# Double Back-Calculation Approach to Deal with Input Saturation in Cascade Control Problems

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**Abstract.** This paper presents a solution to the saturation problem in cascade control schemes. When cascade control approaches work in linear mode without saturation influence, important improvements can be achieved in industrial control loops. The effect of disturbances and/or nonlinear actuator behaviours on the main process variables can be considerably reduced. However, when saturation arises in the inner loop, these improvements cannot be reached and even sometimes the saturated cascade control scheme gives worse results than a single control loop. Thus, this work analyzes this situation and introduces an alternative solution to solve this problem and to reduce the impact of the saturation effect.

**Keywords:** Cascade control · Anti-windup · Back-calculation · PID control · Process control

## 1 Introduction

Cascade control is a very common control structure in process control [1,2]. There are two different cascade control approaches, series and parallel, but the series one is dominating in process control and is the one treated in this paper [3]. There are two major reasons for the wide use of cascade control. The first one is that it provides a fast and efficient compensation of load disturbances entering the inner loop with respect to a single control loop. The second one is that it provides an easy way to compensate for nonlinearities in the inner loop, typically nonlinear actuators [8]. However, a problem with the cascade control structure is that antiwindup is non-trivial and must be treated properly.

Integrator windup may occur in all controllers with integral action when a signal in the control loop becomes saturated. This is a well-known problem, and practically all PID controllers are equipped with antiwindup features to avoid windup when the control signal saturates. However, windup may also occur for


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other reasons, e.g. when limiters or selectors are used outside the controller or the controller block. In cascade control, windup may occur in the outer controller because of limitations in the inner control loop. These limitations may be caused by mode switches, from automatic to manual mode or from external to internal setpoint. However, the most common reason is that the control signal of the inner controller gets saturated. This is the problem treated in this paper.

When the control signal of the inner controller becomes saturated, integral action in the outer controller must be inhibited to avoid windup. This is a problem that has not been well studied in the literature and only a few contributions mentioned this issue [4–6]. Ad-hoc solutions can be found to solve the saturation problem in specific practical applications. One approach that is sometimes used in industry is to make the control signal in the outer loop tracking the process output of the inner loop when the inner loop saturates. This requires some logic information to perform this mode switch. Some other solutions are based on reference governor approaches, where the outer loop is based on MPC or a similar control algorithm that deals with constraints [7].

In this paper, we propose an alternative solution that is based on back calculation. Back calculation is one of the most common antiwindup methods used in PID controllers [2]. It has the advantages that no logical signal or mode switches are needed, and that a tracking-time constant can be set to tune the properties of the antiwindup. In the following sections, the problem is explained in more detail, the proposed solution is presented, and finally it is compared with other solutions.

## 2 Cascade Control

Figure 1 shows the classical cascade control approach that is used in this paper to analyze the problem. The control approach is composed by two control loops, the inner loop and the outer loop. The signals, controller parameters, and process transfer functions in the control scheme are represented with sub-indexes 1 and 2 to refer the outer and the inner loop, respectively. So, the process outputs are represented by $y_j$, the control signals by $u_j$ and the set-points by $r_j$; where $j = \{1, 2\}$. Notice that the control signal of the outer control loop, $u_1$ is the set-point value for the inner loop, $u_1 = r_2$. A load disturbance in the inner loop, $d$ has been also considered.

In both control loops, a PI controller is used as feedback controller that is represented by the following transfer function:

$$C_j(s) = K_j \left( \frac{sT_{ij} + 1}{T_{ij}s} \right) \tag{1}$$

where $K_j$ is the proportional gain and $T_{ij}$ is the integral time. Notice that as the paper is focused on the analysis of the saturation problem, the derivative term is omitted for the sake of simplicity.

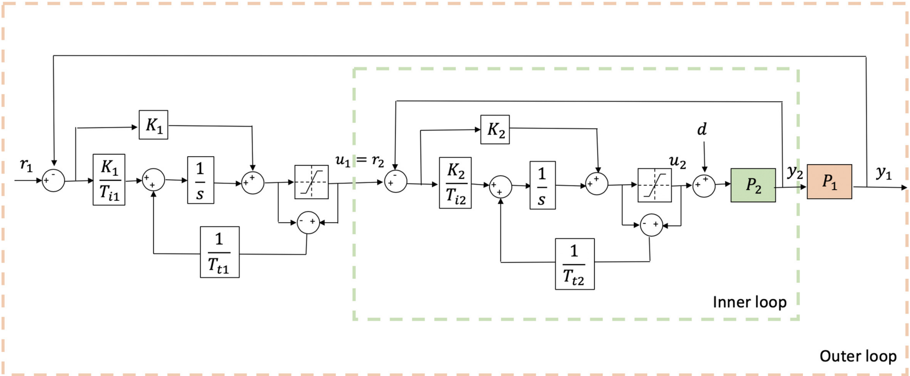


**Fig. 1.** Cascade control scheme with classical back-calculation approach.

Both PI controllers are implemented with an antiwindup scheme based on the back-calculation approach. So, the corresponding tracking constants, $T_{tj}$ are included to each PI control algorithm such as shown in Fig. 1.

The process dynamics, $P_1(s)$ and $P_2(s)$, with the following transfer functions have been considered:

$$P_1(s) = \frac{K_{p1}e^{-sL_1}}{sT_1 + 1}, \quad P_2(s) = \frac{K_{p2}e^{-sL_2}}{s} \tag{2}$$

The process dynamics in the inner loop is an integrator with delay in order to better show the effect of the saturation problem.

## 3 Saturation Problem Solutions

The saturation problem in a control loop arises when the control variable reaches the limits of the actuator. When this happens, the feedback loop is broken and the actuator will remain at its limit regardless the control error. So, when the integral action is used in both control loops within a cascade control scheme, it is necessary to have an approach to reduce the saturation effect or windup phenomena.

Such as commented above, this issue has not received so much attention in the literature. The main problem with the saturation in a cascade control architecture appears when the inner loop is saturated. In that case, the windup effect occurs in the inner loop and is propagated to the outer loop. Thus, anti-windup techniques, such as the back-calculation solution (see Fig. 1), can be included in the inner loop to reduce the saturation effect. This solution allows to reduce the saturation time in the inner loop, but the windup effect is still transmitted to the outer loop and the control performance of the main process variable is deteriorated.

The key point in this problem is that the outer loop has not information about the saturation in the inner loop. Therefore, the solution comes by somehow

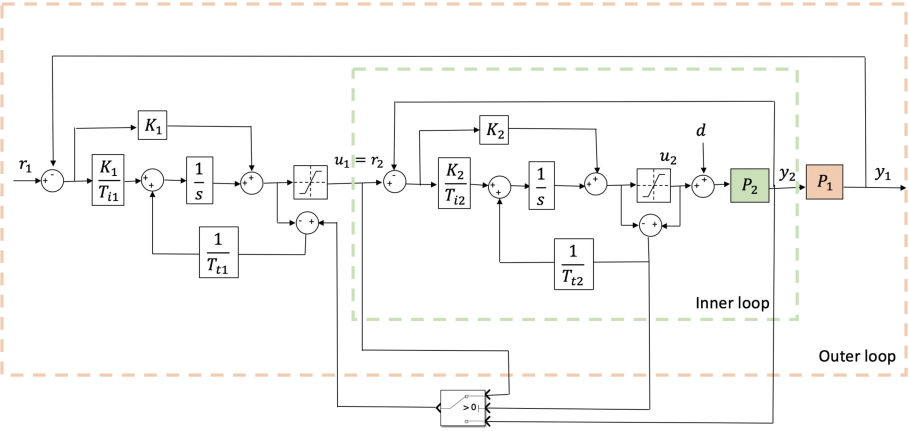


**Fig. 2.** Cascade control scheme with the industrial switching approach.

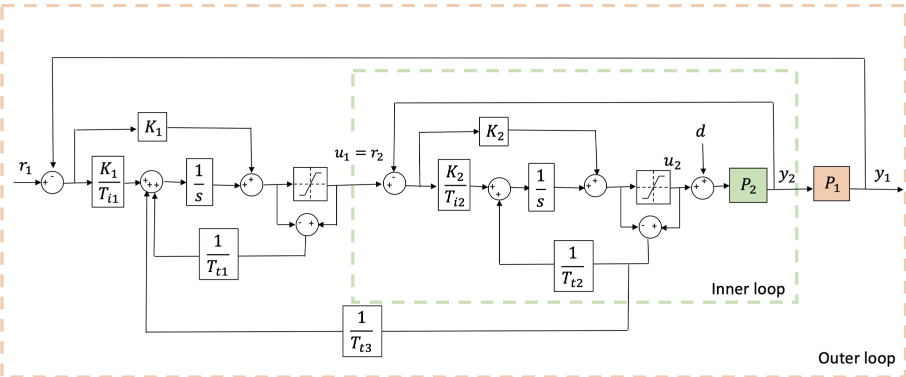


**Fig. 3.** Cascade control scheme with the proposed double back-calculation approach.

notifying the outer loop when the inner loop goes into saturation. In this sense, a common industrial approach is shown in Fig. 2. A switch mode is added to the outer loop. Now, the tracking signal in the outer loop will be switched between the control signal of the outer loop, $u_1$, and the process output of the inner loop, $y_2$. When, the inner loop is not saturated, the tracking signal will be $u_1$ to keep the anti-windup properties in the outer loop. Otherwise, when the inner loop saturates, the outer-loop tracking signal is switched to $y_2$, what will make the control error in the inner loop zero and thus reducing the saturation effect [2]. Note that this approach requires logic that takes care of the switching.

In this paper, a new solution to the problem is proposed, which is shown in Fig. 3. The idea is to avoid using any logical signal, and just to modify the classical back-calculation scheme to deal with the windup problem. Specifically, the proposed approach consists in adding a new tracking signal to the outer loop within the back-calculation control scheme. Thus, the integral term in the outer loop will be modified when any of both control signals, $u_1$ or $u_2$, are saturated. The important issue in this approach is that a new tuning parameter, $T_{t3}$, is available as an extra tracking parameter, that will allow to look for a tradeoff between the saturation time in the inner loop and the performance of the process output of the outer loop (the main process variable).

## 4 Results

This section presents a simulation study to show the saturation problem in cascade control and the analysis of the different solutions described in the previous section. For this study, the following process transfer functions are considered:

$$P_1(s) = \frac{1e^{-10s}}{100s+1}, \quad P_2(s) = \frac{2e^{-2s}}{s} \tag{3}$$

The SIMC tuning method has been selected to tune the PI controllers in both control loops [9]. So, the inner PI controller is first tuned for a closed-loop time constant of 10 s, what results in the following controller parameters: $K_1 = 0.04167$ and $T_{i1} = 48$. Once the inner loop is designed, the outer controller is tuned considering the closed-loop inner dynamics plus the dynamics of $P_2(s)$. Then, the SIMC method is used to tune the PI controller of the outer loop for a closed-loop time constant of 90 s, obtaining $K_2 = 0.9804$ and $T_{i2} = 100$ as controller parameters.

In the following sections, simulation results for the tracking and load disturbance rejection cases are evaluated for the proposed example.

### 4.1 Reference Tracking Example

The saturation problem is first excited by applying a large setpoint change in the outer loop. For this first study, the saturation limits for the control signal in the inner loop were set to $[-1\ 0.04]$, in order to show the simulation results clearly. Figure 4 shows a comparison for the cascade control scheme without antiwindup and the control approach shown in Fig. 1 where the basic back-calculation solution was considered. As observed, the saturation problem in the inner loop generates the windup effect that is propagated to the outer loop. When the back-calculation approach is used, the saturation time is considerably reduced and the performance of the process output is improved. Table 1 shows performance results using the IAE (Integral Absolute Error) for the process outputs and the saturation time for both control approaches. For the back-calculation approach, a tracking time constant $T_{t2} = 0.05$ was used and no better results were obtained for smaller values. Therefore, no further improvement can be reached with this approach.

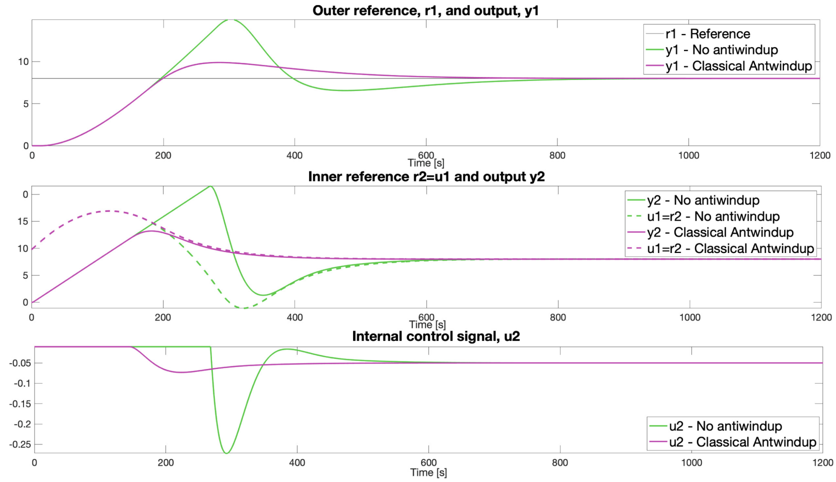


**Fig. 4.** Saturation problem for a large setpoint change in $r_1$. The control schemes without antiwindup approach and with classical back-calculation technique are shown.

**Table 1.** IAE for $y_1$ and $y_2$ and saturation time for $u_2$ for results in Fig. 4.

| | No antiwindup | Classical antiwindup |
|---|---|---|
| IAE y1 | 208.54 | 143.86 |
| IAE y2 | 289.75 | 154.10 |
| Saturation time | 268 s | 144.6 s |

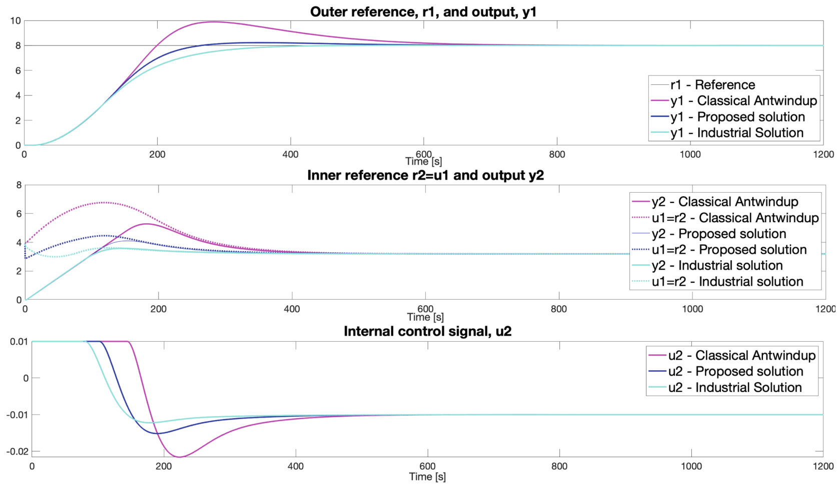


**Fig. 5.** Saturation problem for a large setpoint change in $r_1$. The control schemes with classical back-calculation, industrial solution, and the proposed approach are shown.

Then, the proposed solutions presented in Sect. 3 were simulated and compared with the classical back-calculation scheme. Figure 5 shows the graphical results and Table 2 the performance indices. It can be seen that the industrial solution (control scheme from Fig. 2) and the proposed double back-calculation approach (control scheme from Fig. 3) provide very similar results. Both solutions reduce the saturation time with respect to the classical back-calculation scheme, and the performance of the process outputs are substantially improved. For the proposed approach, the new tracking time constant was set to a value of $T_{t3} = T_{t2}/10 = 0.005$.

Notice that in the case of the industrial solution, there is no chance to modify the response as we can just apply the control scheme. However, with the approach proposed in this paper, a new degree of freedom is available through the tracking time constant $T_{t3}$. Figure 6 shows a new example for three different values of $T_{t3}$ (namely, 0.005, 0.0005, 0.00005) and where the result is compared with the industrial approach. In this figure, only the main process output $y_1$ and the inner control signal $u_2$ are displayed to show the results better. As observed, it is possible to reduce the saturation time below the industrial solution. From the obtained results, it can also be deduced that it is possible to tune $T_{t3}$ to look for a tradeoff between saturation time in the inner loop and performance of the process outputs, being this the main advantage of the new proposed approach.

**Table 2.** IAE for $y_1$ and $y_2$ and saturation time for $u_2$ for results in Fig. 5.

| | Classical Antiwindup | Industrial solution | Proposed solution |
|---|---|---|---|
| IAE y1 | 143.86 | 117.4788 | 111.86 |
| IAE y2 | 154.10 | 44.32 | 66.15 |
| Saturation time | 144.6 s | 78.5 s | 102.1 s |

### 4.2 Disturbance Rejection Example

In this case, the reference in the outer loop is kept constant and a load disturbance is entered in the inner loop. So, the saturation problem arises because of the load disturbance. In this case, the saturation limits for the control signal $u_2$ were modified to $[-2, 2]$ in order to show the results better. The same controller parameters and tracking constants as in the previous examples were used. For the proposed approach, a tracking constant of $T_{t3} = 0.005$ was considered.

Figure 7 shows the graphical simulation results, and the IAE values and the saturation times are given in Table 3. Notice how when the antiwindup scheme is not considered, the performance is considerably deteriorated because of a long saturation time in the inner loop. In this case, it is interesting to see that the classical back-calculation approach and the industrial solution give the same result. However, the proposed control scheme provides much better results reducing the saturation time and improving the performance of the process outputs.

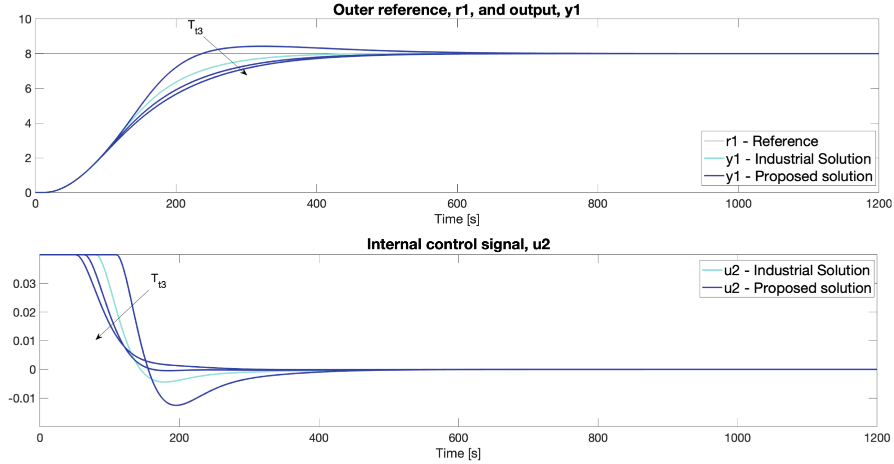


**Fig. 6.** Effect of tracking constant $T_{t3}$ for the proposed control approach.

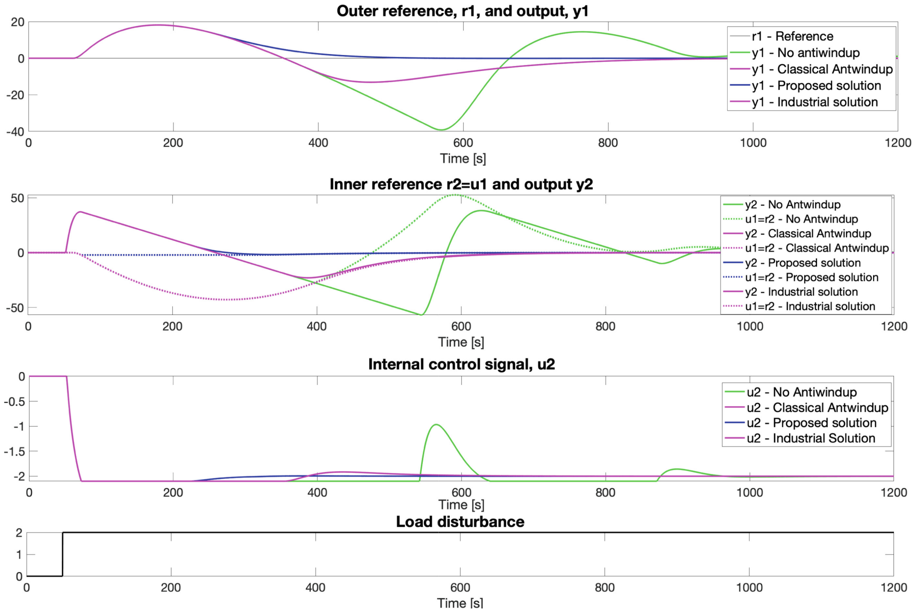


**Fig. 7.** Saturation problem for an incoming load disturbance $d$ in control signal $u_2$. The control schemes without antiwindup, with classical back-calculation technique, industrial solution, and the proposed control approach are shown.

**Table 3.** IAE and saturation time comparisons between the system with no antiwindup, classical antiwindup, industrial solution, and the new proposed solution.

| | No | Classical | Industrial | Proposed |
|---|---|---|---|---|
| IAE y1 | 1248.80 | 674.05 | 674.05 | 383.12 |
| IAE y2 | 2414.70 | 1293.70 | 1293.70 | 457.34 |
| Saturation time | 699.9 s | 281.7 s | 281.7 s | 149.9 s |

## 5 Conclusion

The saturation problem for the cascade control scheme has been evaluated for the setpoint tracking and load rejection cases. It was observed that the performance of the main process variable is considerably deteriorated when the inner loop goes into saturation when no antiwindup is applied. Then, the classical back-calculation approach was used to reduce the saturation effect, but it was demonstrated that the improvement is limited. Therefore, an industrial solution was implemented that is based on adding a switch mode in the tracking signal of the outer loop. This control approach provides the same result as the classical back-calculation scheme for the load disturbance rejection case, and it improves the response for the tracking case. However, there is no possibility to tune the desired response. Finally, a new control approach was introduced that consists in adding an extra tracking term to the outer loop. The new control algorithm obtains better results for the tracking and load disturbance cases, and also it provides a new tuning parameter that allows to a tradeoff between the saturation-time reduction and the performance of the process outputs.

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