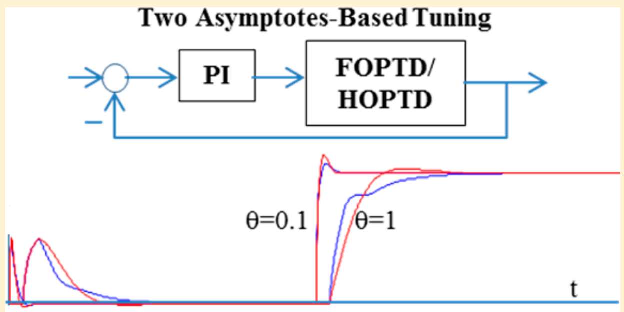
I&EC research
Industrial & Engineering Chemistry Research
Article
Cite This: Ind. Eng. Chem. Res. 2018, 57, 2905−2916
pubs.acs.org/IECR

Simple Proportional Integral Controller Tuning Rules for FOPTD and HOPTD Models Based on Matching Two Asymptotes

Jietae Lee,*[,†] Yongjeh Lee,[‡] Dae Ryook Yang,[‡] and Thomas F. Edgar[§]

[†]Department of Chemical Engineering, Kyungpook National University, Daegu 702-701, Korea
[‡]Department of Chemical and Biological Engineering, Korea University, Seoul 136-713, Korea
[§]Department of Chemical Engineering, University of Texas at Austin, Austin, Texas 78712, United States

ABSTRACT: Many methods are available to tune proportional integral (PI) controllers for first order plus time delay (FOPTD) models of overdamped processes. The two asymptotes for small and large ratios of time delays over time constants are easily calculated. These two asymptotes can be used to evaluate and provide guidelines for the performance and application ranges of PI controller tuning rules. By matching these two asymptotes, a simple analytic tuning rule is suggested. For some overdamped processes whose transfer functions have large zero terms, half-order plus time delay (HOPTD) models are found to yield better results than the FOPTD models. Applying the technique of matching two asymptotes, a simple analytic PI controller tuning rule has also been proposed for the HOPTD models. To apply these tuning rules to high order processes with known transfer functions, model reduction methods to obtain the FOPTD and HOPTD models are investigated. Simulation results for empirical and full models of processes show the performances of the proposed model reduction methods and tuning rules.



1. INTRODUCTION

Proportional integral (PI) controllers are the dominant controllers in the process industries.[1,2] Although they have only two tuning parameters, field tuning requires systematic methods. The Ziegler−Nichols method[3] is one of the earlier successful methods to tune PI controllers. It designs the PI controllers with the ultimate gain and the ultimate period, but it is criticized for providing too-oscillatory closed-loop responses. The Cohen−Coon method[4] uses the first order plus time delay (FOPTD) model of process with three parameters of the overdamped process: steady state gain, time constant, and time delay. The Cohen−Coon method aims for a closed-loop response with quarter decay ratio, which is viewed as too oscillatory.

The iterative continuous cycling method[5,6] designs a PI controller by finding the ultimate gain and the ultimate integral time of the controller successively. The guaranteed gain and phase margin method[7] yields a PI controller such that the closed-loop system meets the given gain and phase margins. When the first order plus time delay model is used, analytic tuning rules in terms of the FOPTD model parameters are obtained. Because design purposes of both methods are not limited to a certain ranges of time delays, both methods can be valid for wide ranges of time delays.

Many analytic tuning rules that use the FOPTD model such as the Cohen−Coon method[4] are available. Optimum PI controller parameters can be obtained by minimizing integral error criteria.[8] For the FOPTD models, they can be fitted by one design parameter, the ratio between time constant and time delay. PI controller tuning relationships are available for various integral error criteria, for example, the integral of the absolute error and integral of the time-weighted absolute error.[1,8] These tuning rules are valid for a certain ranges of time delays due to fitting.

The direct synthesis method[1,9] and the internal model control method[10,11] provide very simple tuning rules for the FOPTD model. The PI controller is designed for the closed-loop system to be a given transfer function with one design parameter, which is the desired closed-loop time constant. Here, the open-loop time constant is canceled by the controller zero and consequently load performances can be very sluggish[1] when the time delay is small compared to the time constant. Skogestad[12] removed the drawback of the internal model control method to tune PI controllers simply by limiting the integral time. It can be ensured by analyzing two asymptotes for small and large time delays that this modification will be effective. Various PI controller tuning rules have been filed by O'Dwyer.[13]

Here two asymptotes of PI controllers are used to evaluate existing tuning rules and to propose new tuning rules for FOPTD and half-order plus time delay (HOPTD) models. Tuning rules for two extreme cases of small and large time delays compared to the time constants can be obtained easily.

Received: September 25, 2017
Revised: January 12, 2018
Accepted: February 12, 2018
Published: February 12, 2018

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905−2916

Table 1. PI Controller Tuning Rules for Pure Delay and Integral Plus Time Delay Processes

process	method[13]	k_c	τ_I	k_I	design parameter
$\exp(-\theta s)$	Astrom−Hagglund	0.195	0.284θ	$0.69/\theta$	$\zeta_0 = 0.707$
		0.135	0.25θ	$0.54/\theta$	$\zeta_0 = 1.0$
	Hansen	0.2	0.3θ	$0.67/\theta$	
	direct synthesis	0.125	0.25θ	$0.5/\theta$	$\tau = 0, \lambda = \theta$
	SIMC[12]	0	0	$0.5/\theta$	$\tau = 0, \lambda = \theta$
$\frac{\exp(-\theta s)}{s}$	Ziegler−Nichols	$0.9/\theta$	3θ	$0.3/\theta^2$	open-loop method
	Tyreus−Luyben	$0.487/\theta$	8.75θ	$0.056/\theta^2$	
	Astrom−Hagglund	$0.35/\theta$	7θ	$0.050/\theta^2$	
	Chien−Fruehauf	$0.556/\theta$	5θ	$0.111/\theta^2$	$\lambda = 2\theta$
	O'Dwyer	$0.357/\theta$	4.3θ	$0.083/\theta^2$	$A_m = 4, \phi_m = 60°$
	Cheng−Yu	$0.524/\theta$	8θ	$0.066/\theta^2$	$A_m = 2.83, \phi_m = 46.1°$
	SIMC[12]	$0.5/\theta$	8θ	$0.063/\theta^2$	

By matching these two tuning rules, simple tuning rules are obtained here. As shown in Lee et al.,[14] the FOPTD model can have difficulty even for some overdamped processes. To overcome such drawbacks of the FOPTD model-based method, the half-order plus time delay (HOPTD) model-based method[14] is suggested. The HOPTD model-based method to tune PI controllers seems to be applicable to almost all overdamped processes robustly. Here, a simple tuning rule for the HOPTD model is also suggested.

2. MOTIVATION AND TWO ASYMPTOTES OF PI CONTROLLER TUNING RULES

Consider the FOPTD model

$$G(s) = k\frac{\exp(-\theta s)}{\tau s + 1} \tag{1}$$

Here k, τ, and θ are the process steady state gain, time constant, and time delay, respectively. Based on this model, the proportional integral (PI) controller

$$C(s) = k_c\left(1 + \frac{1}{\tau_I s}\right) = k_c + k_I\frac{1}{s} \tag{2}$$

is designed. There are many PI controller tuning rules that can be applied to FOPTD processes of eq 1.[1,2,8,13] Some rules are limited applications and some are wider applications.

One of the simplest PI controller tuning rules is[10,11]

$$k_c = \frac{\tau}{k(\lambda + \theta)}$$
$$\tau_I = \tau \tag{3}$$

which can be derived by the direct synthesis method[1] or the internal model control method[10] with the 1/0 Pade approximation of time delay. Here λ is the design parameter representing the closed-loop time constant and its default value is θ. For a large time delay compared to the time constant, its closed-loop performances are excellent. However, as the time delay decreases, its load performances become worse.[1,2] Skogestad[12] has removed this drawback simply by limiting the integral time as

$$\tau_I = \min(\tau, 4(\lambda + \theta)) \tag{4}$$

The closed-loop performances for a small θ are improved considerably. This simple modification of eq 4 provides two asymptotes of $\tau_I = \tau$ for a large θ and $\tau_I = 4(\lambda + \theta)$ for a small θ. Here these two asymptotes are studied to evaluate and design PI controller tuning rules.

When $\theta \gg \tau$, PI controller should be designed for the closed-loop responses to be slow. Hence the process can be approximated as

$$G(s) \approx k\exp(-\theta s) = k\exp(-\tilde{s}), \qquad \tilde{s} = \theta s \tag{5}$$

Tuning rules for this process are given in Table 1, and some closed-loop performances are shown in Figure 1. All tuning rules in Table 1 for the process of eq 5 are acceptable.

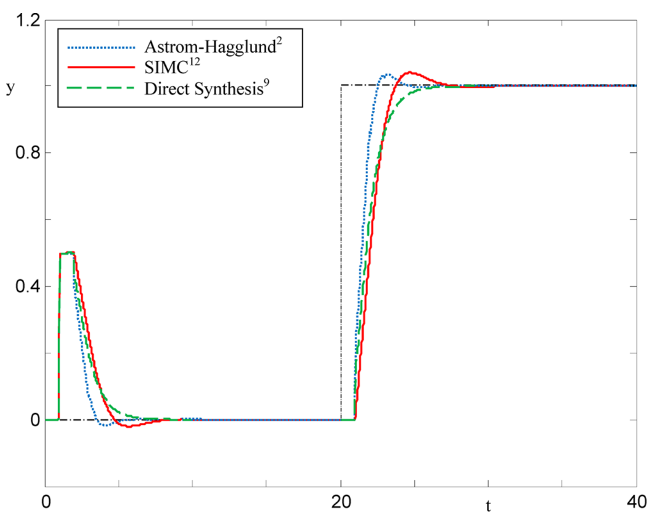


Figure 1. Closed-loop responses of PI controllers for the pure delay process, $G_p(s) = \exp(-s)$.

When $\theta \ll \tau$, PI controller can be designed for the closed-loop responses to be fast. Hence the working frequencies are high and the process can be approximated as

$$G(s) \approx k\frac{\exp(-\theta s)}{\tau s} = \tilde{k}\frac{\exp(-\tilde{s})}{\tilde{s}}, \qquad \tilde{k} = \frac{k\theta}{\tau} \tag{6}$$

Tuning rules for this process are given in Table 1, and some closed-loop performances are shown in Figure 2. Except for the Ziegler−Nichols open-loop method, all tuning rules in Table 1 for the process of eq 6 are acceptable. Here a set-point filter to reduce the overshoot for step set-point changes will be required.

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905−2916

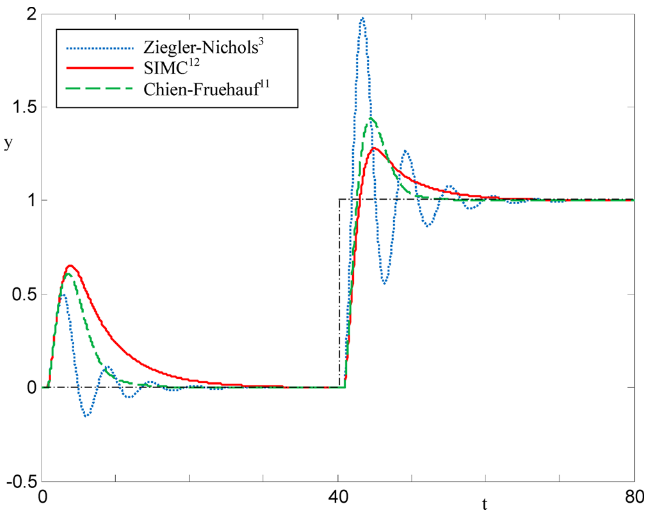


Figure 2. Closed-loop responses of PI controllers for the integral plus time delay process, $G_p(s) = \exp(-s)/s$.

Tuning rules in Table 1 are the desired two asymptotes for small and large time delays over the time constants. Here these two asymptotes are used to evaluate existing tuning rules and to propose new tuning rules by matching them.

3. ASYMPTOTE COMPARISONS

3.1. Ziegler–Nichols (Closed Loop).[3] From the ultimate gain k_u and the ultimate period p_u, Ziegler–Nichols[3] proposed the tuning rule of $k_c = 0.45k_u$ and $\tau_I = p_u/1.2$. Now it is one of the base tuning rules for comparisons.

The ultimate gain and period for $G(s)$ of eq 5 are

$$k_u = 1/k$$
$$p_u = 2\theta \tag{7}$$

and for $G(s)$ of eq 6 are

$$k_u = \frac{\pi}{2\tilde{k}} = \frac{\pi\tau}{2k\theta}$$
$$p_u = 4\theta \tag{8}$$

Hence two asymptotes for the Ziegler–Nichols method[3] are as shown in Table 2. Considering tuning rules in Table 1, τ_I for $\theta \gg \tau$ is too high and closed-loop responses will be sluggish for delay-dominant processes. On the other hand, τ_I for $\theta \ll \tau$ is somewhat small and closed-loop responses will be oscillatory for lag-dominant processes. These are well-known drawbacks of the Ziegler–Nichols method.[2,15] The Ziegler–Nichols method should be used for the moderate range of the time delays compared to the time constants.

3.2. Cohen–Coon.[4] The tuning rule for the Cohen–Coon method[4] is

$$kk_c = \frac{1}{\theta}\left(0.9\tau + \frac{\theta}{12}\right)$$
$$\tau_I = \theta\frac{30\tau + 3\theta}{9\tau + 20\theta} \tag{9}$$

The two asymptotes for this tuning rule are given in Table 2. Considering tuning rules in Table 1, the controller integral time τ_I's for both asymptotes are somewhat low. Hence closed-loop responses will be oscillatory for both delay- and lag-dominant processes. Actually the Cohen–Coon method will provide PI controllers that meet the design concept of quarter decay ratio, which is viewed as too oscillatory.

3.3. ITAE.[1] PI controllers that minimize some error criteria can be designed. The integral of time-multiplied absolute errors (ITAE) method is popular due to its conservative performances.[1] The tuning rule for the set-point change is

$$kk_c = 0.586(\theta/\tau)^{-0.916} \approx 0.586\frac{\tau}{\theta}$$
$$\tau_I = \frac{\tau}{1.030 - 0.165\theta/\tau} \tag{10}$$

Considering tuning rules in Table 1, for a small θ, this tuning rule can be too sluggish due to too-large τ_I. For a large θ, it cannot be used due to negative τ_I.

The tuning rule for the load change is

$$kk_c = 0.859(\theta/\tau)^{-0.977} \approx 0.859\frac{\tau}{\theta}$$
$$\tau_I = \frac{\tau}{0.674(\theta/\tau)^{-0.680}} \tag{11}$$

It shows quite different asymptotes compared to those in Table 1 and cannot be used for both small and large θ's. Drawbacks of these tuning rules are due to the fitting limitations of simpler equations of eqs 10 and 11.

3.4. Iterative Continuous Cycling.[5,6] The characteristic equation for the PI control system stability is

$$1 + \left(k_c + \frac{k_I}{s}\right)\frac{k\exp(-\theta s)}{\tau s + 1} = 0 \tag{12}$$

The iterative continuous cycling method designs k_c first with $k_I = 0$. Find the ultimate gain k_{cu} and let

$$k_c = k_{cu}/\xi \tag{13}$$

Then, under k_c, find the ultimate integral gain k_{Iu} and let

$$k_I = k_{Iu}/\xi \tag{14}$$

The gain margin ξ can be different for k_c and k_I.

With the 1/0 Pade approximation of $\exp(-\theta s) = -\theta s + 1$, the analytic tuning rule can be obtained as

$$kk_c = \frac{\tau}{\xi\theta}$$
$$kk_I = \frac{1}{\xi\theta} + \frac{\tau}{\xi^2\theta^2} \tag{15}$$

For the 1/1 Pade approximation of $\exp(-\theta s) = (-\theta s/2 + 1)/(\theta s/2 + 1)$, the analytic tuning rule can be obtained as ($\xi = 4$ is used)

$$kk_c = \frac{\tau}{2\theta} + 0.25$$
$$kk_I = \frac{3(2\tau + \theta)(2\tau + 5\theta)}{8\theta^2(14\tau + 3\theta)} \tag{16}$$

For the tuning rule of eq 15, with a constant gain margin ξ, it is impossible for both asymptotes to be similar to those in Table 1. The gain margin of $\xi = 2$ for delay-dominant processes and $\xi = 4$ for lag-dominant processes will be useful. The tuning rule of eq 16 has both asymptotes similar to those in Table 1. It can be used for the whole range of time delays.

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905−2916

Table 2. Asymptotes of Various PI Controller Tuning Rules for FOPTD Processes, $G_p(s) = \frac{k\exp(-\theta s)}{\tau s + 1}$

		asymptotes				
		$\theta/\tau \to \infty$		$\theta/\tau \to 0$		
method	tuning rules	kk_c	τ_I (kk_I)	kk_c	τ_I	design parameter
Ziegler–Nichols (closed loop)[3]	$k_c = 0.45k_u$	0.45	1.67θ ($0.27/\theta$)	$0.71\tau/\theta$	3.33θ	
	$\tau_I = p_u/1.2$					
Cohen–Coon[4]	eq 9	0.083	0.15θ ($0.55/\theta$)	$0.9\tau/\theta$	3.33θ	
ITAE[1]	eq 10	$\sim 0.6\tau/\theta$	negative	$\sim 0.6\tau/\theta$	$\sim\tau$	set point
iterative continuous cycling[5]	$kk_c = \frac{\tau}{\xi\theta}$	$\tau/\xi\theta$	τ ($1/\xi\theta$)	$\tau/\xi\theta$	$\xi\theta$	$e^{-\theta s} \approx -\theta s + 1$
	$\tau_I = \left(\frac{1}{\xi\theta} + \frac{1}{\tau}\right)^{-1}$					
	$kk_c = \frac{\tau}{2\theta} + 0.25$	0.25	0.4θ ($0.63/\theta$)	$0.5\tau/\theta$	4.67θ	$e^{-\theta s} \approx \frac{-\theta s/2 + 1}{\theta s/2 + 1}$
	$\tau_I = \theta\frac{28\tau + 6\theta}{6\tau + 15\theta}$					$\xi = 4$
guaranteed gain and phase margins[7]	$kk_c = \frac{\tau}{2.04\theta}$	$0.49\tau/\theta$	τ ($0.49/\theta$)	$0.49\tau/\theta$	5.43θ	$A_m = 3$
	$\tau_I = \left(\frac{1}{5.43\theta} + \frac{1}{\tau}\right)^{-1}$					$\phi_m = \pi/4$ (45°)
	$kk_c = \frac{\tau}{1.99\theta}$	$0.52\tau/\theta$	τ ($0.50/\theta$)	$0.52\tau/\theta$	7.97θ	$A_m = 3$
	$\tau_I = \left(\frac{1}{7.97\theta} + \frac{1}{\tau}\right)^{-1}$					$\phi_m = \pi/3.6$ (50°)
direct synthesis with Taylor series matching[9]	$kk_c = \frac{\tau}{2\theta} + \frac{1}{8}$	0.125	0.25θ ($0.5/\theta$)	$0.5\tau/\theta$	τ	$\lambda = \theta$
	$\tau_I = \tau + 0.25\theta$					
Skogestad internal model control[12,15]	$kk_c = \frac{\tau}{\lambda + \theta}$	$0.5\tau/\theta$	τ ($0.5/\theta$)	$0.5\tau/\theta$	8θ	$\lambda = \theta$
	$\tau_I = \min(\tau, 4(\lambda + \theta))$					
	$kk_c = \frac{\tau + \frac{\theta}{3}}{\lambda + \theta}$	0.167	0.33θ ($0.5/\theta$)	$0.5\tau/\theta$	8θ	$\lambda = \theta$[a]
	$\tau_I = \min\left(\tau + \frac{\theta}{3}, 4(\lambda + \theta)\right)$					
AMIGO[16]	eq 22	0.15	0.3θ ($0.5/\theta$)	$0.35\tau/\theta$	7θ	
	$kk_c = 0.14 + 0.28\frac{\tau}{\theta}$	0.14	0.33θ ($0.42/\theta$)	$0.28\tau/\theta$	7.13θ	pure FOPTD process
	$\tau_I = 0.33\theta + \frac{6.8\tau\theta}{10\theta + \tau}$					

[a]Modified by Grimholt and Skogestad.[15]

3.5. Guaranteed Gain and Phase Margins.[7] Ho et al.[7] proposed PI controller tuning rules based on the given gain and phase margins. They obtained analytic rules with a linear approximation of arctangent function in phase computations as

$$kk_c = \frac{\omega_p \tau}{A_m}$$
$$\tau_I = \left(\omega_p - \frac{4\theta}{\pi}{\omega_p}^2 + \frac{1}{\tau}\right)^{-1} \quad (17)$$
$$\omega_p = \frac{A_m\phi_m + 0.5\pi A_m(A_m - 1)}{({A_m}^2 - 1)\theta}$$

Here A_m and ϕ_m are the design parameters of gain and phase margins, respectively. Typical tuning rules are given in Table 2. Two asymptotes are very similar to those in Table 1, and excellent closed-loop performances regardless of time delays will be obtained.

3.6. Direct Synthesis with Taylor Series Matching.[9] The close- loop transfer function for the process of eq 1 with the controller of eq 2 is

$$G_{cl}(s) = \frac{G(s)\,C(s)}{1 + G(s)\,C(s)} \quad (18)$$

With specifying the closed-loop transfer function as

$$G_{cl}(s) = \frac{\exp(-\theta s)}{\lambda s + 1} \quad (19)$$

the controller $C(s)$ can be obtained as

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905–2916

$$C(s) = G_p(s)^{-1}\frac{G_{cl}(s)}{1 - G_{cl}(s)} = \frac{\tau s + 1}{k}\frac{1}{\lambda s + 1 - \exp(-\theta s)} \tag{20}$$

Various tuning rules according to approximations of the time delay $\exp(-\theta s)$ can be obtained. For example, with the 1/0 Pade approximation of $\exp(-\theta s) = -\theta s + 1$, the tuning rule of eq 3 is derived.

With truncating the Taylor series of eq 20, Lee et al.[9] obtained a PI controller tuning rule

$$\begin{aligned} kk_c &= \frac{\tau}{\lambda + \theta} + \frac{\theta^2}{2(\lambda + \theta)^2} \\ \tau_I &= \tau + \frac{\theta^2}{2(\lambda + \theta)} \end{aligned} \tag{21}$$

Its asymptotes are shown in Table 2 ($\lambda = \theta$ is used). The asymptotes for $\theta \gg \tau$ are similar to those in Table 1. On the other hand, the asymptotes for $\theta \ll \tau$ are quite different from those in Table 1. Actually, when $\theta \ll \tau$, the load responses are very sluggish and some modifications should be applied, for example, introducing the integral process approximation as in Seborg et al.[1] Other tuning rules based on the direct synthesis method and internal model control method also suffer from this drawback.

3.7. Skogestad Internal Model Control (SIMC).[12] Skogestad[12] has removed the drawback of the direct synthesis method and the internal model control method simply by limiting the integral time as in eq 4. This simple modification provides two asymptotes of $\tau_I = \tau$ for a large θ and $\tau_I = 4(\lambda + \theta)$ for a small θ as in Table 2, which are very similar to those in Table 1. Excellent closed-loop performances are obtained for the whole range of time delays. Grimholt and Skogestad[15] proposed a tuning rule slightly improved for a large delay.

3.8. AMIGO.[16] Hagglund and Astrom[16] proposed tuning rules based on the Ms-constrained integral gain optimization (MIGO). They maximize the integral gain k_c under the constraint of peak amplitude ratio of sensitivity function (Ms). One of the approximate explicit tuning rules (AMIGO) is given as

$$\begin{aligned} kk_c &= \begin{cases} 0.35\tau/\theta - 0.6, & \theta < \tau/6 \\ 0.25\tau/\theta, & \tau/6 < \theta < \tau \\ 0.1\tau/\theta + 0.15, & \tau < \theta \end{cases} \\ \tau_I &= \begin{cases} 7\theta, & \theta < 0.11\tau \\ 0.8\theta, & 0.11\tau < \theta < \tau \\ 0.3\theta + 0.5\tau, & \tau < \theta \end{cases} \end{aligned} \tag{22}$$

For FOPTD processes, the tuning rule is further simplified as shown in Table 2. Its two asymptotes are similar to those in Table 1, showing that the tuning rule can be used for the whole range of delays. Tuning rules for PID controllers based on this concept[17] are also available.

4. NEW TUNING RULE FOR THE FIRST ORDER PLUS TIME DELAY (FOPTD) MODEL

The SIMC rule can be interpreted as (the default design parameter of $\lambda = \theta$ is used)

$$\tau_I = \min(\tau, 8\theta) \approx \left(\frac{1}{\tau^p} + \frac{1}{(8\theta)^p}\right)^{-1/p} \tag{23}$$

When p goes to infinity, the approximation of eq 23 is exact.[18] For $p \approx 8$, both terms are nearly the same. The approximation of eq 23 can be interpreted as the matching of two asymptotes of $\tau_I = \tau$ and $\tau_I = 8\theta$. By adjusting p, the bad effect of switching near $\tau = 8\theta$ may also be mitigated. When $p = 1$, the structure of eq 23 is equivalent to those of the guaranteed gain and phase margins method as in Table 2. Utilizing this matching technique, a new tuning rule is developed.

Here a two degree of freedom (2DOF) controller[1,2] as in Figure 3 is considered. The usual PI controller cannot

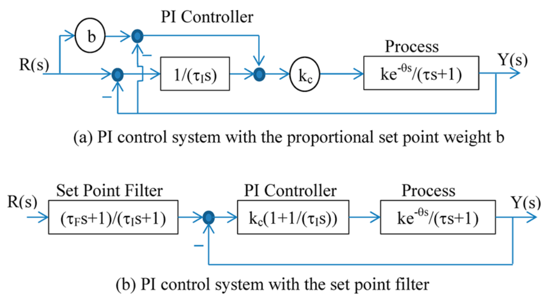


Figure 3. Equivalent 2DOF (two degree of freedom) PI control systems ($\tau_F = b\tau_I$).

guarantee closed-loop performances for both set-point change and load change in the input. Especially, for lag-dominant processes, closed-loop performances for the set-point changes are quite different from those for the load changes in the input.

For delay-dominant processes ($\theta \gg \tau$), a tuning rule based on the internal model control method is effective and used here. The set-point filter is not required. The tuning rule is

$$\begin{aligned} kk_c &= \frac{\tau}{2\theta} \\ \tau_I &= \tau \\ \tau_{sp} &= \tau \end{aligned} \tag{24}$$

For lag-dominant processes ($\theta \ll \tau$), a new tuning rule based on the dominant pole method[2] is considered:

$$\begin{aligned} kk_c &= \frac{\tau}{2\theta} \\ \tau_I &= 4.310\theta \\ \tau_{sp} &= 1.720\theta \end{aligned} \tag{25}$$

It is derived by specifying that a pole of closed-loop system is at $s = -\alpha(1 + j)$. The PI controller parameters are calculated from[2]

$$1 + \left(k_c + \frac{k_I}{s}\right)\frac{k\exp(-\theta s)}{\tau s}\Bigg|_{s=-\alpha(1+j)} = 0 \tag{26}$$

The pole location α is selected to be $1/(2.428\theta)$ so that the proportional gain is $kk_c = \tau/(2\theta)$. The zero time constant τ_{sp} of the set-point filter is set to be $1/|-\alpha(1 + j)|$, which is similar to that suggested in Astrom and Hagglund.[2]

Tuning rules of eqs 24 and 25 are combined as

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905−2916

$$kk_c = \frac{\tau}{2\theta}$$

$$\tau_I = \left(\frac{1}{\tau^2} + \frac{1}{(4.31\theta)^2}\right)^{-1/2} = \tau\frac{1}{\sqrt{1 + \left(\frac{\tau}{4.31\theta}\right)^2}} \quad (27)$$

$$\tau_{sp} = \left(\frac{1}{\tau^2} + \frac{1}{(1.72\theta)^2}\right)^{-1/2} = \tau\frac{1}{\sqrt{1 + \left(\frac{\tau}{1.72\theta}\right)^2}}$$

Here, based on simulation results, $p = 2$ for the matching of two asymptotes is selected. The tuning rule of eq 27 has asymptotes of eqs 24 and 25, which are similar to those in Table 1, and can be used for the whole range of time delays.

Figure 4 shows closed-loop responses for load changes in the process input and step set-point changes. Responses are compared with the modified SIMC tuning rule.[15] For θ = 0.1, the proposed tuning rule shows excellent load and set-point response, compared to the modified SIMC tuning. The modified SIMC method shows load response sluggish a little. It is due to a large integral time of $\tau_I = 8\theta$. However, its choice will be inevitable for the low overshoot in the set-point response. With the 2DOF controller, this can be avoided and here low integral time near $\tau_I = 4.31\theta$ is used without worrying about overshoots. The sluggish load response is relieved and overshoot in the set-point response is suppressed by the set-point filter. For θ = 1 and θ = 5, the proposed tuning rule shows closed-loop responses with a similar shape. The modified SIMC tuning rule shows slow drifts due to the increased integral time. However, such drift may not be a severe disadvantage.

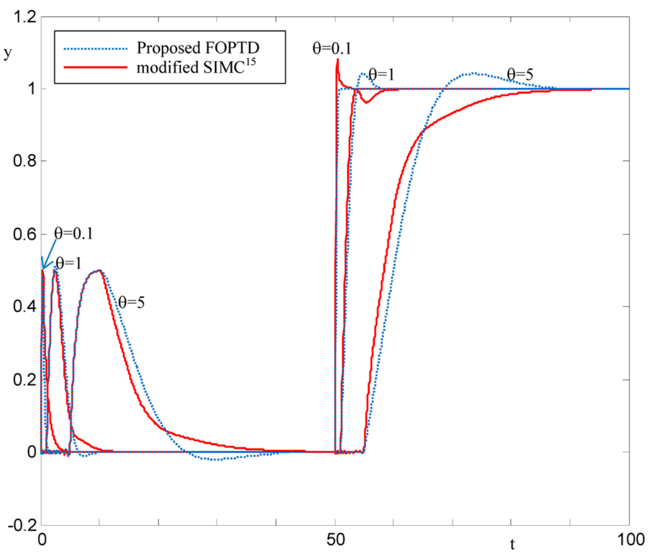


Figure 4. Closed-loop responses of PI controllers for FOPTD processes, $G_p(s) = \exp(-\theta s)/(s + 1)$.

Figure 5 shows several quantitative properties of the proposed tuning rule compared to the modified SIMC tuning. Both tuning rules have similar peak amplitude ratios (Ms) near 1.7, showing their robustness. The values of the integral of absolute errors (IAE) of the proposed tuning rule are higher by about 10% for a large time delay θ. For a small time delay θ, those are increased for the set-point changes and decreased for the load changes. Increments for the set-point changes are due to the set-point filters used in the proposed control system. However, the set-point filters will be inevitable to suppress overshoots in the set-point responses. Except for a very small time delay θ, overshoots in the set-point responses are well below 10% for both tuning rules.

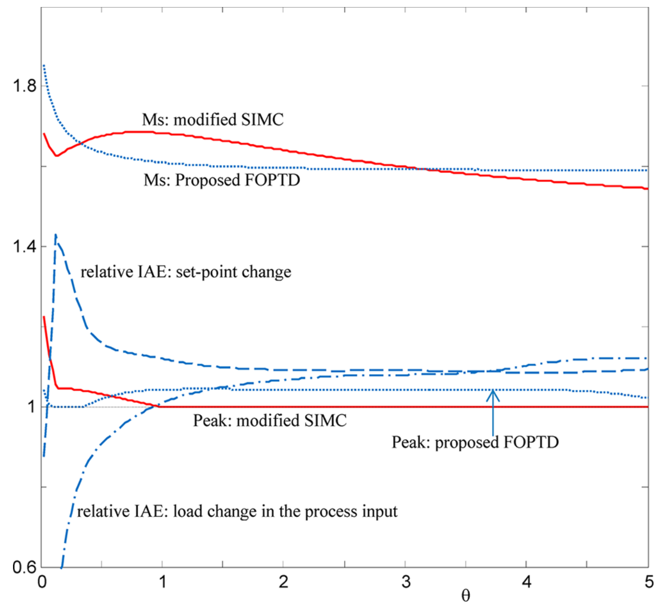


Figure 5. Ms (peak amplitude ratios of the sensitivity function, $S(s) = 1/(1 + G(s)\ C(s))$), relative IAE (integral of absolute errors of the proposed FOPTD method over the modified SIMC method[15]), and peak values in the unit set-point responses for FOPTD processes, $G_p(s) = \exp(-\theta s)/(s + 1)$.

5. NEW TUNING RULE FOR THE HALF-ORDER PLUS TIME DELAY (HOPTD) MODEL

As shown in Lee et al.,[14] the FOPTD model can have trouble even for some overdamped processes. To overcome such drawbacks of the FOPTD model-based method, the half-order plus time delay (HOPTD) model-based method can be used. The HOPTD model-based method to tune PI controllers can be applied to almost all overdamped processes robustly. Consider the HOPTD model[14]

$$G(s) = \frac{k\exp(-\theta s)}{\sqrt{\tau s + 1}} \quad (28)$$

When $\theta \gg \tau$, PI controller should be designed for the closed-loop responses to be slow. Because the Pade approximation for the half-order term can be applied, the process becomes

$$G(s) = \frac{k\exp(-\theta s)}{\sqrt{\tau s + 1}} \approx \frac{k\exp(-\theta s)}{\frac{\tau}{2}s + 1} \quad (29)$$

Applying the SIMC method,[12] we have

$$kk_c = \frac{\tau}{4\theta}$$

$$\tau_I = \frac{\tau}{2} \quad (30)$$

When $\theta \ll \tau$, PI controller can be designed for the closed-loop responses to be fast. Hence the working frequencies are high and the process can be approximated as

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905−2916

$$G(s) = \frac{k\exp(-\theta s)}{\sqrt{\tau s + 1}} \approx \frac{k\exp(-\theta s)}{\sqrt{\tau s}} = \tilde{k}\frac{\exp(-\theta s)}{\sqrt{s}},\quad \tilde{k} = \frac{k}{\sqrt{\tau}} \tag{31}$$

For this lag-dominant model ($\theta \ll \tau$), the dominant pole method[2] as in the above FOPTD model is applied and a tuning rule is obtained as

$$kk_c = 0.4619\sqrt{\frac{\tau}{\theta}},\quad \tau_I = 1.408\theta \tag{32}$$

This tuning rule satisfies[2]

$$1 + \left(k_c + \frac{k_I}{s}\right)\frac{k\exp(-\theta s)}{\sqrt{\tau s}}\bigg|_{s=-\alpha(1+j)} = 0 \tag{33}$$

One of the closed-loop poles is at $s = -\alpha(1 + j)$. Here α is set to $1/(3\theta)$.

Tuning rules of eqs 30 and 32 are combined as

$$kk_c = \left(\left(\frac{4\theta}{\tau}\right)^2 + \left(\frac{1}{0.4619}\sqrt{\frac{\theta}{\tau}}\right)^2\right)^{-1/2} = \frac{\tau}{4\theta}\frac{1}{\sqrt{1 + \frac{\tau}{3.41\theta}}}$$
$$\tau_I = \left(\left(\frac{2}{\tau}\right)^2 + \left(\frac{1}{1.408\theta}\right)^2\right)^{-1/2} = \frac{\tau}{2}\frac{1}{\sqrt{1 + \left(\frac{\tau}{2.82\theta}\right)^2}} \tag{34}$$

Figure 6 shows simulation results. The proposed tuning rule of eq 34 is compared to that in Lee et al.[14] Both closed-loop performances are similar. However, the proposed tuning rule is far simpler. Table 3 shows the proposed tuning rules for FOPTD and HOPTD models.

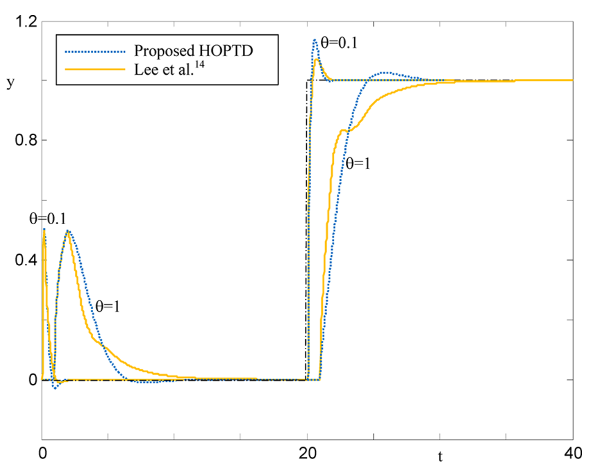


Figure 6. Closed-loop responses of PI controllers for HOPTD processes, $G_p(s) = \frac{\exp(-\theta s)}{\sqrt{s+1}}$.

6. MODIFIED SIMC MODEL REDUCTION METHOD

Processes can often show high order dynamics. When simple graphical identifications are applied, the FOPTD and HOPTD models are obtained and the proposed tuning rules can be applied directly. On the other hand, when elaborate identification methods are applied, process models obtained will be high order. To apply the proposed tuning methods, their orders should be reduced. The SIMC model reduction[13] is one of the simplest model reduction methods and will be useful for PI controller tuning. Here the SIMC model reduction method is slightly modified for better performances.[19] Consider a high order process model of

$$G(s) = k\frac{(\tau_{z1}s + 1)\cdots(\tau_{zm}s + 1)}{(\tau_{p1}s + 1)\cdots(\tau_{pn}s + 1)}\exp(-\theta s),\qquad n > m \tag{35}$$

6.1. FOPTD Model. The FOPTD consists of six steps.

Step 1. Assume the effective time delay $\hat{\theta}$ that will be near the time delay of the reduced FOPTD model.[12]

Step 2. Add all zero time constants with negative τ_z to the time delay.

Step 3. Apply the half-rule of SIMC sequentially from the smallest time constant τ_p (when $\tau_z < \tau_p$, skip this step and do the following step first). For example

$$\begin{aligned}\frac{1}{(s+1)^4} &= \frac{1}{(s+1)^2(s+1)(s+1)}\\ &\approx \frac{\exp(-0.5s)}{(s+1)^2(1.5s+1)} = \frac{\exp(-0.5s)}{(1.5s+1)(s+1)(s+1)}\\ &\approx \frac{\exp(-s)}{(1.5s+1)(1.5s+1)} \approx \frac{\exp(-1.75s)}{2.25s+1}\end{aligned} \tag{36}$$

Step 4. For all pairs of nearest poles and zeros of $(\tau_z s + 1)/(\tau_p s + 1)$, calculate

$$q = \begin{cases} \dfrac{\sqrt{1 + (\tau_z/\hat{\theta})^2}}{\sqrt{1 + (\tau_p/\hat{\theta})^2}}, & \tau_z \geq \tau_p \\[2ex] \dfrac{1 + \tau_z\tau_p/(2\hat{\theta})^2}{1 + {\tau_z}^2/(2\hat{\theta})^2}, & \text{otherwise} \end{cases} \tag{37}$$

Here $\hat{\theta}$ is the assumed effective time delay in the final FOPTD model.

Step 5. Apply the model reduction for the pair with the smallest q as

$$\frac{\tau_z s + 1}{\tau_p s + 1} = \begin{cases} q, & \tau_z \geq \tau_p \\[1ex] \dfrac{1}{q}\dfrac{1}{\frac{\tau_p - \tau_z}{1 + \tau_z\tau_p/(2\hat{\theta})^2}s + 1}, & \text{otherwise}\end{cases} \tag{38}$$

Step 6. Repeat steps 2–5 until the FOPTD model is obtained.

6.2. HOPTD Model. When the process has a large zero term, the above FOPTD model can provide poor tuning results. For such processes, the HOPTD model is considered. The 1/2 Pade approximation of the half-order process[14] is

$$\frac{1}{\sqrt{\tau s + 1}} \approx \frac{\frac{\tau}{2}s + 1}{\left(\frac{\tau}{4 - 2\sqrt{2}}s + 1\right)\left(\frac{\tau}{4 + 2\sqrt{2}}s + 1\right)} \tag{39}$$

Using this relationship, the HOPTD model is obtained as follows.

Step 1. Applying the above model reduction technique for the FOPTD model, obtain

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905–2916

Table 3. Proposed Tuning Rules

		asymptotes			
		$\theta/\tau \to \infty$		$\theta/\tau \to 0$	
process	PI controller	kk_c	τ_I (kk_I)	kk_c	τ_I
$\frac{k\exp(-\theta s)}{\tau s+1}$	$kk_c = \frac{\tau}{2\theta}$	$0.5\tau/\theta$	τ $(0.5/\theta)$	$0.5\tau/\theta$	4.31θ
	$\tau_I = \tau\frac{1}{\sqrt{1+\left(\frac{\tau}{4.31\theta}\right)^2}}$				
	$\tau_{sp} = \tau\frac{1}{\sqrt{1+\left(\frac{\tau}{1.72\theta}\right)^2}}$				
$\frac{k\exp(-\theta s)}{\sqrt{\tau s+1}}$	$kk_c = \frac{\tau}{4\theta}\frac{1}{\sqrt{1+\frac{\tau}{3.41\theta}}}$	$0.25\tau/\theta$	0.5τ $(0.5/\theta)$	$0.46\sqrt{\tau/\theta}$	1.41θ
	$\tau_I = \frac{\tau}{2}\frac{1}{\sqrt{1+\left(\frac{\tau}{2.82\theta}\right)^2}}$				

$$G_2(s) = k\frac{(\tau_{z1}s+1)\exp(-\theta s)}{(\tau_{p1}s+1)(\tau_{p2}s+1)}, \qquad \tau_{p1} > \tau_{z1} > \tau_{p2} \tag{40}$$

Step 2. Apply the Pade approximation of eq 39 with

$$\tau = (4 - 2\sqrt{2})\tau_{p1} \tag{41}$$

Then

$$G_2(s) \approx k\frac{1}{\sqrt{\tau s+1}}\frac{\left(\frac{\tau}{4-2\sqrt{2}}s+1\right)\left(\frac{\tau}{4+2\sqrt{2}}s+1\right)}{\frac{\tau}{2}s+1}\frac{(\tau_{z1}s+1)\exp(-\theta s)}{(\tau_{p1}s+1)(\tau_{p2}s+1)} = k\frac{1}{\sqrt{\tau s+1}}\frac{\tau_{z1}s+1}{\frac{\tau}{2}s+1}\frac{\frac{\tau}{4+2\sqrt{2}}s+1}{\tau_{p2}s+1}\exp(-\theta s) \tag{42}$$

Step 3. Apply the above model reduction technique to the pole–zero pairs and obtain

$$G_2(s) \approx \frac{1}{\sqrt{\tau s+1}}\frac{\hat{k}}{\hat{\tau}s+1}\exp(-\hat{\theta}s) \tag{43}$$

Step 4. Adjust the pole and time delay in eq 43 by applying the half-rule of SIMC

$$G_2(s) \approx \frac{\hat{k}}{\sqrt{(\tau+\hat{\tau})s+1}}\exp(-(\hat{\theta}+\hat{\tau}/2)s) \tag{44}$$

The HOPTD model is used to complement the FOPTD model. When the time delay of the final FOPTD model, θ_{FOPTD}, is much different from the time delay θ of the process model eq 40 as

$$\theta_{FOPTD} > 1.1\theta \tag{45}$$

processes will not be approximated well by the FOPTD model and the HOPTD model is tried.

7. APPLICATIONS

7.1. Process 1.

Consider the process

$$G(s) = \frac{(6s+1)(-2s+1)}{(10s+1)(s+1)^2} \tag{46}$$

From two points that the open-loop step response reaches 28.3 and 63.2% of the new steady state, FOPTD and HOPTD models can be obtained.[14] This simple graphical identification method gives two models of

$$\hat{G}_{FOPTD}(s) = \frac{\exp(-1.594s)}{4.11s+1} \tag{47}$$

$$\hat{G}_{HOPTD}(s) = \frac{\exp(-2.43s)}{\sqrt{8.08s+1}} \tag{48}$$

Because the effective time delay for the FOPTD model is too small, PI controller tuning based on this graphical FOPTD model shows oscillatory closed-loop responses and is not considered for comparisons.

When the exact process model of eq 46 is available, model reduction techniques can be applied to obtain FOPTD and HOPTD models. The SIMC method provides the approximate FOPTD model of

$$G_{SIMC}(s) \approx \frac{\exp(-2s)}{(4s+1)(s+1)^2} \approx \frac{\exp(-3.5s)}{4.5s+1} \tag{49}$$

When the proposed model reduction technique is applied, we have

$$G_{FOPTD}(s) \approx \frac{6s+1}{10s+1}\frac{\exp(-2.5s)}{1.5s+1} \approx \frac{1+\frac{6^2}{(2\cdot 2.5)^2}}{1+\frac{6\cdot 10}{(2\cdot 2.5)^2}}\frac{1}{\frac{10-6}{1+\frac{6\cdot 10}{(2\cdot 2.5)^2}}s+1}\frac{\exp(-2.5s)}{1.5s+1} = \frac{0.718}{1.18s+1}\frac{\exp(-2.5s)}{1.5s+1} \approx \frac{0.718}{2.09s+1}\exp(-3.09s) \tag{50}$$

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905–2916

Table 4. Tuning Results

process		method	k_c	τ_I (k_I)	τ_F	gain margin	phase margin (deg)	Ms[a]
$\exp(-s)$		Astrom−Hagglund	0.195	0.284		2.6	61	1.7
		direct synthesis	0.125	0.25		3.7	68	1.4
		SIMC	0	(0.5)		3.2	61	1.6
$\frac{\exp(-s)}{s}$		Ziegler−Nichols	0.9	3		1.4	16	4.5
		Chien−Fruehauf	0.556	5		2.6	38	2.0
		SIMC	0.5	8		3.0	47	1.7
$\frac{\exp(-\theta s)}{s+1}$	$\theta = 0.1$	modified SIMC[15]	5	0.8		3.1	58	1.6
	$\theta = 0.1$	proposed FOPTD	5	0.3958	0.1692	2.9	44	1.8
	$\theta = 1$	modified SIMC[15]	0.6667	1.3333		2.7	66	1.7
	$\theta = 1$	proposed FOPTD	0.5	0.9741	0.8641	3.1	60	1.6
	$\theta = 5$	modified SIMC[15]	0.2667	2.6667		3.0	70	1.5
	$\theta = 5$	proposed FOPTD	0.1	0.9989	0.9933	3.1	60	1.6
$\frac{\exp(-\theta s)}{\sqrt{s+1}}$	$\theta = 0.1$	Lee et al.[14]	1.6244	0.2021		2.8	67	1.6
	$\theta = 0.1$	proposed HOPTD	1.2611	0.1356		3.4	56	1.5
	$\theta = 1$	Lee et al.	0.5481	1.1817		2.7	78	1.6
	$\theta = 1$	proposed HOPTD	0.2199	0.4712		4.1	64	1.4
$\frac{(6s+1)(-2s+1)}{(10s+1)(s+1)^2}$		modified SIMC[15]	0.8095	5.6667		1.9	82	2.1
		proposed FOPTD	0.3382	2.0647	0.9864	3.9	61	1.4
		proposed HOPTD	0.7168	3.0170		2.0	63	2.0
		HOPTD (graphical ID)	0.5911	2.6127		2.4	61	1.8
$\frac{(11.61s+1)\exp(-3s)}{(18.8s+1)(3.89s+1)}$		modified SIMC[15]	1.0990	10.790		3.6	86	1.4
		proposed FOPTD	0.9327	4.2249	3.5537	3.5	56	1.5
		proposed HOPTD	0.9966	4.2291		3.3	55	1.6
		HOPTD (graphical ID)	0.7326	4.8430		4.7	64	1.3

[a]Peak amplitude ratio of the sensitivity function, $S(s) = 1/(1 + G(s)\,C(s))$.

$$G_{HOPTD}(s) \approx \frac{6s+1}{10s+1}\frac{\exp(-2.5s)}{1.5s+1} \approx \frac{1}{\sqrt{11.7s+1}}$$
$$\frac{6s+1}{5.86s+1}\frac{1.72s+1}{1.5s+1}\exp(-2.5s) \approx \frac{1}{\sqrt{11.7s+1}}$$
$$\frac{\sqrt{1+\left(\frac{6}{2.5}\right)^2}}{\sqrt{1+\left(\frac{5.86}{2.5}\right)^2}}\frac{\sqrt{1+\left(\frac{1.72}{2.5}\right)^2}}{\sqrt{1+\left(\frac{1.5}{2.5}\right)^2}}\exp(-2.5s)$$
$$= \frac{1.06}{\sqrt{11.7s+1}}\exp(-2.5s) \tag{51}$$

Tuning rules are applied to approximate models. Table 4 shows final tuning results. Figure 7 shows closed-loop performances of PI controllers. The proposed tunings based on HOPTD models that are obtained by the graphical identification and the model reduction show better closed-loop responses compared to other tunings.

7.2. Process 2 (Ogunnaike and Ray Column[20]).

Consider the process which is the last diagonal element of the Ogunnaike and Ray column.[20] Here the time delay is increased by 2.

$$G_p(s) = \frac{(11.61s+1)\exp(-3s)}{(18.8s+1)(3.89s+1)} \tag{52}$$

From two points that the open-loop step response reaches 28.3 and 63.2% of the new steady state, FOPTD and HOPTD models can be obtained.[14] This simple graphical identification method gives two models of

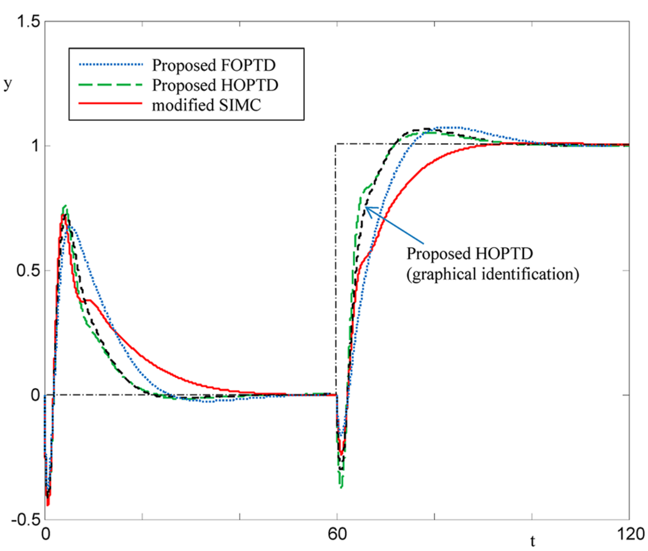


Figure 7. Closed-loop responses of PI controllers for the example processes, $G(s) = \frac{(6s+1)(-2s+1)}{(10s+1)(s+1)^2}$. The modified SIMC method[15] is based on the original model reduction rules of Skogestad.[12]

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905−2916

$$\hat{G}_{\mathrm{FOPTD}}(s) = \frac{\exp(-2.20s)}{9.23s + 1} \tag{53}$$

$$\hat{G}_{\mathrm{HOPTD}}(s) = \frac{\exp(-4.07s)}{\sqrt{18.1s + 1}} \tag{54}$$

Because the effective time delay for the FOPTD model is less than the actual time delay, PI controller tunings based on this graphical FOPTD model show oscillatory closed-loop responses and are not considered for comparisons.

When the exact process model of eq 52 is available, model reduction techniques can be applied to obtain FOPTD and HOPTD models. The SIMC method provides the approximate FOPTD model of

$$G_{\mathrm{SIMC}}(s) \approx \frac{\exp(-3s)}{(7.19s + 1)(3.89s + 1)} \approx \frac{\exp(-4.95s)}{9.14s + 1} \tag{55}$$

When the proposed model reduction technique is applied, we have

$$\begin{aligned} G_{\mathrm{FOPTD}}(s) &\approx \frac{1 + \frac{11.61^2}{(2\cdot 3)^2}}{1 + \frac{11.61\cdot 18.8}{(2\cdot 3)^2}} \frac{1}{\frac{7.19}{1 + \frac{11.61\cdot 18.8}{(2\cdot 3)^2}}s + 1} \frac{\exp(-3s)}{3.89s + 1} \\ &\approx \frac{0.672}{4.40s + 1}\exp(-3.51s) \end{aligned} \tag{56}$$

$$\begin{aligned} &G_{\mathrm{HOPTD}}(s) \\ &\approx \frac{1}{\sqrt{22.0s + 1}} \frac{11.61s + 1}{11.02s + 1} \frac{3.23s + 1}{3.89s + 1}\exp(-3s) \\ &\approx \frac{1}{\sqrt{22.0s + 1}} \frac{\sqrt{1 + \left(\frac{11.61}{3}\right)^2}}{\sqrt{1 + \left(\frac{11.02}{3}\right)^2}} \frac{1 + \frac{3.23^2}{(2\cdot 3)^2}}{1 + \frac{3.23\cdot 3.89}{(2\cdot 3)^2}} \\ &\quad \frac{1}{\frac{3.89 - 3.23}{1 + \frac{3.23\cdot 3.89}{(2\cdot 3)^2}}s + 1}\exp(-3s) \\ &= \frac{1}{\sqrt{22.0s + 1}} \cdot 1.05 \cdot \frac{0.956}{0.489s + 1}\exp(-3s) \\ &\approx \frac{1.00}{\sqrt{22.5s + 1}}\exp(-3.24s) \end{aligned} \tag{57}$$

Tuning rules are applied to these approximate models. Table 4 shows final tuning results. Figure 8 shows closed-loop performances of PI controllers. The proposed tunings based on FOPTD and HOPTD models that are obtained by the model reduction show better closed-loop responses compared to other tunings.

8. CONCLUSION

Two extreme cases of the first order plus time delay (FOPTD) model for small and large time delays over the time constant are the integral plus time delay model and the pure time delay model, respectively. Tuning rules for PI controllers are available for such cases, and tuning rules for the FOPTD models need approach them as the time delay varies. These two asymptotes of PI controller tuning rules are easily calculated and can be used to evaluate the performances and application ranges of PI controller tuning rules, providing guidelines for the selection of PI controller tuning rules. When design parameters are selected

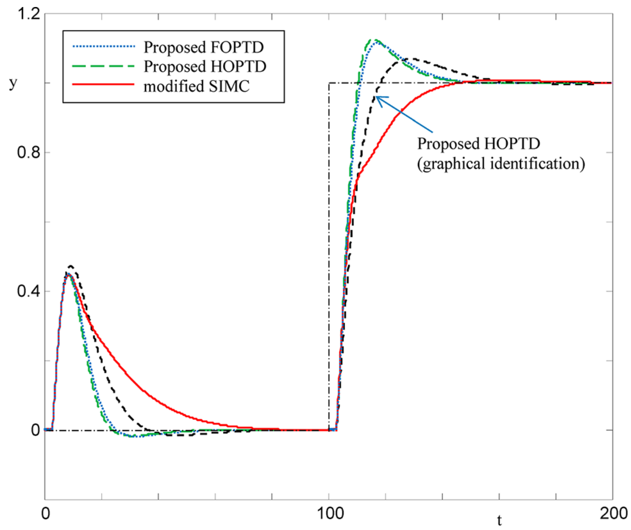


Figure 8. Closed-loop responses of PI controllers for HOPTD processes, $G(s) = \frac{(11.61s + 1)\exp(-3s)}{(18.8s + 1)(3.89s + 1)}$. The modified SIMC method[15] is based on the original model reduction rules of Skogestad.[12]

appropriately, the iterative continuous cycling method, the guaranteed gain and phase margin method, and the Skogestad internal model control method are found to be applied for the whole range of time delays.

By matching two asymptotes for small and large time delays over the time constant, a simple analytic tuning rule for the FOPTD model has been proposed. For some overdamped processes whose transfer functions have large zero terms, the half-order plus time delay (HOPTD) models are found to be better than the FOPTD models. Applying the technique of matching two asymptotes, a simple analytic PI controller tuning rule has also been proposed for the HOPTD models. To apply these tuning rules to high order processes whose transfer functions are given, model reduction methods to obtain the FOPTD and HOPTD models are investigated. Here, the SIMC model reduction rule is modified and some ambiguities in practical applications are removed. Simulation results for empirical and full models of processes are given to show the performances of the proposed model reduction methods and tuning rules.

APPENDIX: DOMINANT POLE TUNING[2]

The closed-loop poles for the process $G(s)$ and PI controller $C(s) = k_c + k_I/s$ are roots of the characteristic equation

$$1 + \left(k_c + \frac{k_I}{s}\right)G(s) = 0 \tag{A1}$$

When a pair of closed-loop poles is specified, PI controller parameters can be determined. These poles can be dominant.

For the FOPTD model of eq 1, with the dominant pole candidates of $s = -\alpha(1 + j)$, we have

$$1 + \left(k_c + \frac{k_I}{s}\right)\frac{k\exp(-\theta s)}{\tau s}\Bigg|_{s=-\alpha(1+j)} = 0 \tag{A2}$$

Then

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905−2916

$$
\begin{aligned}
-kk_c\alpha(1+j) + kk_I &= -\tau\alpha^2(1+j)^2\exp(-\theta\alpha(1+j)) \\
&= -2\tau\alpha^2 j\exp(-\theta\alpha)(\cos(\theta\alpha) - j\sin(\theta\alpha)) \\
&= -2\tau\alpha^2\exp(-\theta\alpha)\sin(\theta\alpha) - 2j\tau\alpha^2\exp(-\theta\alpha)\cos(\theta\alpha)
\end{aligned} \tag{A3}
$$

Equating real and imaginary parts of both sides, we have

$$
\begin{aligned}
-kk_c\alpha + kk_I &= -2\tau\alpha^2\exp(-\theta\alpha)\sin(\theta\alpha) \\
-kk_c\alpha &= -2\tau\alpha^2\exp(-\theta\alpha)\cos(\theta\alpha)
\end{aligned} \tag{A4}
$$

Hence

$$
\begin{aligned}
kk_c &= 2\tau\alpha\exp(-\theta\alpha)\cos(\theta\alpha) \\
kk_I &= -2\tau\alpha^2\exp(-\theta\alpha)\sin(\theta\alpha) + 2\tau\alpha^2\exp(-\theta\alpha)\cos(\theta\alpha) \\
&= 2\tau\alpha^2\exp(-\theta\alpha)(-\sin(\theta\alpha) + \cos(\theta\alpha))
\end{aligned} \tag{A5}
$$

With the choice of $\alpha = 1/(2.428\theta)$, $kk_c = 0.5\tau/\theta$ and $kk_I = 0.116\tau/\theta^2$ ($\tau_I = k_c/k_I = 4.31\theta$).

For the FOPTD model of eq 28, with the dominant pole candidates of $s = -\alpha(1 + j)$, we have

$$
1 + \left(k_c + \frac{k_I}{s}\right)\frac{k\exp(-\theta s)}{\sqrt{\tau s}}\Bigg|_{s=-\alpha(1+j)} = 0 \tag{A6}
$$

Then

$$
\begin{aligned}
&-kk_c\alpha(1+j) + kk_I \\
&= -\sqrt{\tau}\alpha^{3/2}(-1-j)^{3/2}\exp(-\theta\alpha(1+j))
\end{aligned} \tag{A7}
$$

Equating real and imaginary parts of both sides, we have

$$
\begin{aligned}
-kk_c\alpha + kk_I &= -\sqrt{\tau}\alpha^{3/2}\,\mathrm{Re}((-1-j)^{3/2}\exp(-\theta\alpha(1+j))) \\
-kk_c\alpha &= -\sqrt{\tau}\alpha^{3/2}\,\mathrm{Im}((-1-j)^{3/2}\exp(-\theta\alpha(1+j)))
\end{aligned} \tag{A8}
$$

Hence

$$
\begin{aligned}
kk_c &= \sqrt{\tau}\sqrt{\alpha}\,\mathrm{Im}((-1-j)^{3/2}\exp(-\theta\alpha(1+j))) \\
kk_I &= \sqrt{\tau}\alpha^{3/2}(-\mathrm{Re}((-1-j)^{3/2}\exp(-\theta\alpha(1+j))) + \mathrm{Im}((-1-j)^{3/2}\exp(-\theta\alpha(1+j))))
\end{aligned} \tag{A9}
$$

With the choice of $\alpha = 1/(3\theta)$

$$
\begin{aligned}
kk_c &= \sqrt{\frac{\tau}{\theta}}\frac{\mathrm{Im}((-1-j)^{3/2}\exp(-(1+j)/3))}{\sqrt{3}} = 0.4619\sqrt{\frac{\tau}{\theta}} \\
kk_I &= \sqrt{\frac{\tau}{\theta}}\left(\frac{-\mathrm{Re}((-1-j)^{3/2}\exp(-(1+j)/3))}{3^{3/2}\theta} + \frac{\mathrm{Im}((-1-j)^{3/2}\exp(-(1+j)/3))}{3^{3/2}\theta}\right) \\
&= 0.3281\sqrt{\frac{\tau}{\theta}}\frac{1}{\theta}
\end{aligned} \tag{A10}
$$

Hence, $\tau_I = k_c/k_I = 1.408\theta$.

AUTHOR INFORMATION

Corresponding Author

*Tel.: +82-53-950-5620. Fax: +82-53-950-6615. E-mail: jtlee@knu.ac.kr.

ORCID

Jietae Lee: 0000-0001-9667-8905

Notes

The authors declare no competing financial interest.

ACKNOWLEDGMENTS

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2016R1D1A1B03934408).

REFERENCES

(1) Seborg, D. E.; Mellichamp, D. A.; Edgar, T. F.; Doyle, F. J., III. *Process Dynamics and Control*, 3rd ed.; Wiley: Hoboken, NJ, 2011.
(2) Astrom, K. J.; Hagglund, T. *PID Controllers: Theory, Design, and Tuning*, 2nd ed.; Instrument Society of America: Research Triangle Park, NC, 1995.
(3) Ziegler, J. G.; Nichols, N. B. Optimum Settings for Automatic Controllers. *Trans. ASME* **1942**, *64*, 759−768.
(4) Cohen, G. H.; Coon, G. A. Theoretical Considerations of Retarded Control. *Trans. ASME* **1953**, *75*, 827−834.
(5) Seborg, D. E.; Edgar, T. F.; Mellichamp, D. A. *Process Dynamics and Control*; Wiley: Hoboken, NJ, 1989.
(6) Lee, J.; Cho, W.; Edgar, T. F. Multiloop PI controller tuning for interacting multivariable processes. *Comput. Chem. Eng.* **1998**, *22*, 1711−1723.
(7) Ho, W. K.; Hang, C. C.; Cao, L. S. Tuning of PID Controllers Based on Gain and Phase Margin Specifications. *Automatica* **1995**, *31*, 497−502.
(8) Marlin, T. E. *Process Control: Designing Processes and Control Systems for Dynamic Performance*, 2nd ed.; McGraw-Hill: New York, 1995.
(9) Lee, Y.; Park, S.; Lee, M.; Brosilow, C. PID Controller Tuning for Desired Closed-Loop Responses for SI/SO Systems. *AIChE J.* **1998**, *44*, 106−115.
(10) Rivera, D. E.; Morari, M.; Skogestad, S. Internal Model Control 4. PID Controller Design. *Ind. Eng. Chem. Process Des. Dev.* **1986**, *25*, 252−265.
(11) Chien, I. L.; Fruehauf, P. S. Consider IMC Tuning to Improve Controller Performance. *Chem. Eng. Prog.* **1990**, *86*, 33−41.
(12) Skogestad, S. Simple Analytic Rules for Model Reduction and PID Controller Tuning. *J. Process Control* **2003**, *13*, 291−309.
(13) O'Dwyer, A. *Handbook of PI and PID Controller Tuning Rules*, 3rd ed.; Imperial College Press: London, U.K., 2009.

(14) Lee, J.; Lee, Y.; Yang, D. R.; Edgar, T. F. Half Order Plus Time Delay (HOPTD) Models to Tune PI Controllers. *AIChE J.* **2017**, *63*, 601−609.
(15) Grimholt, C.; Skogestad, S. Optimal PI-Control and Verification of the SIMC Tuning Rule. *IFAC Proceedings Volumes* **2012**, *45*, 11−22.
(16) Hagglund, T.; Astrom, A. Revisiting the Ziegler-Nichols Tuning Rules for PI Control. *Asian Journal of Control* **2002**, *4*, 364−380.
(17) Astrom, A.; Hagglund, T. Revisiting the Ziegler-Nichols Step Response Method for PID Control. *J. Process Control* **2004**, *14*, 635−650.
(18) Nayler, A. W.; Sell, G. R. *Linear Operator Theory*; Holt, Rinehart and Winston: New York, 1971.
(19) Lee, J.; Cho, W.; Edgar, T. F. Simple Analytic PID Controller Tuning Rules Revisited. *Ind. Eng. Chem. Res.* **2014**, *53*, 5038−5047.
(20) Ogunnaike, B. A.; Ray, W. H. Multivariable Controller Design for Linear Systems Having Multiple Time Delays. *AIChE J.* **1979**, *25*, 1043−1057.

DOI: 10.1021/acs.iecr.7b03966
Ind. Eng. Chem. Res. 2018, 57, 2905−2916