

TWO-DEGREE OF FREEDOM CONTROLLER TUNING FOR INTEGRAL PLUS TIME DELAY PLANTS

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ABSTRACT. *The paper describes two-degree of freedom analog PI and PID controllers tuning by a multiple dominant pole method for integral plus time delay plants. The effectiveness of the proposed techniques is shown in the example.*

Keywords: Two-degree of freedom controller, Controller tuning, Time delay

1. **Introduction.** The PI and PID controllers tuning for integral plus time delay (IPTD) plants with the transfer function

$$G_P(s) = \frac{k_1}{s} e^{-T_d s} \quad (1)$$

belong to demanding problems (k_1 – the plant gain, T_d – the plant time delay). It is caused by existence of two integrators in the control loop and thereby by the great predisposition to large overshoots, eventually to oscillation and stability loss. From other point of view the use of the controllers with the integral term enables removing the steady-state error caused in the plant input by the disturbance step change.

In case of the plants with a proportional character, the controllers are often tuned from the point of view of a servo control so that regulatory control is a trade-off. If plants have integral character, the trade-off tuning does not guarantee the quality from both points of view. The use of the two-degree of freedom (2DOF) controllers seems to be suitable in these cases.

2. **2DOF Controllers.** The commercial 2DOF PID controller can be described in the complex variable domain by the relation [2]

$$U(s) = r_0 \left\{ bW(s) - Y(s) + \frac{1}{T_I s} [W(s) - Y(s)] + T_D s [cW(s) - Y(s)] \right\} \quad (2)$$

where $U(s)$ – the transform of the manipulated variable $u(t)$, $W(s)$ – the transform of the desired variable $w(t)$, $Y(s)$ – the transform of the controlled variable $y(t)$, r_0 – the controller gain, T_I – the integral time, T_D – the derivative time, b – the set-point weight of the proportional term, c – the set-point weight of the derivative term. Both set-point weights can change in the interval from 0 to 1.

The control system with the 2DOF controller (2) can be transformed in the system in Figure 1 with the input filter with the transfer function

$$G_F(s) = \frac{W'(s)}{W(s)} = \frac{cT_I T_D s^2 + bT_I s + 1}{T_I T_D s^2 + T_I s + 1} \quad (3)$$

and the standard PID controller with the transfer function

$$G_C(s) = r_0 \left(1 + \frac{1}{T_I s} + T_D s \right) = r_0 + \frac{r_{-1}}{s} + r_1 s \quad (4)$$

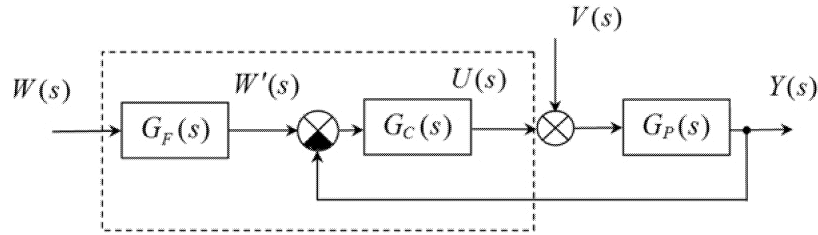


FIGURE 1. Control system with 2DOF controller

Similarly, for $T_D = 0$ from relation (2) the 2DOF PI controller can be obtained and in accordance with Figure 1 the corresponding standard PI controller and the input filter can be obtained too.

It is obvious that for $b = c = 1$ from relation (2) the standard PID controller (4) and for $T_D = 0$ the standard PI controller can be obtained.

3. Control of IPTD Plants. The transfer function of the closed-loop control system with the standard PID controller (4) without the input filter (3) and for the plant (1) is given by the relation

$$G_{w'y}(s) = \frac{Y(s)}{W'(s)} = \frac{k_1 r_0 (T_I T_D s^2 + T_I s + 1)}{T_I s^2 + k_1 r_0 (T_I T_D s^2 + T_I s + 1) e^{-T_d s}} e^{-T_d s} \quad (5)$$

Similarly, for $T_D = 0$ from relation (5) the transfer function of the closed-loop control system with the standard controller PI without the input filter can be obtained.

Most of the known PI and PID controllers tuning methods for IPTD plants give practically acceptable results for the regulatory control but large overshoots for the servo control [1,3,5,7]. These overshoots are caused by the polynomials in the nominators of the close-loop transfer functions, i.e. their stable zeros [see e.g.(5)]. By the use of the 2DOF PID controller (2) for $b = c = 0$ the close-loop transfer function

$$G_{wy}(s) = \frac{Y(s)}{W(s)} = \frac{k_1 r_0}{T_I s^2 + k_1 r_0 (T_I T_D s^2 + T_I s + 1) e^{-T_d s}} e^{-T_d s} \quad (6)$$

can be obtained and similarly, by the use of the 2DOF PI controller [relation (2) for $T_D = 0$] for $b = 0$ the corresponding closed-loop transfer function can be obtained too.

It is obvious that by using of the 2DOF PID and PI controllers the polynomials in the nominators of the closed-loop transfer functions can be fully compensated and so the undesirable large overshoots can be removed for the servo control.

Full removal of the stable zeros from the closed-loop transfer function nominator cases slows down the control process. If the set-point weights of the 2DOF controller can be continuously changed in the interval from 0 to 1, then for their tuning the multiple dominant pole (MDP) method can be used [7].

4. MDP Method. The MDP method supposes the existence of the stable real dominant pole with the multiplicity increased by 1 more than the number of the adjustable controller parameters [4,6,7].

For the characteristic quasipolynomial

$$N(s) = s^2 e^{T_d s} + k_1 (r_1 s^2 + r_0 s + r_{-1}) \quad (7)$$

of the closed-loop control system with the transfer function

$$G_{wy}(s) = \frac{G_C(s)G_P(s)}{1 + G_C(s)G_P(s)} = \frac{k_1 (r_1 s^2 + r_0 s + r_{-1})}{s^2 + k_1 (r_1 s^2 + r_0 s + r_{-1}) e^{-T_d s}} e^{-T_d s}$$

the multiple dominant pole and the PID controller adjustable parameters can be worked out of the system of equations

$$\frac{d^i N(s)}{ds^i} = 0 \quad \text{for } i = 0, 1, 2, 3 \tag{8}$$

From the system of equations (8) the quadruple dominant pole

$$s_4^* = -\frac{3 - \sqrt{3}}{T_d} \tag{9}$$

can be obtained and the PID controller adjustable parameters

$$r_1^* = \frac{\sqrt{3} - 1}{k_1} e^{\sqrt{3}-3}, \quad r_0^* = \frac{6(2\sqrt{3} - 3)}{k_1 T_d} e^{\sqrt{3}-3}, \quad r_{-1}^* = \frac{6(7\sqrt{3} - 12)}{k_1 T_d^2} e^{\sqrt{3}-3}$$

can be successively obtained too.

For $r_1 = 0$ in the characteristic quasipolynomial (7) from the system of equations (8) for $i = 0, 1, 2$ the triple dominant pole

$$s_3^* = -\frac{2 - \sqrt{2}}{T_d} \tag{10}$$

can be obtained and the PI controller adjustable parameters

$$r_0^* = \frac{2(\sqrt{2} - 1)}{k_1 T_d} e^{\sqrt{2}-2}, \quad r_{-1}^* = \frac{2(5\sqrt{2} - 7)}{k_1 T_d^2} e^{\sqrt{2}-2}$$

can be successively obtained too.

After considering the relations

$$T_I^* = \frac{r_0^*}{r_{-1}^*}, \quad T_D^* = \frac{r_1^*}{r_0^*}$$

for both controllers there can be obtained.

PI controller

$$r_0^* = \frac{2(\sqrt{2} - 1)}{k_1 T_d} e^{\sqrt{2}-2} \doteq 0.461 \frac{1}{k_1 T_d}, \quad T_I^* = (2\sqrt{2} + 3) T_d \doteq 5.828 T_d \tag{11}$$

PI controller

$$r_0^* = \frac{6(2\sqrt{3} - 3)}{k_1 T_d} e^{\sqrt{3}-3} \doteq 0.784 \frac{1}{k_1 T_d} \tag{12}$$

$$T_I^* = (2 + \sqrt{3}) T_d \doteq 3.732 T_d, \quad T_D^* = \frac{3 + \sqrt{3}}{18} T_d \doteq 0.263 T_d$$

Since for the MDP method for the standard PID controller is the quadruple dominant pole and for the standard PI controller it is the triple dominant pole, it is possible to achieve the removal of the large overshoot as well as the response acceleration for the servo control by the suitable set-point weights b and c .

For the 2DOF PID controller two poles of (5) can be compensated with two input filter zeros (3) by a suitable choice of the set-point weights b and c . These weights can be obtained via comparison of the coefficients for the same power of the complex variable s , i.e.

$$\left(\frac{1}{|s_4^*|} s + 1 \right)^2 = \frac{2 + \sqrt{3}}{6} T_d^2 s^2 + \frac{3 + \sqrt{3}}{3} T_d s + 1 = c T_I^* T_D^* s^2 + b T_I^* s + 1 \tag{13}$$

After substitution (12) in the relation (13) the set-point weights

$$b = \frac{3 - \sqrt{3}}{3} \doteq 0.423, \quad c = \frac{3 - \sqrt{3}}{2} \doteq 0.634 \tag{14}$$

can be obtained.

Similarly, for the 2DOF PI controller one pole can be compensated with the input filter zero by the suitable choice of the weight b , which can be obtained via comparison of the coefficients for the same power of the complex variable s , i.e.

$$\frac{1}{|s_3^*|}s + 1 = \frac{2 + \sqrt{2}}{2}T_d s + 1 = bT_I^* s + 1 \quad (15)$$

After substitution of the integral time (11) into (15) the weight

$$b = \frac{2 - \sqrt{2}}{2} \doteq 0.293 \quad (16)$$

can be obtained.

5. **Example.** For the IPTD plant

$$G_P(s) = \frac{0.05}{s}e^{-5s}$$

it is necessary for the 2DOF PI and PID controllers to be tuned by the MDP method (the time is in seconds).

Solution: In accordance with the relations (11) and (12), (13) and (14) for $k_1 = 0.05s^{-1}$ and $T_d = 5s$ can be obtained:

2DOF PI controller ($b \doteq 0.29$)

$$r_0^* \doteq 0.461 \frac{1}{k_1 T_d} \doteq 1.84, \quad T_I^* \doteq 5.828 T_d \doteq 29.14s$$

2DOF PID controller ($b \doteq 0.42$, $c \doteq 0.63$)

$$r_0^* \doteq 0.784 \frac{1}{k_1 T_d} \doteq 3.14, \quad T_I^* \doteq 3.732, \quad T_d \doteq 18.66s, \quad T_D^* \doteq 0.263 T_d \doteq 1.32s$$

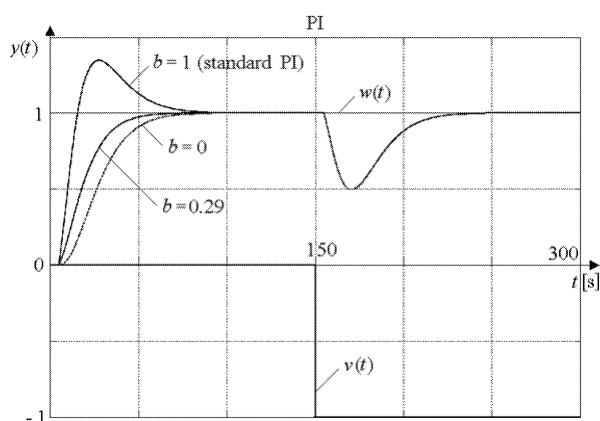


FIGURE 2. Courses of controlled variable for different values of weight b

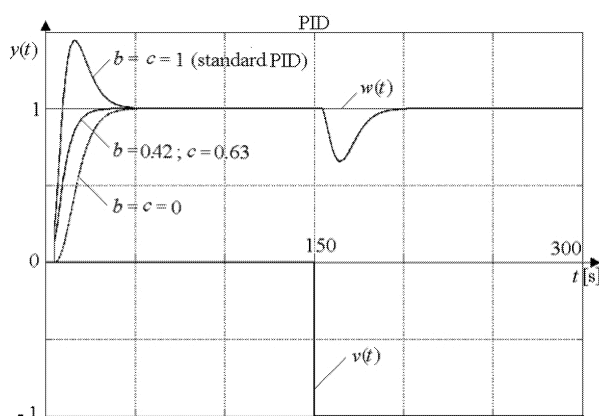


FIGURE 3. Courses of controlled variable for different values of weights b and c

The courses of the controlled variable for the desired variable step at $t = 0s$ and for the disturbance variable step at $t = 150s$ are shown for the different set-point weights in Figure 2 and Figure 3. From both figures it is obvious that by using the 2DOF controllers a removal of undesirable large overshoot for servo control was done. By applying the set-point weights, which are derived in the paper, the servo responses are without overshoots and essentially accelerated. The regulatory responses remained without any changes.

6. Conclusions. In this paper 2DOF PI and PID controller tuning by the multiple dominant pole method is described and the set-point weights are derived. The mentioned approach enables removing the undesirable overshoot of the controlled variable and simultaneously accelerating the servo responses in comparison with an ordinary approach when the zero set-point weights are chosen.

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