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A strategy for simulation and optimization of gas and oil production

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Abstract

A new formulation to easy simulation and optimization of gas and oil production systems is presented in this work. The problem of interest corresponds to the typical daily gas and oil production calculation, i.e. a one-day time period planning. The simulation problem considers determination of oil and gas production for the current topology. The optimization problem is established by considering that the instantaneous production of the gas/oil network must optimally satisfy a given demand. Non-linear behavior and reverse flow is assumed for wells and cubic equations of state are used to estimate thermodynamic properties. The resulting model contains differential and algebraic equations for a given fixed topology of interconnected wells coupled with surface pipeline networks. The model combines *DASSL* to solve the inner set of differential–algebraic equations, and KNITRO to solve both the optimization problem and the overall set of non-linear equations. The good performance of the model and the numerical strategy is illustrated with real cases of study.

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Keywords: Simulation and optimization; Differential and algebraic systems; Production systems; Reverse flow

1. Introduction

Petroleum, either as gas or oil, is a finite and scarce resource upon which modern society is strongly dependent. Hence, mankind is forced to rationalize and optimize its production and consumption to make them safer, more efficient, and cheaper. Indeed, the complexity of gas and oil production systems represents a unique challenge. A typical production infrastructure, either offshore or onshore, can be described as a number of fields containing several reservoirs where wells have been or may be perforated for production or even injection when recovery actions are performed. Surface facilities are also included to, among other purposes, allow well streams interconnectivity via manifolds. In offshore oilfields, gas and oil production is concentrated in different platforms from where it is transported to sales or storage points. A typical well may consist of either a single pipe or two pipes. If the well contains two pipes then one pipe, which is called the tubing line, goes inside the other one, which is called the casing line. The entry point of gas and oil from the reservoir into the well is referred to as the wellbore. Valves represent an important part in production systems to allow individual flowrate control. Thus, the resulting pipeline network includes reservoirs, wells, manifolds, and valves (Fig. 1).

The main target of integrated reservoir management should be to maximize oil and gas recovery within the current economic and technical limits. The target is not easy to achieve since a reasonably good development plan, even for a single well, involves several stages in a traditional approach (Morooka, Guilherme, & Mendes, 2001). Attempts to optimize petroleum production have evolved in different directions. Considering just the reservoir, the optimization problem implies an analysis of the production–injection operation system where geological parameters become variables to optimize (Yang, Zhang, & Gu, 2003). Efforts to optimize based on building intelligent systems has also demonstrated to be a potentially fertile technique to increase petroleum production (Morooka et al., 2001).

On other direction, it was observed that a schedule of operations is typically demanded from the production planning point of view. This analysis leads naturally to establish a multiperiod optimization problem. Linear programming (LP) models have

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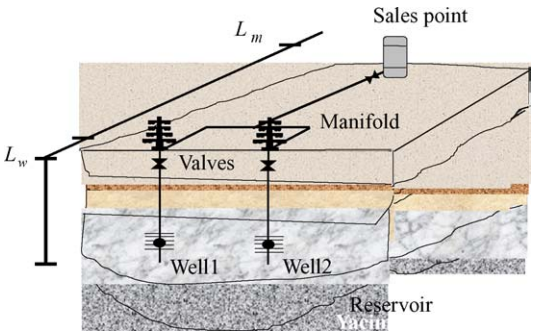

Fig. 1. A two-wells production system.

been used to solve planning of some coordinated operations in petroleum production (Aronofsky & Williams, 1962; Garvin, Crandall, John, & Spellman, 1957; Lee & Aronofsky, 1958). To improve the accuracy of LP, an iterative scheme has been suggested where non-linear simulation and LP are wisely combined (Eeg & Herring, 1997). More recently, a multiperiod mixed-integer linear programming (MILP) model has been proposed for the planning and scheduling of oil investment and operation of offshore facilities (Iyer, Grossmann, Vasantharajan, & Cullick, 1998). They applied piecewise linear approximations to model each source of non-linearity in equations. Though avoiding direct dealing with the difficulty of non-linear equations eases the computational burden, the result is not entirely appropriate.

Mixed-integer non-linear programming (MINLP) has also been applied for oil production: Van den Heever and Grossmann (2000) proposed a MINLP model for offshore oilfield production planning where non-linearities are incorporated. A bilevel decomposition technique and the aggregation of time periods, allowing logic structures, were combined into a so-called aggregation/disaggregation algorithm to deal with the problem. Complex business rules such as tax and royalty calculations were later added to this model (Van den Heever, Grossmann, Vasantharajan, & Edwards, 2000). A further analysis of the problem and an innovative approach based on a Lagrangean decomposition heuristic were presented in Van den Heever, Grossmann, Vasantharajan, and Edwards (2001). In this case, operating conditions are assumed constant across the whole planning horizon. However, it is well known that well and reservoir conditions change as a function of several variables such as the volume of gas and oil extracted/injected and geological parameters (Horne, 1998). Ortiz-Gómez, Rico-Ramírez, and Vázquez-Román (2001) have incorporated this effect in a short-term planning of oil production.

Yet, the daily oil and gas production problem has received little attention. It has been reckoned that significant uplift in gas and oil production can be achieved by means of continuous production system optimization on a daily basis (Heinemann, Lyons, & Tai, 1998). However, only a few papers have been reported to optimize production in the most immediate term. In the past, decisions regarding production considered process simulation where nodal analysis (Beggs, 2003), which is not a robust approach and fail for complex networks, has been typically applied and fluid properties were normally estimated via correlations (Abdel Waly, El-Massry, Darweesh, & El Sallaly, 1996). More recently, Handley-Schachler, McKie, and Quintero (2000) proposed an optimization approach that includes property evaluation based on cubic equation of state but serious simplifications were imposed on the model. This approach has been implemented in a commercial package for both simulation and optimization of gas production. More recently, a model has been developed that includes flow correlations to determine gas-lift allocation where gas injection rates are discretized and solved via special order sets (Kosmidis, Perkins, & Pistikopoulos, 2004).

In the daily gas and oil production problem, petroleum engineers are typically requested to exactly produce as to satisfy a given demand with the existing wells network, having a fixed topology and associated valves to adjust each well production. A difficulty of satisfying the programmed schedule produced by LP, MILP, or MINLP planning schemes arises because the interconnectivity of wells has never been considered during planning stages. Even when productive wells belong to the same reservoir, pressure at each wellbore can be radically different and the associated cost of production may also be different in each well. Hence, individual well productivity is substantially affected because of the interconnectivity. As a result of the interconnectivity, a well might also attempt to inject into another well rather than contributing to the overall production. It is then important to include the prediction of flow direction in each well. It is of course expected that the given demand, rather than maximizing production, be optimally satisfied.

This paper deals with the typical daily gas and oil production calculation. This problem can be visualized as a one-day time period planning. Both simulation and optimization problems of oil and gas production are considered in this paper. The *K*NITRO code (Byrd, Gilbert, & Nocedal, 2000; Waltz, Morales, Orban, & Nocedal, 2003) is applied to solve these problems. This code implements an interior method for non-linear programming that uses trust regions and a merit function to promote convergence. The oil and gas production problem is established in Section 2 to identify all process units involved in this process. Basic conceptual process units are detected from the process description and modeled in Section 3. Section 4 gives the solution strategy to obtain convergence in both simulation and optimization problems. Cases of study are analyzed in Section 5. Finally, our conclusions are contained in Section 6.

2. Problem statement

The daily gas and oil production simulation and optimization problems are established here as follows: for a given configuration of a production system the purpose of simulation is to calculate the contribution of each well in the overall production; the purpose of optimization concerns determination of the contribution of each well when operating to satisfy a given demand but minimizing the total cost. The production system consists of a fixed topology, i.e. a process flowsheet, containing a number of reservoirs where several wells have been already drilled. All wells must be considered ready to produce and are interconnected via surface pipeline facilities where gas and oil is

distributed to sale and/or storage points. Once the wells topology is established, the problem solved in this study answers questions such as if the network can satisfy a given demand to eventually take further appropriate actions. It is assumed that all parameters in the model described later remain constant on a time period of one day. A cost coefficient is associated to each well and the problem is then formulated as minimizing the production cost subject to technical constraints that come from the topology and the constraint because of the given demand. Constraints for the optimization problem include mass, energy and momentum balances, pipes' diameters and valve's parameters to control the flow. Reverse flow is allowed so that injection from one well into another well is an unknown to determine. Neither drilling nor close down of wells is considered in this study. The system must not produce more than the demand to avoid storage problems, i.e. it becomes an equality constraint. It is felt here that incorporating the demand constraint is a more realistic problem since petroleum production is often limited by international agreements. Next section contains a description of the mathematical model.

3. Mathematical models

To ease both network description and the solution procedure, two main types of conceptual process units are proposed: *well* and *manifold* (Fig. 1). The conceptual *well* represents the pipe connecting the reservoir to allow gas and oil flow from the reservoir to the surface where it is connected to a *manifold*. Since a *well* has already been drilled and connected in one side to a reservoir, it has only one point left for connection that corresponds to the outlet point of the pipe. The *manifold* then represents a pipe where several pipes, *wells* or *manifolds*, can be interconnected and in where the mass may be transported certain distance. For simplicity, we assume that the *manifold* may interconnect several *wells* or *manifolds* at the entry (input) point but just another *manifold* may be connected at the outlet point (output) of the *manifold*. A pipe has always an input point and one output point associated so that a calculated negative flow means that the flow direction is opposite to the expected one. There is also a valve associated to each *well* or *manifold*. The underlining idea is that the valve in the *well/manifold* may not only represent the so called "choke" valve of wells but also can it be used for the purpose of production control. Since both conceptual units *well* and *manifold* contain two common parts, pipe and valve, the following subsections start by describing the model with assumptions for each common part. Models for whole conceptual units are described latter. For the sake of simplicity in representation, subindexes in several variables are only used during the conceptual units description.

3.1. Pipe

Considering a homogeneous mass flowrate $\dot{m}$ flowing across a pipe segment with diameter D, inclined a θ angle, and total length L (Fig. 2), then it has been shown that applying momentum and energy balance the resulting equations are

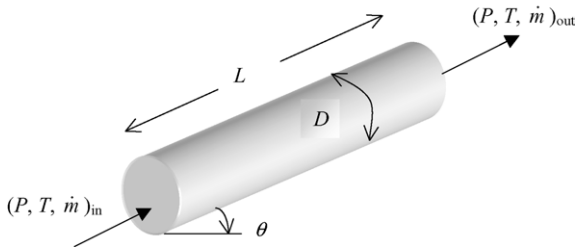

Fig. 2. A pipe segment.

(Vázquez-Román, 1998):

$$\frac{\mathrm{d}P_{\mathrm{pipe}}}{\mathrm{d}L} = -\rho g \sin\theta - \frac{f\rho v^2}{2D} - \rho v \frac{\mathrm{d}v}{\mathrm{d}L} \tag{1a}$$

$$\frac{\mathrm{d}\dot{m}H_{\mathrm{pipe}}}{\mathrm{d}L} = -\dot{m} g \sin\theta - \frac{\mathrm{d}Q}{\mathrm{d}L} - \dot{m} v \frac{\mathrm{d}v}{\mathrm{d}L} \tag{1b}$$

where P is the pressure, ρ the fluid density, g the acceleration due to gravity, f the friction factor, v the fluid velocity, H the enthalpy, and Q is the heat transfer.

According to Gibbs, given the mass and two independent properties it is possible to determine the complete thermodynamic state. Eqs. (1a) and (1b) suggest that pressure and enthalpy represent the most natural independent variables for the thermodynamic state evaluations. Thus, it is convenient to express the velocity variable in (1a) and (1b) as a function of these two variables, i.e. $v = v(P, H)$ so that,

$$\frac{\mathrm{d}v}{\mathrm{d}L} = \left(\frac{\partial v}{\partial P}\right)_H \mathrm{d}P + \left(\frac{\partial v}{\partial H}\right)_P \mathrm{d}H \tag{2}$$

In addition, velocity can be expressed in terms of flowrate, density, and the cross-sectional area of the pipe A,

$$v = \frac{\dot{m}}{\rho A} \tag{3}$$

Eqs. (1a), (1b), (2), and (3) can be combined for the pipe to produce,

$$\left[1 + \frac{\dot{m}}{Ag_c}\left(\frac{\partial v}{\partial P}\right)_H\right]\frac{\mathrm{d}P_{\mathrm{pipe}}}{\mathrm{d}L} + \left[\frac{\dot{m}}{Ag_c}\left(\frac{\partial v}{\partial H_{\mathrm{pipe}}}\right)_P\right]\frac{\mathrm{d}H_{\mathrm{pipe}}}{\mathrm{d}L} = -\frac{\dot{m}g}{Avg_c}\sin\theta - \frac{f\dot{m}v}{2ADg_c} \tag{4a}$$

$$\left[\dot{m}v\left(\frac{\partial v}{\partial P}\right)_H\right]\frac{\mathrm{d}P_{\mathrm{pipe}}}{\mathrm{d}L} + \left[1 + \dot{m}v\left(\frac{\partial v}{\partial H_{\mathrm{pipe}}}\right)_P\right]\frac{\mathrm{d}H_{\mathrm{pipe}}}{\mathrm{d}L} = -\dot{m}g\sin\theta - \frac{\mathrm{d}Q}{\mathrm{d}L} \tag{4b}$$

Assuming constant average values in selected terms, Eqs. (4a) and (4b) have been often integrated to determine outlet conditions in a pipe (Vázquez-Román, 1998). Furthermore, energy balance (Eq. (4b)) is normally over simplified and it is even removed. These integrations, with constant values in most involved variables, have generated several models consisting of pure algebraic equations. These models tend to contain

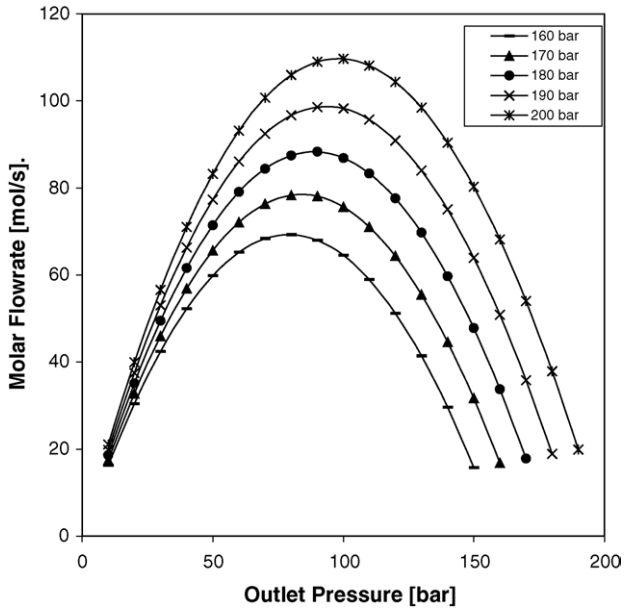

Fig. 3. Results with algebraic pressure drop models.

quadratic terms in flowrate, or even a fractional order, which introduce numerical difficulties because of inconsistencies such as the existence of multiple numerical solutions. Fig. 3 shows this behavior when using the average velocity model proposed in Vázquez-Román (1998) for several pressures using a single pipe with inlet temperature fixed to 410 K and mixture composition as given in Table 1. Several correlations based on this type of simplified integration that are currently used to calculate the pressure drop in wells can be found in Beggs (2003). In this paper, the above equations are incorporated in the model without further simplifications and the result indicates that some difficulties such as multiple solutions tend to disappear. All required properties such as density or temperature are obtained when the thermodynamic state is calculated.

Heat transfer in production systems is another complex problem. The best assumption to make for heat transfer coefficients around the wellbore is to consider them as buried pipelines. Methods to estimate heat transfer coefficients can be found, for instance, in Incropera and De Witt (1996). The overall heat transfer coefficients in wells range from 0.5 to 50 Btu/(h ft^2 R) though the most typical values are around 2 Btu/(h ft^2 R). Wellbore completion geometry, diameter, materials, and geologic information along flow path could be used to estimate the coefficients in real cases. However, trial and error is normally applied to fit values for each well. Wells experience high overall coefficients because of convection in high permeability water sands. It is clear though that convective effect should be greater when the seawater surrounds the pipe. Experience with commercial programs indicates that different values in the heat transfer coefficient are required to achieve the same answer. This tends to occur when results are very sensitive to temperature. The best solution is to segment the well according to the geologic model. To apply this solution in this approach, the conceptual *well* should be connected to a series of interconnected *manifolds* where one *manifold* is connected to another one having different properties in the surroundings. The heat transfer equation can easily be inserted in Eq. (1b) so that no extra equation has to be considered for the model.

Table 1
Molar composition for a testing mixture

Species	mol%
Methane	86.74
Ethane	10.56
Propane	1.19
n-Butane	0.17
Isobutane	0.12
n-Pentane	0.04
Isopentane	0.04
n-Hexane	0.03
Nitrogen	1.03
Carbon dioxide	0.08

The friction factor required in Eqs. (4a) and (4b) is an extremely complex surface physics phenomenon. Empirically based correlations to "fit" the observed behavior have been developed using the friction factor as a function of the relative roughness of the pipe wall and the Reynolds number (Re). The Re is a dimensionless number which measures the "smoothness" of the flow: a low Re indicates smooth or *laminar* flow, and a high number, indicates mixed or *turbulent* flow. The following equations are often used for friction factor (Perry, Green, & Maloney, 1988):

$$f = \frac{64}{Re}, \quad \text{if } Re < 700 \tag{5a}$$

$$f^{1/2} = \left[1.74 - 2\log\left(\frac{\varepsilon}{D} + \frac{18.7}{Ref^{1/2}}\right)\right]^{-1}, \quad \text{if } Re > 3000 \tag{5b}$$

$$f = \frac{64}{Re} + 1.008 \times 10^{-5}(Re - 700), \quad \text{if } 700 < Re < 3000 \tag{5c}$$

It is worth to noting that these correlations are discontinuous at the two boundaries (Re = 700 and 3000) although in most cases pipes are operated in turbulent conditions. However, the above correlations have been slightly modified to ensure that the friction factor is continuously differentiable, prevent friction factor blow up, to improve smoothness at small Re and continuity at intermediate values. The main problem with Eqs. (5a)–(5c) appears when it is applied to two-phase systems because it is difficult to assign a universal definition for the viscosity in the Re. The most commonly used form due to McAdams, Woods, and Heroman (1942) is used for this purpose here. In addition, absolute values for Re in Eqs. (5a)–(5c) are used to allows reverse flow calculations.

Having the capability of evaluating the thermodynamic state and the friction factor, the pipe model essentially provides two differential equations. Thus, Eqs. (4a) and (4b) can be integrated to produce the pressure drop and enthalpy change in the pipe for given inlet conditions, mass flowrate, and all parameters such

as pipe diameter, pipe length, and heat transfer coefficient. The result is,

$$\Delta P_{\text{pipe}} = \int_0^L \frac{b_1 a_{22} - b_2 a_{12}}{a_{11}a_{22} - a_{12}a_{21}}\,\mathrm{d}L \tag{6}$$

$$\Delta H_{\text{pipe}} = \int_0^L \frac{b_1 a_{21} - b_2 a_{11}}{a_{11}a_{22} - a_{12}a_{21}}\,\mathrm{d}L \tag{7}$$

where

$$a_{11} = 1 + \frac{\dot{m}}{Ag_c}\left(\frac{\partial v}{\partial P}\right) \tag{8a}$$

$$a_{12} = \frac{\dot{m}}{Ag_c}\left(\frac{\partial v}{\partial H}\right) \tag{8b}$$

$$a_{21} = \dot{m}v\left(\frac{\partial v}{\partial P}\right)_H \tag{8c}$$

$$a_{22} = 1 + \dot{m}v\left(\frac{\partial v}{\partial H}\right)_P \tag{8d}$$

$$b_1 = -\frac{\dot{m}g}{Avg_c}\sin\theta - \frac{f\dot{m}v}{2ADg_c} \tag{8e}$$

$$b_2 = -\dot{m}g\sin\theta - \frac{\mathrm{d}Q}{\mathrm{d}L} \tag{8f}$$

It is not possible to obtain analytical solutions to Eqs. (6) and (7) since all terms in these equations change with length. Hence, they have to be integrated numerically. The given initial conditions could be the pressure and enthalpy at inlet point or even measurable properties such as pressure and temperature in which case calculation of the thermodynamic state would produce enthalpy among other variables.

3.2. Valves

Valves perform an important part in production systems. In particular, the so-called choke valves produce effects in flow that deserve a separated analysis. Flow across a choke is divided in two groups: subcritical and critical. Subcritical flow occurs when the associated mixture velocity at the choke throat is less than the sonic velocity. This is also referred to as the condition when the ratio of the downstream pressure to the upstream pressure is greater than the critical pressure ratio. Critical flow occurs when the associated mixture velocity in the throat equals the sonic velocity. The practical implication is that the flowrate depends on the downstream pressure when subcritical flow occurs at the choke throat, whereas this dependency is radically eliminated in critical flow. It means that after achieving critical flow, downstream pressure could be decreased further but the volumetric flowrate would remain constant. Supercritical condition cannot occur in reality. Choke valves are strategically placed in wells to avoid the blow up of the formation.

Most flow simulators do not reproduce this effect and either aborts the iteration or calculates a lower flowrate value because of the above-mentioned inconsistency in simplified integrated models where multiple solutions may appear. In this work, an isentropic process is considered in all valves and the pressure drop, ΔP_{valve}, is modeled using a simplified linear expression (Smith & Corripio, 2006):

$$\Delta P_{\text{valve}} = \frac{C_{\text{valve}}\dot{m}}{\text{Ap}} \tag{9}$$

where C_{valve} is a constant parameter and Ap is the aperture of valve which has a restricted value $\text{Ap} \in [0, 1]$. Using the aperture of valve allows that flow becomes nil when the valve is closed though there may be a pressure difference between the downstream and upstream pressures. In some cases, the C_V can be used to fit experimental information. An important feature of this equation is that it can be used without any change for reverse flow calculation.

In addition, speed of sound is evaluated to detect critical flow. From physics, the speed of sound c is obtained from thermodynamic properties (see, for instance, Smith, Van Ness, & Abbot, 1996). The maximum flowrate, $\dot{m}_s$, is then calculated from,

$$\dot{m}_s = -VA\left(\frac{\partial P}{\partial V}\right)_S \tag{10}$$

where V is the molar volume, P the pressure, and S is the entropy.

For the sake of simplicity, it is assumed here that valves operate under pure Joule–Thompson effect, i.e. the enthalpy is the same at valve inlet and outlet. For given inlet pressure and enthalpy conditions at valve inlet point and a positive flowrate, the outlet pressure is then calculated with Eq. (9) while the enthalpy remains constant. If the flow is reverse then the known pressure and enthalpy correspond to the outlet point conditions and Eq. (9) can be used to calculate the pressure at the so defined entry point.

It must be observed that Eq. (9) is valid only for subcritical flow. When critical flow is detected then the calculation is reduced to determine the energy content at outlet stream, i.e. the thermodynamic state. Hence, the valve model, as well as the pipe model, demands the evaluation of thermodynamic states. Next section contains a description of how the evaluation of thermodynamic states is visualized in this model.

3.3. Thermodynamic state

During production system operations, the state of aggregation of fluid mixtures can be liquid, gas or a mixed liquid–gas as a result of the various pressure and temperature conditions. An equation of state is then required to model the thermodynamic behavior for both liquid and gas pure phases and mixtures. Using cubic equations of state is very attractive because they can be applied in all fluid phases and they also cover a sufficient range of pressure. The main disadvantage is that it aggregates extra numerical difficulties because of the highly non-linear equations added to the phase equilibrium problem. Several methods have been developed to solve the phase equilibrium problem. The Gibbs energy function is the most widely thermodynamic function used to determine the equilibrium state at given temperature and pressure. However, other energy functions could be more conveniently used for a different combination of independent variables such as pressure and volume.

In general, methods can be classified as those using direct minimization of the Gibbs energy function or those based on satisfying the necessary conditions for the minimum. Seider, Gautam, and White (1980) present a good review of these approaches. More recently, global optimization strategies have been applied to guarantee that the obtained numerical solution corresponds to the overall minimum. McDonald and Floudas (1995a, 1995b) applied activity coefficient models to address global optimization. Harding and Floudas (2000) analyze the most conventional cubic equations of state to provide convexity by reformulating each non-linear expression and hence to obtain the global minimum.

It is clear from above that calculating the thermodynamic state is a challenging task. If the state calculation were directly incorporated into the production model then the possibility of failure is high. Geoffrion (1972) has observed while solving optimization problems that some non-linearity difficulties may ease by subdivisions. Hence, the calculation of the thermodynamic state is considered here as a *black box* to overcome the numerical difficulties that cubic equations of state could introduce in the model. Thus, an ad hoc method is used to produce appropriate values in all required variables when given two independent properties, pressure and enthalpy or temperature, and the molar fraction of the system. A general cubic equation of state (Poling, Prausnitz, & O'Connell, 2001) has been programmed to calculate the thermodynamic states when up to two phases are in equilibrium. The resulting subroutine was named *TState*. The following subsections describe how the above models are used in the present approach for the so-called here conceptual *wells* and *manifolds*.

3.4. Conceptual well

The conceptual *well* model is divided here in three main parts as indicated above: mass flowing from the reservoir to the wellbore, flowrate in the pipe, and the valve associated to the outlet point of the pipe. The main assumption considered here is that the system operates under steady-state conditions because the reservoir and pipe dynamics are irrelevant for the gas and oil production problem in a daily basis.

Gas and oil flowrate from the reservoir to the well depends on several factors such as geological properties of well surroundings like fractures, permeability, thickness, porosity, etc. Modeling these factors is a considerable difficult task and it is beyond our goals. It is a normal practice to indirectly evaluate them and incorporate their effects in terms of productivity indexes for each well. Thus, well production generates a pressure drop between the reservoir pressure and the flowing pressure at wellbore, ΔP_{rw}, given by (Horne, 1998):

$$\Delta P_{\mathrm{rw}} = P^{\mathrm{w}} - P^{\mathrm{r}} = \frac{141.2qB\mu}{kh}\left(\frac{1}{2}\left[\ln\frac{0.000264kt}{\Phi\mu c_{\mathrm{t}} r_{\mathrm{B}}^2} + 0.80907\right]\right) \quad (11)$$

where B, μ, k, h, c_{t}, r_{B}, and Φ are experimentally determined parameters associated to the reservoir and the well, P^{r} the reservoir pressure, P^{w} the wellbore or bottom hole flowing pressure, q the volumetric production flow, and t is the operation time that starts when all parameters were determined. The original reference should be consulted for appropriate units.

Since all parameters and operation time are known for the daily production problem then Eq. (11) can be simplified as:

$$\Delta P_{\mathrm{rw}} = c_1 \dot{m} \quad (12)$$

where c_1 is the result of combining the parameters from Eq. (11). Lastly, Eq. (12) indicates that pressure drop can be approximated in the short time as a linear function of flowrate. In addition, each well has a maximum production flow, which may depend on geological properties. This maximum is normally determined from experimental work. Experience indicates however that the maximum production for a well will rather depend on the choke valve performance.

To evaluate the change of enthalpy from the reservoir to the wellbore, it is assumed here that an isothermal process is carried out. Then, the change of enthalpy is given by,

$$\Delta H_{\mathrm{rw}} = H(P^{\mathrm{wellbore}}, T^{\mathrm{r}}, \mathbf{x}) - H(P^{\mathrm{r}}, T^{\mathrm{r}}, \mathbf{x}), \quad (13)$$

where P^{wellbore} is the pressure at wellbore, P^{r}, T^{r}, and $\mathbf{x}$ the reservoir pressure, temperature, and molar fraction, respectively, $H(P^{\mathrm{r}}, T^{\mathrm{r}}, \mathbf{x})$ then the enthalpy evaluated at reservoir conditions, and $H(P^{\mathrm{wellbore}}, T^{\mathrm{r}}, \mathbf{x})$ is the enthalpy evaluated at wellbore conditions.

The equations for pipes and valves are incorporated to Eqs. (12) and (13) to produce the overall *well* model. The total pressure drop in the *well* represents the difference between the pressure at the reservoir and the outlet pressure, $P_{\mathrm{outlet},w}$. For a given flowrate, the total pressure drop in well w is calculated as,

$$\Delta P_w = P_{\mathrm{outlet},w} - P_w^{\mathrm{r}} = \Delta P_{\mathrm{rw},w} + \Delta P_{\mathrm{pipe},w} + \Delta P_{\mathrm{valve},w}, \quad \forall w \in W \quad (14)$$

where W is the set of conceptual *wells* w, ΔP_w the total pressure drop, $\Delta P_{\mathrm{rw},w}$ the pressure drop from the reservoir to the wellbore that is calculated with Eq. (12), $\Delta P_{\mathrm{pipe},w}$ the pressure drop in the pipe of the *well* from the wellbore to the valve that is calculated by simultaneous solution of Eqs. (6) and (7), and $\Delta P_{\mathrm{valve},w}$ is the pressure drop in the valve calculated with Eq. (9).

Similarly, the total change in enthalpy for a *well*, ΔH_w, is obtained as follows:

$$\Delta H_w = H_{\mathrm{outlet},w} - H_w^{\mathrm{r}} = \Delta H_{\mathrm{rw},w} + \Delta H_{\mathrm{pipe},w} + \Delta H_{\mathrm{valve},w}, \quad \forall w \in W \quad (15)$$

where H_w^{r} is the enthalpy at reservoir conditions, $H_{\mathrm{outlet},w}$ the enthalpy at outlet point, $\Delta H_{\mathrm{rw},w}$ the enthalpy change from the reservoir to the wellbore that is calculated with Eq. (13), $\Delta H_{\mathrm{pipe},w}$ the enthalpy change in the pipe that is calculated by simultaneous solution of Eqs. (6) and (7), and $\Delta H_{\mathrm{valve},w}$ is the enthalpy change in the valve that is considered nil.

3.5. Conceptual manifold

The conceptual *manifold* allows that several pipes become interconnected but it is itself a pipe where a valve is incorporated to control flowrate. As above indicated, mixture of all inlet streams occurs at the inlet extreme point of this pipe. Then, the inlet pressure of a *manifold* is equivalent to the outlet pressure for each interconnected conceptual *well* or even another *manifold*. Thus, a manifold would provide as many equations of same type as interconnected units:

$$P_{\text{inlet},m} = P_{\text{outlet},j}, \quad \forall m \in M \ \text{ and } \ \forall j \in J_m \tag{16}$$

where M is the set of *manifolds* m, J_m the set of conceptual units j connected at the inlet point of *manifold* m, $P_{\text{inlet},m}$ the pressure at inlet point of conceptual manifold m, and $P_{\text{outlet},j}$ is the outlet pressure of the conceptual unit j.

Mass and energy balances are also required at inlet of each manifold:

$$x_{i,m}\dot{m}_m = \sum_j x_{i,j}\dot{m}_j, \quad i = 1, \ldots, c, \quad \forall m \in M \ \text{ and } \ \forall j \in J_m \tag{17a}$$

$$\dot{m}_m H_{\text{inlet},m} = \sum_j \dot{m}_j H_{\text{outlet},j}, \quad i = 1, \ldots, c, \quad \forall m \in M \ \text{ and } \ \forall j \in J_m \tag{17b}$$

where c is the number of chemical species, $x_{i,j}$ the molar fraction of species i in conceptual unit j, $\dot{m}_j$ the total molar rate in conceptual unit j, $H_{\text{outlet},j}$ the enthalpy at outlet point of conceptual unit j, and $H_{\text{inlet},m}$ is the enthalpy at inlet of conceptual unit m.

The overall pressure drop in the *manifold* m, ΔP_m, from the inlet to the outlet is,

$$\begin{aligned}\Delta P_m &= P_{\text{outlet},w} - P_{\text{inlet},w} \\ &= \Delta P_{\text{pipe},m} + \Delta P_{\text{valve},m}, \quad \forall m \in M\end{aligned} \tag{18}$$

where $\Delta P_{\text{pipe},m}$ is the pressure drop in the pipe of the manifold from the inlet point to the valve that is calculated by simultaneous solution of Eqs. (6) and (7) and $\Delta P_{\text{valve},m}$ is the pressure drop in the valve that is calculated with Eq. (9). Indeed, the effect of pressure drop in the valve can be not only fitted but also cancelled by setting the parameter C_{valve} to zero. This might particularly be desirable when a *manifold* interconnect several *manifolds*.

The overall change of enthalpy for the *manifold* m, ΔH_m, is obtained from,

$$\begin{aligned}\Delta H_m &= H_{\text{outlet},m} - H_{\text{inlet},m} \\ &= \Delta H_{\text{pipe},m} + \Delta H_{\text{valve},m}, \quad \forall m \in M\end{aligned} \tag{19}$$

where $H_{\text{outlet},m}$ is the enthalpy at outlet conditions, $H_{\text{inlet},m}$ the enthalpy at inlet conditions, $\Delta H_{\text{pipe},m}$ the change of enthalpy within the pipe that results from simultaneous solution of Eqs. (6) and (7), and $\Delta H_{\text{valve},m}$ is the change of enthalpy in the valve that is assumed nil.

4. Solution strategy

The KNITRO code (Byrd et al., 2000; Waltz et al., 2003) is applied to solve both simulation and optimization problems. This code implements an interior method for non-linear programming that uses trust regions and a merit function to promote convergence. A detailed description of the algorithm is given in Waltz et al. (2003). In this work, gradients are approximated with forward finite differences and the Hessian is approximated with the BFGS method. Even the resulting set of non-linear equations is solved in KNITRO where, to specify a non-linear equations problem, the objective function is specified as zero and all equations are considered as equality constraints. Numerical integration of Eqs. (6) and (7) is performed by applying the code *DASSL* (Brenan, Campbell, & Petzold, 1989) to the set of differential equations.

To start, a production system has to be described in terms of conceptual *wells* and *manifolds*. A dialog-based computer program, called *WellNet*, was developed to easy this description. Fig. 4 shows this program where the topology of a production system consisting on eight *wells* and two *manifolds* has been incorporated. Each *well* is represented with a circle, whereas each *manifold* is represented with a rectangle. Associated parameters to each conceptual unit are defined in descriptive dialogs that are activated by clicking in the representative circle or rectangle.

WellNet provides facilities to define the chemical species in the mixture via a database that contains several pure chemical species. Were it required, these properties could easily be modified to use other values. In addition, several cubic equations of state could also be selected. *WellNet* was programmed in Visual Studio C++ 6.0 and all cases presented here were tested in a PC Pentium 4, 2.40 GHz.

Once the topology of a production system and all required information are appropriately incorporated, users should select what problem to solve. Next sections describe both simulation and optimization problems.

4.1. The simulation problem

Several simulation cases are possible to evaluate in *WellNet*. In all simulation problems, the topology and physical parameters such as the aperture of valves remain constant. The most typical case in production systems consists of determining all flowrates for given conditions at reservoir and sales or storage points. Two types of process simulation can be identified in this work: standalone and overall simulation.

In a standalone situation for *wells*, at least the molar composition of the flowing mixture and aperture of the valve must be given as well as all parameters, shown in the model above, such as length, diameter, etc. An analysis of degrees of freedom based in the most simplified model indicates eight unknowns and four equations to result in four degrees of freedom (Table 2). More equations are involved in the model but they do not modify the actual value for degrees of freedom. In fact, all of them have been coded to configure the subroutine *TState* that calculates the whole thermodynamic state. Eqs. (3), (4a), and (4b) from

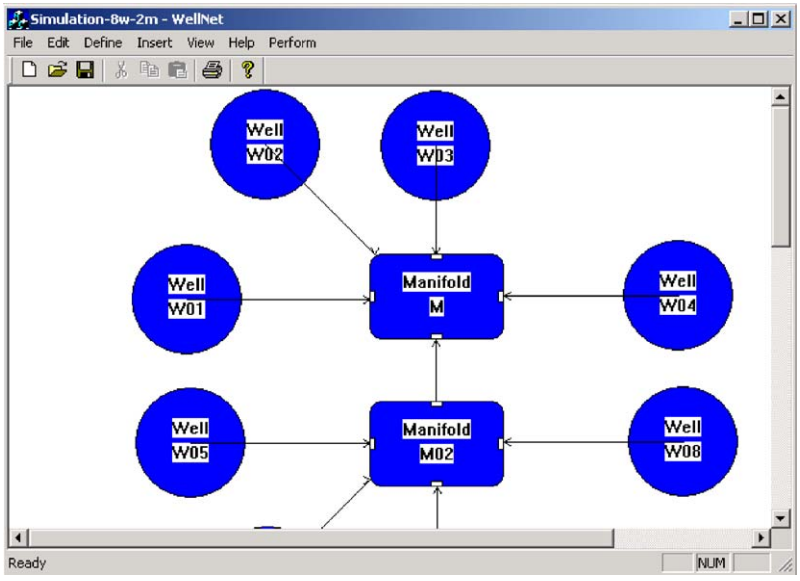

Fig. 4. *WellNet* interface.

Table 2
Degrees of freedom in *wells* for standalone simulation

Equation		Probable unknowns
1	$P_{r,w} - P_{outlet,w} - c_{1,w}\dot{m}_w - \int_0^L \left(\frac{b_1a_{22}-b_2a_{12}}{a_{11}a_{22}-a_{12}a_{21}}\right)_w dL - \frac{C_{valve,w}\dot{m}_w}{Ap_w} = 0$	$P_{r,w}$, $P_{outlet,w}$, Ap_w, $\dot{m}_w$
2	$\dot{m}_w(H_{outlet,w} - H_{inlet,w}) - \int_0^L \left(\frac{b_1a_{21}-b_2a_{11}}{a_{11}a_{22}-a_{12}a_{21}}\right)_w dL = 0$	$\underline{H}_{outlet,w}$, $\underline{H}_{inlet,w}$
Subroutines in *TState*		
3	$T_{outlet,w} = T_{outlet,w}(P_{outlet,w}, \underline{H}_{outlet,w}, \mathbf{x}_w)$	$T_{outlet,w}$
4	$T_{inlet,w} = T_{inlet,w}(P_{r,w}, \underline{H}_{inlet,w}, \mathbf{x}_w)$	$T_{inlet,w}$

Number of equations: 4; number of unknowns: 8; degrees of freedom: 4.

Table 2 are also included in this code. Thus, *TState* calculates all thermodynamic states required to calculate the residuals for Eqs. (1) and (2) from Table 1, which are used to converge the simulation problem using *KNITRO*.

A typical scenario for a current standalone simulation in a *well* consists in determining the outlet pressure and temperature for given flowrate, and inlet temperature and pressure. The thermodynamic state at the reservoir can be calculated for given experimental pressure, temperature, and molar composition at the reservoir. Then, the thermodynamic state at the wellbore can be calculated assuming an isothermal process and the pressure calculated with Eq. (12) for a given flowrate. The pipe model is then applied for the given flowrate and the thermodynamic state at the wellbore to calculate the pressure drop and final enthalpy for a given length. Simultaneous solution to Eqs. (6) and (7) requires several calculations of thermodynamic states at different lengths that are carried out in *TState*. The valve model is finally solved to calculate the thermodynamic state at the *well* outlet. If the flowrate is unknown then the outlet (surface) pressure should be defined and an iterative procedure could be used to calculate the flowrate. The procedure is repeated under *KNITRO* control until convergence is achieved.

Another typical standalone simulation implies exchanging the flowrate with the outlet pressure in the above problem so that

Table 3
Degrees of freedom in *manifolds* for standalone simulation

Equation		Probable unknowns
1	$P_{inlet,m} - P_{outlet,m} - \int_0^L \left(\frac{b_1a_{22}-b_2a_{12}}{a_{11}a_{22}-a_{12}a_{21}}\right)_m dL - \frac{C_{valve,m}\dot{m}_m}{Ap_m} = 0$	$P_{inlet,m}$, $P_{outlet,m}$, Ap_m, $\dot{m}_m$
2	$\dot{m}_m(H_{outlet,m} - H_{inlet,m}) - \int_0^L \left(\frac{b_1a_{21}-b_2a_{11}}{a_{11}a_{22}-a_{12}a_{21}}\right)_m dL = 0$	$\underline{H}_{outlet,m}$, $\underline{H}_{inlet,m}$
Subroutines in *TState*		
3	$T_{outlet,m} = T_{outlet,m}(P_{outlet,m}, \underline{H}_{outlet,m}, \mathbf{x}_m)$	$T_{outlet,m}$
4	$T_{inlet,m} = T_{inlet,m}(P_{r,m}, \underline{H}_{inlet,m}, \mathbf{x}_m)$	$T_{inlet,m}$

Number of equations: 4; number of unknowns: 8; degrees of freedom: 4.

the flowrate becomes an unknown while the pressure is given. In this case, *TState* would again calculate the inlet thermodynamic state and Eqs. (1) and (2) from Table 1 are used to converge flowrate and outlet temperature.

In all standalone simulations, *WellNet* calculates the maximum flowrate, according to Eq. (10), to report this value and flagging on if the demanded or calculated flowrate results bigger than the critical value. Both residuals for Eqs. (1) and (2) from Table 1 and *TState* are prepared to use negative flowrate to allow reverse flow.

The standalone simulation for a *manifold* is similar to the *well*. Table 3 indicates also four degrees of freedom to result in a set of options similar to the standalone simulation for *wells*. It is important in this case that the thermodynamic state at inlet point be previously calculated and then the similarity to *wells* is maintained. Two cases are thus implemented within *WellNet* for standalone simulation of *manifolds*: in the first case, Eqs. (1) and (2) from Table 3 are solved to determine the outlet pressure and enthalpy and then the thermodynamic state. In a second case, the flowrate becomes an unknown, whereas the outlet pressure is given. Critical flowrate is also flagged on when it appears.

In the overall simulation, one or several wells are interconnected with one or several *manifolds*. The mixture effect at inlet point of manifolds given by Eqs. (17a) and (17b) must be incorporated here. Inclusions of these equations allow calculation of those variables related to the inlet thermodynamic state in *manifolds*. These variables correspond to those that were given during the standalone simulation. Besides equations related to thermodynamic states, the system is considered to generate two equations, and hence two degrees of freedom, per each conceptual unit.

Calculation of critical flow and its effect becomes an important issue for the overall simulation. An option to model this effect is by introducing auxiliary variables in Eqs. (14) and (18) to extend their applicability to critical flow and to allow that their residuals be nil in those cases where the outlet pressure is lower than the calculated value when using critical flow. However, it was observed that KNITRO allows solving non-linear equations with dynamic bounds on the variables. Thus, the maximum flowrate, calculated with (10), was incorporated in *WellNet* as a bound for the variable. Then, a simple conditional was programmed to make the residuals in Eqs. (14) and (18) nil when the flow is critical and the real outlet pressure is lower than the one calculated with these equations. Section 5 contains examples where some simulation situations are analyzed.

4.2. The optimization problem

The optimization problem attached to the operational problem can be formulated in several forms. In general terms, the objective function consists of maximizing profits, minimizing costs, or a combination of these purposes. For instance, when oil production comes from different reservoirs then the objective function may include a term where the oil production in a well is affected by the price of the type of oil that is produced in each well (Kosmidis et al., 2004). In this work, the objective function consists of associating a producing-cost coefficient for each *well* to minimize the production cost as in Ortiz-Gómez et al. (2001) but reducing the expression to a single period. The objective function is then established as,

$$\min \sum c_w \dot{m}_w, \quad \forall w \in W \tag{20}$$

where c_w is the cost coefficient in US$/mol for *well* w to obtain the objective function in US$/day.

The cost of production for gas in Mexico is around US$ 0.50 per billion BTU at standard conditions and it is almost a constant for each well. The cost of production for oil depends on the well since some may produce naturally but others may require artificial lift. This cost is normally reported in US$/barrel. Obviously, a simple procedure is applied to convert the cost of oil and gas production into the appropriate units required in Eq. (20).

Gas and oil production are subject to reservoir and surface constraints, i.e. reservoir deliverability in a well and surface pressure. In addition, a well is interconnected in a platform with any other well producing from the same reservoir or even to wells producing from different reservoirs and coming from another platform. Mass and energy balances are also required in mixing points. The equations related to these constraints have been already described in the simulation problem and they are directly extracted in *WellNet* from the declared conceptual units and their interconnectivity.

In addition, gas and oil production in the world is regulated by international laws and agreements. Hence, the production must always satisfy a given demand so that the imposed constraint must be an equality equation:

$$\sum \dot{m}_w = d, \quad \forall w \in W \tag{21}$$

where d is the given demand. Molar flowrate is again considered here but volumetric flow could easily be used instead.

It can be observed that flowrates are the most natural variables for the optimization problem. Since valve apertures are used to control flowrates in the model proposed here, valve apertures are used as the optimization variables. In principle, this choice is consistent with the real system since valves associated to wells are physical means to flow control. Valve apertures clearly contain specific bounds [0, 1], which can be enforced to satisfy throughout the optimization process within KNITRO.

The optimization problem consists then in using the objective function, Eq. (20), to satisfy a given demand (Eq. (21)) subject to the equations of the model for *wells* and *manifolds* (Tables 2 and 3), where aperture of valves for *wells* are the unknown variables. Any standalone optimization for *wells* would then produce information on the possibility of satisfying the given demand but there is no standalone simulation for manifolds in *WellNet*.

The overall optimization problem contains the following equations: the objective function (Eq. (20)); demand constraint (Eq. (21)); two equations and the *TState* code for each well (Eqs. (14) and (15)); $(c + 3)$ equations and the *TState* code for each manifold (Eqs. (17a), (17b), (18), and (19)). The result is an

Table 4
Molar composition in well CHUC 62

Species	mol%
Methane	34.029
Ethane	11.138
Propane	7.565
n-Butane	3.756
Isobutane	1.313
n-Pentane	1.326
Isopentane	1.838
n-Hexane	3.010
C7+	32.456
Nitrogen	0.230
Carbon dioxide	2.446
H_2S	0.893

objective function that is certainly linear and a set of constraints that are highly non-linear. If the cheapest resource could satisfy the demand then the solution is rather trivial since the optimum value would obviously be to keep production with just one *well*. However, when several interconnected wells are needed to satisfy the demand the answer is not so simple because of interconnectivity effects. There have been real cases of study where one well may even attempt to inject into another one. In other words, the maximum production of a group of interconnected *wells* is not equivalent to the sum of the maximum production of each individual *well*. This mistake in the model occurs very often during multiperiod planning where interconnectivities are not included in the model. Next section contains a description of some real cases of study where the proposed model has been applied.

5. Illustrative examples

As indicated above, a prototype called *WellNet* has been programmed using C++ Microsoft Visual Studio 6.0 in order to test our approach. It allows incorporation of all conceptual *wells* and *manifolds* having the system memory as the limitation. *K*NITRO was incorporated into the Visual Studio project via a conventional library. The code has not been optimized and may take too much time to solve a given problem because of a continuous memory verification that has been programmed. However, it has been sufficient for our research purposes. The main user interface in *K*NITRO is shown in Fig. 4.

Several examples have been solved to highlight the scope of the proposed approach. Some of them are presented below to show not only convergence properties but also feasibility of the model. All examples were calculated in a 700 MHz PC with 390 kB RAM. Though several cubic equations of state could be selected in our prototype for thermodynamic properties estimation, the Peng–Robinson equation has been used in most examples given below.

5.1. Case of study I

The first case of study corresponds to a single Mexican well called CHUC 62. Table 4 shows the reported molar fraction for the mixture in this well. Experimental production flowrate and outlet conditions are also known. Other thermodynamic experimental values have been omitted but Peng–Robinson and Soave cubic equations of state seem to predict well those properties. To detect the accuracy and robustness of the proposed formulation, it was decided to simulate the well in *WellNet* for the known flowrate to compare the outlet pressure with the experimental value.

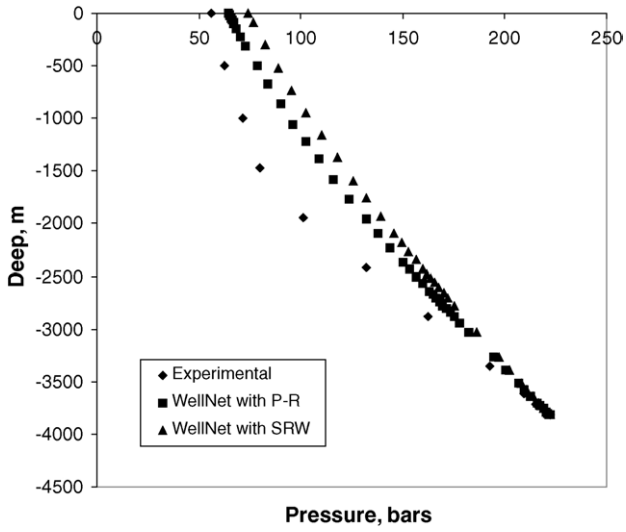

Fig. 5. Pressure profile in well CHUC 62.

The numerical evidence indicates that, for the given well production, the error in the outlet pressure estimation compared to experimental data was within 15%. The pressure profile obtained in *WellNet*, when thermodynamic properties were estimated with Peng–Robinson and Soave, is shown in Fig. 5. It is convenient to show these results in a *P–T* graph to detect the two-phase zone. Fig. 6 shows the predicted *P–T* thermodynamic behavior for this mixture using Peng–Robinson and the estimated pressure profile. It is observed that the error increases when the mixture separates into two phases in equilibrium. This separation occurs at around 3000 m deep.

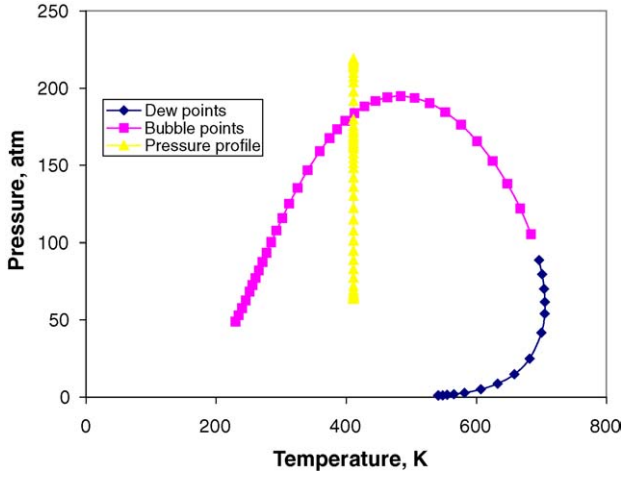

Fig. 6. *P–T* and pressure profile in well CHUC 62.

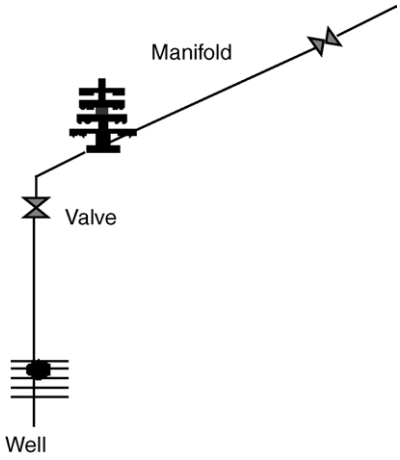

Fig. 7. Case of study II.

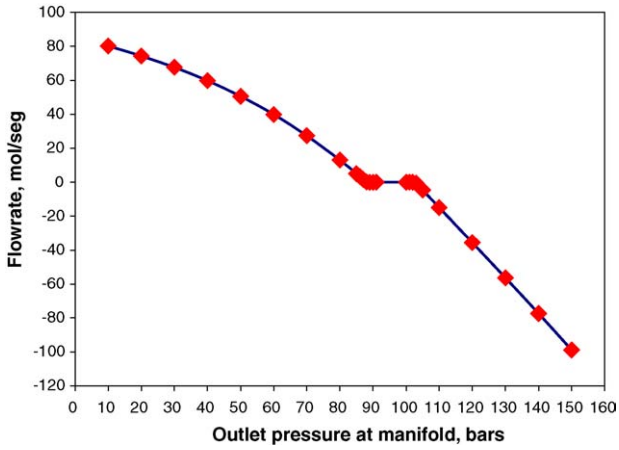

Fig. 8. Production behavior in case of study II.

5.2. Case of study II

An 800 m deep vertical well connected to a 50 m length *manifold* is considered as a second case of study. Fig. 7 shows this system that is a real case situated in the north area of Mexico and the well is named Cañas 2. The system was modeled assuming that conditions of 95 bar and 340 K remains constant at wellbore. The pipe diameter is 0.073 m and composition of the mixture is given in Table 5. The numerical experiment purpose in this case of study was to test the model capability to include reverse flow calculation. The outlet pressure at the *manifold* was then modified and flowrate was calculated in each simulation. Convergence was indeed achieved in all cases in less than 10 iterations and 30 evaluations of the residuals. However, it was observed that convergence around the nil flow took too much time to achieve the solution. The difficulty was in solving the differential set of Eqs. (6) and (7) where the integration step tends to be too small. The solution taken to avoid this numerical situation was to introduce a disjunctive equation where the flowrate is enforced to be nil when, in any iteration, the suggested flow is [−0.5,,0.5] in mole. Thus, Fig. 8 shows the final results where it can be seen that reverse flow may occur when outlet pressure is greater than 100 bar. Optimization of this system is allowed in *WellNet* but the problem is too small. The result however is consistent and provides the aperture of the valve to satisfy a given demand.

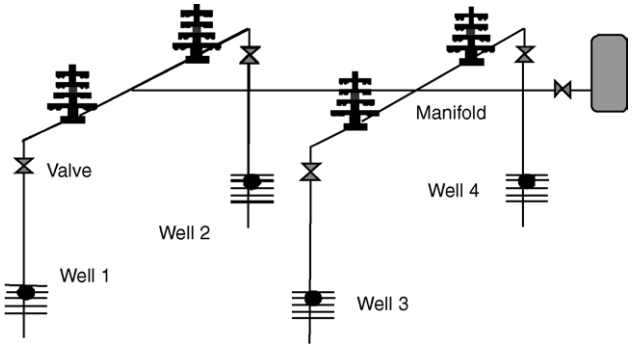

Fig. 9. Case of study III: four *wells* and one *manifold*.

5.3. Case of study III

The third case of study consists of a gas field having four vertical wells with natural flow and one *manifold* (Fig. 9). Since all of them are producing from the same field, then the overall composition of the mixture is the same (Table 6). Each pipe in the well has a diameter of 0.073 m, and lengths of 1355, 1800, 1700, and 1390 m, respectively. The *manifold* is 2000 m length and 0.073 m diameter and the outlet pressure is 40 bar. The process simulation indicates that the gas production would be 2315, 3632, 3853, and 2007 MSCF/day, respectively. It took 8 iterations and 56 residual evaluations in *WellNet*.

Table 5
Molar composition in well Cañas 2

Species	mol%
Methane	65.33
Ethane	5.68
Propane	6.61
n-Butane	2.08
Isobutane	0.41
n-Pentane	0.38
n-Hexane	0.02
Nitrogen	18.34
Carbon dioxide	1.15

Table 6
Molar composition in case of study III

Species	mol%
Methane	86.74
Ethane	10.56
Propane	1.19
n-Butane	0.17
Isobutane	0.12
n-Pentane	0.08
n-Hexane	0.03
Nitrogen	1.03
Carbon dioxide	0.08

Considering the same topology, it was decided to modify the outlet pressure at the *manifold* to verify the possibility of reverse flow. When the outlet pressure in the *manifold* is considered 50 bar, the gas production is −643, 5360, 5589, and −405 MSCF/day. These results indicate that two wells would stop producing and possible would receive mass injected from the two other wells. In this case, it took 7 iterations and 49 evaluations to achieve the solution.

The same topology is considered to show the optimization capability. A demand of 11520 MSCF/day was established and the cost of production associated to wells was 11.4, 12.6, 12.3, and 11.4 expressed in Mexican pesos per MSCF, respectively. The optimal result indicates that valves in wells 1, 3, and 4 should be totally open, whereas the valve in well 2 should be 80% open. In this case, it took 16 iterations and 114 evaluations to achieve the solution. This result is as expected since it goes well with the logical analysis of producing first with the cheapest resource, then with next cheapest resource until the demand is achieved. It indicates that the production system was not badly designed. However, it should also be clear from above that increasing the outlet pressure might produce reverse flow in which case the optimization answer is not so simple.

To continue with testing the accuracy and robustness of the model and numerical approach, several numerical experiments were carried out. Number of iterations, gradient and function evaluations, and time of convergence were registered. Same mixture composition used in the above three cases of study were used with up to 20 wells and 5 manifolds in different topologies and different conditions to force reverse flow. It was observed that same conclusions could be obtained from these experiments or from the already discussed cases of study. The main disadvantage observed in the proposed model is that, when production in one of the wells becomes nil or reverse, the time required for convergence may take more than a day, which is obviously unacceptable.

6. Conclusions

An innovative model has been proposed in this paper for simulation and optimization of gas and oil production systems. The proposed approach integrates the whole process from reservoirs to surface facilities where all units are rigorously modeled. Several features such as reverse flow and detection of critical flow have been incorporated in the model. In principle, the resulting model represents a very difficult set of differential–algebraic equations. However, separating thermodynamic state calculations yields great results in terms of iterations and time of convergence. Results obtained in several cases where this algorithm has been applied demonstrate the robustness of this approach. However, numerical evidence indicates that solving the set of differential algebraic equations may be impractical though they do not generate multiple solutions.

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References

Abdel Waly, A. A., El-Massry, Y., Darweesh, T. A., & El Sallaly, M. (1996). Network model for an integrated production system applied to the Zeit Bay field, Egypt. *Journal of Petroleum Science and Engineering*, *15*, 57–68.

Aronofsky, J. S., & Williams, A. C. (1962). A use of linear programming and mathematical models in underground production. *Management Science*, *8*, 394–407.

Beggs, H. D. (2003). *Production optimization using NODAL™ analysis* (2nd ed.). Tulsa, OK: OGCI Inc.

Brenan, K. E., Campbell, S. L., & Petzold, L. R. (1989). *Numerical solution of initial-value problems in differential–algebraic equations*. New York: North-Holland.

Byrd, R. H., Gilbert, J. Ch., & Nocedal, J. (2000). A trust region method based on interior point techniques for nonlinear programming. *Mathematical Programming*, *89*(1), 149–185.

Eeg, O. S., & Herring, T. (1997). *Combining linear programming and reservoir simulation to optimize asset value*. Paper SPE 37446.

Garvin, W. W., Crandall, H. W., John, J. B., & Spellman, R. A. (1957). Applications of linear programming in the oil industry. *Management Science*, *3*(4), 407–430.

Geoffrion, A. M. (1972). Generalized Benders decomposition. *Journal of Optimization Theory and Applications*, *10*(4), 237–260.

Handley-Schachler, S., McKie, C., & Quintero, N. (2000). *New mathematical techniques for the optimization of oil and gas production systems*. Paper SPE 65161.

Harding, S. T., & Floudas, C. A. (2000). Phase stability with cubic equations of state: Global optimization approach. *AIChE Journal*, *46*, 1422.

Heinemann, R. F., Lyons, S. L., & Tai, W. (1998). Next generation reservoir optimization. *World Oil*, *219*(1), 47–54.

Horne, R. N. (1998). *Modern well test analysis* (2nd ed.). Palo Alto, CA, USA: Petroway Inc.

Incropera, F. I., & De Witt, D. P. (1996). *Introduction to heat transfer* (3rd ed.). John Wiley & Sons.

Iyer, R. R., Grossmann, I. E., Vasantharajan, S., & Cullick, A. S. (1998). Optimal planning and scheduling of offshore oil field infrastructure investment and operations. *Industrial and Engineering Chemistry Research*, *37*, 1380.

Kosmidis, V. D., Perkins, J. D., & Pistikopoulos, E. N. (2004). Optimization of well oil rate allocations in petroleum fields. *Industrial and Engineering Chemistry Research*, *43*, 3513.

Lee, A. S., & Aronofsky, J. S. (1958). A linear programming model for scheduling crude oil production. *Journal of Petroleum Technology*, *10*(7), 51–54.

McAdams, W. H., Woods, W. K., & Heroman, L. C. (1942). Vaporization inside horizontal tubes, 2: benzene–oil mixtures. *Transactions of ASME*, *64*, 193.

McDonald, C. M., & Floudas, C. A. (1995a). Global optimization for the phase stability problem. *AIChE Journal*, *41*, 1798.

McDonald, C. M., & Floudas, C. A. (1995b). Global optimization and analysis for the Gibbs free energy function using the UNIFAC Wilson and ASOG equations. *Industrial and Engineering Chemistry Research*, *34*, 1674.

Morooka, C. K., Guilherme, I. R., & Mendes, J. R. P. (2001). Development of intelligent systems for well drilling and petroleum production. *Journal of Petroleum Science and Engineering*, *32*, 191–199.

Ortiz-Gómez, A., Rico-Ramírez, V., & Vázquez-Román, R. (2001). In R. Gani & S. B. Jorgensen (Eds.), *Mixed-integer multiperiod model for the planning of oilfield production. ESCAPE-11* (pp. 907–912). Denmark: Pergamon Press.

Perry, R. H., Green, D., & Maloney, J. O. (1988). *Perry's chemical engineers' handbook* (6th ed.). McGraw-Hill International Editions.

Poling, B. E., Prausnitz, J. M., & O'Connell, J. P. (2001). *The properties of gases and liquids* (5th ed.). McGraw-Hill.

Seider, W. D., Gautam, R., & White, C. W. (1980). Computation of phase and chemical equilibrium: A review. *American Society Symposium Series*, *124*, 115–134.

Smith, C. A., & Corripio, A. B. (2006). *Principles and practices of automatic process control* (3rd ed.). John Wiley & Sons, Inc.

Smith, J. M., Van Ness, H. C., & Abbot, M. M. (1996). *Introduction to chemical engineering thermodynamics* (5th ed.). McGraw-Hill, Inc.

Van den Heever, S. A., & Grossmann, I. E. (2000). An iterative aggregation/disaggregation approach for the solution of a mixed-integer nonlinear oilfield infrastructure planning model. *Industrial and Engineering Chemistry Research*, *39*, 1955–1971.

Van den Heever, S. A., Grossmann, I. E., Vasantharajan, S., & Edwards, K. (2000). Integrating complex economic objectives with the design and planning of offshore oilfield infrastructures. *Computers and Chemical Engineering*, *24*, 1049–1055.

Van den Heever, S. A., Grossmann, I. E., Vasantharajan, S., & Edwards, K. (2001). A Lagrangean decomposition heuristic for the design and planning of offshore hydrocarbon field infrastructures with complex economic objectives. *Industrial and Engineering Chemistry Research*, *40*, 2857–2875.

Vázquez-Román, R. (1998). A model based on average velocity for gas production pipes simulation. *Computers and Chemical Engineering*, *22*, 307–S314.

Waltz, R. A., Morales, J. L., Orban, D., & Nocedal, J. (August 2003). *KNITRO-direct: A hybrid interior algorithm for non-linear optimization*. Technical report 2003-6. Evanston, IL, USA: Optimization Technology Center, Northwestern University.

Yang, D., Zhang, O., & Gu, Y. (2003). Integrated optimization and control of the production–injection operation systems for hydrocarbon reservoirs. *Journal of Petroleum Science and Engineering*, *37*, 57–69.