Nonlinear model based/model predictive control with constraints and with/without nonlinear observer

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Abstract

A literature survey from the area of nonlinear model predictive control (MPC) is presented. After a brief review of the main characteristics of linear MPC algorithms various classes of nonlinear MPC algorithms based on state-space models, such as nonlinear extensions of dynamic matrix control (DMC), Newton-type algorithms and nonlinear programming (NLP) based algorithms are discussed. These algorithms are often combined with algorithms for on-line state and parameter estimation. Recursive and moving horizon-based approaches are briefly reviewed. Various input/output models for nonlinear MPC, such as neural network models, fuzzy models, Volterra models, Hammerstein models and polynomial NARX models are also discussed. The same optimization strategies as is used for state-space models can usually be used for nonlinear input/output models. Finally, some features with various constraint handling strategies are discussed.

1 Introduction

The purpose of this report is to present a literature survey from the area of nonlinear model based/model predictive process control. The report is part of the background material to be used for the detailed definition of the INCOOP project.

Chemical manufacturing processes present many challenging control problems, including: nonlinear dynamic behaviour; multivariable interactions between manipulated and controlled variables; unmeasured state variables; unmeasured and frequent disturbances; high-order and distributed processes; uncertain and time-varying parameters; unmodelled dynamics; constraints on manipulated and state variables; and (variable) deadtime on inputs and measurements. Further, reliable measurements of important variables to be controlled, such as quality related variables, are often difficult to obtain on-line.

A number of control approaches and algorithms that are able to handle some of the above process characteristics have been presented in the academic literature in resent years. Bequette (1991a) gives a review of various approaches, such as: internal model approaches; differential geometric approaches; reference system synthesis techniques, including internal decoupling and generic model control; model predictive control approaches; and also various special and ad hoc approaches. Many of these approaches are not able to handle the various process characteristics and requirements met in industrial applications, and some of the approaches can only be applied for special classes of models.

Nonlinear model predictive control appear to be the only general approaches which can handle most of the common process characteristics and industrial requirements in a satisfactory way. It also seems to be the approaches which are most suitable for the development of general and application independent software, which is essential for the development of cost-effective applications. For the above reasons this survey will focus on nonlinear model predictive control approaches presented in the open literature.

Algorithms for nonlinear model predictive control are often extensions of linear model predictive control algorithms. For continuity the main characteristic features of linear model predictive control are briefly discussed.

2 Linear model predictive control

Model predictive control (MPC) refers to a class of algorithms that compute a sequence of manipulated variable adjustments in order to optimize the future behaviour of the plant. Qin and Badgwell (1996) provides an overview of commercially available MPC technology, based primarily on data provided by MPC vendors. Most of these commercial products are based on empirical and linear models of the plant, and will be called linear MPC technology in this report.

Qin and Badgwell (1996) starts their survey with a brief history of industrial MPC technology. Some of the early technologies are: IDCOM (Richalet et al., 1978), DMC (Cutler and Ramaker, 1980), QDMC (Garcia and Morshedi, 1986), and IDCOM-M (Grosdidier et al., 1988; Froisy and Matsko, 1990). Other technologies are also referred to. Further, Qin and Badgwell have recently surveyed five MPC vendors in order to assess the current status of commercial MPC technology. This status is briefly summarized below, see Qin and Badgwell (1996) for more details and a thorough discussion:

- Most commercial MPC technologies are based on linear input-output models; e.g., impulse response models, step response models, transfer function models, etc. These models are usually identified from plant test data. The identification procedure and particularly the design of plant tests is crucial for the performance of MPC algorithms.
- At each sampling time the MPC algorithm must determine which inputs can be manipulated and which outputs should be controlled. Some algorithms include a controllability supervisor which decides, based on the current set of available inputs and outputs, which outputs can be independently controlled. The selection can be based on the condition number of the plant gain matrix; a list of controllability priorities is then used to determine which outputs to drop from the problem if an ill-conditioned set is encountered. Other strategies for removal of ill-conditioning is discussed by Qin and Badgwell (1996).

- The process intputs are determined from the optimization of a quadratic performance objective over a finite prediction horizon, weighting outputs, inputs and input moves. The performance objective can also be multi-objective: a quadratic objective for the outputs, and, if there is extra degrees of freedom, an objective for the inputs.
- Future outputs are specified by setpoints, reference trajectories, zones or funnels.
- Input and output constraints. The output constraints can be hard or soft. Hard output constraints are ranked in order of priority, and low priority constraints may be dropped in order to recover from an infeasible solution.

In the survey by Qin and Badgwell (1996) MPC applications performed by each of the surveyed vendors are also summarized by application area. More than 2200 applications are reported.

An interesting discussion of industrial applications of MPC, which focus more on the application projects than on the control algorithms, is presented by Richalet (1993).

As mentioned above, most of the commercial MPC technologies are based on linear input-output models. Integral action in these controllers is usually achieved by estimating a constant output disturbance which is added to the predicted outputs. Muske and Rawlings (Muske and Rawlings 1993a, b; Rawlings and Muske, 1993) present a more general approach based on a linear state-space formulation for the plant model. Unstable as well as stable models can then be represented. Output feedback is addressed in the well-established framework of the linear quadratic state-estimation problem. This framework allows the flexibility to handle nonsquare systems, noisy inputs and outputs, and input, output, and state disturbances. This formulation subsumes the integral control schemes currently in industrial use. Muske and Rawlings also introduce an infinite prediction horizon, which has the advantage that nominal stability is guaranteed for any choice of weights in the optimization criterion.

Lee et al. (1994) also discuss the relation between state-space and input-output models for model predictive control.

3 Nonlinear model predictive control based on state-space models

Nonlinear model predictive control (NMPC) algorithms are often extensions of the linear MPC (LMPC) algorithms discussed in the previous section. An advantage with this is that new commercial NMPC technologies can partly be based on the well-proven LMPC technologies currently in use. This section gives a brief survey of published NPMC algorithms based on nonlinear state-space models, which are usually derived from first-principles.

3.1 Nonlinear extensions of Dynamic Matrix Control

Nonlinear extensions of Dynamic Matrix Control (DMC) (Cutler and Ramaker, 1980) and Quadratic Dynamic Matrix Control (QDMC) (Garcia and Morshedi, 1986) have been proposed by several researchers (García, 1984; Peterson et al., 1989, 1992; Lee and Ricker, 1994; Gattu and Zafiriou 1992, 1995).

In nonlinear quadratic dynamic matrix control (NLQDMC), proposed by García (1984), a nonlinear model is used to compute the effect of past manipulated variables on the predicted output. A linear model, obtained by linearizing the nonlinear model at each sampling time, is used to compute the manipulated variable values. An advantage with this approach is that only one quadratic program is solved at each sampling time.

The algorithm proposed by Peterson et al. (1989, 1992) extends the linear DMC algorithm by extending the disturbance vector to account also for the effect of nonlinearities in the prediction horizon, which are determined from a nonlinear state-space model. No observer is included. The future control moves are determined from an iteration, where the control moves are calculated from the ordinary DMC formula, and a new "disturbance vector" is calculated from the nonlinear model. The step response matrix used in DMC calculations are calculated by linearizing the nonlinear model at a given steady state or by perturbing the nonlinear model by a unit step. Conditions for convergence of the iterations are stated and proved. The algorithm can trivially be extended to a nonlinear QDMC algorithm by determining the control moves from the solution of a quadratic program.

Gattu and Zafiriou (1992, 1995) extended the NLQDMC algorithm proposed by García (1984) by incorporating state estimation using a steady-state Kalman filter gain computed at each sampling time. This algorithm can be used both for nonlinear input-output models and state-space models. Disturbance rejection and offset free tracking is addressed in a general setting utilizing concepts from filtering theory. Various kinds of disturbance models can be incorporated in the formulation. A nonlinear model is used for predicting the effects of past inputs. Future input moves, however, are calculated from a linear model, by solving av quadratic program at each sampling time. The computational burden is then comparable to the ordinary QDMC algorithm. In the case of a nonlinear state-space model, the linear model is obtained by linearizing the nonlinear model around the current state estimate. In the case of an input-output model the nonlinear model is also linearized, and a minimal state-space realization of the linear input-output model is constructed. This linear model is then used both for observer design and optimization of the input moves.

A nonlinear QDMC algorithm has been tested on the Tennessee Eastman Challenge process by Ricker and Lee (1995).

3.2 Newton-type algorithms

A Newton-type nonlinear MPC algorithm has been developed by Biegler and co-workers (Li and Biegler 1988, 1989; de Oliveira and Biegler, 1995). Their approach is to linearize a nonlinear state-space model around a nominal trajectory determined by the input sequence computed at the previous sampling time. Impulse or step response models are determined from the linear and time-varying state-space models obtained from the linearization. Alternatively, the impulse or step response models can be determined directly by perturbing the nonlinear model. A new input sequence is computed by solving a quadratic program once over the time horizon, followed by a line search where the quadratic optimization criterion is computed based on the nonlinear model. Through the action of the line search, global convergence of the method can be enforced as long as the method exhibits descent directions. Sufficient conditions for global convergence and stability are developed by Li and Biegler (1989). The development assumes that the states are available. State estimation is not considered in this work.

de Oliveira and Biegler (1994b) extends the Newton-type algorithm by introducing trust region concepts originating from nonlinear programming. This extension is well suited to deal with ill-conditioning of the predictive equations.

The optimization method used by Biegler and co-workers can be considered a special form of a sequential quadratic programming (SQP) strategy. However, one major iteration (quadratic program solution, followed by a line search) at each sampling time is usually considered sufficient (Li and Biegler 1989). It should also be noted that the Hessian of the optimization criterion is computed in this Newton-type algorithm, it is not just estimated from past objective function evaluations, which is usually done in SQP algorithms.

3.3 Nonlinear programming based algorithms

Several researchers have proposed nonlinear MPC algorithms which utilize general nonlinear programming (NLP) techniques for solving the optimization (Jang et al., 1987; Brengel and Seider, 1989; Eaton and Rawlings, 1990; Patwardhan et al., 1990; Bequette, 1991b; Balchen et al., 1992; Ali and Zafiriou, 1993). These techniques are often used in conjunction with nonlinear, continuous state-space models. Their main differences are in the way that the model equations are solved and in the specific optimization method that is utilized. SQP algorithms are often used for solving these NLP problems. These algorithms are computationally more demanding than the previously referred nonlinear MPC algorithms.

4 State and parameter estimation

The field of identification and estimation is usually divided into off-line and on-line methods. Off-line methods generally utilize batch processing of the data and are concerned with identification of time-invariant model parameters. The problem is often formulated as a least-squares optimization with the parameters chosen to minimize the sum of the squared errors between the measurements and model predictions. On-line methods require recursive processing of the data and are concerned with time-varying parameters and states. Algorithms are developed from stochastic formulations.

4.1 Recursive state and parameter estimation

The extended Kalman filter (EKF) is by far the most popular algorithm for on-line state and parameter estimation in practical applications. Several extensions and modifications of the EKF algorithm have been developed, such as the iterated EKF, Gaussian second-order filter, and the statistically linearized filter (Jazwinski, 1970; Gelb, 1974; Anderson and Moore, 1979). Ljungquist and Balchen (1994) discuss and compare the EKF algorithm with a recursive Gauss-Newton algorithm which computes an approximation to the maximum likelihood estimate (RPEM) (Ljung and Söderström, 1983). The methods are also compared with a new method based on a line-search strategy. The latter algorithm is shown through simulations to have improved performance under certain conditions. Gudi et al. (1995) develop a multirate formulation of the iterated EKF which can handle measurements of process outputs that arrive at different sampling rates. Measurement delays associated with some of the process outputs are included in the system description by addition of delayed states. Schei (1997) has developed a method where the covariance matrices in the EKF and other nonlinear recursive estimation algorithms are determined directly from the nonlinear model, by perturbing the stochastic variables in the nonlinear model in a way that depends on the covariances for these variables. The method is shown to increase the accuracy and robustness of the EKF algorithm. It is also an advantage that a symbolic linearization of the nonlinear model do not have to be derived. The method is well suited for the development of general estimation software to be interfaced with arbitrary models, which may be configured from a model library. Numerous applications of the EKF and similar algorithms have been presented in the literature, e.g. Ricker and Lee (1995) use the EKF algorithm for on-line state and parameter estimation for the Tennessee Eastman challenge process.

4.2 Moving horizon-based approaches for least-squares estimation

Several authors have derived moving horizon based approaches to on-line state and parameter estimation, which are extensions of the least-squares batch estimation algorithms (e.g., Jang et al. 1986, 1987). These algorithms are based on a finite data window of past measurements. The decision variables in these algorithms are often initial states and parameters, which are assumed to be constant over the horizon defined by the data window. Hence, it is assumed that the initial states and parameters completely determine the value of all future states over the prediction horizon. Robertson et al. (1996) present a more general formulation of the moving horizon estimator, where states and parameters, and not just process outputs, are assumed to be influenced by stochastic disturbances. This approach is, however, computationally much more demanding for a long data window. Robertson et al. (1996) give an informative overview of different moving horizon-based approaches, and make comparisons with recursive methods such as EKF.

5 Nonlinear model predictive control based on input/output models

In this section we will summarize some approaches to nonlinear MPC based on input/output models which are typically identified from plant data. The methods we review are mainly categorized in terms of their nonlinear model representation since this is their main characteristic difference; the formulation and solution of the optimization problem are typically similar, also to linear MPC and some approaches to nonlinear MPC based on state-space models. In particular, Gattu and Zafiriou (1995) provides a nonlinear extension of QDMC to general input/output and state-space models in a common framework, see also section 3.1.

5.1 Neural Network Models

Black-box process identification using various kinds of neural networks has been extensively studied and a wide range of commercial software is available. Temeng et al. (1995) and Saint-Donat et al. (1991) rely on the common sigmoidal neural networks and variations of the well-known back-propagation algorithm to identify NARX models. Nonlinear MPC based on NLP is implemented, and the experimental results of Temeng et al. (1995) illustrate the applicability of the approach on an industrial packed bed reactor. The model was identified using designed experiments where a single input was manipulated at a time using pseudo-random test signals.

Another experimental application of neural networks in model predictive control (C3 splitter distillation column) is described by Turner et al. (1996). A feedforward neural network with static non-linearities interconnected with linear filters is identified from experimental data using least-squares NLP. 80% of the project time was associated with data pre-processing and analysis, with the final 20% includes model identification and controller implementation. The on-line optimization was performed using a gradient method.

5.2 Fuzzy and Multiple Linear Models

Fuzzy model are based on a smooth interpolation (fuzzy inference) between various pieces of data and models, often multiple local linear models valid in different operating regimes. Nonlinear MPC based on multiple linear models typically belong to one of two categories; Nakamori et al. (1992) and Chow et al. (1995) suggest to use the multiple linear model “database” to provide easy computation of a linearized model at each sample by interpolation (fuzzy inference). QP can then be applied at each sample to compute the control input. In Foss et al. (1995) the full nonlinear fuzzy model based on an interpolation of multiple local linear models is applied for on-line dynamic optimization using NLP. A somewhat different approach is taken by Foss and Qin (1997) where it is suggested to interpolate the solutions of multiple linear MPC optimizations to approximate nonlinear MPC. Identification technology for fuzzy models is well developed and considerably simpler than for neural networks since it relies on multiple linear models, at least when the operating regimes of the local linear models can be determined using process knowledge, Johansen and Foss (1997). A multiple model database can also be derived from a nonlinear first principles model.

5.3 Volterra Models

Volterra series can be viewed as natural extensions of linear FIR models to nonlinear FIR models by introducing cross-products and polynomials of the inputs up to some order. This makes them particularly interesting for nonlinear MPC, since linear MPC frequently makes use of FIR models. However, predictors based on Volterra series are inherently nonlinear with respect to the inputs and therefore requires the MPC optimization problem to be solved by NLP. Volterra models can be obtained as computationally efficient approximations to other models, or directly identified from experimental data, see Doyle III et al. (1995), Maner et al. (1996) and Genceli and Nickolaou (1995).

5.4 Hammerstein Models

Hammerstein models are composed of a static nonlinear element at the input, followed by a linear dynamic element. Many processes can be accurately modelled using such simple models, Fruzzetti et al. (1997). Such models are particularly well suited for nonlinear MPC since linear MPC can be applied directly simply by transforming the input signal by inverting the static nonlinearity.

5.5 Polynomial NARX models

Black-box identification of NARX models based on polynomial nonlinearities is a fairly well developed technology, based on structural identification to determine the significant polynomial terms. An experimental study of nonlinear MPC based on such a model of a wastewater neutralization process is described by Proll and Karim (1994). Polynomial NARX model identification was carried out using data from designed experiments with pseudo-random test signals. Although the MPC optimization problem is non-quadratic and possibly non-convex in general, the polynomial structure of the model can be applied to compute the global optimum in some very special cases, Sriniwas and Arkun (1997).

5.6 Discussion

Most linear MPC technologies are based on empirical input/output models, and the model representation, experiment design and identification methodology is often seen as an integral part of a MPC technology. The positive experience with such models in linear MPC suggests that empirical input/output models are strong candidates also for nonlinear MPC; the step from linear to nonlinear will be smaller when the model representations are similar, input/output models are typically more computationally efficient than first principles models, and empirical models can often be found at a significantly lower cost than first principles models.

When identifying empirical input/output models, the name of the game is to have complete and accurate experimental data, in particular if one attempts to find nonlinear models. This makes nonlinear black-box modelling significantly more expensive than linear black-box modelling. Integration of empirical and first principles models is therefore desired in order to make efficient use of available process data and knowledge. Some approaches are described in Psichoigos and Ungar (1992), Thompson and Kramer (1994), Tulleken (1993), Johansen and Foss (1997).

Even though an identified nonlinear model may be sufficiently accurate as a predictor, it is often observed that the Jacobian may lead to a poor approximation to the linearized dynamics of the system, e.g. Saint-Donat et al. (1991). Most nonlinear MPC algorithms rely on computation of gradients or linearizations, and therefore require an accurate Jacobian to ensure fast convergence. Special care must typically be given during identification to ensure this.

6 Constraint handling

Hard input and output constraints are handled within the general framework of MPC algorithms based on the solution of a quadratic program or a general NLP problem. However, it is well known that hard output constraints can cause problems for two reasons: the optimization may become infeasible, and some of the constraints must then be relaxed or eliminated; in the case of a feasible solution the system may become unstable due to active output constraints (de Oliveira and Biegler, 1994a). To cope with these problems and extend the constraint handling capabilities of MPC, soft constraints are introduced by adding slack variables to the inequality constraints, and then adding the slack variables as a penalty to the objective function to be minimized. de Oliveira and Biegler (1994a) compare the use of quadratic and linear penalty formulations with the use of hard constraints. It is shown that the use of a linear penalty leads to preferable stability properties compared to using quadratic penalty as well as hard constraints in the optimization. Moreover, if the hard constrained controller is stable, the linear penalty strategy requires only finite penalty parameters to match the solution of the controller with hard output constraints. This characteristic allows better control of the errors resulting from constraint relaxation using soft constraints.

Muske and Rawlings (1993a) propose an alternative approach. This approach relies on the removal of the constraints that become infeasible in the beginning of the prediction horizon.

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