11

Interaction

11.1 Introduction

So far we have focused on control of simple loops with one sensor, one actuator, and one controller. In practical applications, a control system can have many loops, sometimes thousands. In spite of this, a large control system can often be dealt with loop by loop since the interaction between the loops is negligible. There are, however, situations when there may be considerable interaction between different control loops. A typical case is when several streams are blended to obtain a desired mixture. In such a case it is clear that the loops interact. Other cases are control of boilers, paper machines, distillation towers, chemical reactors, heat exchangers, steam distribution networks, drive systems, and systems for air-conditioning. Processes that have many control variables and many measured variables are called multi-input multi-output (MIMO) systems. Because of the interactions it may be difficult to control such systems loop by loop.

A reasonably complete treatment of multivariable systems is far outside the scope of this book. In this chapter we will briefly discuss some issues in interacting loops that are of particular relevance for PID control. Section 11.2 gives simple examples that illustrate what may happen in interacting loops. In particular it is shown that controller parameters in one loop may have significant influence on dynamics of other loops. Bristol's relative gain array, which is a simple way to characterize the interactions, is also introduced. The problem of pairing inputs and outputs is discussed, and it is shown that the interactions may generate zeros of a multivariable system. In Section 11.3 we present a design method based on decoupling, which is a natural extension of the tuning methods for single-input single-output systems. Section 11.4 presents problems that occur in drive systems with parallel motors. The chapter ends with a summary and references.

11.2 Interaction of Simple Loops

In this section we will illustrate some effects of interaction in the simplest case

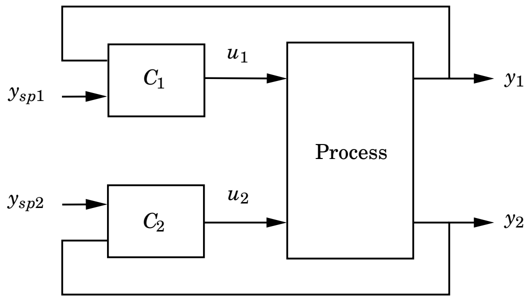

Figure 11.1 Block diagram of a system with two inputs and two outputs (TITO).

of a system with two inputs and two outputs. Such a system is called a TITO system. The system can be represented by the equations

$$\begin{aligned} Y_1(s) &= p_{11}(s)U_1(s) + p_{12}U_2(s) \\ Y_2(s) &= p_{21}(s)U_1(s) + p_{22}U_2(s), \end{aligned} \tag{11.1}$$

where $p_{ij}(s)$ is the transfer function from the j:th input to the i:th output. The transfer functions p_{11}, $p_{12}(s)$, $p_{21}(s)$, and p_{22} can be combined into the matrix

$$P(s) = \begin{pmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{pmatrix}, \tag{11.2}$$

which is called the transfer function or the matrix transfer function of the system. Some effects of interaction will be illustrated by an example.

Example 11.1—Effects of Interaction

Consider the system described by the block diagram in Figure 11.1. The system has two inputs and two outputs. There are two controllers, the controller C_1 controls the output y_1 by the input u_1 and C_2 controls the output y_2 by the input u_2. One effect of interaction is that the tuning of one loop can influence the other loop. This is illustrated in Figure 11.2, which shows a simulation of the first loop when C_1 is a PI controller and $C_2 = k_2$ is a proportional controller.

The example shows that the gain of the second loop has a significant influence on the behavior of the first loop. The response of the first loop is good when the second loop is disconnected, $k_2 = 0$, but the system becomes more sluggish when the gain of the second loop is increased. The system is unstable for $k_2 = 0.8$.

Simple analysis gives insight into what happens. In the particular case the system is described by

$$\begin{aligned} Y_1(s) &= \frac{1}{(s+1)^2}U_1(s) + \frac{2}{(s+1)^2}U_2(s) \\ Y_2(s) &= \frac{1}{(s+1)^2}U_1(s) + \frac{1}{(s+1)^2}U_2(s). \end{aligned}$$

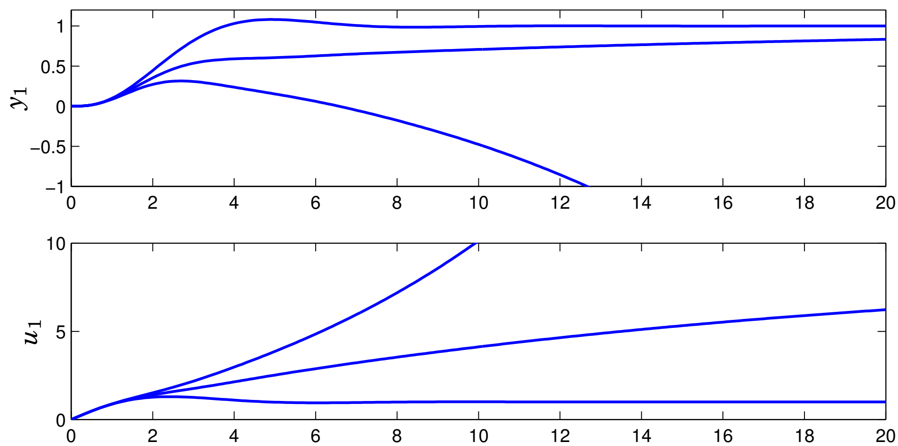

Figure 11.2 Simulation of responses to steps in set points for loop 1 of the system in Figure 11.1. Controller C_1 is a PI controller with gains $k_1 = 1$, $k_i = 1$, and the C_2 is a proportional controller with gains $k_2 = 0$, 0.8, and 1.6.

The feedback in the second loop is $U_2(s) = -k_2 Y_2(s)$. Introducing this in the second equation gives

$$U_2(s) = -\frac{k_2}{s^2 + 2s + k_2 + 1} U_1(s),$$

and insertion of this expression for $U_2(s)$ in the first equation gives

$$Y_1(s) = g_{11}^{cl}(s) U_1(s) = \frac{s^2 + 2s + 1 - k_2}{(s+1)^2(s^2 + 2s + 1 + k_2)} U_1(s).$$

This equation shows clearly that the gain k_2 in the second loop has a significant effect on the dynamics relating u_1 and y_1. The static gain is

$$g_{11}^{cl}(0) = \frac{1 - k_2}{1 + k_2}.$$

Notice that the gain decreases as k_2 increases and that the gain becomes negative for $k_2 > 1$. □

The example indicates that there is a need to have some way to determine if interactions may cause difficulties. A simple measure of interaction will now be discussed.

Bristol's Relative Gain Array

A simple way to investigate the effect of the interaction is to investigate how the static process gain of one loop is influenced by the gains in the other loops. Consider first the system with two inputs and two outputs shown in Figure 11.1. We will investigate how the static gain in the first loop is influenced by the controller in the second loop. To avoid making specific assumptions about the controller, Bristol assumed that the second loop was in perfect control, meaning that the output of the second loop is zero. It then follows from (11.1) that

$$\begin{aligned} Y_1(s) &= p_{11}(s)U_1(s) + p_{12}U_2(s) \\ 0 &= p_{21}(s)U_1(s) + p_{22}U_2(s). \end{aligned}$$

Eliminating $U_2(s)$ from the first equation gives

$$Y_1(s) = \frac{p_{11}(s)p_{22}(s) - p_{12}(s)p_{21}(s)}{p_{22}(s)} U_1(s).$$

The ratio of the static gains of loop 1 when the second loop is open and when the second loop is closed is thus

$$\lambda = \frac{p_{11}(0)p_{22}(0)}{p_{11}(0)p_{22}(0) - p_{12}(0)p_{21}(0)}. \tag{11.3}$$

Parameter λ is called *Bristol's interaction index* for TITO systems. Notice that the index refers to static conditions. In practice this can also be interpreted as interaction for low-frequency signals. There is no interaction if $p_{12}(0)p_{21}(0) = 0$, which implies that $\lambda = 1$. Small or negative values of λ indicate that there are interactions. Consider, for example, the system in Example 11.1 where the interaction index is $\lambda = -1$, which indicates that interactions pose severe difficulties.

The interaction index can be generalized to systems with many inputs and many outputs. The idea is to compare the static gains for one output when all other loops are open with the gains when all other outputs are zero. The result can be summarized in *Bristol's relative gain array* (RGA) which is defined as

$$R = P(0). * P^{-T}(0), \tag{11.4}$$

where $P(0)$ is the static gain of the system, $P^{-T}(0)$ the transpose of the inverse of $P(0)$, and $.*$ denotes component-wise multiplication of matrices. The element r_{ij} is the ratio between the open-loop and closed-loop static gains from the input signal u_j to the output y_i. It can be shown that the matrix R is symmetric and that all rows and columns sum to one. Notice that Bristol's relative gain array only captures the behavior of the process at low frequencies.

For the system (11.1) the relative gain array becomes

$$R = \begin{pmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{pmatrix}, \tag{11.5}$$

where λ is the interaction index (11.3). There is no interaction if $\lambda = 1$. This means that the second loop has no impact on the first loop and vice versa. If λ is between 0 and 1 the closed loop has higher gain than the open loop. The effect is most severe for $\lambda = 0.5$. If λ is larger than 1 the closed loop has lower gain than the open loop. When λ is negative the gain of the first loop changes sign when the second loop is closed. The effect of the interactions is thus severe.

Pairing

To control a system loop by loop we must first decide how the controllers should be connected, i.e., if y_1 in Figure 11.1 should be controlled by u_1 or u_2. This is called the *pairing problem*.

The relative gain array can be used as a guide for pairing. There is no interaction if $\lambda = 1$. If $\lambda = 0$ there is also no interaction, but the loops should be interchanged. The loops should be interchanged when $\lambda < 0.5$. If $0 < \lambda < 1$ the gain of the first loop increases when the second loop is closed, and if $\lambda > 1$ the closed-loop gain is less than the open-loop gain. Bristol recommended that pairing should be made so that the corresponding relative gains are positive and as close to one as possible. Pairing of signals with negative relative gains should be avoided. If the gains are outside the interval $0.67 < \lambda < 1.5$, decoupling can improve the control significantly. We illustrate pairing with an example.

EXAMPLE 11.2—PAIRING OF SIGNALS

Consider the system in Example 11.1. The static gain matrix is

$$P(0) = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}.$$

Its inverse is

$$P^{-1}(0) = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix},$$

and the relative gain array becomes

$$R = P(0).*P^{-T}(0) = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} .* \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix},$$

which means that $\lambda = -1$. The pairing rule says that y_1 should be paired with u_2.

When $u_1 = -k_2 y_2$ the relation between u_2 and y_1 becomes

$$Y_1(s) = g_{12}^{cl}(s) U_2(s) = \frac{2s^2 + 4s + 2 + k_2}{(s+1)^2(s^2 + 2s + 1 + k_2)} U_2(s),$$

and the static gain is

$$g_{12}^{cl}(0) = \frac{2 + k_2}{1 + k_2}.$$

The gain decreases with increasing k_2, but it is never negative for $k_2 > 0$. There is interaction but not as severe as for the pairing of y_1 with u_1. The properties of the closed-loop system are illustrated in Figure 11.3. A comparison with Figure 11.2 shows that there is a drastic reduction in the interaction when the inputs are switched. □

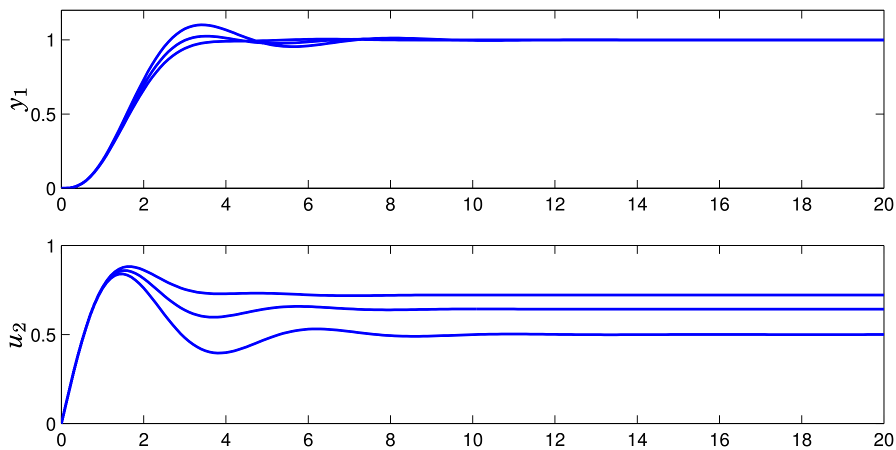

Figure 11.3 Simulation of responses to a step in the set point for y_1 of the system in Figure 11.1 when the loops are switched so that the controller for y_1 is $U_2 = C_1(s)(Y_{sp1} - Y_1)$ and the controller for y_2 is $u_1 = -k_2 y_2$ with $k_2 = 0$, 0.8, and 1.6. The controller C_1 is a PI controller with gains $k_1 = 1$, $k_i = 1$.

Multivariable Zeros

In Section 4.3 we found that right-half plane zeros imposed severe restriction on the achievable performance. For single-input single-output systems the zeros can be found by inspection. For multivariable systems zeros can, however, also be created by interaction. One definition of zeros that also works for multivariable systems is that the zeros are the poles of the inverse system. The zeros of the system (11.1) are given by

$$\det P(s) = p_{11}(s)p_{22}(s) - p_{12}(s)p_{21}(s) = 0. \tag{11.6}$$

Zeros in the right half plane are of particular interest because they impose limitations on the achievable performance. We illustrate this with an example.

EXAMPLE 11.3—ROSENBROCK'S SYSTEM
Consider a system with the transfer function

$$P(s) = \begin{pmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{pmatrix} = \begin{pmatrix} \dfrac{1}{s+1} & \dfrac{2}{s+3} \\ \dfrac{1}{s+1} & \dfrac{1}{s+1} \end{pmatrix}. \tag{11.7}$$

The dynamics of the subsystems are very benign. There are no dynamics limitations in control of any individual loop. The relative gain array is

$$R = \begin{pmatrix} 1 & 2/3 \\ 1 & 1 \end{pmatrix} .* \begin{pmatrix} 3 & -3 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix},$$

which shows that there are significant interactions. Using the rules for pairing we find that it is reasonable to pair u_1 with y_1 and u_2 with y_2. Since $\lambda > 1.5$

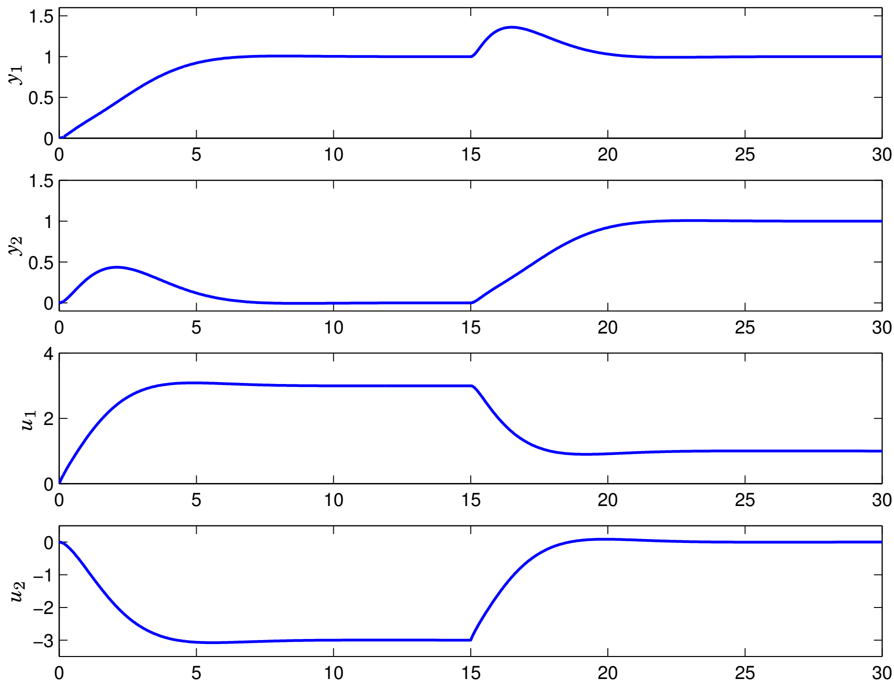

Figure 11.4 Step responses of the process (11.7) with PI control of both loops. Both PI controllers have gains $k = 2$ and $k_i = 2$. A step in y_{sp1} is first applied at time 0, and a step in y_{sp2} is then applied at time 15.

we can expect difficulties because of the interaction. It follows from (11.6) that the zeros of the system are given by

$$\det P(s) = \frac{1}{s+1}\left(\frac{1}{s+1} - \frac{2}{s+3}\right) = \frac{1-s}{(s+1)^2(s+3)} = 0.$$

There is a zero at $s = 1$ in the right half plane, and we can therefore expect difficulties when control loops are designed to have bandwidth larger than $\omega_0 = 1$.

Consider, for example, the problem of controlling the variable y_1. If the second loop is open we can achieve very fast response with a PI controller. When the second loop is closed there will, however, be severe performance limitations due to the interactions, and the control loop has to be detuned. Figure 11.4 shows responses obtained with controllers having gains $k = 2$ and $k_i = 2$ in both loops. In the figure we have first made a unit step in the set point of the first controller and then a set-point change in the second controller. The figure shows that there are considerable interactions. The system becomes unstable if the gain is increased by a factor of 3. □

Example 11.3 illustrates that an innocent-looking multivariable system may have zeros in the right half plane. The opposite is also possible, as is illustrated by the next example.

EXAMPLE 11.4—BENEFICIAL INTERACTION
Consider the system

$$P(s) = \begin{pmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{pmatrix} = \begin{pmatrix} \dfrac{s-1}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \\ \dfrac{-6}{(s+1)(s+2)} & \dfrac{s-2}{(s+1)(s+2)} \end{pmatrix}. \tag{11.8}$$

The system has the relative gain array

$$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

which indicates that y_1 should be paired with u_1 and that y_2 should be paired with u_2. The multivariable system has no zeros. We thus have the interesting situation that there are severe limitations to control either the first or the second loop individually because of the right-half plane zeros in the elements p_{11} and p_{22}. Since the multivariable system does not have any right-half plane zeros it is possible to control the multivariable system with high bandwidth. This is illustrated in Figure 11.4, where both loops are controlled with PI controllers having gains $k = 100$ and $k_i = 2000$. Notice the fast response of the system. One difficulty is, however, that the system becomes unstable if one of the loops is broken. □

11.3 Decoupling

Decoupling is a simple way to deal with the difficulties created by interactions between loops. The idea is to design a controller that reduces the effects of the interaction. Ideally, changes in one set point should only affect the corresponding process output. This can be accomplished by a precompensator that mixes the signals sent from the controller to the process inputs. The details will be given for systems with two inputs and two outputs, but the method can be applied to signals with many inputs and many outputs.

Assume that the process has the transfer function (11.2) and that $P(0)$ is nonsingular. We first introduce a static decoupler $u = D\bar{u}$, where D is a constant matrix

$$D = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}.$$

The transfer function from $\bar{u}$ to y is then given by $P(s)D$. The choice

$$D = P^{-1}(0) = \frac{1}{\det P(0)} \begin{pmatrix} p_{22}(0) & -p_{12}(0) \\ -p_{21}(0) & p_{11}(0) \end{pmatrix} \tag{11.9}$$

makes $P(0)D$ the identity matrix. The system $P(s)D$ is thus statically decoupled, and the coupling is small for low frequencies. The coupling remains small if the system is controlled by decoupled controllers, provided that the bandwidths of the control loops are sufficiently small.

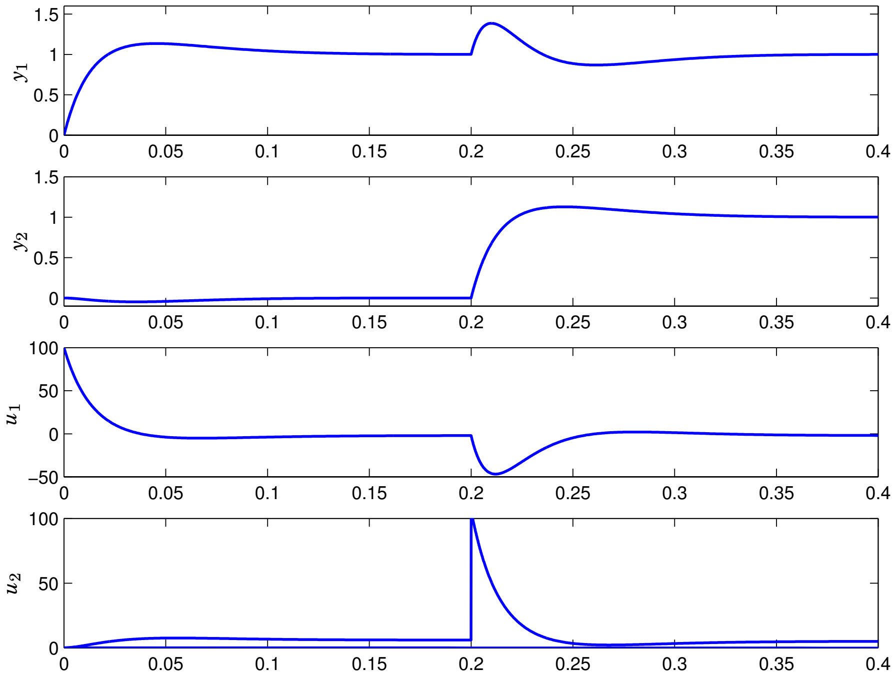

Figure 11.5 Step responses of PI control of the process (11.8) when both loops are closed. PI controllers with gains $k = 100$ and $k_i = 2000$ are used in both loops.

Assuming that the controllers are PID controllers we find that the statically decoupled controller is described by

$$\begin{pmatrix} U_1(s) \\ U_2(s) \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \begin{pmatrix} \bar{c}_1(s)Y_{sp1}(s) - c_1(s)Y_1(s) \\ \bar{c}_2(s)Y_{sp2}(s) - c_2(s)Y_2(s) \end{pmatrix},$$

where U is the control signal, Y the process output, and Y_{sp} the set point. The controllers are PID controllers with set-point weighting, hence,

$$c_i = k_{Pi} + \frac{k_{Ii}}{s} + k_{Di}s, \quad \bar{c}_i = b_i k_{Pi} + \frac{k_{Ii}}{s},$$

where b_i is the set-point weight. The set-point weights influence the interaction between the loops. Choosing $b_i = 0$ gives the smallest interaction.

The Decoupled System

The transfer function of the decoupled system is $Q(s) = P(s)D$, where

$$
\begin{aligned}
q_{11}(s) &= \frac{p_{11}(s)p_{22}(0) - p_{12}(s)p_{21}(0)}{\det P(0)}\\
q_{12}(s) &= \frac{p_{12}(s)p_{11}(0) - p_{12}(0)p_{11}(s)}{\det P(0)}\\
q_{21}(s) &= \frac{p_{21}(s)p_{22}(0) - p_{21}(0)p_{22}(s)}{\det P(0)}\\
q_{22}(s) &= \frac{p_{22}(s)p_{11}(0) - p_{21}(s)p_{12}(0)}{\det P(0)}.
\end{aligned}
$$

It follows from the construction that $Q(0)$ is the identity matrix. A Taylor series expansion of the transfer function $Q(s)$ for small $|s|$ gives

$$
Q(s) \approx \begin{pmatrix} 1 & \kappa_{12}s \\ \kappa_{21}s & 1 \end{pmatrix}
$$

for some constants κ_{12} and κ_{21}. For low frequencies ω, the diagonal elements of $Q(s)$ are equal to one, and the off-diagonal elements are proportional to s. If the bandwidth of the decentralized PID controller is sufficiently low, the off-diagonal terms will thus be small, and the system will be approximately decoupled. The closed-loop system can be described by

$$
\begin{pmatrix} 1+q_{11}c_1 & q_{12}c_2 \\ q_{21}c_1 & 1+q_{22}c_2 \end{pmatrix} Y = \begin{pmatrix} q_{11}\bar{c}_1 & q_{12}\bar{c}_2 \\ q_{21}\bar{c}_1 & q_{22}\bar{c}_2 \end{pmatrix} Y_{sp},
$$

where the dependency on s is suppressed to simplify the notation. This equation can be written as

$$
Y = \bar{H} Y_{sp},
$$

where

$$
\begin{aligned}
\bar{h}_{11} &= \frac{q_{11}\bar{c}_1(1+q_{22}c_2) - q_{12}q_{21}\bar{c}_1c_2}{(1+q_{11}c_1)(1+q_{22}c_2) - q_{12}q_{21}c_1c_2}\\
\bar{h}_{12} &= \frac{q_{12}\bar{c}_2(1+q_{22}c_2) - q_{12}q_{22}\bar{c}_2c_2}{(1+q_{11}c_1)(1+q_{22}c_2) - q_{12}q_{21}c_1c_2}\\
\bar{h}_{21} &= \frac{q_{21}\bar{c}_1(1+c_1q_{11}) - q_{11}q_{21}c_1\bar{c}_1}{(1+q_{11}c_1)(1+q_{22}c_2) - q_{12}q_{21}c_1c_2}\\
\bar{h}_{22} &= \frac{q_{22}\bar{c}_2(1+q_{11}c_1) - q_{12}q_{21}c_1\bar{c}_2}{(1+q_{11}c_1)(1+q_{22}c_2) - q_{12}q_{21}c_1c_2}.
\end{aligned}
$$

Since we designed the controllers so that the interactions are small, the term $q_{12}q_{21}$ is smaller than $q_{11}q_{22}$. The matrix $\bar{H}$ can then be approximated by

$$
\bar{H} \approx H = \begin{pmatrix} \dfrac{q_{11}\bar{c}_1}{1+q_{11}c_1} & \dfrac{q_{12}\bar{c}_2}{1+q_{11}c_1} \\ \dfrac{q_{21}\bar{c}_1}{1+q_{22}c_2} & \dfrac{q_{22}\bar{c}_2}{1+q_{22}c_2} \end{pmatrix}.
$$

The diagonal elements of H are the same as for SISO control design. The standard methods for design of PI controllers presented in Chapters 6 and 7 can be used to find the controllers c_1 and c_2. By analysing the off-diagonal elements we can estimate how severe the interactions are. The controllers may have to be detuned to make sure that the interactions are tolerable. The interaction can be reduced arbitrarily by making the control loops sufficiently slow. The interaction analysis also gives the performance loss due to the interaction. If much performance is lost it is advisable to consider other design methods.

Estimating Effects of Interaction

A simple way to estimate the effects of the interactions will now be developed. The off-diagonal elements of H are given by

$$h_{12} = \frac{q_{12}\bar{c}_2}{1+q_{11}c_1}$$
$$h_{21} = \frac{q_{21}\bar{c}_1}{1+q_{22}c_2}.$$

Notice that $q_{11}(0) = q_{22}(0) = 1$ and that $q_{12} \approx \kappa_{12}s$ and $q_{21}(s) \approx \kappa_{21}s$ for small s. Since the controllers have integral action, we have for small s

$$h_{12}(s) \approx \frac{\kappa_{12}k_{I2}s}{k_{I1}}, \qquad h_{21}(s) \approx \frac{\kappa_{21}k_{I1}s}{k_{I2}}.$$

The interaction is thus very small at low frequencies, and we can thus guarantee that the interaction is arbitrarily small by having sufficiently slow controllers. To estimate the maximum of the interaction, we observe that

$$h_{12} = q_{12}\bar{c}_2 S_1, \quad h_{21} = q_{21}\bar{c}_1 S_2,$$

where $S_1 = (1+q_{11}c_1)^{-1}$ and $S_2 = (1+q_{22}c_2)^{-1}$ are the sensitivity functions for the loops when the interaction is neglected. A crude estimate of the interaction terms is thus

$$\max_\omega |h_{12}(i\omega)| \approx |\kappa_{12}|k_{I2}M_{s1}$$
$$\max_\omega |h_{21}(i\omega)| \approx |\kappa_{21}|k_{I1}M_{s2},$$

where M_{s1} and M_{s2} are the maximum sensitivities of the individual loops and where we have also used the estimate

$$q_{12}(s) \approx \kappa_{12}s, \quad q_{21}(s) \approx \kappa_{21}s$$

and

$$\bar{c}_1 \approx k_{I1}/s, \quad \bar{c}_2 \approx k_{I2}/s.$$

The interaction can thus be captured by the interaction indices

$$\kappa_1 = |\kappa_{12}k_{I2}|M_{s1}, \quad \kappa_2 = |\kappa_{21}k_{I1}|M_{s2}. \tag{11.10}$$

The index κ_1 describes how the second loop influences the first loop, and κ_2 describes how the first loop influences the second loop. Note that the term κ_{12} depends on the system and the integral gain k_{I2} in the second loop. Interaction can thus be reduced by making the integral gains lower. The estimates are not precise because of the approximations made. They are not reliable when there is a significant difference in the bandwidths of the loops.

Examples

The design method will be illustrated by two examples. We will start by investigating Rosenbrock's system.

EXAMPLE 11.5—ROSENBROCK'S SYSTEM
Consider the system in Example 11.3 where the process has the transfer function (11.7). We have

$$D = P^{-1}(0) = \begin{pmatrix} 1 & 2/3 \\ 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & -2 \\ -3 & 3 \end{pmatrix}.$$

If we introduce static decoupling, the compensated transfer function becomes

$$Q(s) = \begin{pmatrix} \dfrac{3(1-s)}{(s+1)(s+3)} & \dfrac{4s}{(s+1)(s+3)} \\ 0 & \dfrac{1}{s+1} \end{pmatrix} \approx \begin{pmatrix} 1-7s/3 & 4s/3 \\ 0 & 1-s \end{pmatrix}.$$

The interaction is given by $\kappa_{12} = 4/3$ and $\kappa_{21} = 0$. Since $\kappa_{21} = 0$, interaction gives no performance limitations for the second loop. There are, however, limitations because of the right half-plane zero at $s = 1$. Designing a PI controller that maximizes integral gain subject to the constraints that the maximum sensitivity M_{s1} and the maximum complementary sensitivity M_{p1} are less than 1.6, gives $k_{P1} = 0.2975$ and $k_{I1} = 0.3420$.

Since $\kappa_{12} = 4/3$ there are constraints on the design of the first loop because of the coupling. Requiring that the coupling κ_1 be less than 0.5 and the maximum sensitivity M_{s2} be less than 1.6, we find that the integral gain of the second loop k_{I2} must be less than $\kappa_1/(\kappa_{12}M_{s1}M_{s2}) = 0.23$. To design a PI controller, we use a placement procedure where the fast process pole $s = -1$ is canceled. The gain in the second loop is then $k_{P2} = 0.23$.

Figure 11.6 shows the frequency responses of h_{11}, h_{12}, and h_{22}. The largest magnitude of the term h_{12} is 0.26, which is half of the estimated value. The reason for the discrepancy is that the the simple estimate $q_{12} \approx \kappa_{12}s$ overestimates the term.

Figure 11.7 shows simulations of set point responses for the closed-loop system. The solid lines show the responses for controllers with set-point weighting $b_1 = 0$ and $b_2 = 0$. The dashed line shows the responses for controllers with error feedback. The plots show the proposed design with set-point weighting ($b_1 = b_2 = 0$). A unit step in the set point of the first controller is applied at time $t = 0$, and a step in the set point of the second controller is then applied at time $t = 20$. Figure 11.7 shows the step responses for a controller without set-point weighting. The figure clearly indicates the advantage of set-point weighting for multivariable systems. The reason why there is such a large difference is that the control signal is much smoother with set-point weighting.

The effect of set-point weighting is illustrated also in Figure 11.6, which shows the frequency response of the closed-loop system with (solid) and without (dashed) set-point weighting. The interaction increases considerably when no set-point weighting is applied. □

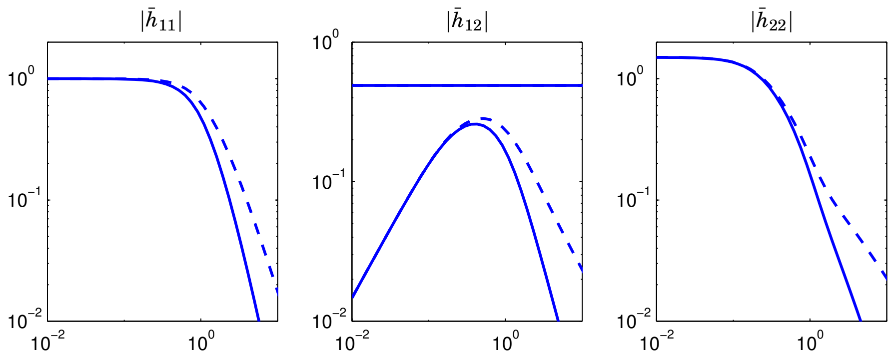

Figure 11.6 Frequency responses of the closed-loop system with set-point weighting (dashed) and without (solid). Note that without set-point weighting the interaction $|\bar{h}_{12}(i\omega)|$ is larger and extends to higher frequencies.

Distillation columns are typical industrial processes where interaction is significant. The next example deals with such a case.

EXAMPLE 11.6—THE WOOD–BERRY BINARY DISTILLATION COLUMN

The Wood–Berry binary distillation column is a multivariable system that has been studied extensively. A simple model of the system is given by the transfer function

$$P(s) = \begin{pmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21.0s+1} \\ \dfrac{6.60e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{pmatrix}.$$

Designing a static decoupler we find that

$$Q(s) = P(s)P^{-1}(0) \approx \begin{pmatrix} 1-11.7s & -12.31s \\ -0.5138s & 1-17.3s \end{pmatrix}.$$

Hence, $\kappa_{12} = -12.31$ and $\kappa_{21} = -0.5138$. Designing PI controller for the diagonal elements by maximizing integral gain subject to the robustness constraint $M_s = 1.6$ gives $k_1 = 2.3481$, $k_{i1} = 1.5378$, $k_2 = 0.5859$, and $k_{i2} = 0.2978$. The sensitivity frequencies are $\omega_{s1} = 0.30$ and $\omega_{s2} = 0.11$. Notice that the second loop is slower than the first loop. We have $\kappa_1 = 5.8$ and $\kappa_2 = 1.26$, which indicates that the interaction imposes constraints on the achievable performance and it is necessary to detune the controllers. This is illustrated by the dashed curves in the simulation shown in Figure 11.8. To reduce the interactions we will detune the controllers by decreasing the integral gains. As a first attempt we will reduce both integral gains by a factor of four. This implies that the integrated error for load disturbances is four times larger than for an uncoupled loop. Using the simple gain reduction rule developed in Section 7.9 we find that the proportional gains should then be reduced by a factor of two; see (7.27). The solid lines in Figure 11.8 show that the responses give a significant reduction of the interactions. The interaction can be reduced further at the price of lowered performance. □

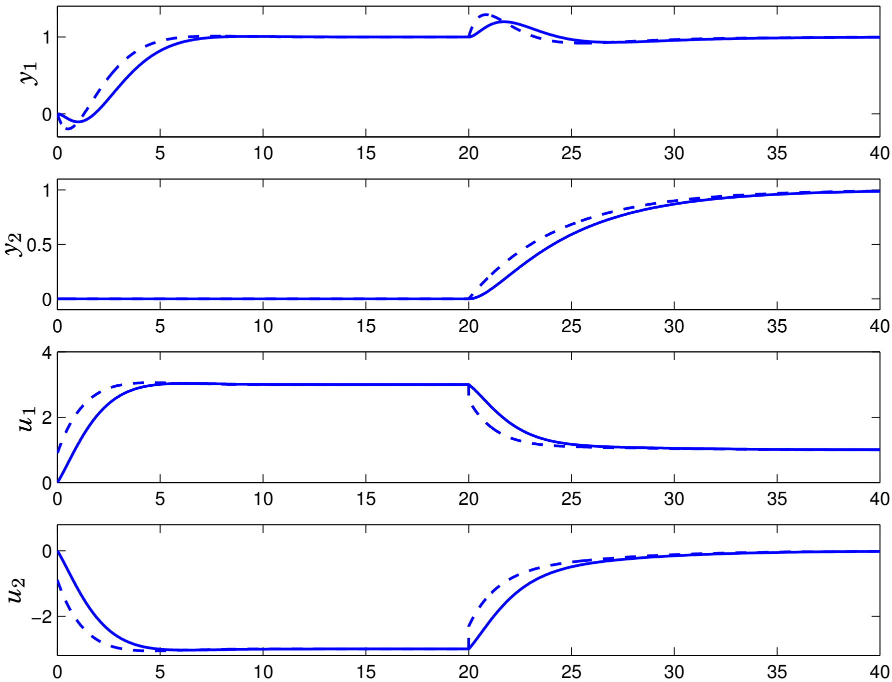

Figure 11.7 Simulation of the design method applied to Rosenbrock's system. The figure shows the response of the outputs to steps in the command signals. The PI controllers have gains $k_{P1} = 0.30$, $k_{I1} = 0.34$, $k_{P2} = 0.23$, $k_{I2} = 0.23$. The dashed lines show results with error feedback, and the solid lines show results with zero set-point weights.

11.4 Parallel Systems

Systems that are connected in parallel are quite common, particularly in drive systems. Typical examples are motors that are driving the same load, power systems, and networks for steam distribution. Control of such systems requires special consideration. To illustrate the difficulties that may arise we will consider the situation with two motors driving the same load. A schematic diagram of the system is shown in Figure 11.9.

Let ω be the angular velocity of the shaft, J the total moment of inertia, and D the damping coefficient. The system can then be described by the equation

$$J\frac{d\omega}{dt} + D\omega = M_1 + M_2 - M_L, \tag{11.11}$$

where M_1 and M_2 are the torques from the motors and M_L is the load torque.

Proportional Control

Assume each motor is provided with a proportional controller. The control

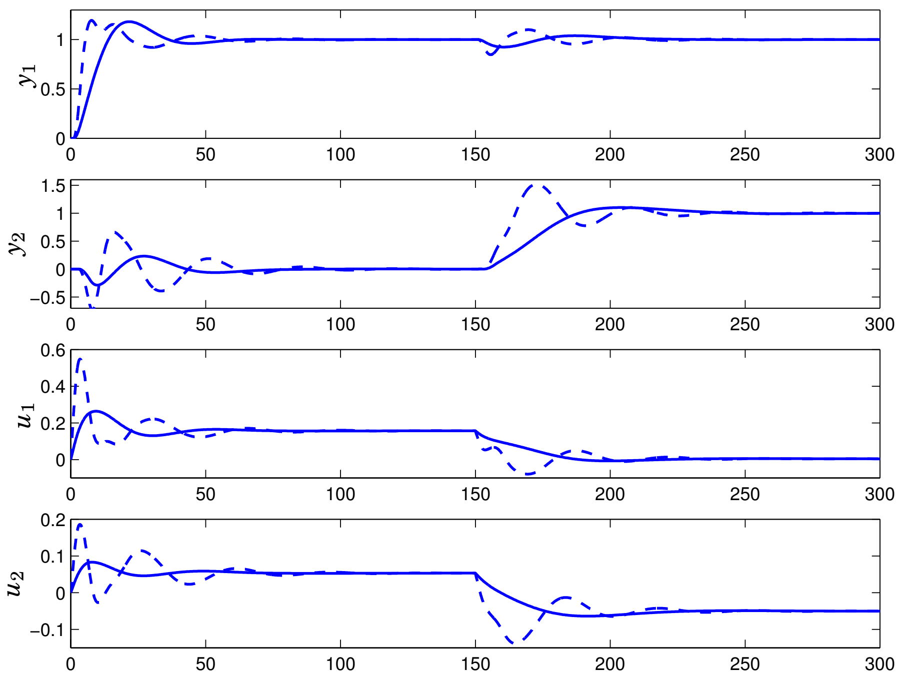

Figure 11.8 Simulation of decoupling control of Wood-Berry's distillation column. The figure shows the response of the outputs to steps in the command signals. The dashed curves show responses with PI controllers having gains $k_{P1} = 2.348$, $k_{I1} = 1.537$, $k_{P2} = 0.586$, and $k_{I2} = 0.298$. The solid lines show responses with detuned PI controllers. The gains are $k_{P1} = 1.119$, $k_{I1} = 0.384$, $k_{P2} = 0.293$, and $k_{I2} = 0.0745$. The set-point weights are zero in all cases.

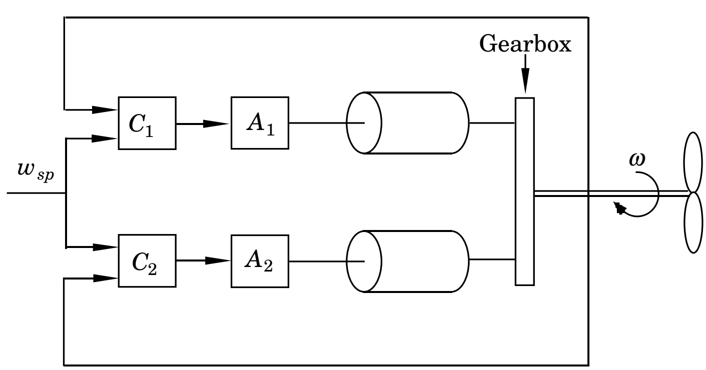

Figure 11.9 Schematic diagram of two motors that drive the same load.

strategies are then

$$\begin{aligned} M_1 &= M_{10} + K_1(\omega_{sp} - \omega) \\ M_2 &= M_{20} + K_2(\omega_{sp} - \omega). \end{aligned} \tag{11.12}$$

In these equations the parameters M_{10} and M_{20} give the torques provided by each motor when $\omega = \omega_{sp}$ and K_1 and K_2 are the controller gains. It follows from (11.11) and (11.12) that

$$J\frac{d\omega}{dt} + (D + K_1 + K_2)\omega = M_{10} + M_{20} - M_L + (K_1 + K_2)\omega_{sp}.$$

The closed-loop system is, thus, a dynamical system of first order. After perturbations, the angular velocity reaches its steady state with a time constant

$$T = \frac{J}{D + K_1 + K_2}.$$

The response speed is thus given by the sum of the damping and the controller gains. The stationary value of the angular velocity is given by

$$\omega = \omega_0 = \frac{K_1 + K_2}{D + K_1 + K_2}\omega_{sp} + \frac{M_{10} + M_{20} - M_L}{D + K_1 + K_2}.$$

This implies that there normally will be a steady-state error. Similarly, we find from (11.12) that

$$\frac{M_1 - M_{10}}{M_2 - M_{20}} = \frac{K_1}{K_2}.$$

The ratio of the controller gains will indicate how the load is shared between the motors.

Proportional and Integral Control

The standard way to eliminate a steady-state error is to introduce integral action. In Figure 11.10 we show a simulation of the system in which the motors have identical PI controllers. The set point is changed at time 0. A load disturbance in the form of a step in the load torque is introduced at time 10, and a pulse-like measurement disturbance in the second motor controller is introduced at time 20. When the measurement error occurs the balance of the torques is changed so that the first motor takes up much more of the load after the disturbance. In this particular case the second motor is actually breaking. This is highly undesirable, of course.

To understand the phenomenon we show the block diagram of the system in Figure 11.11. The figure shows that there are two parallel paths in the system that contain integration. This is a standard case where observability and controllability is lost. Expressed differently, it is not possible to change the signals M_1 and M_2 individually from the error. Since the uncontrollable state is an integrator, it does not go to zero after the disturbance. This means that the torques can take on arbitrary values after disturbance. For example, it may happen that one of the motors takes practically all the load, clearly an undesirable situation.

How to Avoid the Difficulties

Having understood the reason for the difficulty, it is easy to modify the controller as shown in Figure 11.12. In this case only one controller with integral

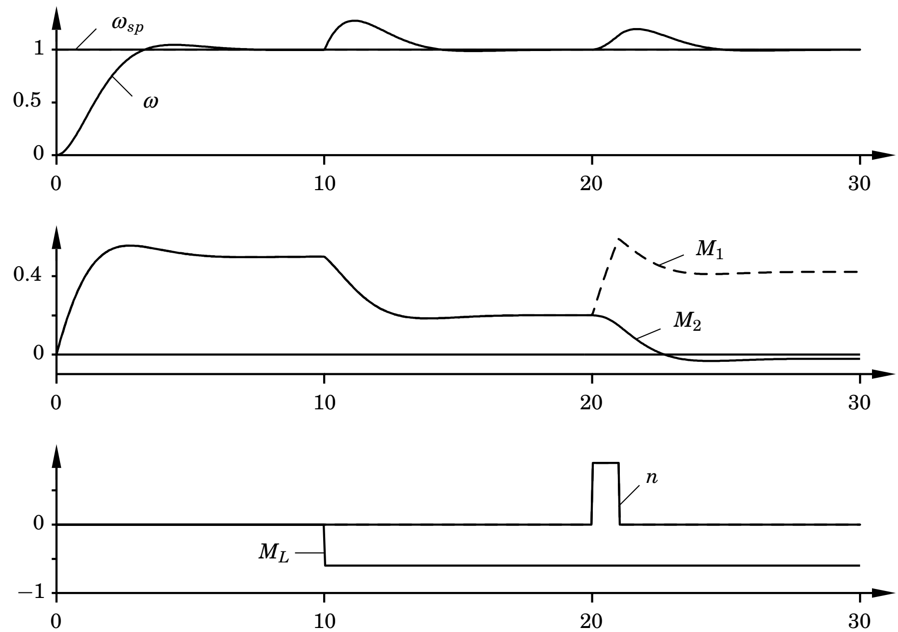

Figure 11.10 Simulation of a system with two motors with PI controllers that drive the same load. The figure shows set point ω_{sp}, process output ω, control signals M_1 and M_2, load disturbance M_L, and measurement disturbance n.

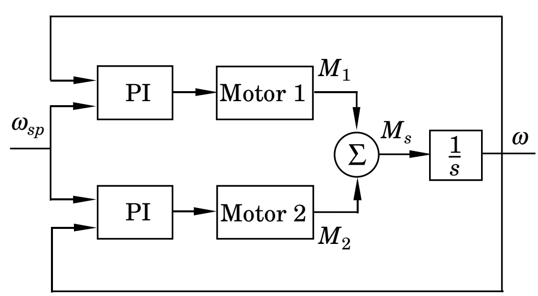

Figure 11.11 Block diagram for the system in Figure 11.10.

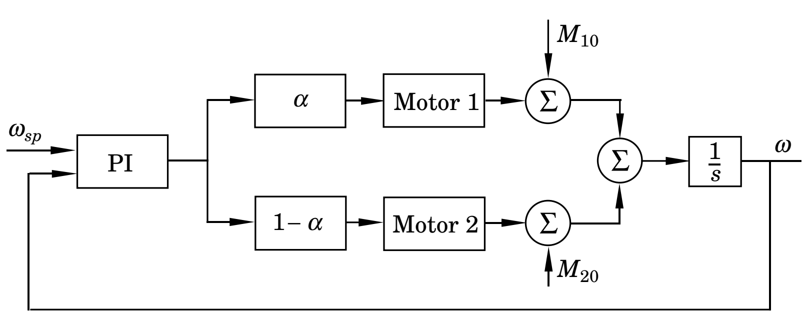

Figure 11.12 Block diagram of an improved control system.

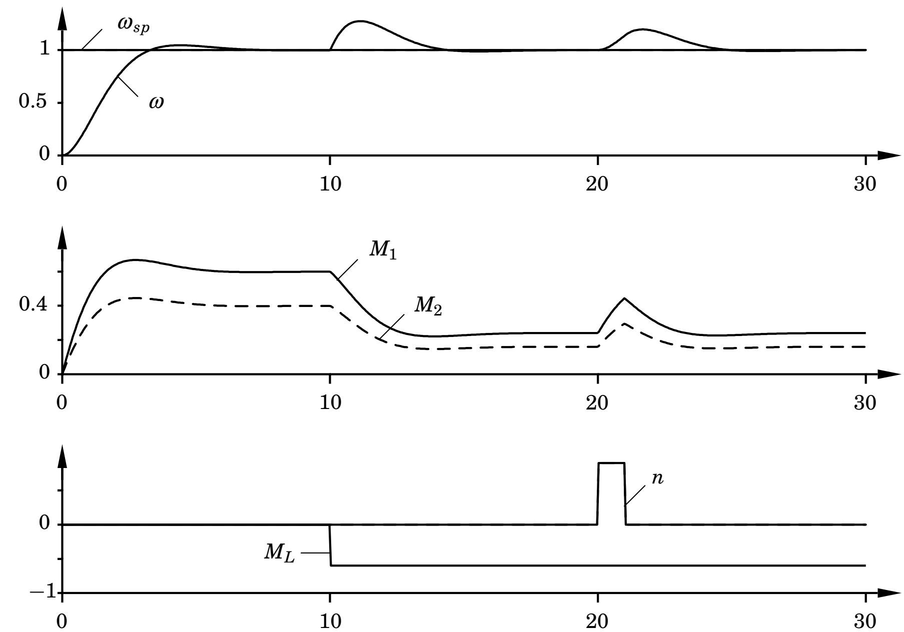

Figure 11.13 Simulation of the system with the modified controller. The figure shows set point ω_{sp}, process output ω, control signals M_1 and M_2, load disturbance M_L, and measurement disturbance n.

action is used. The output of this drives proportional controllers for each motor. A simulation of such a system is shown in Figure 11.13. The difficulties are clearly eliminated.

The difficulties shown in the examples with two motors driving the same load are accentuated even more if there are more motors. Good control in this case can be obtained by using one PI controller and distributing the outputs of this PI controller to the different motors, each of which has a proportional controller. An alternative is to provide one motor with a PI controller and let the other have proportional control. To summarize, we have found that there may be difficulties with parallel systems having integral action. The difficulties are caused by the parallel connection of integrators that produce unstable subsystems that are neither controllable nor observable. With disturbances these modes can change in an arbitrary manner. The remedy is to change the control strategies so there is only one integrator.

11.5 Summary

Even if a large control system may have many sensors and many actuators it can often be controlled by simple controllers of the PID type. This is particularly easy when there is little interaction in the system. In this chapter we have presented simple measures of interaction. They can be used to judge if the control problem can be solved using simple loops. Bristol's relative gain array can also be used to find pairs of inputs and outputs that are suitable for single-loop control. A simple design method that can be applied to systems with interaction has also been presented. This method combines static decoupling with the methods for design of single-loop controllers presented earlier in the book. Control of drive systems with parallel motors has also been discussed. For such systems there are particular problems with controllers having integral action.

11.6 Notes and References

Some fundamental issues related to interaction in systems are treated in [Rijnsdorp, 1965a; Rijnsdorp, 1965b; McAvoy, 1983]. The relative gain array was introduced in [Bristol, 1966]. It has been used widely and successfully in the process industries [Shinskey, 1981; McAvoy, 1983]. The most well-known results on the RGA are that a plant with large or negative elements in its RGA is difficult to control and that input and output variables should be paired such that the diagonal elements of the RGA are as close as possible to unity [Grosdidier *et al.*, 1985; Skogestad and Morari, 1987]. The RGA is based on the static gain of the process; an extension to account for dynamics is given in [McAvoy, 1983] . An alternative measure called the steady-state interaction indices was developed in [Chang and Davison, 1987] and it may provide a more accurate representation. Static and dynamic decoupling are treated in many textbooks in process control, e.g., [Seborg *et al.*, 2004]. Recent contributions to the design of decoupled PID control include the work by [Adusumilli *et al.*, 1998]. Detuning for multi-variable PID control, as discussed in the paper, was treated in a heuristic setting by [Niederlinski, 1971]. The particular method presented in Section 11.3 is based on [Åström *et al.*, 2002], other methods for design of non-interacting systems are given in [Yuzu *et al.*, 2002] and [Wang *et al.*, 2003]. Control of systems with strong interaction between many loops requires techniques that are very different from those discussed in this chapter; see [Cutler and Ramaker, 1980] and [Seborg *et al.*, 1986]. Multivariable systems are treated in standard textbooks on process control such as [Luyben, 1990; Marlin, 2000; Bequette, 2003; Seborg *et al.*, 2004]. There are also books that focus on multivariable systems: see [Shinskey, 1981; Skogestad and Postlethwaite, 1996].