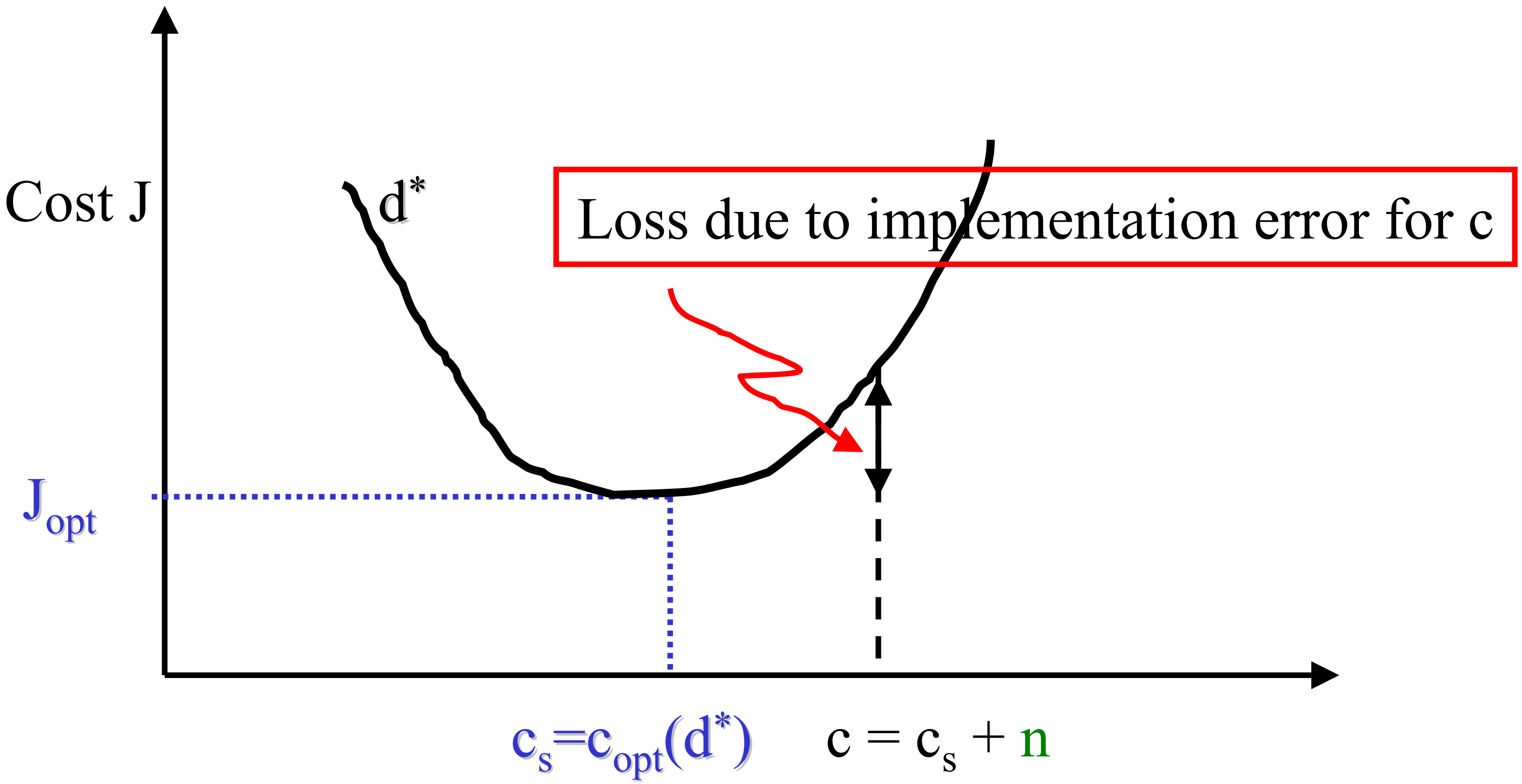


Problem no. 1: Disturbance d

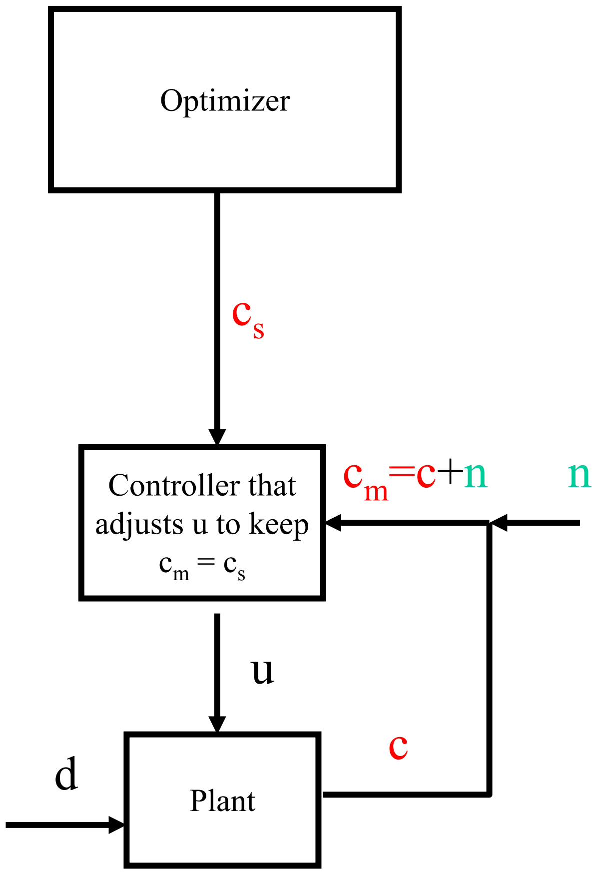


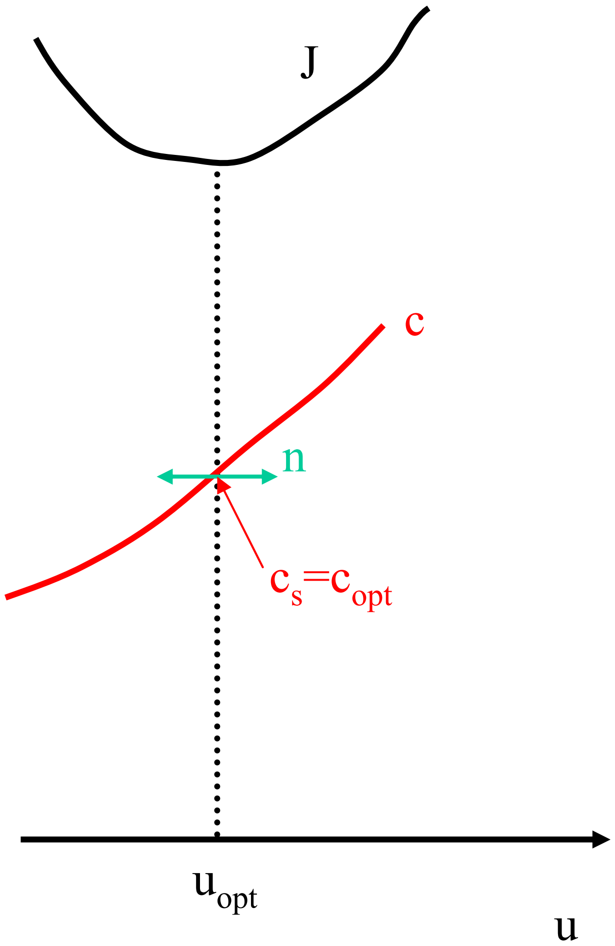
⇒ Want c_{opt} independent of d

Problem no. 2: Implementation error n



⇒ Want n small and ”flat” optimum





⇒ Want c sensitive to u ("large gain")

Which variable c to control?

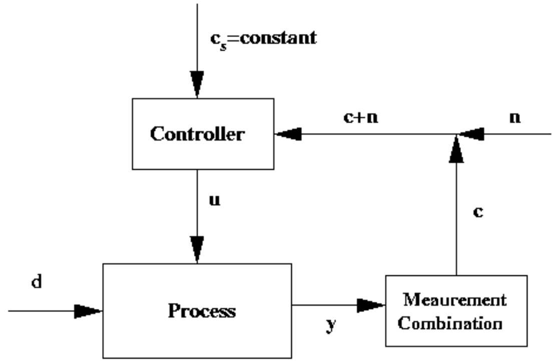
- *Define optimal operation:* Minimize cost function J

- *Each candidate variable c:*

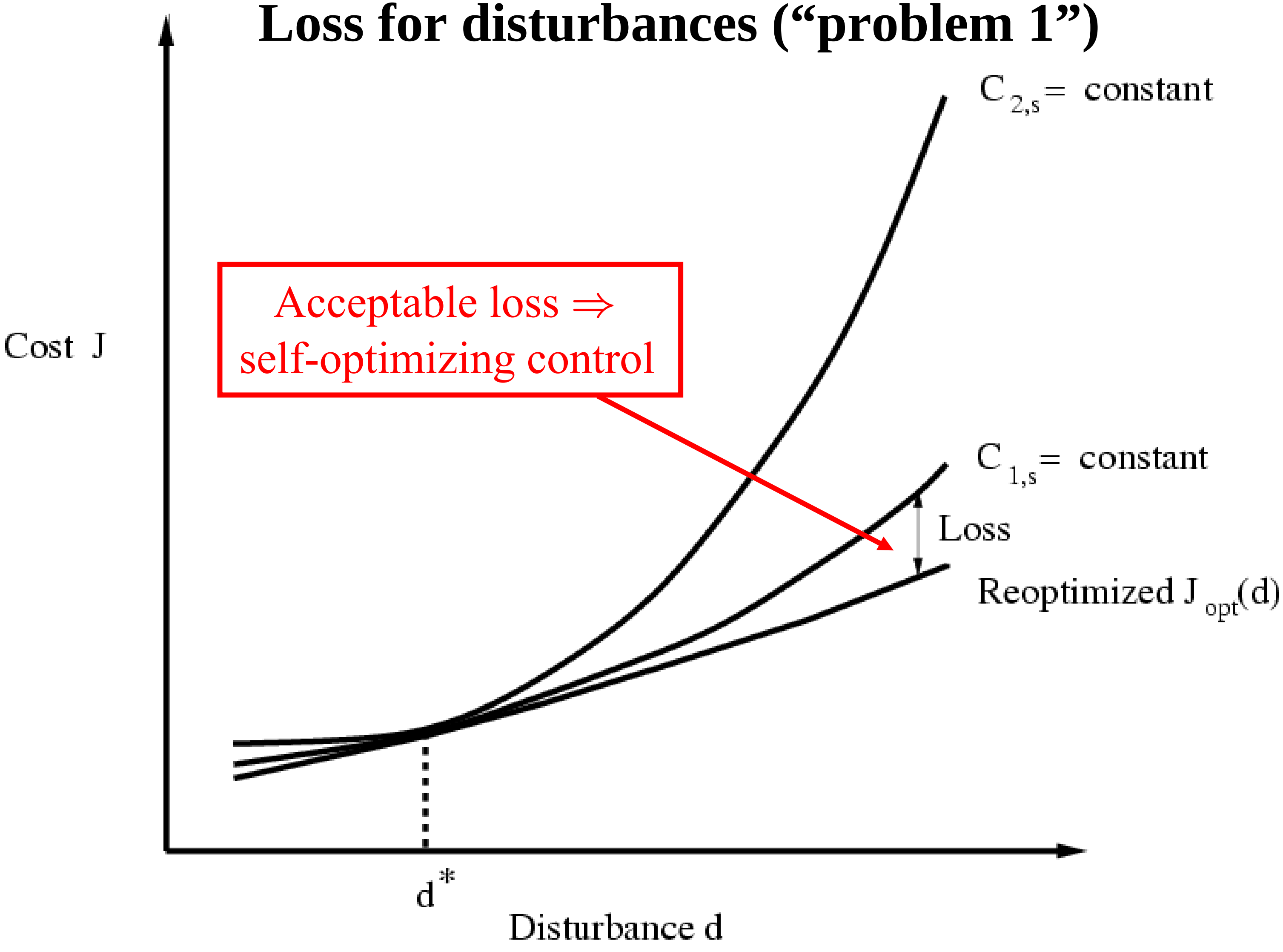
  With constant setpoints c_s compute loss L for expected disturbances d and implementation errors n

$$L(d) = J(c_s + n, d) - J_{opt}(d)$$

- Select variable c with smallest loss



Constant setpoint policy:
Loss for disturbances (“problem 1”)



Good candidate controlled variables c (for self-optimizing control)

Requirements:

- The *optimal value* of c should be *insensitive* to disturbances (avoid problem 1)
- c should be easy to measure and control (rest: avoid problem 2)
- The *value* of c should be *sensitive* to changes in the degrees of freedom
  (Equivalently, J as a function of c should be flat)
- For cases with more than one unconstrained degrees of freedom, the selected controlled variables should be independent.

Singular value rule *(Skogestad and Postlethwaite, 1996):*
Look for variables that maximize the minimum singular value of the appropriately scaled steady-state gain matrix G *from u to c*

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Examples self-optimizing control

- Marathon runner
- Central bank
- Cake baking
- Business systems (KPIs)
- Investment portfolio
- Biology
- Chemical process plants: Optimal blending of gasoline

Define optimal operation (J) and look for ”magic” variable (c) which when kept constant gives acceptable loss (self-optimizing control)

Self-optimizing Control – Marathon

- Optimal operation of Marathon runner, J=T
  - Any self-optimizing variable c (to control at constant setpoint)?

Self-optimizing Control – Marathon

- Optimal operation of Marathon runner, J=T
  - Any self-optimizing variable c (to control at constant setpoint)?
    - c_1 = distance to leader of race
    - c_2 = speed
    - c_3 = heart rate
    - c_4 = level of lactate in muscles

Further examples

- Central bank. J = welfare. c=inflation rate (2.5%)
- Cake baking. J = nice taste, c = Temperature (200C)
- Business, J = profit. c = ”Key performance indicator (KPI), e.g.
  - Response time to order
  - Energy consumption pr. kg or unit
  - Number of employees
  - Research spending

  Optimal values obtained by ”benchmarking”
- Investment (portofolio management). J = profit. c = Fraction of investment in shares (50%)
- Biological systems:
  - ”Self-optimizing” controlled variables c have been found by natural selection
  - Need to do ”reverse engineering” :
    - Find the controlled variables used in nature
    - From this identify what overall objective J the biological system has been attempting to optimize

Looking for "magic" variables to keep at constant setpoints.
How can we find them?

- Consider available measurements y, and evaluate loss when they are kept constant ("brute force"):

$c = y_i$: Single measurements, e.g. pressure, temperature, composition

$c = \frac{y_i}{y_j}$: Combinations of measurements (e.g flow ratios)

- More general: Find optimal linear combination (matrix H):

$$c = h_1 y_1 + h_2 y_2 + \ldots + h_n y_n = Hy$$

Optimal measurement combination (Alstad)

$$\Delta c = H \Delta y$$

- *Basis*: Want optimal value of c independent of disturbances ⇒
  - $\Delta c_{opt} = 0 \cdot \Delta d$
- Find optimal solution as a function of d: $u_{opt}(d)$, $y_{opt}(d)$
- Linearize this relationship: $\Delta y_{opt} = F \Delta d$
  - F – sensitivity matrix
- Want:

$$\Delta c_{opt} = H \Delta y_{opt} = H F \Delta d = 0$$

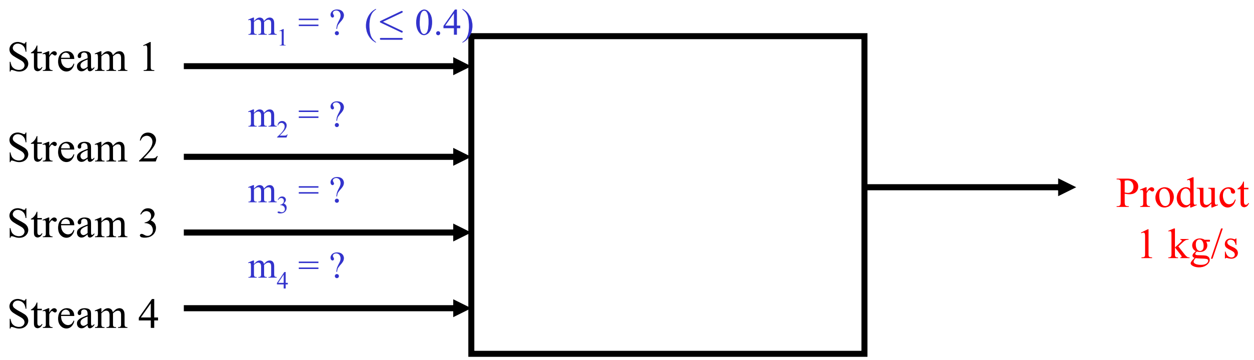
- To achieve this for all values of Δ d:

$$HF = 0 \quad \Rightarrow \quad H \in \mathcal{N}(F^T)$$

- Always possible if

$$\# y \geq \# u + \# d$$

Example: Optimal blending of gasoline



Stream 1	99 octane	0 % benzene	$p_1 = (0.1 + m_1)$ \$/kg
Stream 2	105 octane	0 % benzene	$p_2 = 0.200$ \$/kg
Stream 3	95 → 97 octane	0 % benzene	$p_3 = 0.120$ \$/kg
Stream 4	99 octane	2 % benzene	$p_4 = 0.185$ \$/kg
Product	> 98 octane	< 1 % benzene	

Disturbance

Optimal solution

- Degrees of freedom

  $u_o = (m_1\ m_2\ m_3\ m_4)^T$

- Optimization problem: Minimize

  $J = \sum_i p_i m_i = (0.1 + m_1)\, m_1 + 0.2\, m_2 + 0.12\, m_3 + 0.185\, m_4$

  subject to

  $m_1 + m_2 + m_3 + m_4 = 1$

  $m_1 \geq 0$; $m_2 \geq 0$; $m_3 \geq 0$; $m_4 \geq 0$

  $m_1 \leq 0.4$

  $99\, m_1 + 105\, m_2 + 95\, m_3 + 99\, m_4 \geq 98$ (octane constraint)

  $2\, m_4 \leq 1$ (benzene constraint)

- Nominal optimal solution ($d^* = 95$):

  $u_{0,opt} = (0.26\ \ 0.196\ \ 0.544\ \ 0)^T \Rightarrow J_{opt}=0.13724$ \$

- Optimal solution with d=octane stream 3=97:

  $u_{0,opt} = (0.20\ \ 0.075\ \ 0.725\ \ 0)^T \Rightarrow J_{opt}=0.13724$ \$

- 3 active constraints ⇒ 1 unconstrained degree of freedom

Implementation of optimal solution

- Available "measurements": $y = (m_1\ m_2\ m_3\ m_4)^T$
- Control active constraints:
  - Keep $m_4 = 0$
  - Adjust one (or more) flow such that $m_1+m_2+m_3+m_4 = 1$
  - Adjust one (or more) flow such that product octane = 98
- Remaining unconstrained degree of freedom
  1. $c=m_1$ is constant at 0.126 ⇒ Loss = 0.00036 \$
  2. $c=m_2$ is constant at 0.196 ⇒ Infeasible (cannot satisfy octane = 98)
  3. $c=m_3$ is constant at 0.544 ⇒ Loss = 0.00582 \$
- Optimal combination of measurements

  $c = h_1\ m_1 + h_2\ m_2 + h_3\ m_a$

  From optimization: $\Delta\ m_{opt} = F\ \Delta\ d$ where sensitivity matrix $F = (-0.03\ \ -0.06\ \ 0.09)^T$

  Requirement: $HF = 0 \Rightarrow$

  $-0.03\ h_1 - 0.06\ h_2 + 0.09\ h_3 = 0$

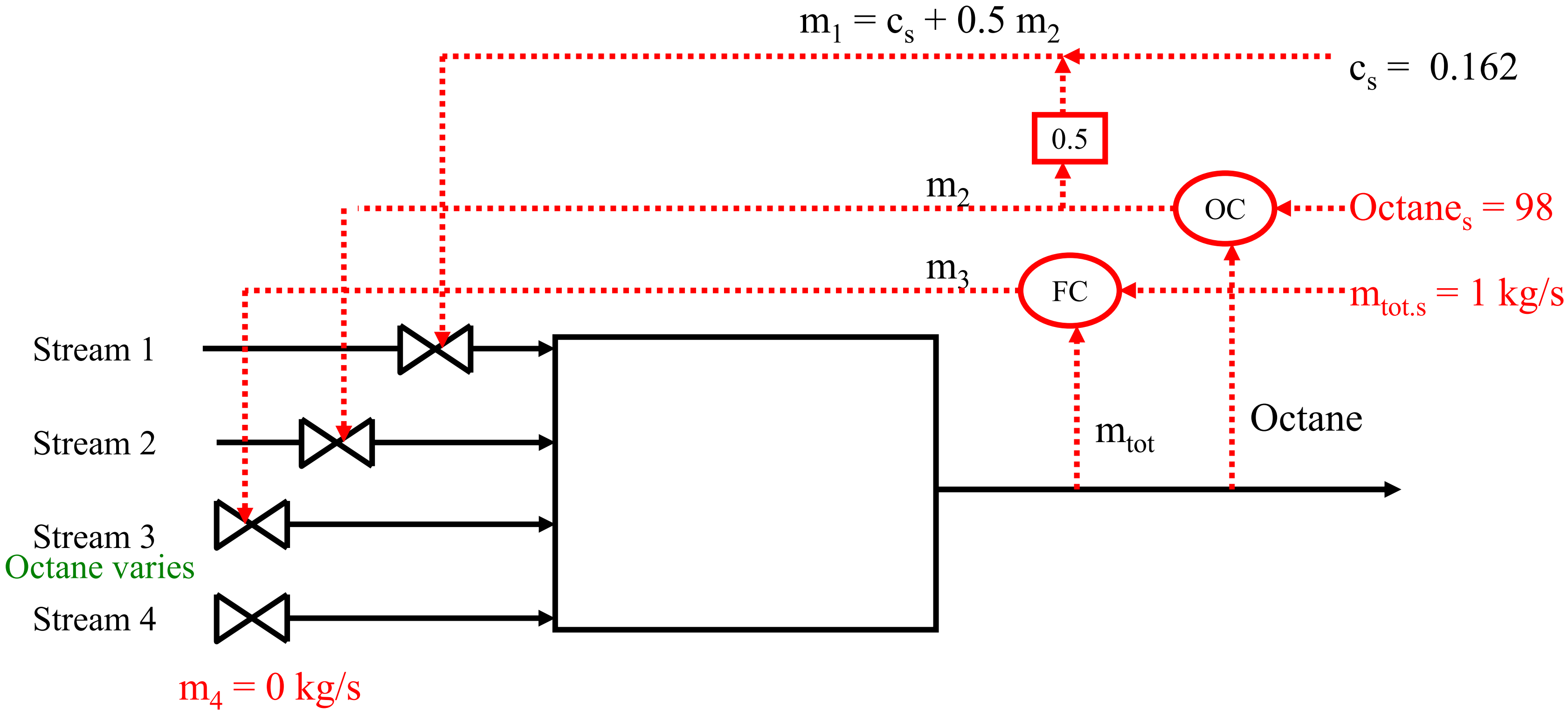
  This has infinite number of solutions (since we have 3 measurements and only ned 2):

  $c = m_1 - 0.5\ m_2$ is constant at 0.162 ⇒ Loss = 0

  $c = 3\ m_1 + m_3$ is constant at 1.32 ⇒ Loss = 0

  $c = 1.5\ m_2 + m_3$ is constant at 0.83 ⇒ Loss = 0
- Easily implemented in control system

Example of practical implementation of optimal blending



- Selected "self-optimizing" variable: $c = m_1 - 0.5\, m_2$
- Changes in feed octane (stream 3) detected by octane controller (OC)
- Implementation is optimal provided active constraints do not change
- Price changes can be included as corrections on setpoint c_s

Conlusion

- Operation of most real system: Constant setpoint policy ($c = c_s$)
  - Central bank
  - Business systems: KPI's
  - Biological systems
  - Chemical processes
- *Goal:* Find controlled variables c such that constant setpoint policy gives acceptable operation in spite of uncertainty

  ⇒ Self-optimizing control
- *Method:* Evaluate loss $L = J - J_{opt}$
- Optimal linear measurement combination:

  $\Delta c = H \Delta y$ where $HF=0$