ON AGGRESSIVENESS OF PI CONTROL

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Abstract: The aggressiveness of a PI controller is defined and a quantitative characterization is given in relation to the ratio of the proportional and integral actions of the controller. This concept provides simple analytic design relations for tuning PI controllers. It is illustrated by simulation results obtained with a test batch of processes representative of industrial applications and by control of a real water tank. Such results show the link between the aggressiveness of the controller and the minimum of the Integral-Time-Absolute-Error (ITAE) performance index.

Keywords: Controller design; Integral criteria; Aggressiveness of PI controller; Tuning methods

1. INTRODUCTION

PI control provides adequate performance in a vast majority of applications (Shinskey 1995). Nevertheless, despite continual advances in control theory (Åström and Hägglund 1995) there remains some interest for research on PI control. The Web of Science refers about more than 400 papers which were published on this topics since 1980. More than a hundred of rules for tuning of PI controllers can be found in the literature ; see e.g. (Seborg *et al.* 1989, Åström and Hägglund 1995, Shinskey 1995). An extensive compilation of the tuning rules has been published by (O'Dwyer 2003).

However, when applying a selected rule, control operators often have difficulties to predict the resulting controller functioning, so that tuning a PI controller requires some subsequent investigations. PI controller tuning therefore becomes rather subjective than optimal and operators a priori ask the following key questions: will the PI controller tuning produce satisfactory closed loop responses, will it fulfill the required performance specifications, or how will it prevent actuators? In some cases a partial answer on these questions is known ; Ziegler-Nichols tuning, for example, often leads to a rather oscillatory response to set–point changes (Liu and Daley 2001). However, generally there is no such easy answer.

PI tuning relations guarantying some predetermined property can be used as a possible solution to this "information embargo". For example, Åström and Hägglund suggested to tune PI controllers with a predetermined maximum sensitivity of the closed loop (Åström and Hägglund 1995), and they give tuning relations for two predetermined values of the latter. Here, another property is proposed for the design of PI con-

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trollers ; it is called "aggressiveness" of the controller since it depends on the relative amount of the two terms of the controller, with a clear intuitive interpretation. High aggressiveness means impetuous control with a highly dominant proportional action. Low aggressiveness means soft control with the leading role being left to the integral term. A particular case is the so-called "balanced tuning" where the proportional action and the integral action are equal *in the average*. Balanced tuning was proposed in (Klán and Gorez 2000), and from simulation results obtained for a batch of typical processes given in (Åström and Hägglund 1995), it provides robust control with smooth responses having no or negligibly small overshoot. Here, the aggressiveness of a PI controller in a given application is characterized by a specific coefficient determining the relative amount of the P and I actions. Then, tuning rules can be proposed, which take into account the aggressiveness desired for the PI control loop. This allows the designer to select a priori the control aggressiveness in a given application, for example with a view to actuators protection.

Concept of the aggressiveness is detailed for a first-order plus dead time (FOPDT) model because of its adequate representation of the dynamics of many overdamped processes. This typical model has the transfer function

$$G(s) = \frac{K_P}{Ts+1}\exp(-sL), \tag{1}$$

where L and T are the apparent dead time and the apparent time constant of the process, and K_P is the process gain. As pointed in (Åström and Hägglund 1995), the difficulty of controlling a given process can be characterized by the *normalized dead time* of (1), that is to say the ratio $\tau = L/(L+T)$ or $\tau = L/T_{ar}$, where $T_{ar} = L+T$ is the average residence time.

The paper is organized as follows. Section 2 introduces the characterization of the P and I control actions, and of the aggressiveness of a PI controller. Section 3 proposes some types of control based on different values of the aggressiveness and new design relations for tuning the controller with predetermined aggressiveness. It establishes some link between the controller aggressiveness and the minimization of the ITAE performance index. Section 4 gives some related results based on experiments with a real water tank. Conclusions are given in Section 5.

2. CHARACTERIZATION OF THE PI CONTROL AGGRESSIVENESS

2.1 Characterizing the actions of a PI controller

Usually, a PI control law is expressed as

$$u(t) = K\,[e(t) + \frac{1}{T_I}\int_0^t e(\tau)\;d\tau], \tag{2}$$

where u is the controller output variable, e denotes the error signal resulting from the difference between the controller setpoint and the process output, and K and T_I are the proportional gain and the integral time constant of the controller, respectively. The control law can also be expressed in the so-called velocity form (Åström and Hägglund 1995)

$$\dot{u}(t) = K\,[\dot{e}(t) + \frac{1}{T_I}e(t)]. \tag{3}$$

Clearly the well-known performance index (Seborg *et al.* 1989)

$$\text{ITAE} = \int_0^\infty t\,|e(t)|\;dt, \tag{4}$$

which is often used for obtaining well-damped closed-loop responses, can be related to the second term of (3) ; thus, it can be used to characterize the integral term of the PI controller. A similar performance index was introduced in (Klán and Gorez 2000) to characterize the proportional control action as follows :

$$\text{ITAD} = T_I\int_0^\infty t\,|\dot{e}(t)|\;dt, \tag{5}$$

with the acronym ITAD being used to mean the integral of time × the absolute value of the time derivative of the error signal. When multiplied by the common factor K/T_I the two performance indices (5) and (4) can be viewed as measures of the performance used in the P and I control actions. In (Klán and Gorez 2000) it is shown that *balanced tuning* where the ITAE performance index is minimized under the constraint of equal values of ITAE and ITAD provides well-damped closed-loop responses.

2.2 Defining and characterizing the aggressiveness of a PI controller

Fig. 1 shows the closed-loop responses to a step change of the set–point, with PI control of a given process and different settings of the controller parameters ; the values of the latter and that of

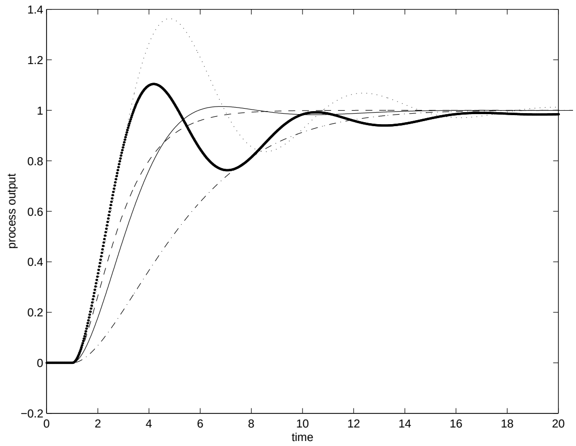


Fig. 1. Step response (broken line) and closed-loop responses of the process $G(s) = \frac{e^{-s}}{(s+1)^2}$ with PI control and different controller settings :

Ziegler–Nichols	thick line
Cohen–Coon	dotted line
Åström–Hägglund	dash–dotted lines
Balanced tuning	full line

Table 1. PI control of the test process $G(s) = \frac{e^{-s}}{(s+1)^2}$

PI settings	T_I	K	ITAE	ITAD	PIA
Ziegler–Nichols	3.84	1.22	13.50	32.70	2.4
Cohen–Coon	1.67	0.96	14.70	21.10	1.4
Balanced tuning	1.76	0.55	6.55	6.68	1.0
Åström–Hägglund	1.10	0.21	17.60	5.85	0.3

the resulting ITAE and ITAD indices are given in Table 1. It turns out that Ziegler-Nichols and Cohen-Coon controller tunings provide fast poorly damped responses. Clearly, these controllers have a high gain resulting into an "aggressive" control action, as indicated by the value of the ITAD index which is much higher than that of the ITAE index. At the opposite, the Åström–Hägglund design relations for a maximum sensitivity $M_s = 1.4$ lead to a slow overdamped response ; this controller tuning is clearly not aggressive, with a small controller gain and a value of ITAD much lower than that of ITAE. As for balanced tuning of the controller, with ITAD $\approx$ ITAE, it provides a smooth response with negligible overshoot and the lowest settling time ; besides, the closed-loop response of the process is very close to its natural step response. Furthermore, it can be observed that the design relations proposed by Ziegler-Nichols, Cohen-Coon and Åström–Hägglund lead to values of the ITAE index which are close to each other and almost twice as big as that provided by balanced tuning with minimization of the ITAE index.

These observations suggest that the aggressiveness of a PI controller can be characterized by the ratio

$$\mathrm{PIA} = \frac{\mathrm{ITAD}}{\mathrm{ITAE}}, \qquad (6)$$

where the acronym PIA means the Proportional-Integral-Aggressiveness of the PI controller. Values of PIA higher than 1.0 mean aggressive control with dominant P action resulting into fast and often underdamped responses, as shown by Fig. 1 and Tab. 1 (PIA = 2.4 with Ziegler–Nichols tuning relations, PIA = 1.4 for Cohen-Coon tuning). On the contrary, low values of PIA indicate slow and often overdamped responses, for example PIA = 0.3 with Åström–Hägglund tuning for $M_s = 1.4$. Eventually, a value of PIA close to 1.0 is obtained by balanced tuning where none of the two terms of the controller is dominant, and it provides responses with almost no overshoot. The responses from these experiments indicate a possible role of the aggressiveness in PI control. Whilst aggressively tuned PI controllers should tend towards underdamped control, the nonaggressive ones relate to overdamped control. However, there exists a compromise between them in balancing both P and I actions resulting in the $PIA = 1.0$.

The previous conclusions are valid for the given process, which is not very difficult to control ($\tau = 0.54$). In order to check if they hold for other processes, different sets of tuning relations have been used for PI control of the test batch of typical process transfer functions proposed in (Åström and Hägglund 1995). Therefore, the values of the PIA index obtained for PI control of various processes of the test batch, with different settings of the PI controllers, are plotted with respect to the process normalized dead time in Fig. 2.

Fig. 2 shows values of the PIA index obtained for different tunings of the PI controller. It turns out that except for balanced tuning, the aggressiveness of PI control decreases as the difficulty of controlling the process increases. This dependency is close to a step function for Ziegler–Nichols tuning, it is linear for Cohen–Coon tuning and exponential for both Åström–Hägglund tunings. In order to characterize a degree of aggressiveness, two groups of processes can be considered: processes which are easy to control ($\tau \leq 0.5$) and processes difficult to control ($0.5 < \tau < 0.8$).

In the first group, Ziegler-Nichols tuning is the most aggressive one, being followed by Cohen–Coon tuning and balanced tuning with Åström–Hägglund tunings. This confirms the known tendency of Ziegler–Nichols tuning to oscillatory responses to step changes of the set–point. Cohen-Coon and the other tunings are not too aggressive. Therefore their step responses are better damped.

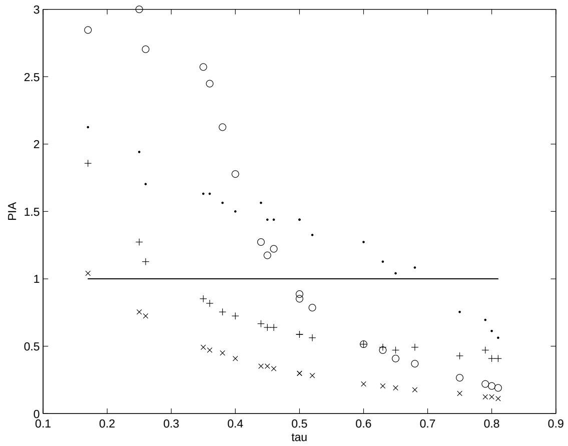


Fig. 2. PIA index as a function of the normalized dead time for the processes of the test batch and different controller settings.

Ziegler–Nichols	○
Cohen–Coon	·
Åström–Hägglund with $M_s = 2$	+
Åström–Hägglund with $M_s = 1.4$	×
Balanced tuning	broken line

The second group tunings are much more balanced. The most aggressiveness is reached with Cohen–Coon tuning and balanced tuning followed by the other tunings. Actually, the aggressiveness of the latter tunings is almost the same, since the I term is dominant in such controls ; then, the controller tends to be careful in control of such processes with $\tau > 0.5$.

The previous results indicate how the PIA index can be selected, for example in order to reduce the ITAE performance index, depending on the difficulty of controlling the process. Clearly, the tuning resulting into too big or too small PIA values are not good potential candidates for ITAE reduction. They tend towards too underdamped or overdamped control responses with high values of the ITAE performance index. The designer may choose a priori a middle-sized aggressiveness of the PI control loop, hence tuning the controller in order to reduce the ITAE performance index under the constraint of a given PIA value. A simple analytic approach for that purpose is proposed in the next section.

3. DESIGN RULES FOR PI CONTROL WITH PREDETERMINED AGGRESSIVENESS

3.1 Balanced PI control

Balanced tuning is characterized by the predetermined aggressiveness $PIA = 1.0$ or ITAD = ITAE. It provides responses close to the natural dynamics of the process to be controlled (Klán and Gorez 2000). For the three-parameter process (1) the control error after a step in the controller set–point is

$$e(t) = K_P e^{-(t-L)/T} \tag{7}$$

for $t \geq L$, with $e(t) = K_P$ for $0 \leq t < L$. The balance condition for PI control ITAE = ITAD then leads to an explicit design relation for setting the controller integral time constant as (Klán and Gorez 2004)

$$T_I = \frac{T_{ar}^2 + T^2}{2T_{ar}} = T_{ar}\,\frac{1+(1-\tau)^2}{2}, \tag{8}$$

where $T_{ar} = L + T$ and $\tau = L/T_{ar}$ are the average residence time and the normalized dead time of (1). In particular, if the controlled process reduces to a simple time lag ($L = 0$) the integral time constant of the controller will cancel that of the process, while in the case of pure time delay ($T = 0$) the relation (8) leads to $T_I = L/2$.

Another condition is needed to obtain a relation for the controller gain K. Since balanced tuning provides closed-loop step responses close to the process step response, this condition can be formulated as follows: keep the closed-loop average residence time equal to that of the process. Then, the controller gain K will be selected to ensure this condition. For any stable transfer function $F(s)$, the average residence time is given by $T_{ar} = -F'(0)/F(0)$ (Åström and Hägglund 1995). Then, for the closed-loop transfer function relating the process output to the controller set–point (Klán and Gorez 2004)

$$\frac{K_P K e^{-sL}(T_I s + 1)}{T_I s(Ts+1) + K_P K e^{-sL}(T_I s + 1)} \tag{9}$$

the average residence time is given by T_I/K_l, where $K_l = KK_P$ is the loop gain. The previous condition on the equality of the closed-loop and open-loop average residence times, $T_I/K_l = T + L$ provides to the following explicit design relation for setting the controller gain:

$$K = \frac{1}{K_P}\frac{T_I}{T+L} = \frac{1}{K_P}\frac{1+(1-\tau)^2}{2}. \tag{10}$$

Again, in the particular cases $L = 0$ and $T = 0$, the controller gain will be equal to $1/K_P$ and $0.5/K_P$, respectively. As for the mid-range case $\tau = 0.5$, that is to say equal apparent dead time and time lag, (8) and (10) yield $T_I = 1.25L$ and $K = 0.625/K_P$.

According to the natural dynamics of the three-parameter model (1), the PIA after a step in the controller set–point is

$$\mathrm{PIA} = T_I\,\frac{2}{T_{ar}[1+(1-\tau)^2]} \tag{11}$$

Since the latter term in the right–hand side of (11) depends on the process characteristics but not upon the controller tuning, the aggressiveness is related to the integral constant only. This is not surprising because of the balance condition ITAE = ITAD. In particular, PIA = T_I/T for $\tau = 0$ and PIA = $2T_I/L$ for $\tau = 1$. This agrees with equations (8) and (10) for PIA = 1.0.

3.2 General aggressiveness

In order to select a predetermined aggressiveness in control applications, one can formulate a general balance

$$\text{ITAD} = \text{PIA} \times \text{ITAE}, \qquad (12)$$

where PIA is some predetermined value. However, how to select the latter in a current control application? If PIA = 1.0, it is possible to use the relations (8) and (10) for tuning a PI controller. In order to investigate other predetermined PIA values, simulation runs have been performed on a test batch of 17 processes proposed in (Åström and Hägglund 1995), with the process normalized dead time ranging from 0.2 to 0.8. Various design rules have been used for tuning the PI controller and the value of the ITAE performance index for set–point step changes has been recorded in each case. The average of the ITAE values obtained with the given set of tuning rules for the 17 processes is given in Tab. 2. The latter shows that the lowest values of the ITAE performance index are generally obtained with the design rules predetermining aggressiveness about 1.0. An optimal value of the latter is PIA = 1.38 where the proportional performance slightly overcomes the integral one. This clearly establishes a link between a proper selection of the controller aggressiveness and the minimization of the ITAE performance index. An aggressiveness value about 1.0 provides a reasonable quality of control together with a good robustness.

Based on the predetermined aggressiveness, different types of control can be distinguished, for example

- Aggressive control PIA=4.0
- Balanced control PIA=1.0
- Soft control PIA=0.4 .

PI controllers have been tuned with a view to these three types of control, and also for very soft control (PIA=0.05), for the four following typical processes whose normalized dead time values cover the full range of the control difficulty:

Table 2. Average ITAE performance index at predetermined PIA for different settings of PI controller parameters

PIA	Tuning method	*ITAE*
1/9	Min ITAE	105
1/4	Min ITAE	83.4
1/3	Min ITAE	71.1
1/2	Min ITAE	56.1
1.0	Min ITAE	29.8
1.0	Balanced	31.7
1.04	Min ITAE (no overshoot)	25.2
1.38	Min ITAE (global)	20.0
1.5	Min ITAE	24.2
2.0	Min ITAE	24.6
2.33	Min ITAE	26.5
4.0	Min ITAE	40.2

$$\frac{1}{(1+3s)(1+0.7s)(1+0.16s)(1+0.04s)} \qquad \tau = 0.2$$

$$\frac{e^{-0.8s}}{(1+1.6s)^2} \qquad \tau = 0.4$$

$$\frac{1-s}{(1+s)^3} \qquad \tau = 0.6$$

$$\frac{1}{(1+0.13s)^{30}} \qquad \tau = 0.8\ .$$

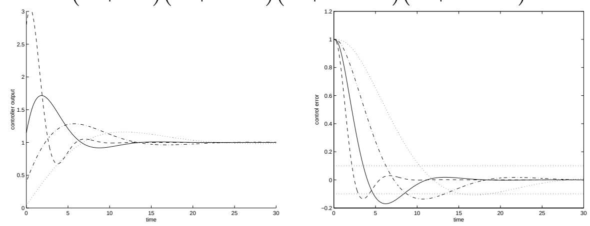
The closed-loop step responses are shown in Fig. 3. They illustrate aggressive, balanced and soft control strategies.

4. EXPERIMENTAL RESULTS

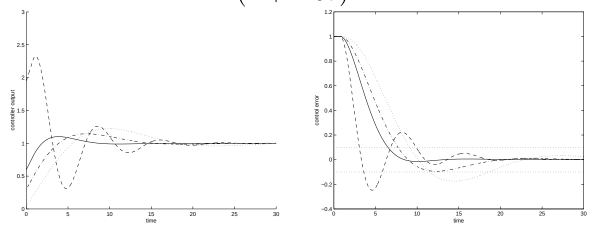
According to the previous sections the balanced tuning methodology provides good results in relation to the ITAE criterion. Therefore, it has been tested on a laboratory process including two interconnected water tanks, with a view to the control of the water level in the second tank.

From measurements on a step response record, the following values were obtained for the parameters of a FOPDT model of the process : $K_P = 10.6$, $T = 190s$, $L = 71.7s$, hence an average residence time $T_{ar} = 261.7s$ and a normalized dead time $\tau = 0.27$. Then the design relations (8) and (10) for balanced tuning provide the following values for a PI controller : $K = 0.071$, $T_I = 198.9s$. These values can be compared to that, $K = 0.225$, $T_I = 239s$, obtained by the classical Ziegler–Nichols relations ; the latter can be used for designing controllers for processes with such a low normalized dead time, but as shown by the comparison of the parameter values they will lead to a fast aggressive control. This is confirmed by the experimental results presented in Fig. 4 : the right-hand side plots show the water level in the second tank, and the left-hand side plot the

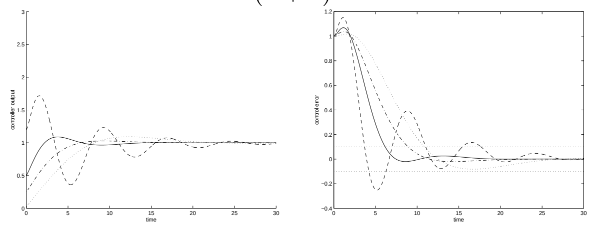
$$\frac{1}{(1+3s)(1+0.7s)(1+0.16s)(1+0.04s)}$$



$$\frac{e^{-0.8s}}{(1+1.6s)^2}$$



$$\frac{1-s}{(1+s)^3}$$



$$\frac{1}{(1+0.13s)^{30}}$$

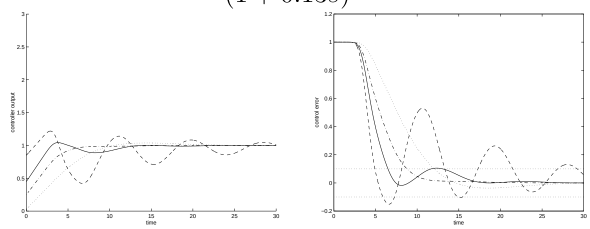


Fig. 3. Closed-loop step responses with PI controller tuned for min ITAE and predetermined aggressiveness: $PIA = 4.0$ (broken lines), $PIA = 1.0$ (full lines), $PIA = 0.4$ (dash–dotted lines), $PIA = 0.05$ (dotted lines). Left–side: controller outputs, right–side: process outputs.

actuating variable, with broken lines for Ziegler–Nichols settings and full lines for balanced tuning. These plots confirm the expectation that Ziegler–Nichols tuning is very aggressive, due to the high value of the controller gain : this aggressiveness leads to relatively fast closed-loop responses, with more than 10% overshoot. Balanced tuning provides much smoother control without any overshoot. There is also a major difference in the activity of the actuating variable, balanced tuning resulting in very few variations of the latter, hence minimizing the energy needed for control.

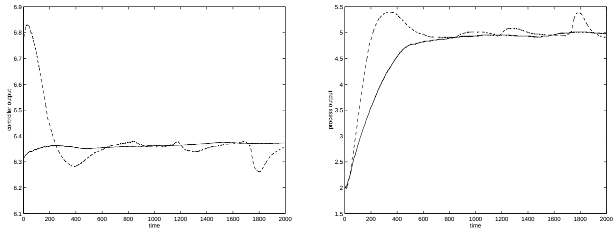


Fig. 4. Closed-loop responses for level control in a 2–tank process, with PI controller tuned via Ziegler–Nichols relations (broken lines) or for balanced tuning (full lines).

5. CONCLUSIONS

The concept of PI control aggressiveness has been introduced with a quantitative characterization in relation to the ratio of the proportional and integral actions of the controller. This new original concept allows the designer of the control system to select the aggressiveness of the control loop, in other words, to choose between soft but slow control, fast but hard control, or balanced control. In each case the designer is provided with new design rules for tuning the controller. This is illustrated by simulation results, which also establish some link with the minimization of the ITAE performance index.

REFERENCES

Åström, K.J. and T. Hägglund (1995). *PID controllers: Theory, Design and Tuning.* Instrument Society of America, Research Triangle Park. NC, U.S.A.

Klán, P. and R. Gorez (2000). Balanced tuning of PI controllers. *European Journal of Control* **6**, 541–550.

Klán, P. and R. Gorez (2004). Simple analytic rules for balanced tuning of pi controllers. In: *Control System Design: A Proceedings volume from the 2^{n}d IFAC Conference.* pp. 47–52. Elsevier LTD.

Liu, G.P. and S. Daley (2001). Optimal tuning PID control for industrial systems. *Control Engineering Practice* **9**, 1185–1194.

O'Dwyer, A. (2003). *Handbook of PI and PID Controller Tuning Rules.* Imperial College Press.

Seborg, D., T. Edgar and T. Mellichamp (1989). *Process Dynamics and Control.* John Wiley & Sons.

Shinskey, F.G. (1995). *Process Control Systems: Application, Design and Tuning.* Mc Graw–Hill.