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Simple Analytic PID Controller Tuning Rules Revisited

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ABSTRACT: The SIMC method by Skogestad (*J. Process Control* **2003**, *13*, 291−309) to tune the PID controller is revisited, and a new method (K-SIMC) is proposed. The proposed K-SIMC method includes modifications of model reduction techniques and suggestions of new tuning rules and set point filters. Effects of such modifications are illustrated through simulations for a wide variety of process models. The proposed modifications permit the SIMC method to be applied with more confidence.

INTRODUCTION

The proportional−integral−derivative (PID) controllers are the main controllers used in industry.[1,2] There are many tuning rules to determine the three tuning parameters systematically.[3] However, poorly tuned PID controllers are often found in industry. One of the keys to overcome this is that the tuning rules should be simple and applicable to a wide range of processes.[4]

Simple analytic tuning rules can be derived from the direct synthesis (DS) method[5−7] or equivalently the internal model control (IMC) method.[8−10] By prescribing the desired closed-loop transfer function and the IMC filter, the controller can be designed analytically using direct synthesis. Since the resulting controller is complicated, it is further simplified into the PID controller form. For this, various approximation methods such as the Pade approximation of process time delay and the Taylor series expansion are used along with pole-zero cancellations usually. Chien and Fruehauf[9] applied the original IMC method by Rivera et al.[8] to various process models and provided hints to solve the problem that, when the desired closed-loop time constant is small compared to the open-loop time constant, the load response can be sluggish. The problem of sluggish load responses appearing in methods based on the pole-zero cancellations can be mitigated by making the leading process pole not be canceled by the controller zero.[6,7,9,10] However, it makes the design procedure and the resulting tuning rules for the PID controller more complicated.

Very simple tuning rules for the PID controller known as the SIMC method have been proposed by Skogestad,[4] which consists of two steps. The first step is to obtain an approximate first-order or second-order model from a complicated process model. The second step is to design PI and PID controllers from the reduced first-order and second-order models by applying the IMC method, respectively. He resolves the sluggish load responses of the IMC method simply by limiting the controller integral time. The SIMC method has been improved further[11,12] and applied to real experimental processes,[13] supporting its utility. Here, the SIMC method is re-examined.

MODEL REDUCTION

One of the key contributions of the SIMC method[4] is the model reduction technique to obtain approximate models of

$$G(s) = \begin{cases} \dfrac{k e^{-\theta s}}{\tau_1 s + 1}, \text{ for PI controller} \\ \dfrac{k e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}, \text{ for PID controller} \end{cases} \tag{1}$$

The applicable model reduction rules are listed in Table 1. In the paper, some features of the SIMC method are retained but some are modified.

First, the famous half rule of the SIMC method is considered. The SIMC method uses the reduction

$$\frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)} \approx \frac{e^{-(\tau_2/2)s}}{(\tau_1 + \tau_2/2)s + 1} \tag{2}$$

For $\tau_2 \ll \tau_1$, a simple approximation of $e^{-\tau_2 s}/(\tau_1 s + 1)$ is better than the above approximation of eq 2 in the phase angle, resulting in better control systems for some processes. A new approximation rule including this case is investigated. The Taylor series expansions of both transfer functions in eq 2 are identical up to the term s. The approximation whose Taylor series expansions are identical up to the term s^2 can be obtained as

$$\frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)} \approx \frac{e^{-\theta s}}{\tau s + 1}$$
$$\theta = \tau_1 + \tau_2 - \sqrt{\tau_1^2 + \tau_2^2}, \quad \tau = \sqrt{\tau_1^2 + \tau_2^2} \tag{3}$$

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Table 1. Model Reduction Rules[a]

dynamics	method	condition	approximate model
$\frac{\prod_j(-T_{j0}s+1)}{\prod_i(\tau_{i0}s+1)}e^{-\theta_0 s}$ (τ_{i0}'s are in the descending order)			$\frac{1}{\tau_1 s+1}e^{-\theta s}$
	Rule 1a	first order	$\tau_1 = \tau_{10} + 0.5\tau_{20}^2/\tau_{10}$ $\theta = \theta_0 + \tau_{20}(1 - 0.5\tau_{20}/\tau_{10}) + \sum_{i\geq 3}\tau_{i0} + \sum_j T_{j0} + h/2$
	Rule 1b	second order	$\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}e^{-\theta s}$ $\tau_1 = \tau_{10}$ $\tau_2 = \tau_{20} + 0.5\tau_{30}^2/\tau_{20}$ $\theta = \theta_0 + \tau_{30}(1 - 0.5\tau_{30}/\tau_{20}) + \sum_{i\geq 4}\tau_{i0} + \sum_j T_{j0} + h/2$
$\frac{T_0 s+1}{(\tau_{0a}s+1)(\tau_{0b}s+1)}$ $\tau_{0a} \geq T_0 \geq \tau_{0b}$	Rule 2a (SIMC)	$T_0 \leq \sqrt{\tau_{0a}\tau_{0b}}$ and $T_0 \leq 1.6\tau_{0b}$	$\frac{T_0 s+1}{\tau_0 s+1}\frac{1}{\tau_1 s+1}$, $\tau_0 = \tau_{0b}$, $\tau_1 = \tau_{0a}$
	Rule 2b (SIMC)	otherwise	$\frac{T_0 s+1}{\tau_0 s+1}\frac{1}{\tau_1 s+1}$, $\tau_0 = \tau_{0a}$, $\tau_1 = \tau_{0b}$
$\frac{T_0 s+1}{\tau_0 s+1}$	Rule 3a	$T_0 \geq \tau_0$ or $\tau_0 \geq T_0 \geq 5\lambda$	$k = \frac{\sqrt{1+(T_0/\lambda)^2}}{\sqrt{1+(\tau_0/\lambda)^2}}$
	Rule 3b	$5\lambda \geq \tau_0 \geq T_0$	$\frac{1+T_0^2/(2\lambda)^2}{1+\tau_0 T_0/(2\lambda)^2}\frac{1}{\frac{\tau_0 - T_0}{1+\tau_0 T_0/(2\lambda)^2}s+1}$
	Rule 3c	$T_0 \geq 5\lambda \geq T_0$	apply Rules 3a and 3b to $\frac{5\lambda s+1}{\tau_0 s+1}\frac{T_0 s+1}{5\lambda s+1}$
$\frac{1}{s}$	Rule 4 (SIMC)		$\frac{1}{s} \Leftrightarrow \frac{q}{qs+1}$: sufficiently large q

[a]The variable h is the sampling period and λ is the tuning parameter representing the desired closed-loop time constant.

It is simplified as

$$\theta = \tau_1 + \tau_2 - \sqrt{\tau_1^2 + \tau_2^2} \approx \tau_1(1 + \tau_2/\tau_1 - \sqrt{1 + (\tau_2/\tau_1)^2})$$
$$\approx \tau_1(1 + \tau_2/\tau_1 - (1 + 0.5(\tau_2/\tau_1)^2)) = \tau_2 - \frac{1}{2}\frac{\tau_2^2}{\tau_1}$$
$$\tau = \tau_1 + (\tau_2 - \theta) = \tau_1 + \frac{1}{2}\frac{\tau_2^2}{\tau_1} \tag{4}$$

The proposed Rules 1a and 1b in Table 1 implement this model reduction technique. Here a factor τ_2/τ_1 is multiplied to the half rule of $\tau_2/2$. This reduction rule is equal to the SIMC method when $\tau_1 = \tau_2$ and, as τ_2 decreases, θ approaches τ_2.

Second, we simplify the SIMC rules to treat the positive numerator time constants. Rules T1, T1a, T1b, and T2 of SIMC are unified as

$$\frac{T_0 s + 1}{\tau_0 s + 1} \approx k = \frac{\sqrt{1 + (T_0/\lambda)^2}}{\sqrt{1 + (\tau_0/\lambda)^2}} = \left|\frac{T_0 s + 1}{\tau_0 s + 1}\right|_{s=j/\lambda} \tag{5}$$

Here λ is the design parameter representing the desired closed-loop time constant. It is usually set to the effective delay, θ. Equation 5 is equal to Rules T1 and T2 for large T_0 and τ_0 ($\gg\lambda$) and the Rule T1b for small T_0 and τ_0 ($\ll\lambda$). For intermediate values of T_0 and τ_0, it is different from those of the SIMC method. Simulations show that eq 5 provides similar or better results than the SIMC rules. Equation 5 selects the approximation gain which is equal to the amplitude ratio at the frequency, $\omega = 1/\lambda$.

We also change Rule T3 of SIMC slightly. The SIMC method uses $(T_0 s + 1)/(\tau_0 s + 1) \approx 1/((\tau_0 - T_0)s + 1)$ for $5\lambda \geq \tau_0 \geq T_0$. Here we propose the approximation

$$\frac{T_0 s + 1}{\tau_0 s + 1} \approx \frac{k}{\tau s + 1}$$
$$k = \frac{1 + T_0^2/(2\lambda)^2}{1 + \tau_0 T_0/(2\lambda)^2}, \ \tau = \frac{\tau_0 - T_0}{1 + \tau_0 T_0/(2\lambda)^2} \tag{6}$$

Equation 6 is such that both models are equal at $s = j/(2\lambda)$, i.e., $(T_0 s + 1)/(\tau_0 s + 1) = k/(\tau s + 1)$ at $s = j/(2\lambda)$. For small T_0 ($\ll 2\lambda$), it is equal to the Rule T3 of SIMC. The SIMC approximation is accurate for frequencies near zero. However, it will be inaccurate for important frequencies to ensure the stability and performance of the control system. The proposed one is accurate for frequencies around $\omega = 1/(2\lambda)$.

The approximation of the integrating element in Table 1 is a technique used very often to derive tuning rules for integrating processes.[10] Here we use it explicitly. The parameter q should be large enough but is problem-dependent.

■ TUNING RULES

PI Controller. The basic tuning rule for the first order plus time delay process of eq 1 is

$$C(s) = k_c\left(1 + \frac{1}{\tau_I s}\right)$$

$$k_c = \frac{\tau_1}{k(\lambda + \theta)},\ \tau_I = \min(\tau_1, 5\lambda) \quad (7)$$

It can be derived from Figure 1

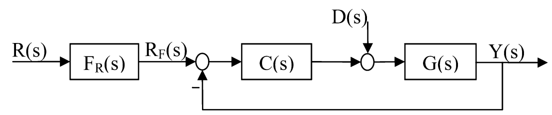

Figure 1. Feedback control system with the set point filter.

$$\frac{Y(s)}{R_F(s)} = \frac{G(s)C(s)}{1 + G(s)C(s)} = \frac{1}{\lambda s + 1}e^{-\theta s} \quad (8)$$

We limit the integral time τ_I by 5λ for better load responses. The SIMC method uses $4(\lambda + \theta)$ instead of 5λ, and because the SIMC limit $4(\lambda + \theta)$ shows somewhat sluggish load responses for a large τ_1, rules in the form of $c(\lambda + \theta)$ have been studied by several authors.[12−14] Since λ is usually near the value of θ, our τ_I can be smaller than that of SIMC, resulting in more aggressive control. The overshoot for the step set point change can be large. So, when $\tau_1 > 5\lambda$, we use the set point filter for the controller of eq 7.

$$F_R(s) = \frac{2.5\lambda(1 + 5\lambda/\tau_1)s + 1}{5\lambda s + 1} \quad (9)$$

Instead of the proposed limit 5λ, different limits based on users' experiences and preferences can be used.

Lag-Dominant Processes. When the process time constant τ_1 is large, the process behaves similar to a delayed integrating process as

$$G(s) = \frac{k}{\tau_1 s + 1}e^{-\theta s} = \frac{k/\tau_1}{s + 1/\tau_1}e^{-\theta s} \approx \frac{k/\tau_1}{s}e^{-\theta s} = \frac{k'}{s}e^{-\theta s} \quad (10)$$

The controller tuning rule for this delayed integrating process is

$$k_c = \frac{1}{k'(\lambda + \theta)},\ \tau_I = 5\lambda \quad (11)$$

The closed loop transfer function of this control system is

$$\begin{aligned}\frac{Y(s)}{R_F(s)} &= \frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{\frac{1}{k'(\lambda+\theta)}\frac{5\lambda s+1}{5\lambda s}\frac{k'}{s}e^{-\theta s}}{1 + \frac{1}{k'(\lambda+\theta)}\frac{5\lambda s+1}{5\lambda s}\frac{k'}{s}e^{-\theta s}}\\ &= \frac{(5\lambda s + 1)e^{-\theta s}}{(\lambda + \theta)(5\lambda)s^2 + (5\lambda s + 1)e^{-\theta s}}\\ &\approx \frac{(5\lambda s + 1)e^{-\theta s}}{(\lambda + \theta)(5\lambda)s^2 + (5\lambda s + 1)(-\theta s + 1)} = \frac{(5\lambda s + 1)e^{-\theta s}}{5\lambda^2 s^2 + (5\lambda - \theta)s + 1}\\ &= \frac{(5\lambda s + 1)e^{-\theta s}}{(\sqrt{5}\lambda s + 1)^2 + (5\lambda - 2\sqrt{5}\lambda - \theta)s} \approx \frac{(5\lambda s + 1)e^{-\theta s}}{(\sqrt{5}\lambda s + 1)^2}\end{aligned} \quad (12)$$

The lead term $(5\lambda s + 1)$ causes the overshoot of about 41% for the step set point change. To reduce such overshoot, we use the set point filter as

$$F_R(s) = \frac{2.5\lambda s + 1}{5\lambda s + 1} \approx \frac{\sqrt{5}\lambda s + 1}{5\lambda s + 1} \quad (13)$$

With this set point filter, a first order like response between $R(s)$ and $Y(s)$ can be obtained.

As the process time constant τ_1 increases, the proposed control system eq 7 with the set point filter eq 9 yields responses that approach to that of the integrating process.

Delay-Dominant Processes. When the process time constant is very small, the controller of eq 7 becomes a pure integral controller:

$$C(s) \approx \frac{1}{k(\lambda + \theta)s} \quad (14)$$

Although its control performance is not unsatisfactory, it may be inconvenient to implement since the controller gain is zero. For better performances with a nonzero controller gain, the improved SIMC method[11] uses $\tau_1 + \theta/3$ instead of τ_1 for the controller integral time. Similar results can be obtained by implementing the model reduction technique of Rule 3a in Table 1 as, for $\tau_1 < 0.3\theta$,

$$\begin{aligned}G(s) &= \frac{k}{\tau_1 s + 1}e^{-\theta s} = \frac{0.3\theta s + 1}{\tau_1 s + 1}\frac{k}{0.3\theta s + 1}e^{-\theta s}\\ &\approx \frac{\sqrt{1 + (0.3\theta/\lambda)^2}}{\sqrt{1 + (\tau_1/\lambda)^2}}\frac{k}{0.3\theta s + 1}e^{-\theta s}\end{aligned} \quad (15)$$

and applying the tuning rule of eq 7. Table 2 summarizes PI controller tuning rules.

Table 2. PI Controller Tuning Rules

process	method	k_c	τ_I	$F_R(s)$
$\frac{ke^{-\theta s}}{\tau s + 1}$	DCLR	$\frac{\tau_I}{k(\lambda + \theta)}$	$\tau + \frac{\theta^2}{2(\lambda + \theta)}$	
	SIMC	$\frac{\tau}{k(\lambda + \theta)}$	$\min(\tau, 4(\lambda + \theta))$	
	improved SIMC	$\frac{\tau + \theta/3}{k(\lambda + \theta)}$	$\min(\tau + \theta/3, 4(\lambda + \theta))$	
	proposed	$\frac{\tau}{k(\lambda + \theta)}$	$\min(\tau, 5\lambda)$	$\frac{2.5\lambda(1 + 5\lambda/\tau)s + 1}{5\lambda s + 1}$ if $\tau > 5\lambda$
$\frac{k'e^{-\theta s}}{s}$	SIMC	$\frac{1}{k'(\lambda + \theta)}$	$4(\lambda+\theta)$	
	proposed	$\frac{1}{k'(\lambda + \theta)}$	5λ	$\frac{2.5\lambda s + 1}{5\lambda s + 1}$

Table 3. PID Controller Tuning Rules

process	method	k_c	τ_I	τ_D	$F_R(s)$
$\dfrac{ke^{-\theta s}}{\tau_1 s + 1}$	proposed[a]	$\dfrac{\tau_1}{k(\lambda + \theta)}$	$\min(\tau_1, 5\lambda)$	$\max((\theta - \lambda)/2, 0)$	$\dfrac{2.5\lambda(1 + 5\lambda/\tau_1)s + 1}{5\lambda s + 1}$ if $\tau_1 > 5\lambda$
$\dfrac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	SIMC[a]	$\dfrac{\tau_1}{k(\lambda + \theta)}$	$\min(\tau_1, 4(\lambda + \theta))$	τ_2	
	proposed[a]	$\dfrac{\tau_1}{k(\lambda + \theta)}$	$\min(\tau_1, 5\lambda)$	$\tau_2 + \max((\theta - \lambda)/2, 0)$	$\dfrac{2.5\lambda(1 + 5\lambda/\tau_1)s + 1}{5\lambda s + 1}$ if $\tau_1 > 5\lambda$
$\dfrac{ke^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}$	DS[b]	$\dfrac{2\zeta\tau}{k(\lambda + \theta)}\left(1 + \dfrac{1}{2\zeta\tau s} + \dfrac{\tau}{2\zeta}s\right)$			
	proposed[b]	$\dfrac{\tau^2/\tau_D}{k(\lambda + \theta)}\left(1 + \dfrac{1}{\tau_I s} + \tau_D s\right)$ $\tau_I = \min(2\zeta\tau, 7.07\lambda)$, $\tau_D = \min\left(\dfrac{\tau}{2\zeta}, 3.54\lambda\right)$			$\dfrac{(2.5\lambda(1 + 5\lambda/\tau)s + 1)^2}{(5\lambda s + 1)^2}$
$\dfrac{k''e^{-\theta s}}{s^2}$	SIMC[a]	$\dfrac{1}{k''4(\lambda + \theta)^2}\left(1 + \dfrac{1}{4(\lambda + \theta)s}\right)(1 + 4(\lambda + \theta)s)$			
	proposed[b]	$\dfrac{1}{k''3.54\lambda(\lambda + \theta)}\left(1 + \dfrac{1}{7.07\lambda s} + 3.54\lambda s\right)$			$\dfrac{(2.5\lambda s + 1)^2}{(5\lambda s + 1)^2}$

[a]Series-form PID: $C(s) = k_c(1 + 1/(\tau_I s))(1 + \tau_D s)$. [b]Parallel-form PID: $C(s) = k_c(1 + 1/(\tau_I s) + \tau_D s)$.

PID Controller. The direct synthesis (DS) method[5] or the internal model control method[8] for the second order plus time delay process of eq 1 is[9]

$$C(s) = k_c\left(1 + \frac{1}{\tau_I s}\right)(1 + \tau_D s)$$

$$k_c = \frac{\tau_1}{k(\lambda + \theta)},\ \tau_I = \tau_1,\ \tau_D = \tau_2 \tag{16}$$

For the PID controller, the design parameter λ can be chosen to be smaller than θ. As λ decreases, in addition to increasing k_c, it will be better to increase the derivative time as well. This can be obtained by approximating the process as

$$G(s) = \frac{ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} = \frac{ke^{-\eta\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)e^{(1-\eta)\theta s}}$$

$$\approx \frac{ke^{-\eta\theta s}}{(\tau_1 s + 1)((\tau_2 + (1 - \eta)\theta)s + 1)} \tag{17}$$

Applying the DS method, we have

$$k_c = \frac{\tau_1}{k(\tilde{\lambda} + \eta\theta)},\ \tau_I = \tau_1,\ \tau_D = \tau_2 + (1 - \eta)\theta \tag{18}$$

Comparing eq 18 with $\tilde{\lambda} = \eta\theta$ to eq 16, we have

$$2\eta\theta = \lambda + \theta \tag{19}$$

Equation 18 becomes, for $\lambda < \theta$,

$$k_c = \frac{\tau_1}{k(\lambda + \theta)},\ \tau_I = \tau_1,\ \tau_D = \tau_2 + 0.5(\theta - \lambda) \tag{20}$$

As in the SIMC method, for better load responses, the tuning rule 20 is modified as

$$k_c = \frac{\tau_1}{k(\lambda + \theta)},\ \tau_I = \min(\tau_1, 5\lambda),$$

$$\tau_D = \tau_2 + \max(0.5(\theta - \lambda), 0) \tag{21}$$

The set point filter of eq 9 is also used for this PID controller. For faster responses, this PID controller can be applied to the first order plus time delay process by letting $\tau_2 = 0$. These tuning rules are given in Table 3. Tuning rule 21 is used only for $\tau_2 + 0.5(\theta - \lambda) \leq 5\lambda$, and for a larger τ_2, the following tuning rule will be used.

For a general second order plus time delay process of

$$G(s) = \frac{ke^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{22}$$

a new tuning rule is derived by applying the concept of the SIMC method. The PID controller for which the desired closed loop transfer function is the same as eq 8 is

$$C(s) = \frac{1}{k(\lambda + \theta)}\frac{\tau^2 s^2 + 2\zeta\tau s + 1}{s}$$

$$= \frac{2\zeta\tau}{k(\lambda + \theta)}\left(1 + \frac{1}{2\zeta\tau s} + \frac{\tau}{2\zeta}s\right)$$

$$= \frac{2\zeta\tau}{k(\lambda + \theta)}\left(1 + \frac{1}{2\zeta\tau s} + \frac{\tau}{2\zeta}s\right) \tag{23}$$

As in the above PI controller, to improve load responses, the integral and derivative times in eq 23 are limited ($\tau = 5$, $\zeta = 1/\sqrt{2}$):

$$C(s) = \tilde{k}_c\left(1 + \frac{1}{\tilde{\tau}_I s} + \tilde{\tau}_D s\right)$$

$$\tilde{k}_c = \frac{\tau^2/\tau_D}{k(\lambda + \theta)}$$

$$\tilde{\tau}_I = \min(2\zeta\tau, 5\sqrt{2}\lambda) = \min(2\zeta\tau, 7.07\lambda)$$

$$\tilde{\tau}_D = \min\left(\frac{\tau}{2\zeta}, \frac{5\lambda}{\sqrt{2}}\right) = \min\left(\frac{\tau}{2\zeta}, 3.54\lambda\right) \tag{24}$$

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When $\tau > 5\lambda$, extending the set point filter of eq 9, we use

$$F_R(s) = \frac{(2.5\lambda(1 + 5\lambda/\tau)s + 1)^2}{(5\lambda s + 1)^2} \tag{25}$$

By increasing the process time constant, we can obtain the PID controller tuning rule for the double integrating process with time delay. For delayed double integrating processes, set point filters are also required to reduce the overshoot for the set point change.

Table 3 summarizes the proposed PID controller tuning rules. For delayed integrating processes with additional poles and zeros, PID controllers can be designed by applying model reduction techniques in Table 1 and tuning rules in Tables 2 and 3. Table 4 summarizes tuning procedures for delayed integrating processes:

Table 4. Proposed Approximations for $G(s) = k'(Ts + 1)e^{-\theta s}/(s(\tau_2 s + 1))$

method	condition		approximation
proposed	With Rule 4 for the integrator, obtain $\frac{k' q(Ts+1)e^{-\theta s}}{(qs+1)(\tau_2 s+1)}$ and apply model reduction techniques. Then design the PID controller. Following are some cases.		
	$T = 0$	$\tau_2 < 5\lambda$	$C(s)$ is for $\frac{k' 20\lambda e^{-\theta s}}{(20\lambda s+1)(\tau_2 s+1)}$
		$\tau_2 > 5\lambda$	$C(s)$ is for $\frac{k' 20\lambda e^{-\theta s}}{20\lambda\tau_2 s^2 + (20\lambda + \tau_2)s + 1}$
	$T < 1.6\tau_2$		Obtain $\frac{\tilde{k}e^{-\theta s}}{s(\tilde{\tau}_2 s+1)}$ by applying Rule 3 to $(Ts + 1)/(\tau_2 s + 1)$ and then design $C(s)$
	$T > 1.6\tau_2$		Obtain $\frac{\tilde{k}e^{-\theta s}}{(\tilde{\tau}_1 s+1)(\tau_2 s+1)}$ by applying Rule 3 to $q(Ts + 1)/(qs + 1)$ and then design $C(s)$

$$G(s) = \frac{(T_0 s + 1)e^{-\theta s}}{s(\tau_0 s + 1)} \approx \frac{q(T_0 s + 1)e^{-\theta s}}{(qs + 1)(\tau_0 s + 1)} \tag{26}$$

SIMULATIONS

To illustrate performance of these modifications, the proposed method is applied to various process models and compared with the SIMC method. Controller parameters with the robustness indexes (gain margin (GM), phase margin (PM), and sensitivity peak (Ms)) are given in Table 5.

Example 1 (First Order Plus Time Delay Process). Consider a first order plus time delay process of eq 1. The proposed method uses a smaller value than the SIMC method in limiting the controller integral time. Its effect for the integrating process has been shown already in Figure 3 of the SIMC paper (when $\lambda = \theta$ is used, the proposed method uses $\tau_I = 5\theta$, while the SIMC method used $\tau_I = 8\theta$ for the integrating process).[4] The proposed method shows better load responses. On the other hand, it shows an overshoot of about 41% without the set point filter. This overshoot can be reduced to 5% by the set point filter of eq 13. The limit on the controller integral time is based on the user's preference. Without the set point filter, the SIMC one is a good tradeoff between load and set point responses but, with the set point filter, the proposed correlation will yield better results.

As the process time constant goes to zero, both controller gain and integral time of the SIMC method become zero. To avoid this, the improved SIMC method[11] uses $\tau_1 + \theta/3$ for the controller integral time. The controller integral time of the desired closed loop response (DCLR) method[6] approaches $\tau_1 + \theta/4$ as the process time constant goes to zero. This problem can also be resolved by the model reduction technique as shown in eq 15. In fact, control performances of the SIMC method (pure integral control), the DCLR method, the improved SIMC (I-SIMC) method, and the proposed method (K-SIMC) with the model reduction technique of eq 15 are similar.

Figure 2 shows control performances for

$$G(s) = \frac{e^{-s}}{0.2s + 1} \tag{27}$$

The SIMC method and the proposed method of eq 15 with $\lambda = \theta = 1$ show very similar responses. For $\lambda = 0.6 < \theta$, the SIMC method increases just the PI controller gain which shows oscillatory responses. On the other hand, the proposed PID controller of eq 21 shows excellent responses that are fast and less oscillatory.

Example 2 (Second Order Plus Time Delay Process). To illustrate the proposed tuning rule of eq 21, the following process is considered.

$$G(s) = \frac{e^{-2s}}{(s + 1)(0.7s + 1)} \tag{28}$$

For this process, the PI controller is designed by applying the model reduction rule and the tuning rule of the SIMC method. Two PID controllers are designed by the SIMC method with $\lambda = 2$ and $\lambda = 1.2$. The proposed method is the same as the SIMC method except for $\lambda = 1.2$. Figure 3 compares closed-loop performances, which shows improved results for the proposed method of eq 21.

Example 3 (Second Order Underdamped Process). The direct synthesis method and the internal model control method result in pole and zero cancellations. Hence they suffer from poor load responses for small time delay processes. The SIMC method overcomes this simply by limiting the integral time and the technique can also be applied to the overdamped second order plus time delay processes. Here, it is extended to the underdamped second order plus time delay process. To illustrate this, the underdamped process,

$$G(s) = \frac{e^{-s}}{100s^2 + 10s + 1} \tag{29}$$

is considered. Because the SIMC method does not consider underdamped processes, we compare the proposed method with the original direct synthesis (DS) method[5,9] as in Table 3. The damping factor of this process is 0.5, and the open-loop system shows oscillatory responses. These oscillatory responses appear directly in the load responses of the DS method. The proposed method can suppress the oscillatory load response as shown in Figure 4. The proposed method without the set point filer can suffer from the somewhat large overshoot for the step set point change. The set point filter reduces this overshoot effectively.

Example 4 (Delayed Integrating Process). Consider the process

$$G(s) = \frac{e^{-s}}{s(as + 1)} \tag{30}$$

Table 5. PID Controller Parameters Calculated

ex	$G(s)$	method	λ	k_c	τ_I	τ_D	GM	PM	Ms	$F_R(s)$
1	$\frac{e^{-s}}{0.2s+1}$	SIMC	1	0.1	0.2		3.1	62°	1.6	
		SIMC	0.6	0.125	0.2		2.5	54°	1.8	
		proposed	1	0.1465	0.3		3.3	65°	1.5	
		proposed	0.6	0.125	0.2	0.2	2.9	61°	1.6	
2	$\frac{e^{-2s}}{(s+1)(0.7s+1)}$	SIMC	2.35	0.2128	1.35		4.0	69°	1.4	
		SIMC	2	0.25	1	0.7	3.1	61°	1.6	
		SIMC	1.2	0.3125	1	0.7	2.5	54°	1.8	
		proposed	1.2	0.3125	1	1.1	2.4	60°	1.8	
3	$\frac{e^{-s}}{100s^2+10s+1}$	DS	1	5	10	10	3.1	61°	1.6	
		proposed[a]	1	14.14	7.071	3.536	2.9	39°	1.8	$(3.75s+1)^2/(5s+1)^2$
4	$\frac{e^{-s}}{s(as+1)}$ $a = 0.4$	SIMC	1	0.5	8	0.4	2.9	47°	1.7	
		proposed	1.4	0.3571	7		3.1		1.8	$(3.5s+1)/(7s+1)$
		proposed	1	0.5	5	0.4	2.8	39°	1.8	$(2.5s+1)/(5s+1)$
		proposed	0.6	0.625	5	0.6	2.2	42°	2.0	$(2s+1)/(4s+1)$
	$a = 20$	SIMC	1	0.5	8	20	2.9	47°	1.7	
		SIMC	1	1.25	8	8	2.8	39°	1.8	
		proposed[a]	1	2.8284	7.071	3.536	2.8	33°	1.9	$(2.78s+1)^2/(5s+1)^2$
5	$\frac{e^{-s}}{s^2}$	SIMC	1	0.0625	8	8	2.8	33°	2.0	
		proposed[a]	1	0.1414	7.071	3.536	2.7	27°	2.1	$(2.5s+1)^2/(5s+1)^2$
6	$\frac{0.3s+1}{(s+1)^2(0.1s+1)}$	SIMC	0.05	35	0.4		∞	25°	2.4	
		proposed	0.095	51.784	0.475	0.0354	∞	47°	1.3	$(0.35s+1)/(0.475s+1)$
	$\frac{0.5s+1}{(s+1)^2(0.1s+1)}$	SIMC	0.05	21	0.4		∞	34°	1.9	
		proposed	0.095	20.880	0.475		∞	36°	1.8	$(0.35s+1)/(0.475s+1)$
7	$\frac{5(1.6s+1)e^{-s}}{(20s+1)(s+1)}$	SIMC	1	1.25	8		3.7	68°	1.4	
		proposed	1	1.4991	5		3.0	58°	1.6	$(3.125s+1)/(5s+1)$
8	$\frac{(s+1)e^{-s}}{(0.2s+1)^2}$	SIMC	1	0.1	0.2		2.0	81°	2.0	
		I-SIMC	1	0.2667	0.533		1.4	91°	3.6	
		proposed	1	0.0721	0.2		2.8	85°	1.6	
9	$\frac{(2s+1)e^{-s}}{(5s+1)(0.1s+1)}$	SIMC	1.05	1.4524	3.05		1.7	91°	2.3	
		SIMC	2.1	0.9683	3.05		2.6	88°	1.6	
		proposed	1.1	0.7466	0.9784		3.1	61°	1.5	
10	$\frac{(s+1)e^{-s}}{s(as+1)}$ $a = 0.2$	proposed	1.2	0.2901	2.449		3.3	40°	1.5	
	$a = 0.7$	proposed	1	0.4316	5		2.9	47°	1.6	

[a]Parallel-form PID: $C(s) = k_c(1 + 1/(\tau_I s) + \tau_D s)$.

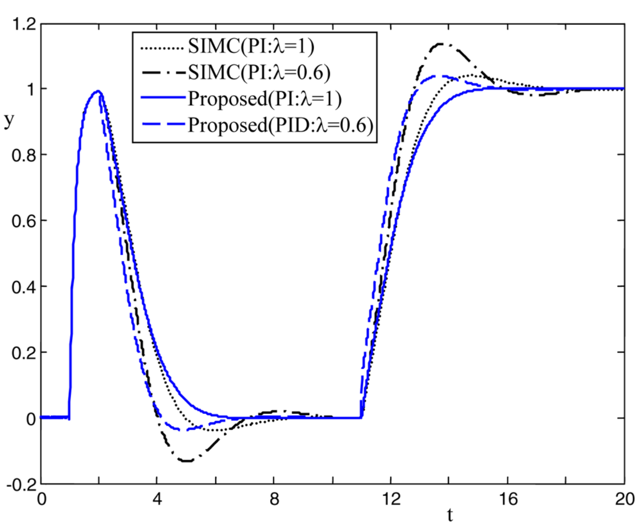

Figure 2. Control responses for the process $G(s) = \exp(-s)/(0.2s + 1)$.

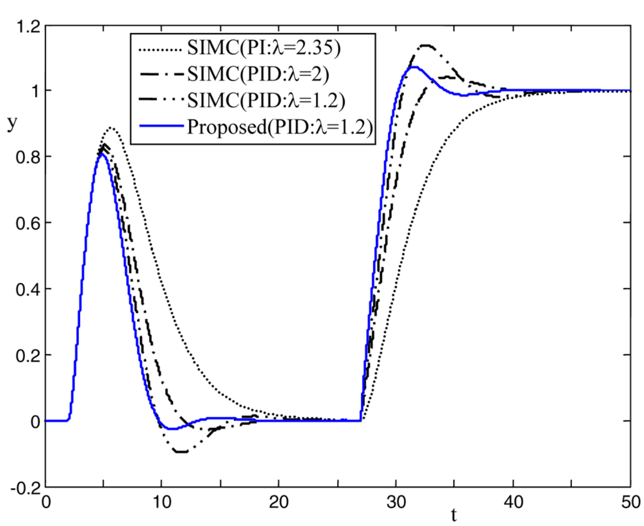

Figure 3. Control responses for the process $G(s) = \exp(-2s)/((s + 1)(0.7s + 1))$.

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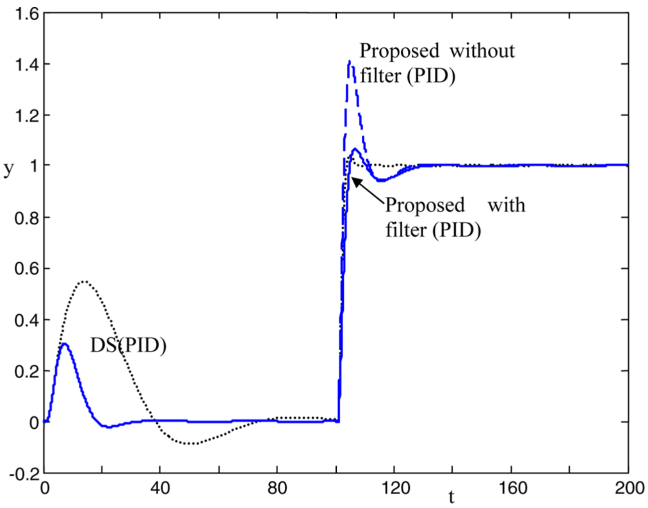

Figure 4. Control responses for the process $G(s) = \exp(-s)/(100s^2 + 10s + 1)$ (load step size: 5).

For this process of $a = 0.4$, PID controllers by the SIMC method ($\lambda = \theta$) and the proposed method ($\lambda = \theta$ and 0.6θ) can be designed. In addition to PID controllers, the PI controller can also be designed with the approximation of

$$G(s) = \frac{e^{-s}}{s(0.4s + 1)} \approx \frac{qe^{-s}}{(qs + 1)(0.4s + 1)} \approx \frac{qe^{-(1+0.4)s}}{qs + 1} \approx \frac{e^{-1.4s}}{s} \tag{31}$$

Figure 5 shows closed-loop responses of one PI controller and three PID controllers. The proposed controller can be designed from

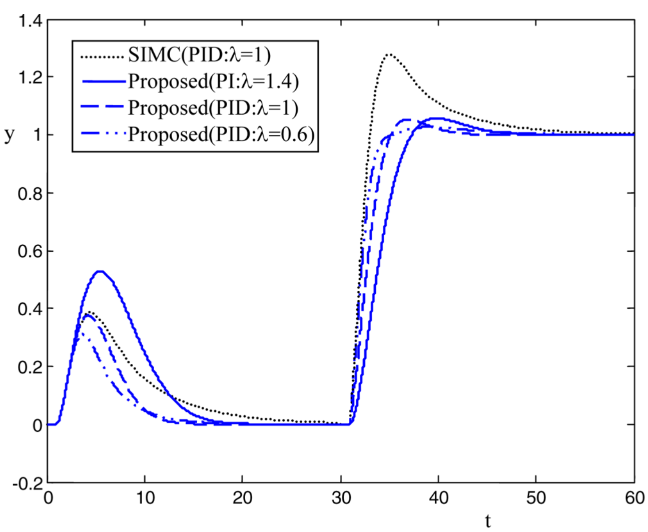

Figure 5. Control responses for the process $G(s) = \exp(-s)/(s(0.4s + 1))$ (load step size = 0.2).

$$G(s) = \frac{e^{-s}}{s(0.4s + 1)} = \frac{e^{-\eta s}}{s(0.4s + 1)e^{(1-\eta)s}} \approx \frac{e^{-\eta s}}{s((1.4 - \eta)s + 1)} \tag{32}$$

where $\lambda = 2\eta - 1$ (eq 19). We can see that the PI controller is also acceptable.

As a increases, the process approaches the delayed double integrating process. For $a = 20$, the series-form PID controller by the SIMC method ($\lambda = 1$) is $k_c = 0.5$, $\tau_I = 8$, and $\tau_D = 20$. However, because the derivative time is too large, it may be better to be $k_c = 1.25$, $\tau_I = 8$, and $\tau_D = 8$ with limiting τ_D. The proposed method uses an approximation (Table 4)

$$G(s) = \frac{e^{-s}}{s(20s + 1)} \approx \frac{qe^{-s}}{(qs + 1)(20s + 1)} = \frac{qe^{-s}}{20qs^2 + (20 + q)s + 1} \tag{33}$$

Figure 6 shows control performances. For the proposed method, $q = 20$ is used. The SIMC method with $\tau_D = 20$

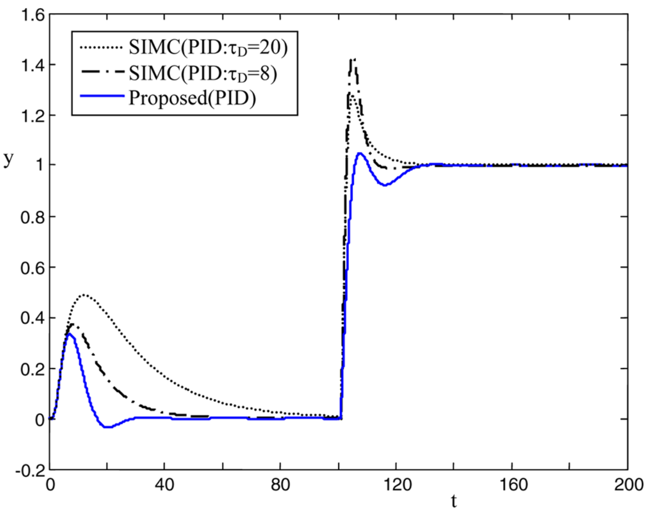

Figure 6. Control responses for the process $G(s) = \exp(-s)/(s(20s + 1))$.

shows sluggish load responses, and that with $\tau_D = 8$ shows a large overshoot. The proposed method does not have such problems.

Example 5 (Delayed Double Integrating Process). As the process time constant increases, the second order plus time delay process becomes the double integrating process with delay. Control performances are compared for the double integrating process:

$$G(s) = \frac{e^{-s}}{s^2} \tag{34}$$

Figure 7 shows closed-loop performances. The SIMC method shows a large overshoot for the step set point change. The SIMC method also needs the set point filter to reduce the overshoot for delayed integrating processes. The proposed method with the set point filter shows excellent responses for both load and set point changes.

Example 6. To illustrate the proposed model reduction rule of eq 4 modifying the SIMC half rule, we consider the process

$$G(s) = \frac{10(as + 1)}{(s + 1)^2(0.1s + 1)} \tag{35}$$

For $a < 0.25$, the SIMC method reduces the process as

$$G(s) = 10\frac{as + 1}{s + 1}\frac{1}{(s + 1)(0.1s + 1)} \approx \frac{10ae^{-0.05s}}{1.05s + 1} \tag{36}$$

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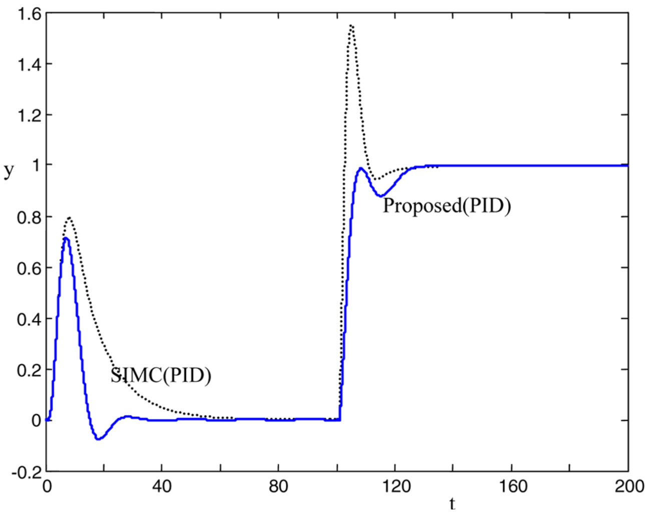

Figure 7. Control responses for the process $G(s) = \exp(-s)/s^2$ (load step size = 0.1).

On the other hand, the proposed method reduces the process as ($\lambda = 0.095$)

$$G(s) = \frac{10(as+1)}{s+1}\frac{1}{(s+1)(0.1s+1)} \approx \begin{cases} 10\frac{\sqrt{1+(a/0.095)^2}}{\sqrt{1+(1/0.095)^2}}\frac{e^{-0.095s}}{1.005s+1}, & a \geq 0.475 \\ 4.8224\frac{as+1}{0.475s+1}\frac{e^{-0.095s}}{1.005s+1}, & a < 0.475 \end{cases} \tag{37}$$

Figure 8 shows closed-loop performances. The SIMC method shows large overshoots for the step set point change. These large overshoots are due to the small effective time delay in the reduced model eq 36.

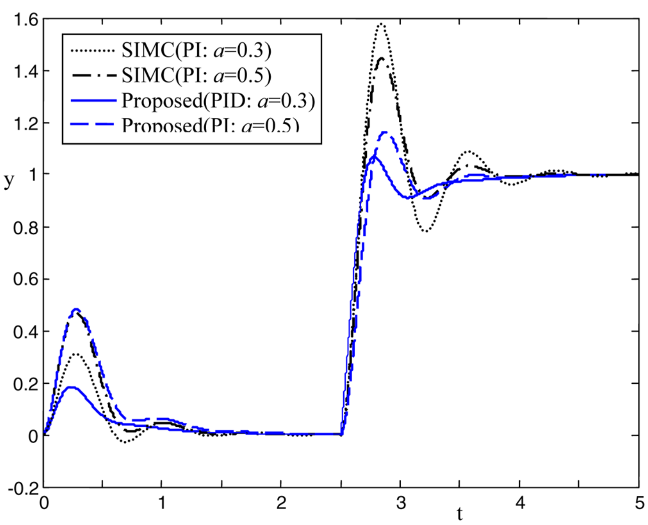

Figure 8. Control responses for the process $G(s) = (as + 1)/((s + 1)^2 (0.1s + 1))$ (load step size = 10).

Example 7. The proposed method is applied to the process

$$G(s) = \frac{(1.6s+1)e^{-s}}{(20s+1)(s+1)} \tag{38}$$

This process is selected to check the validity of the Rule T1 of SIMC. The SIMC method designs the PI controller from the approximation of

$$G(s) = \frac{1.6s+1}{s+1}\frac{e^{-s}}{20s+1} \approx 1.6\frac{e^{-s}}{20s+1} \tag{39}$$

The proposed method with $\lambda = 1$ uses the approximation of

$$G(s) = \frac{1.6s+1}{s+1}\frac{e^{-s}}{20s+1} \approx \frac{\sqrt{1+(1.6/1)^2}}{\sqrt{1+(1/1)^2}}\frac{e^{-s}}{20s+1} = \frac{1.3342e^{-s}}{20s+1} \tag{40}$$

The process gain by the proposed method is less (about 17%) than that of the SIMC method. Skogestad[4] worried that the process gain by the Rule T1 becomes too large and the resulting control system becomes sluggish. Our approximation may relieve this concern. Figure 9 shows the responses of both methods. The proposed model reduction method provides the PI controller that is a little less conservative.

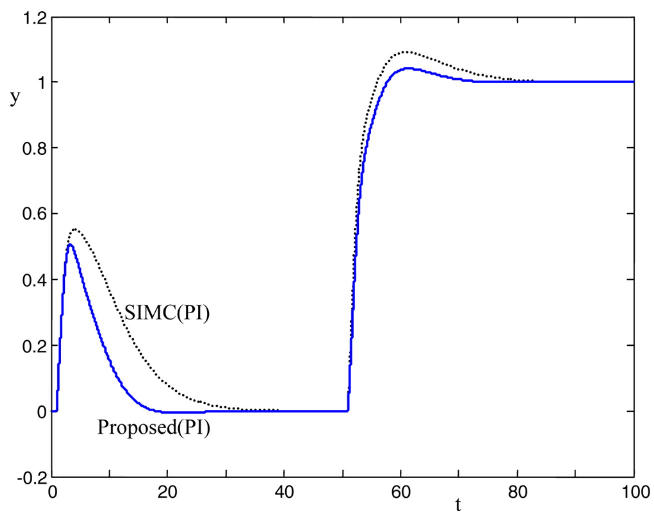

Figure 9. Control responses for the process $G(s) = 5(1.6s + 1)\exp(-s)/((20s + 1)(s + 1))$.

Example 8. Consider the process

$$G(s) = \frac{(s+1)e^{-s}}{(0.2s+1)^2} \tag{41}$$

This process is selected to check the validity of the Rule T1b of SIMC. The SIMC method designs the PI controller from the approximation of

$$G(s) = \frac{(s+1)e^{-s}}{(0.2s+1)^2} \approx \frac{e^{-s}}{0.2s+1} \tag{42}$$

The proposed method with $\lambda = 1$ uses the approximation of

$$G(s) = \frac{(s+1)e^{-s}}{(0.2s+1)^2} \approx \frac{\sqrt{1+1}}{\sqrt{1+0.04}}\frac{e^{-s}}{0.2s+1} = \frac{1.3868e^{-s}}{0.2s+1} \tag{43}$$

The process gain of the proposed method is greater (41%) than that of the SIMC method. Hence the proposed method will

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provide a more conservative (lower) controller gain. Figure 10 shows responses of the SIMC method and I-SIMC method[11]

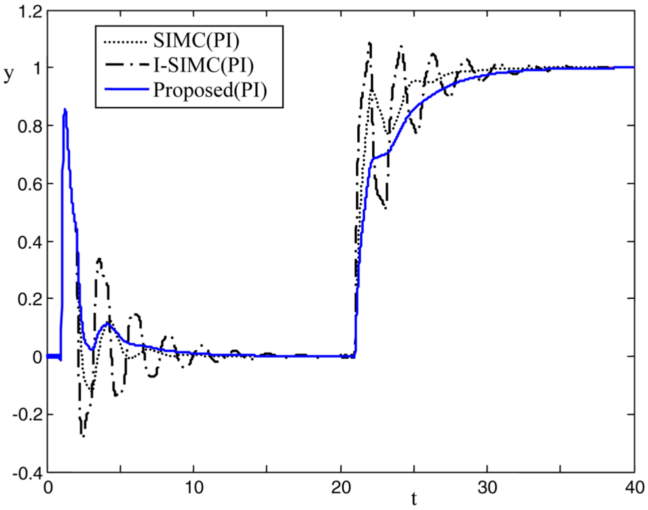

Figure 10. Control responses for the process $G(s) = \exp(-s)(s + 1)/(0.2s + 1)^2$ (load step size = 0.4).

for the model of eq 41 and the proposed K-SIMC method for the model of eq 42. We can see that closed-loop responses of the SIMC and I-SIMC methods are too oscillatory. Robustness indexes in Table 5 also show problems with the SIMC method. However, for this process, the oscillatory responses of the SIMC method can be avoided by increasing the design parameter λ.

Example 9. Consider the process

$$G(s) = \frac{(2s + 1)e^{-s}}{(5s + 1)(0.1s + 1)} \tag{44}$$

This process can check the validity of Rule T3 of SIMC. The SIMC method designs the PI controller by approximating the process as

$$G(s) = \frac{(2s + 1)e^{-s}}{(5s + 1)(0.1s + 1)} \approx \frac{e^{-s}}{((5 - 2)s + 1)(0.1s + 1)} \approx \frac{e^{-1.05s}}{3.05s + 1} \tag{45}$$

The proposed method uses the approximation of, with letting $\lambda = 1.1$,

$$\begin{aligned} G(s) &= \frac{(2s + 1)e^{-s}}{(5s + 1)(0.1s + 1)} \\ &\approx \frac{1 + 4/2.2^2}{1 + 10/2.2^2} \frac{e^{-s}}{\left(\frac{5-2}{1+10/2.2^2}s + 1\right)(0.1s + 1)} \\ &= \frac{0.5957e^{-s}}{(0.9784s + 1)(0.1s + 1)} \approx \frac{0.5957e^{-1.095s}}{0.9835s + 1} \end{aligned} \tag{46}$$

Figure 11 shows closed-loop responses of both methods. The SIMC method with $\lambda = 1.05$ shows somewhat oscillatory responses with a high Ms of 2.3 (Table 5). Responses are made to be less oscillatory by increasing λ to 1.5 but become sluggish. Responses of the proposed method are excellent.

Example 10. Consider the process

$$G(s) = \frac{(s + 1)e^{-s}}{s(as + 1)} \tag{47}$$

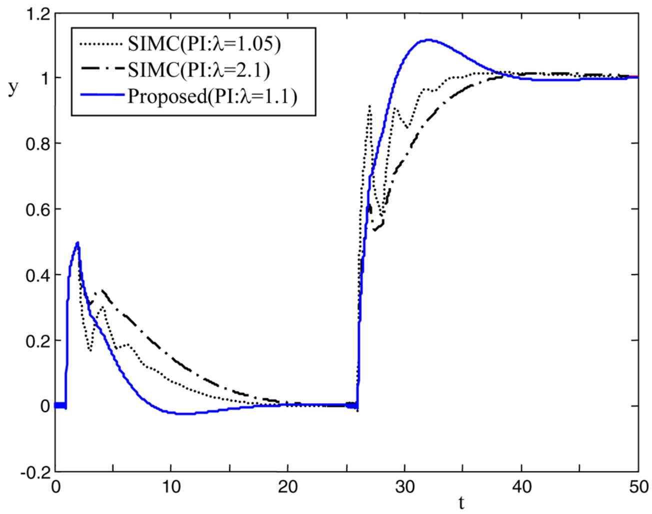

Figure 11. Control responses for the process $G(s) = \exp(-s)(2s + 1)/((5s + 1)(0.1s + 1))$.

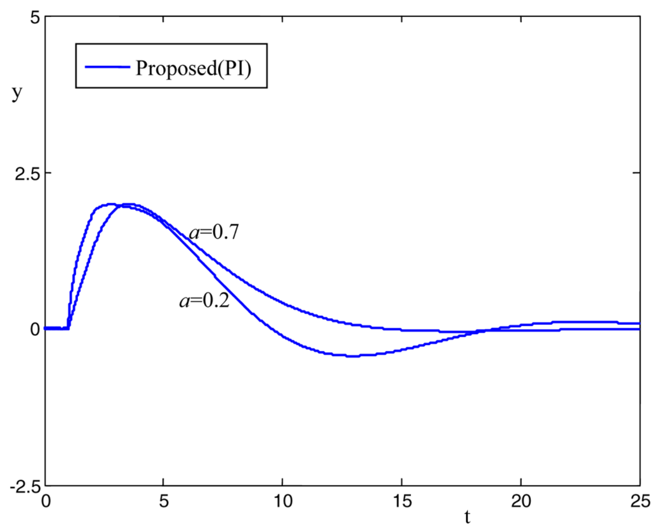

Figure 12. Load responses for the process $G(s) = \exp(-s)(s + 1)/(s(as + 1))$ (load step size = 0.2).

For $a = 0.2$, the proposed method (K-SIMC) uses the approximation of, with letting $\lambda = 1.2$ and applying our model reduction rules in Table 1,

$$\begin{aligned} G(s) &= \frac{(s + 1)e^{-s}}{s(0.2s + 1)} \approx \frac{q(s + 1)e^{-s}}{(qs + 1)(0.2s + 1)} \\ &= \frac{q(6s + 1)}{(qs + 1)} \frac{(s + 1)e^{-s}}{(6s + 1)(0.2s + 1)} \\ &\approx \frac{q\sqrt{1 + (6/1.2)^2}}{\sqrt{1 + (q/1.2)^2}} \frac{1 + 1/2.4^2}{1 + 6/2.4^2} \frac{e^{-s}}{\left(\frac{6-1}{1+6/2.4^2}s + 1\right)(0.2s + 1)} \\ &\approx \frac{q}{q/1.2} \frac{2.9311e^{-s}}{(2.449s + 1)(0.2s + 1)} \approx \frac{3.5173e^{-1.192s}}{2.457s + 1} \end{aligned} \tag{48}$$

Here q is assumed to be very large. For $a = 0.7$, with letting $\lambda = 1$,

$$G(s) = \frac{(s+1)e^{-s}}{s(0.7s+1)} \approx \frac{(s+1)qe^{-s}}{(0.7s+1)(qs+1)} \approx \frac{\sqrt{1+1^2}}{\sqrt{1+0.7^2}} \frac{qe^{-s}}{qs+1} = \frac{1.1586e^{-s}}{s} \quad (49)$$

For this process, Table 5.2 in the improved SIMC method[11] can be misleading to design unstable control systems. The additional condition of $T \geq 5\theta$ in addition to $T \geq \tau_2$ should be added. The proposed K-SIMC method shows stable responses for both a values of 0.2 and 0.7 as shown in Figure 12.

■ CONCLUSION

Model reduction techniques and tuning rules in the SIMC method are reconfirmed and some are modified. The proposed modifications are summarized below:

[1] The half rule of the SIMC method is slightly modified.
[2] Three model reduction rules of Rule T1, T1a, T1b, and T2 of the SIMC method are combined.
[3] The model reduction rule of Rule T2 of the SIMC method is refined.
[4] A method to treat the process integrating element is included in the model reduction techniques.
[5] A tuning rule to improve PID controllers for a design parameter less than the effective process time delay is proposed.
[6] Set point filters corresponding to limits on the controller integral and derivative times are suggested. It is shown that limits on the controller integral and derivative times work well for underdamped second-order processes.

The modifications between [1] and [4] are given in Table 1 and those of [5] and [6] are in Tables 2 and 3. The proposed modifications can be used for the SIMC method to be applied with more confidence.

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Notes

The authors declare no competing financial interest.

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