Evaluation of the Decentralized Closed-Loop Integrity for Multivariable Control Systems

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This paper presents a novel approach for evaluating the decentralized closed-loop integrity (DCLI) of multivariable control systems. Through application of the left−right factorization to the decomposed relative interaction array, the relative interaction to a particular loop from other loops is represented by element summation of the decomposed relative interaction sequence. The maximum interactions from other loops under different combinations and sequences are determined by the maximum values of the decomposed relative interaction sequence according to the failure index. Consequently, the necessary and sufficient conditions for the DCLI of an individual loop under both single- and multiple-loop failure are provided. Also, a simple and effective algorithm for verifying the DCLI for multivariable processes is developed. The usefulness of the proposed approach is illustrated by two classical examples.

1. Introduction

Despite the availability of sophisticated methods for designing centralized control systems, decentralized control remains popular in many industrial applications for the following reasons:[1,2]

1. Hardware simplicity: The cost of implementation of a decentralized control system is significantly lower than that of a centralized controller. A centralized control system for an $n \times n$ plant consists of n! individual single-input single-output transfer functions, which significantly increases the complexity of the controller hardware. Furthermore, if the controlled and/or manipulated variables are physically far apart, a full controller could require numerous expensive communication links.

2. Design and tuning simplicity: Decentralized controllers involve far fewer parameters, resulting in a significant reduction in the time and cost of tuning.

3. Flexibility in operation: A decentralized structure allows operating personnel to restructure the control system by bringing subsystems in and out of service individually, which allows the system to handle changing control objectives during different operating conditions.

The flexibility to bring subsystems in and out of service is very important also for the situations when actuators or sensors in some subsystems fail. The characteristic of failure tolerance is that, without readjustment to the other parts of the control system, stability can be preserved in the case of any sensor failure and/or actuator failure.[3] The relative gain array (RGA),[4,5] Niederlinski index (NI),[6] and block relative gain[7] are widely used for eliminating pairing that produces unstable closed-loop systems under failure conditions.[8−11] Chiu and Arkun[12] introduced the concept of decentralized closed-loop integrity (DCLI), which requires that the decentralized control structure should be stabilized by a controller having integral action and should maintain its nominal stability in the face of failures in its sensors and/or actuators. A number of necessary or sufficient conditions for DCLI were also developed.[12,13] However, the necessary and sufficient conditions for DCLI are still not available. Morari and co-workers[10,14] defined decentralized integral controllability (DIC) to address the operational issues, which consider the failure tolerance as a subproblem. Physically, a decentralized integral controllable system allows the operator to reduce the controller gains independently to zero without introducing instability (as a result of positive feedback). Some necessary and/or sufficient conditions for DIC were developed.[10,11,15,16] Even using only the steady-state gain information, however, the calculation[17] to verify the DIC is very complicated especially for a high dimension system, which still is an open problem.

In this paper, through application of the left−right (LR) factorization to the decomposed relative interaction array (DRIA), the relative interaction (RI) is converted to the summation of a series of values as presented by the decomposed relative interaction sequence (DRIS). The DRIS provides important insights into the cause−effect results of loop interaction due to the fact that the interactions are transferable through interactive loops. Subsequently, the maximum decomposed relative interaction factor (DRIF) of the loop providing the maximum interaction among the remaining loops is determined and used to analyze the RI of an individual loop in the face of single- or multiple-loop failure. Consequently, the necessary and sufficient conditions for the DCLI of an individual loop under both single- and multiple-loop failure are provided. Also, a simple and effective algorithm for verifying the DCLI for multivariable processes is developed.

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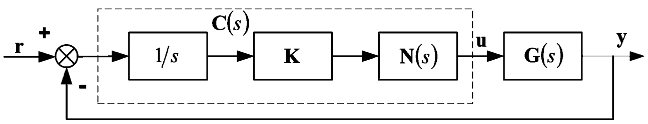

Figure 1. Decentralized integral control of multivariable systems.

The structure of this paper is as follows. In section 2, we revisit the DCLI as well as the necessary conditions provided by RGA and NI. In section 3, through application of the LR factorization to DRIA, the RI is transformed to the DRIS. The necessary and sufficient conditions for an individual loop possessing single- and multiple-loop failure tolerance are derived based on the DRIS and are given in section 4. An effective algorithm for verifying the DCLI of multivariable processes is developed in section 5. The usefulness of the novel approach is illustrated by two classical examples in section 6. The paper closes with our conclusions and future works in section 7.

2. Preliminaries

Throughout this paper, it is assumed that the system is square ($n \times n$), open-loop-stable, strictly proper, nonsingular at steady state, and under a decentralized control configuration, as shown in Figure 1. Here, $\mathbf{G}(s)$ is the transfer function matrix of the plant, and its steady-state gain matrix and individual elements are represented by $\mathbf{G}(0)$ (or simply $\mathbf{G}$) and g_{ij}, respectively. The decentralized controller $\mathbf{C}(s)$ can be decomposed into $\mathbf{C}(s) = \mathbf{N}(s)\,\mathbf{K}/s$, where $\mathbf{N}(s)$ is the transfer function matrix of the dynamic compensator, which is diagonal and stable and does not contain integral action, and $\mathbf{K} = \mathrm{diag}\{k_i\}$, where $i = 1, 2, ..., n$.

When loop failures of an arbitrary loop in system $\mathbf{G}(s)$ are investigated, all possible scenarios of the other $n - 1$ loops in any failure order have to be considered, which are as many as $(n - 1)!$. To effectively reflect these failed possibilities, we define a failure index M, which consists of $n - 1$ different integers: $M = \{i_1, ..., i_m, ..., i_{n-1}\}$, where $m, i_m \in [1, n - 1]$.

The following definitions and theorems for loop pairing and DCLI for multivariable control systems are needed in our development.

Definition 2.1.[4] The relative gain for variable pairing $\mathbf{y}_i-\mathbf{u}_j$ is defined as the ratio of two gains representing, first, the process gain in an isolated loop and, second, the apparent process gain in the same loop when all other loops are closed

$$\lambda_{ij} = \frac{(\partial \mathbf{y}_i/\partial \mathbf{u}_j)_{\mathbf{u}_{l\neq j}\mathrm{constant}}}{(\partial \mathbf{y}_i/\partial \mathbf{u}_j)_{\mathbf{y}_{k\neq i}\mathrm{constant}}} = g_{ij}[\mathbf{G}^{-1}]_{ji}$$

and RGA, $\Lambda(\mathbf{G})$, in matrix form is defined as

$$\Lambda(\mathbf{G}) = [\lambda_{ij}] = \mathbf{G} \otimes \mathbf{G}^{-\mathrm{T}}$$

where $\otimes$ is the Hadamard product and $\mathbf{G}^{-\mathrm{T}}$ is the transpose of the inverse of $\mathbf{G}$.

Definition 2.2.[6] Given a multivariable process $\mathbf{G}(s)$, NI is defined as

$$\mathrm{NI}(\mathbf{G}) = \det[\mathbf{G}]/\prod_{i=1}^{n} g_{ii}$$

The pairing rules based on RGA and NI are that manipulated and controlled variables in a decentralized control system should be paired in such a way that (i) the paired RGA elements are closest to 1.0, (ii) NI is positive, (iii) all paired RGA elements are positive, and (iv) large RGA elements should be avoided.

Definition 2.3.[18] The RI for loop pairing $\mathbf{y}_i-\mathbf{u}_j$ is defined as the ratio of two elements: the increment of the process gain after all other control loops are closed and the apparent gain in the same loop when all other control loops are open

$$\phi_{ij,n-1} = \frac{(\partial \mathbf{y}_i/\partial \mathbf{u}_j)_{\mathbf{y}_{k\neq i}\mathrm{constant}} - (\partial \mathbf{y}_i/\partial \mathbf{u}_j)_{\mathbf{u}_{l\neq j}\mathrm{constant}}}{(\partial \mathbf{y}_i/\partial \mathbf{u}_j)_{\mathbf{u}_{l\neq j}\mathrm{constant}}} = \frac{1}{\lambda_{ij}} - 1 \quad (1)$$

where the subscript $ij,n-1$ indicates that the RI is from the other $n - 1$ loops to individual loop $\mathbf{y}_i-\mathbf{u}_j$.

Even though the interpretations of RI and RGA are different, they are, nevertheless, equivalent because one can be derived from another through simple transformation of coordinates. Therefore, the properties of the RI can be easily derived from the RGA.[18-21] Similarly to the RGA-based loop pairing rule,[4] one can obtain the following loop pairing rule in terms of the RI as

$$\phi_{ij,n-1} \to 0 \quad \text{and} \quad \phi_{ij,n-1} > -1 \quad (2)$$

However, both RI- and RGA-based pairing rules do not offer any suggestions on the reverse effect of individual loop and loop-by-loop interactions, which may lead to undesirable loop pairing. To solve this problem, He and Cai decomposed the RI as DRIA to give important insights into the cause-effect results of loop interactions.[22]

Definition 2.4.[22] The DRIA of an $n \times n$ system is given as

$$\Psi_{ij,n-1} = \Delta\mathbf{G}_{ij,n-1} \otimes [\mathbf{G}^{ij}]^{-\mathrm{T}} \quad (3)$$

with its klth element

$$\psi_{ij,kl} = \phi_{ij,kl}\lambda_{kl}^{ij} \quad (4)$$

where $\mathbf{G}^{ij}$ is the transfer function matrix $\mathbf{G}$ with its ith row and jth column removed.

$$\Delta\mathbf{G}_{ij,n-1} = -\frac{1}{g_{ij}} g_{\bullet j}^{ij} g_{i\bullet}^{ij} \quad (5)$$

is the incremental process gain matrix of subsystem $\mathbf{G}^{ij}$ when loop $\mathbf{y}_i-\mathbf{u}_j$ is closed, $\phi_{ij,kl}$ is the RI between loop $\mathbf{y}_i-\mathbf{u}_j$ and loop $\mathbf{y}_k-\mathbf{u}_l$, λ_{kl}^{ij} is the relative gain of loop $\mathbf{y}_k-\mathbf{u}_l$ in subsystem $\mathbf{G}^{ij}$, and vectors $g_{i\bullet}^{ij}$ and $g_{\bullet j}^{ij}$ are the ith row and the jth column of $\mathbf{G}$ with the ijth element, g_{ij}, removed.

Using DRIA, the RI, $\phi_{ij,n-1}$, can be decomposed according to the following theorem:

Theorem 2.1.[22] For an arbitrary nonzero element g_{ij} of $\mathbf{G}$, the corresponding $\phi_{ij,n-1}$ is the sum of all elements of $\Psi_{ij,n-1}$

$$\phi_{ij,n-1} = ||\Psi_{ij,n-1}||_{\Sigma} = \sum_{k=1,k\neq i}^{n} \sum_{l=1,l\neq j}^{n} \psi_{ij,kl}$$

where $||\mathbf{A}||_\Sigma$ is the summation of all elements in a matrix $\mathbf{A}$.

In the design of a decentralized control system, it is desirable to choose input/output pairings such that the system possesses the property of DCLI, which is defined as follows:

Definition 2.5.[12] A stable plant is said to be DCLI if it can be stabilized by a stable decentralized controller, which contains integral action as shown in Figure 1, and if it remains stable after failure occurs in one or more of the feedback loops.

The necessary conditions for a system to be DCLI is given as follows:

Theorem 2.2 (Necessary Conditions for DCLI[12,13]). Given an $n \times n$ stable process $\mathbf{G}(s)$, the closed-loop system of decentralized feedback structure possesses DCLI only if

$$[\Lambda(\mathbf{G}_m)]_{ii} > 0, \quad \forall\, m = 1, ..., n;\, i = 1, ..., m \tag{6}$$

or

$$\mathrm{NI}[\mathbf{G}_m] > 0, \quad \forall\, m = 1, ..., n \tag{7}$$

where $\mathbf{A}_m$ is an arbitrary $m \times m$ principal submatrix of $\mathbf{A}$.

In theorem 2.2, either RGA or NI can be used as a necessary condition to examine the DCLI of decentralize control systems. However, the necessary and sufficient conditions for DCLI with respect to single- and multiple-loop failure are still unknown.

3. DRIS

We first reveal the relationship between DRIA and RGA, which is fundamental for the remaining developments.

Lemma 3.1. For an arbitrary loop $\mathbf{y}_i-\mathbf{u}_i$ in system $\mathbf{G}$, the relationship between elements of $\Psi_{ii,n-1}$ and elements of Λ satisfies

$$\frac{\lambda_{il}}{\lambda_{ii}} = \sum_{k=1,k\neq i}^{n} \psi_{ii,kl}, \quad \forall\, i, l = 1, ..., n;\, l \neq i \tag{8}$$

and

$$\frac{\lambda_{ki}}{\lambda_{ii}} = \sum_{l=1,l\neq i}^{n} \psi_{ii,kl}, \quad \forall\, i, k = 1, ..., n;\, k \neq i \tag{9}$$

Because the relationship provided by eq 9 is similar to that provided by eq 8, only the relationship given by eq 8 is proved here.

Proof. Because

$$\frac{\lambda_{il}}{\lambda_{ii}} = \frac{(-1)^{i+l} g_{il} \det[\mathbf{G}^{il}]/\det[\mathbf{G}]}{(-1)^{i+i} g_{ii} \det[\mathbf{G}^{ii}]/\det[\mathbf{G}]} = \frac{(-1)^{i+l} g_{il} \det[\mathbf{G}^{il}]}{(-1)^{i+i} g_{ii} \det[\mathbf{G}^{ii}]}$$

$$= (-1)^{l-i} \frac{g_{il}}{g_{ii}} \frac{\sum_{k=1,k\neq i}^{n} [(-1)^{k+i-1} g_{ki} \det[(\mathbf{G}^{ii})^{kl}]]}{\det[\mathbf{G}^{ii}]}$$

$$= \sum_{k=1,k\neq i}^{n} \left[-\frac{g_{il} g_{ki}}{g_{ii}} \frac{(-1)^{k+l} \det[(\mathbf{G}^{ii})^{kl}]}{\det[\mathbf{G}^{ii}]} \right]$$

where$(\mathbf{G}^{ii})^{kl}$ is the transfer function matrix $\mathbf{G}$ with its ith and kth rows and ith and lth columns removed, using eq 4, we obtain

$$\frac{\lambda_{il}}{\lambda_{ii}} = \sum_{k=1,k\neq i}^{n} (\phi_{ii,kl} \lambda_{kl}^{ii}) = \sum_{k=1,k\neq i}^{n} \psi_{ii,kl}$$

Remark 1. Lemma 3.1 presents an important relationship between the elements of DRIA and those of RGA. By definition of the RGA number[23]

$$\text{RGA number} = ||\Lambda - \mathbf{I}||_{\text{sum}} = \sum_{i=1}^{n} [\lambda_{ii} |1 - 1/\lambda_{ii}| + \sum_{l=1,l\neq i}^{n} |\lambda_{il}/\lambda_{ii}|]$$

It is obvious that both $\lambda_{ii} \to 1$ and $|\lambda_{il}/\lambda_{ii}| \to 0$ are desired. As indicated by theorem 2.1 and lemma 3.1, this is consistent with the expectation that RI, ϕ_{ii}, and all elements of DRIA have smaller values. Furthermore, a smaller element $\psi_{ii,kl}$ means less interaction either between loop $\mathbf{y}_i-\mathbf{u}_i$ and loop $\mathbf{y}_k-\mathbf{u}_l$ or between loop $\mathbf{y}_k-\mathbf{u}_l$ and all of the other loops in subsystem $\mathbf{G}^{ii}$. Therefore, the DRIA provides more information than RGA, and selecting loop pairings that have smaller elements of DRIA is more effective than using the RGA-based loop pairing rules.

Using the LR matrix factorization method[24] to DRIA, $\Psi_{ii,n-1}$ can be factorized as

$$\Psi_{ii,n-1} = \mathbf{L}_{ii,n-1} \times \mathbf{R}_{ii,n-1} \tag{10}$$

where $\mathbf{L}_{ii,n-1}$ is a $n - 1 \times n - 1$ lower triangular matrix with its diagonal elements equal to unity and $\mathbf{R}_{ii,n-1}$ is a $n - 1 \times n - 1$ upper triangular matrix. Then, we have the following lemma:

Lemma 3.2. Given a subsystem $\mathbf{G}^{ii}$ of $\mathbf{G}$, if its first $n - m - 1$ loops are removed, then the RI to loop $\mathbf{y}_i-\mathbf{u}_i$ from the remaining m loops is the sum of all elements of the matrix that is produced by the submatrices $\mathbf{L}_{ii,m}$ and $\mathbf{R}_{ii,m}$

$$\phi_{ii,m} = ||\Psi_{ii,m}||_\Sigma = ||\mathbf{L}_{ii,m} \times \mathbf{R}_{ii,m}||_\Sigma \tag{11}$$

Proof. According to the LR factorization algorithm, the DRIA $\Psi_{ii,n-1}$ can be factorized step-by-step. The first step is given as

$$\Psi_{ii,n-1} = \begin{pmatrix} \psi_{ii,11} & \psi_{ii,12} & \cdots & \psi_{ii,1n} \\ \psi_{ii,21} & \psi_{ii,22} & \cdots & \psi_{ii,2n} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{ii,n1} & \psi_{ii,n2} & \cdots & \psi_{ii,nn} \end{pmatrix}_{n\neq i}$$

$$= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ \psi_{ii,21}/\psi_{ii,11} & 1 & & 0 \\ \vdots & & \ddots & \\ \psi_{ii,n1}/\psi_{ii,11} & 0 & & 1 \end{pmatrix}_{n\neq i} \times \begin{pmatrix} \psi_{ii,11} & \psi_{ii,12} & \cdots & \psi_{ii,1n} \\ 0 & & & \\ \vdots & & \tilde{\Psi}_{ii,n-2} & \\ 0 & & & \end{pmatrix}_{n\neq i} \tag{12}$$

Chart 1

$$\begin{aligned}\phi^M_{ii,n-1} &= \|\boldsymbol{\Psi}^M_{ii,n-1}\|_\Sigma = \|\mathbf{L}^M_{ii,n-1} * \mathbf{R}^M_{ii,n-1}\|_\Sigma \\ &= \underbrace{\begin{pmatrix} 1 & \cdots & 1 \end{pmatrix}}_{n-1} * \mathbf{L}^M_{ii,n-1} * \mathbf{R}^M_{ii,n-1} \times \underbrace{\begin{pmatrix} 1 & \cdots & 1 \end{pmatrix}}_{n-1}^T \\ &= \underbrace{\begin{pmatrix} \sum[\mathbf{L}^M_{ii,n-1}]_{\bullet 1} & \cdots & \sum[\mathbf{L}^M_{ii,n-1}]_{\bullet n-1} \end{pmatrix}}_{n-1} * \underbrace{\begin{pmatrix} \sum[\mathbf{R}^M_{ii,n-1}]_{1\bullet} & \cdots & \sum[\mathbf{R}^M_{ii,n-1}]_{n-1\bullet} \end{pmatrix}}_{n-1}^T \\ &= \sum_{p=1}^{n-1} [\sum[\mathbf{L}^M_{ii,n-1}]_{\bullet p}] * [\sum[\mathbf{R}^M_{ii,n-1}]_{p\bullet}] \\ &= \sum_{p=1}^{n-1} s^M_{ii,p}.\end{aligned}$$

where

$$\tilde{\boldsymbol{\Psi}}_{ii,n-2} = \begin{pmatrix} \psi_{ii,22} - \psi_{ii,12}\psi_{ii,21}/\psi_{ii,11} & \cdots & \psi_{ii,2n} - \psi_{ii,1n}\psi_{ii,21}/\psi_{ii,11} \\ \vdots & \ddots & \vdots \\ \psi_{ii,n2} - \psi_{ii,12}\psi_{ii,n1}/\psi_{ii,11} & \cdots & \psi_{ii,nn} - \psi_{ii,1n}\psi_{ii,n1}/\psi_{ii,11} \end{pmatrix}_{n\neq i}$$

In eq 12, the vectors $[1, \psi_{ii,21}/\psi_{ii,11}, ..., \psi_{ii,n1}/\psi_{ii,11}]^T$ and $[\psi_{ii,11}, \psi_{ii,12}, ..., \psi_{ii,1n}]$ are the first column and the first row of triangular matrices $\mathbf{L}_{ii,n-1}$ and $\mathbf{R}_{ii,n-1}$, respectively. On the basis of eq 4, the klth element $\tilde{\psi}_{ii,kl}$ of $\tilde{\Psi}_{ii,n-2}$ can be simplified as

$$\begin{aligned}\tilde{\psi}_{ii,kl} &= \psi_{ii,kl} - \psi_{ii,1l}\psi_{ii,k1}/\psi_{ii,11} \\ &= -\frac{g_{il}g_{ki}}{g_{ii}} \times \frac{\det[(\mathbf{G}^{ii})^{11}]\det[(\mathbf{G}^{ii})^{kl}] - \det[(\mathbf{G}^{ii})^{1l}]\det[(\mathbf{G}^{ii})^{i1}]}{\det[\mathbf{G}^{ii}]} \\ &= -\frac{g_{il}g_{ki}}{g_{ii}}\frac{\det[((\mathbf{G}^{ii})^{11})^{kl}]}{\det[\mathbf{G}^{ii}]} = \phi_{ii,kl}\lambda^{ii(11)}_{kl} = \psi^{11}_{ii,kl}\end{aligned}$$

where the superscript 11 means that loop $\mathbf{y}_1-\mathbf{u}_1$ is removed. Obviously,

$$\begin{aligned}\tilde{\boldsymbol{\Psi}}_{ii,n-2} &= [\boldsymbol{\Psi}_{ii,n-1} - [\boldsymbol{\Psi}_{ii,n-1}]_{\bullet 1} \times [\boldsymbol{\Psi}_{ii,n-1}]_{1\bullet}/[\boldsymbol{\Psi}_{ii,n-1}]_{11}]^{11} \\ &= \boldsymbol{\Psi}^{11}_{ii,n-2} \end{aligned} \quad (13)$$

is the DRIA of loop $\mathbf{y}_i-\mathbf{u}_i$ in subsystem $\mathbf{G}^{11}$. Therefore, on the basis of theorem 2.1,

$$\phi_{ii,n-2} = ||\boldsymbol{\Psi}_{ii,n-2}||_\Sigma = ||\mathbf{L}_{ii,n-2} \times \mathbf{R}_{ii,n-2}||_\Sigma$$

The above relationship can be applied straightforward to all loops of subsystem $\mathbf{G}^{ii}$. Consequently, the result given by eq 10 is obtained. If the top-left corner element of the matrix is not equal to zero, the similar factorization step can be continued loop-by-loop up to m to result in eq 11.

Remark 2. According to eq 12, because the top-left corner elements of DRIA such as $\psi_{ii,11}$ are applied in the denominator in every step of factorization, they must not be equal to zero, which requires that all elements of $\mathbf{G}$ not be equal to zero. In practice, this problem can be easily solved by setting those zero elements to a very small value, say 1×10^{-9}. Our simulation results show that using very small values to replace those zero elements does not avert the outcomes of the failure tolerance property.

Theorem 3.1. Supposing that the control configuration of system $\mathbf{G}$ has been selected, for an arbitrary failure index M, the RI to loop $\mathbf{y}_i-\mathbf{u}_i$ from the other $n-1$ loops can be represented by the summation of $n-1$ elements

$$\phi^M_{ii,n-1} = \sum_{p=1}^{n-1} s^M_{ii,p} \quad (14)$$

and

$$s^M_{ii,p} = \sum[\mathbf{L}^M_{ii,n-1}]_{\bullet p} \times \sum[\mathbf{R}^M_{ii,n-1}]_{p\bullet} \quad (15)$$

where $\bullet p$ and $p\bullet$ indicate the pth column and the pth row of the matrix, respectively.

Proof. Equation 14 can be derived straightforward from eq 11 because of Chart 1.

Definition 3.1. For individual loop $\mathbf{y}_i-\mathbf{u}_i$ in system $\mathbf{G}$

$$\mathbf{S}^M_{ii} = \{s^M_{ii,1}, ..., s^M_{ii,p}, ..., s^M_{ii,n-1}\} \quad (16)$$

and its individual element $s^M_{ii,p}$ are defined as DRIS and DRIF to failure index M, respectively. To explain the physical meaning of DRIF, we analyze an arbitrary element in DRIS, say $s^M_{11,1}$, as an example. If all control loops of subsystem $(\mathbf{G}^{11})^{22}$ are closed, the subsystem including loop $\mathbf{y}_1-\mathbf{u}_1$ and loop $\mathbf{y}_2-\mathbf{u}_2$ is given as

$$\tilde{\mathbf{G}}_{11-22} = \begin{pmatrix} \tilde{g}_{11} & \tilde{g}_{12} \\ \tilde{g}_{21} & \tilde{g}_{22} \end{pmatrix}$$

where $\sim$ indicates that subsystem $(\mathbf{G}^{11})^{22}$ is closed and

$$\tilde{g}_{11} = g_{11}/\lambda^{22}_{11} = g_{11}/[g_{11}\det[(\mathbf{G}^{22})^{11}]/\det[\mathbf{G}^{22}]] = \det[\mathbf{G}^{22}]/\det[(\mathbf{G}^{22})^{11}]$$

$$\tilde{g}_{12} = g_{12}/\lambda^{21}_{12} = g_{12}/[g_{12}\det[(\mathbf{G}^{21})^{12}]/\det[\mathbf{G}^{21}]] = \det[\mathbf{G}^{21}]/\det[(\mathbf{G}^{22})^{11}]$$

$$\tilde{g}_{21} = g_{21}/\lambda^{12}_{21} = g_{21}/[g_{21}\det[(\mathbf{G}^{12})^{21}]/\det[\mathbf{G}^{12}]] = \det[\mathbf{G}^{12}]/\det[(\mathbf{G}^{22})^{11}]$$

$$\tilde{g}_{22} = g_{22}/\lambda_{22}^{11} = g_{22}/[g_{22}\det[(\mathbf{G}^{11})^{22}]/\det[\mathbf{G}^{11}]] = \det[\mathbf{G}^{11}]/\det[(\mathbf{G}^{22})^{11}]$$

Now, if loop $\mathbf{y}_2-\mathbf{u}_2$ is also closed, the incremental RI to loop $\mathbf{y}_1-\mathbf{u}_1$ can be obtained as

$$\begin{aligned}\tilde{\phi}_{11,22} &= -\frac{\tilde{g}_{12}\tilde{g}_{21}}{g_{11}\tilde{g}_{22}}\\ &= -\frac{1}{g_{11}}\frac{\det[\mathbf{G}^{12}]\det[\mathbf{G}^{21}]/(\det[(\mathbf{G}^{22})^{11}])^2}{\det[\mathbf{G}^{11}]/\det[(\mathbf{G}^{22})^{11}]}\\ &= -\frac{1}{g_{11}}\frac{\det[\mathbf{G}^{12}]\det[\mathbf{G}^{21}]}{\det[\mathbf{G}^{11}]\det[(\mathbf{G}^{22})^{11}]}\\ &= [g_{12}\det[\mathbf{G}^{12}]/\det[\mathbf{G}]\, g_{21}\det[\mathbf{G}^{21}]/\det[\mathbf{G}]]/\\ &\qquad[-g_{12}g_{21}/(g_{11}g_{22})g_{22}\det[(\mathbf{G}^{22})^{11}]/\\ &\qquad\det[\mathbf{G}^{11}](g_{11}\det[\mathbf{G}^{11}]/\det[\mathbf{G}])^2]\\ &= \frac{1}{\psi_{22}^{11}}\frac{\lambda_{12}}{\lambda_{11}}\frac{\lambda_{21}}{\lambda_{11}}\end{aligned}$$

Then, from lemma 3.1

$$\begin{aligned}\tilde{\phi}_{11,22} &= \frac{(\psi_{11,22}+\psi_{11,23}+...+\psi_{11,2n})(\psi_{11,22}+\psi_{11,32}+...+\psi_{11,n2})}{\psi_{11,22}}\\ &= s_{11,1}^{M}\end{aligned}$$

which suggests the following:

1. If subsystem $(\mathbf{G}^{11})^{22}$ is closed, closing loop $\mathbf{y}_2-\mathbf{u}_2$ will make the RI to loop $\mathbf{y}_1-\mathbf{u}_1$ increase by a value of $s_{11,1}^{M}$.

2. Reversely, if loop $\mathbf{y}_2-\mathbf{u}_2$ is taken out of service, the RI to loop $\mathbf{y}_1-\mathbf{u}_1$ will decrease by a value of $s_{11,1}^{M}$.

To generalize the above explanation, we conclude that, for an arbitrary loop $\mathbf{y}_i-\mathbf{u}_i$, if the first $p-1$ loops of subsystem $\mathbf{G}^{ii}$ have already been taken out of service, the removal of the pth loop will decrease $\phi_{ii,n-p}^{M}$ by a value of $s_{ii,p}^{M}$.

The significances of the development in this section are as follows:

1. The RI, $\phi_{ii,n-1}^{M}$, to individual control loop $\mathbf{y}_i-\mathbf{u}_i$ from all other $n-1$ control loops is decomposed as the DRIS, $\mathbf{S}_{ii}^{M}$, according to the failure index M, such that the interaction from an arbitrary loop of the remaining closed loops is represented by the DRIF.

2. The DRIF, $s_{ii,p}^{M}$, indicates the interaction to individual control loop $\mathbf{y}_i-\mathbf{u}_i$ from the pth control loop of the remaining $n-p$ closed loops, which means that when the pth control loop is put in or taken out of service, the corresponding DRIF should be added to or subtracted from the overall interaction RI.

3. In terms of DRIS, not only the interactions between the individual loop and the remaining loops but also the interactions to this individual loop from any combination of loops taken out of service can be reflected precisely.

4. Tolerance to Loop Failures

From eqs 1 and 2, both large values and values close to -1 of RI imply significant interaction among individual loops. Because we are investigating the property of loop failure tolerance, only the lower boundary (-1) is considered. By selecting the maximum DRIF from all possible values, we can determine a failure index $\bar{M}$ corresponding DRIS $\mathbf{S}_{ii}^{\bar{M}}$ of individual loop $\mathbf{y}_i-\mathbf{u}_i$ as

$$\mathbf{S}_{ii}^{\bar{M}} = \{s_{ii,p}^{\bar{M}}|s_{ii,p}^{\bar{M}} = \max\{s_{ii,p}^{M}\}, p = 1, 2, ..., n-1\} \quad (17)$$

Therefore, taking the pth loop out of service according to failure index $\bar{M}$ will result in

$$\phi_{ii,n-p-1}^{\bar{M}} = \min\{\phi_{ii,n-p-1}^{M}\} \quad (18)$$

The value of RI is closest to -1, implying that the particular combination of loop failures has the most significant effect on the DCLI. On the basis of eqs 2 and 18 and theorem 2 of ref 3, we now provide the necessary and sufficient conditions if individual loop $\mathbf{y}_i-\mathbf{u}_i$ is DCLI to single-loop failure.

Theorem 4.1. For decentralized control multivariable process $\mathbf{G}$, individual loop $\mathbf{y}_i-\mathbf{u}_i$ is DCLI to single-loop failure if and only if

$$\phi_{ii,n-2}^{\bar{M}} > -1 \quad (19)$$

or

$$s_{ii,1}^{\bar{M}} < 1/\lambda_{ii} \quad (20)$$

Proof. *Sufficient:* In the case of an arbitrary loop failure, eqs 14 and 17 give the RI to individual loop $\mathbf{y}_i-\mathbf{u}_i$ as

$$\phi_{ii,n-2}^{M} = \sum_{p=2}^{n-1} s_{ii,p}^{M} = \phi_{ii,n-1}^{M} - s_{ii,1}^{M} \geq \phi_{ii,n-1}^{\bar{M}} - s_{ii,1}^{\bar{M}} = \phi_{ii,n-2}^{\bar{M}}$$

Obviously, when eq 19 holds, inequality $\phi_{ii,n-2}^{M} > -1$ holds. Therefore, the sign of the steady-state gain for individual loop $\mathbf{y}_i-\mathbf{u}_i$ does not change in the face of single-loop failure.

Necessary: Because individual loop $\mathbf{y}_i-\mathbf{u}_i$ possesses single-loop failure tolerance, the sign of its steady-state loop gain does not change in the face of any single-loop failure.

$$\phi_{ii,n-2}^{M} > -1, \quad \forall M \Rightarrow \phi_{ii,n-2}^{\bar{M}} > -1$$

Then, according to eqs 1 and 14, inequality 20 can be obtained.

Similar to single-loop failure, on the basis of eqs 2 and 18 and theorem 2 of ref 3, the necessary and sufficient conditions if individual loop $\mathbf{y}_i-\mathbf{u}_i$ is DCLI for multiple-loop failure are given as follows.

Theorem 4.2. For decentralized control multivariable process $\mathbf{G}$, individual loop $\mathbf{y}_i-\mathbf{u}_i$ is DCLI to multiple-loop failure if and only if

$$\phi_{ii,m_{\min}}^{M} > -1 \quad (21)$$

where

$$\phi_{ii,m_{\min}}^{M} = \min\{\sum_{p=n-m_{\min}}^{n-1} s_{ii,p}^{\bar{M}}|m_{\min} = 1, ..., n-1\} \quad (22)$$

Proof. *Sufficient:* In the case of $n-m-1$ loop failure, eqs 14 and 17 show that the RI to individual loop $\mathbf{y}_i - \mathbf{u}_i$ from the remaining m loops is

$$\phi_{ii,m}^{M} = \sum_{p=n-m}^{n-1} s_{ii,p}^{M} = \phi_{ii,n-1}^{M} - \sum_{p=1}^{n-m-1} s_{ii,p}^{M} \geq \phi_{ii,n-1}^{\bar{M}} - \sum_{p=1}^{n-m_{\min}-1} s_{ii,p}^{\bar{M}} = \phi_{ii,m_{\min}}^{M}$$

Obviously, when eq 21 holds, inequality $\phi_{ii,m}^{M} > -1$ always holds. Therefore, the sign of the steady-state gain for individual loop $\mathbf{y}_i - \mathbf{u}_i$ does not change in the face of multiple-loop failure.

Necessary: Because individual loop $\mathbf{y}_i - \mathbf{u}_i$ possesses multiple-loop failure tolerance, the sign of its steady-state loop gain does not change in the face of any single-loop failure.

$$\phi_{ii,m}^{M} > -1, \quad \forall M \Rightarrow \phi_{ii,m_{\min}}^{M} > -1$$

Remark 3. The significances of theorems 4.1 and 4.2 are as follows:

1. The necessary and sufficient conditions for both single- and multiple-loop failure tolerance are provided.
2. In the case where two or more control structures are DCLI, the one with $\phi_{ii,m_{\min}}^{M} \to 0$ should be preferred.
3. Single-loop failure is a special case of multiple-loop failure.

5. Pairings Algorithm for DCLI

In subsystem $\mathbf{G}^{ii}$, the DRIF $s_{ii,p}^{M}$ may have as many as $n - p$ possible values according to different failure sequences of the remaining $n - p$ loops. Therefore, to find either $\phi_{ii,m_{min}}^{\bar{M}}$ or $\phi_{ii,n-2}^{\bar{M}}$, one is required first to determine the index $\bar{M}$ and then to calculate the DRIS $\mathbf{S}_{ii}^{\bar{M}}$, where $s_{ii,p}^{M}$ can be determined by the first row and column of $\Psi_{ii,n-p}^{M}$ (eqs 12 and 15).

$$s_{ii,p}^{M} = \sum_{k=1}^{n-p} [\Psi_{ii,n-p}^{M}]_{1k} \sum_{k=1}^{n-p} [\Psi_{ii,n-p}^{M}]_{k1} / [\Psi_{ii,n-p}^{M}]_{11}$$

However, there is no need to arrange elements of DRIA $\Psi_{ii,n-p}^{M}$ $n - p$ times to calculate $s_{ii,p}^{\bar{M}}$ because the elements of DRIA are permutation-independent (eqs 3 and 5). In fact, once the DRIA $\Psi_{ii,n-p}$ for $p = 1$ has been obtained (eq 3), the DRIF $s_{ii,p}^{\bar{M}}$ can be directly calculated from

$$[s_{ii,p}^{\bar{M}}, l] = \max\{\mathrm{diag}[(\sum_{k=1}^{n-p} [\Psi_{ii,n-p}]_{\bullet k} \sum_{k=1}^{n-p} [\Psi_{ii,n-p}]_{k\bullet}) \odot \Psi_{ii,n-p}]\} \quad (23)$$

where function max[**A**] finds the maximum diagonal element of matrix **A** and provides its row number l in matrix $\Psi_{ii,n-p}$ and diag[**A**] is a diagonal matrix containing the diagonal elements of matrix **A**. $\odot$ indicates element-by-element division.

For checking the DCLI of individual loop $\mathbf{y}_i - \mathbf{u}_i$ against the failure of $p + 1$ loops, the DRIA $\Psi_{ii,n-p-1}$ can be recursively calculated as (eqs 12 and 13)

$$\Psi_{ii,n-p-1} = [\Psi_{ii,n-p} - [\Psi_{ii,n-p}]_{\bullet l} \times [\Psi_{ii,n-p}]_{l\bullet} / [\Psi_{ii,n-p}]_{ll}]^{ll} \quad (24)$$

and the DRIF $s_{ii,p+1}^{\bar{M}}$ can be calculated by applying DRIA $\Psi_{ii,n-p-1}$ to eq 23.

Therefore, for individual loop $\mathbf{y}_i - \mathbf{u}_i$ of $n \times n$ system **G**, after $\Psi_{ii,n-1}$ is obtained, its DRIS $\mathbf{S}_{ii}^{\bar{M}}$ can be calculated by using iterative eqs 23 and 24 $n - 2$ times, which requires only one matrix inverse of $n - 1$ order to calculate the DRIA; as shown by eq 3, the computational load is much reduced compared with that of permutation methods.

For a given multivariable process $\mathbf{G}(s)$, its control configuration can be obtained based on its steady-state transfer function matrix $\mathbf{G}(0)$ by using a loop-pairing criterion such as the one developed in ref 22. After all elements in matrix $\mathbf{G}(0)$ have been rearranged to place the gains of control loops in the diagonal position, the proposed method can be used to verify DCLI of the selected control configuration, and an algorithm is given as follows.

Algorithm 1.

Step 1. Calculate $\Psi_{ii,n-1}$ of loop $\mathbf{y}_i - \mathbf{u}_i$ by eqs 3 and 5.

Step 2. Obtain $s_{ii,p}^{\bar{M}}$ and $\mathbf{S}_{ii}^{\bar{M}}$ of loop $\mathbf{y}_i - \mathbf{u}_i$ by eqs 23 and 24.

Step 3. Verify single-loop failure tolerance by eq 19.

Step 4. Obtain $\phi_{ii,m_{\min}}^{\bar{M}}$ to loop $\mathbf{y}_i - \mathbf{u}_i$ from the other loops by eq 22.

Step 5. Verify multiple-loop failure tolerance by referring to eq 21.

Step 6. Repeat the previous five steps loop-by-loop until any one loop fails or all loops pass.

Step 7. End.

The procedure for the determination of DCLI for a decentralized control system is illustrated by the flowchart shown in Figure 2.

6. Case Study

6.1. Example 1. Consider the following 4×4 process[14] with the process steady-state transfer function matrix given by

$$\mathbf{G}(0) = \begin{pmatrix} 8.72 & -15.80 & 2.98 & 2.81 \\ 6.54 & -20.79 & 2.50 & -2.92 \\ -5.82 & -7.51 & -1.48 & 0.99 \\ -7.23 & 7.86 & 3.11 & 2.92 \end{pmatrix}$$

To verify DCLI to single-loop failure of the first loop $\mathbf{y}_1 - \mathbf{u}_1$ by using a RGA-based criterion, three alternatives have to be tested, namely, calculation of RGAs of subsystems $\mathbf{G}^{22}$, $\mathbf{G}^{33}$, and $\mathbf{G}^{44}$ for single-loop failure of $\mathbf{y}_2 - \mathbf{u}_2$, $\mathbf{y}_3 - \mathbf{u}_3$, and $\mathbf{y}_4 - \mathbf{u}_4$, respectively. Furthermore, to verify DCLI to multiple-loop failure, an additional three RGAs need to be calculated. Consequently, six inverse matrices have to be performed.

From application of algorithm 1 and with one matrix inverse, the DRIA of control loop $\mathbf{y}_1 - \mathbf{u}_1$ is obtained, and DRIS is calculated through a series of vector operations. The results for DCLI to single- and multiple-loop failures are listed in Table 1.

Initially, when all control loops in subsystem $\mathbf{G}^{11}$ are closed, the RI $\phi_{11,3}^{\bar{M}} = 1.4142 > -1$, implying that there is no sign change before and after subsystem $\mathbf{G}^{11}$ has been closed. Following Table 1, DCLI information of loop $\mathbf{y}_1 - \mathbf{u}_1$ can be obtained as follows:

1. Loop $\mathbf{y}_4 - \mathbf{u}_4$ provides the maximum interaction, and if it fails, the RI of loop $\mathbf{y}_1 - \mathbf{u}_1$ will decrease in a value of $s_{11,1}^{\bar{M}} = 2.4095$ and is $\phi_{11,2}^{\bar{M}} = -0.9953 > -1$, and for

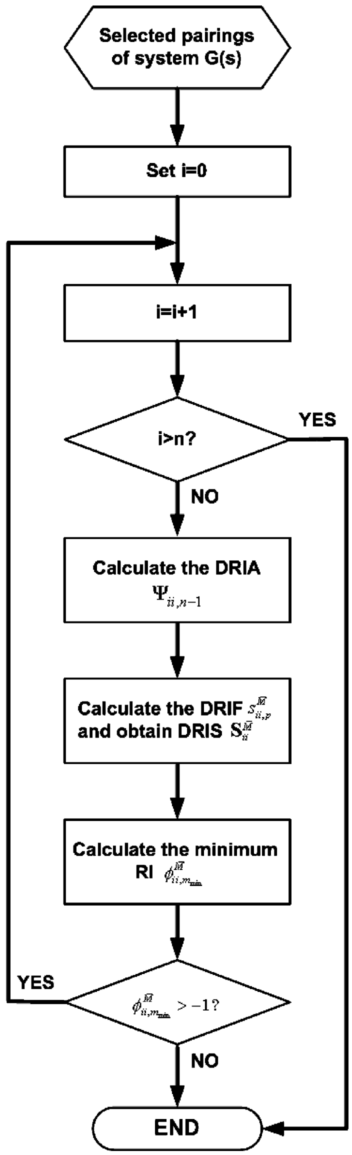

Figure 2. Flowchart for determining DCLI.

Table 1. DCLI Verification of Control Loop y_1-u_1[a]

loop	RI	DRIS	failed loop	DCLI SLF	DCLI MLF
$\mathbf{y}_1-\mathbf{u}_1$	$\phi^{\bar{M}}_{11,3} = 1.4142$			yes	no
	↓	$s^{\bar{M}}_{11,1} = 2.4095$	$\mathbf{y}_4-\mathbf{u}_4$		
	$\phi^{\bar{M}}_{11,2} = -0.9953$				
	↓	$s^{\bar{M}}_{11,2} = 0.3486$	$\mathbf{y}_2-\mathbf{u}_2$		
	$\phi^{\bar{M}}_{11,1} = -1.3439$				
	↓	$s^{\bar{M}}_{11,3} = -1.3439$	$\mathbf{y}_3-\mathbf{u}_3$		
	0				

[a] SLF = single-loop failure. MLF = multiple-loop failure.

loop $\mathbf{y}_1-\mathbf{u}_1$, the sign of its loop gain does not change for any single-loop failure. Therefore, loop $\mathbf{y}_1-\mathbf{u}_1$ is DCLI for single-loop failure.

2. Loop $\mathbf{y}_2-\mathbf{u}_2$ provides the maximum interaction among the two remaining loops after loop $\mathbf{y}_4-\mathbf{u}_4$ has already been taken out of service. If loop $\mathbf{y}_2-\mathbf{u}_2$ fails, the RI of loop $\mathbf{y}_1-\mathbf{u}_1$ will decrease in a value of $s^{\bar{M}}_{11,2} = 0.3486$ and is $\phi^{\bar{M}}_{11,1} = -1.3439 < -1$. Hence, the process gain of loop $\mathbf{y}_1-\mathbf{u}_1$ will change its sign, and it is not DCLI when both $\mathbf{y}_4-\mathbf{u}_4$ and $\mathbf{y}_2-\mathbf{u}_2$ fail (implying that $\mathbf{G}$ is not DCLI for multiple-loop failure).

Table 2. DCLI Verification of Control Loop y_1-u_1[a]

loop	RI	DRIS	failed loop	DCLI SLF	DCLI MLF
$\mathbf{y}_1-\mathbf{u}_1$	$\phi^{\bar{M}}_{11,3} = 1.1237$			yes	yes
	↓	$s^{\bar{M}}_{11,1} = 0.6885$	$\mathbf{y}_2-\mathbf{u}_2$		
	$\phi^{\bar{M}}_{11,2} = 0.4352$				
	↓	$s^{\bar{M}}_{11,2} = 1.4309$	$\mathbf{y}_3-\mathbf{u}_3$		
	$\phi^{\bar{M}}_{11,1} = -0.9957$				
	↓	$s^{\bar{M}}_{11,3} = -0.9957$	$\mathbf{y}_4-\mathbf{u}_4$		
	0				

[a] SLF = single-loop failure. MLF = multiple-loop failure.

Table 3. DCLI Verification of the Other Three Control Loops[a]

loop	RI	DRIS	failed loop	DCLI SLF	DCLI MLF
$\mathbf{y}_2-\mathbf{u}_2$	$\phi^{\bar{M}}_{11,3} = 1.2873$	$s^{\bar{M}}_{22,1} = 0.7416$	$\mathbf{y}_1-\mathbf{u}_1$	yes	yes
	$\phi^{\bar{M}}_{11,2} = 0.5458$	$s^{\bar{M}}_{22,2} = 0.2418$	$\mathbf{y}_3-\mathbf{u}_3$		
	$\phi^{\bar{M}}_{11,1} = 0.3039$	$s^{\bar{M}}_{22,3} = 0.3039$	$\mathbf{y}_4-\mathbf{u}_4$		
$\mathbf{y}_3-\mathbf{u}_3$	$\phi^{\bar{M}}_{11,3} = 1.4765$	$s^{\bar{M}}_{33,1} = 0.8086$	$\mathbf{y}_4-\mathbf{u}_4$	yes	yes
	$\phi^{\bar{M}}_{11,2} = 0.6679$	$s^{\bar{M}}_{33,2} = 0.2620$	$\mathbf{y}_1-\mathbf{u}_1$		
	$\phi^{\bar{M}}_{11,1} = 0.4059$	$s^{\bar{M}}_{33,3} = 0.4059$	$\mathbf{y}_2-\mathbf{u}_2$		
$\mathbf{y}_4-\mathbf{u}_4$	$\phi^{\bar{M}}_{11,3} = 0.7498$	$s^{\bar{M}}_{44,1} = 0.5713$	$\mathbf{y}_3-\mathbf{u}_3$	yes	yes
	$\phi^{\bar{M}}_{11,2} = 0.1785$	$s^{\bar{M}}_{44,2} = 1.1742$	$\mathbf{y}_2-\mathbf{u}_2$		
	$\phi^{\bar{M}}_{11,1} = -0.9957$	$s^{\bar{M}}_{44,3} = -0.9957$	$\mathbf{y}_1-\mathbf{u}_1$		

[a] SLF = single-loop failure. MLF = multiple-loop failure.

To show how DCLI is pairing-dependent, reconfigure the control structure using the loop pairing criterion proposed in ref 22 as follows:

$$\mathbf{G}(0) = \begin{pmatrix} 2.81 & -15.80 & 8.72 & 2.98 \\ -2.92 & -20.79 & 6.54 & 2.50 \\ 0.99 & -7.51 & -5.82 & -1.48 \\ 2.92 & 7.86 & -7.23 & 3.11 \end{pmatrix}$$

The DCLI results of $\mathbf{y}_1-\mathbf{u}_1$ are listed in Table 2.

From Table 2, we observe the following:

1. When all control loops in subsystem $\mathbf{G}^{11}$ are closed, the RI $\phi^{\bar{M}}_{11,3} = 1.1237 > -1$, implying that there is no sign change before and after subsystem $\mathbf{G}^{11}$ has been closed.

2. Loop $\mathbf{y}_1-\mathbf{u}_1$ can tolerate any single-loop failure because the minimal $\phi^{\bar{M}}_{11,2} = 0.4352 > -1$.

3. Loop $\mathbf{y}_1-\mathbf{u}_1$ can tolerate any double-loop failure because the minimal $\phi^{\bar{M}}_{11,1} = -0.9957 > -1$.

4. Because $\phi^{\bar{M}}_{11,3} > \phi^{\bar{M}}_{11,2} > 0$, if loop $\mathbf{y}_2-\mathbf{u}_2$ fails, interaction between loop $\mathbf{y}_1-\mathbf{u}_1$ and the remaining loops will be smaller.

5. If loop $\mathbf{y}_3-\mathbf{u}_3$ also fails, interaction between loop $\mathbf{y}_1-\mathbf{u}_1$ and loop $\mathbf{y}_4-\mathbf{u}_4$ becomes significant for $\phi^{\bar{M}}_{11,1} = -0.9957 \rightarrow -1$, implying that the equivalent process gain of loop $\mathbf{y}_1-\mathbf{u}_1$ will undergo a big change in the case of where either loop $\mathbf{y}_4-\mathbf{u}_4$ is closed first in system $\mathbf{G}$ or loop $\mathbf{y}_2-\mathbf{u}_2$ and loop $\mathbf{y}_3-\mathbf{u}_3$ fail first in closed subsystem $\mathbf{G}$.[11]

Using algorithm 1, DRIS of the other three control loops can be obtained and are listed in Table 3; all control loops are DCLI to both single- and multiple-loop failure.

6.2. Example 2. Consider the 4 × 4 distillation column studied by Chiang and Luyben (CL column).[25]

Table 4. DCLI Verification of the CL Column[a]

loop	RI	DRIS	failed loop	DCLI SLF	DCLI MLF
$\mathbf{y}_1-\mathbf{u}_1$	$\phi_{11,3}^{\bar{M}} = -0.5233$	$s_{11,1}^{\bar{M}} = 0.1783$	$\mathbf{y}_4-\mathbf{u}_4$	yes	yes
	$\phi_{11,2}^{\bar{M}} = -0.7017$	$s_{11,2}^{\bar{M}} = 0.0000$	$\mathbf{y}_3-\mathbf{u}_3$		
	$\phi_{11,1}^{\bar{M}} = -0.7017$	$s_{11,3}^{\bar{M}} = -0.7017$	$\mathbf{y}_2-\mathbf{u}_2$		
$\mathbf{y}_2-\mathbf{u}_2$	$\phi_{11,3}^{\bar{M}} = -0.2490$	$s_{22,1}^{\bar{M}} = 0.4527$	$\mathbf{y}_4-\mathbf{u}_4$	yes	yes
	$\phi_{11,2}^{\bar{M}} = -0.7017$	$s_{22,2}^{\bar{M}} = 0.0000$	$\mathbf{y}_3-\mathbf{u}_3$		
	$\phi_{11,1}^{\bar{M}} = -0.7017$	$s_{22,3}^{\bar{M}} = -0.7017$	$\mathbf{y}_1-\mathbf{u}_1$		
$\mathbf{y}_3-\mathbf{u}_3$	$\phi_{11,3}^{\bar{M}} = -0.3394$	$s_{33,1}^{\bar{M}} = -0.1074$	$\mathbf{y}_1-\mathbf{u}_1$	yes	yes
	$\phi_{11,2}^{\bar{M}} = -0.2320$	$s_{33,2}^{\bar{M}} = -0.2320$	$\mathbf{y}_4-\mathbf{u}_4$		
	$\phi_{11,1}^{\bar{M}} = 0.0000$	$s_{33,3}^{\bar{M}} = 0.0000$	$\mathbf{y}_2-\mathbf{u}_2$		
	$\phi_{11,3}^{\bar{M}} = 1.6000$	$s_{44,1}^{\bar{M}} = 1.5672$	$\mathbf{y}_2-\mathbf{u}_2$		
$\mathbf{y}_4-\mathbf{u}_4$	$\phi_{11,2}^{\bar{M}} = 0.0328$	$s_{44,2}^{\bar{M}} = 0.0122$	$\mathbf{y}_1-\mathbf{u}_1$	yes	yes
	$\phi_{11,1}^{\bar{M}} = 0.0206$	$s_{44,3}^{\bar{M}} = 0.0206$	$\mathbf{y}_3-\mathbf{u}_3$		

[a] SLF = single-loop failure. MLF = multiple-loop failure.

The steady-state transfer function matrix is given as follows:

$$\mathbf{G}(0) = \begin{pmatrix} 4.45 & -7.4 & 0 & 0.35 \\ 17.3 & -41 & 0 & 9.2 \\ 0.22 & -4.6 & 3.6 & 0.042 \\ 1.82 & -34.5 & 12.2 & -6.92 \end{pmatrix}$$

When the zero elements in $\mathbf{G}(0)$ are set to 1×10^{-9} to make the zero interaction a microinteraction, the maximum DRIF and DRIS of all control loops are obtained as listed in Table 4.

Obviously, as Table 4 indicates, all four control loops are DCLI to multiple-loop failure.

7. Conclusion

In this paper, a novel approach for evaluating DCLI for multivariable control systems was proposed. The DRIS was introduced to represent the RI to a particular loop from other loops. The maximum DRIF was used to find the maximum interaction from the remaining loops among all possible failure indexes. Consequently, the necessary and sufficient conditions for DCLI of an individual loop under both single- and multiple-loop failure were provided. A simple and effective algorithm for verifying DCLI for multivariable control systems was developed. Two classical examples were used to illustrate the effectiveness of the proposed approach. Because DRIS provides more detailed information of interactions among loops, it can be used to design robust multiloop controllers for multivariable processes. This topic is currently under investigation, and the results will be reported later.

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