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Time-domain performance limitations arising from decentralized architectures and their relationship to the RGA

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Predominantly, control theory deals with centralized (unrestricted) architectures. However, in practice, decentralized architectures are often preferred. The reasons for this preference are manyfold and include ease of understanding, maintainability, cabling issues and others. The aim of the current paper is to gain insight into the fundamental performance limitations that arise from the use of a decentralized architecture. These fundamental limitations can guide the design of decentralized controllers and offer insight into the performance loss incurred by the use of a restricted architecture. An interesting feature of the results is that they depend, *inter-alia*, on the relative gain array (RGA). This gives new insight into this standard tool for assessing input–output pairings in decentralized control architectures.

1. Introduction

Practical control systems often utilize restricted control architectures. The reason for this choice are manyfold and include familiarity, ease of design, intuitive appeal and tuning issues. For this reason, there has been ongoing interest in the design of decentralized control systems (see for example Bristol (1966), Mayne (1973), Güçlü and Özgüler (1986), Bryant and Yeung (1994), Hovd and Skogestad (1994), Sourlas and Manousiousthakis (1995), Savkin and Petersen (1998), Goodwin *et al*. (1999), Yuz and Goodwin (2003), Salgado and Conley (2004)). This line of work owes much to the insights provided by Bristol (1966) regarding the RGA.

Unfortunately, there are relatively few systematic design procedures for decentralized control systems. One reason is that these design problems are typically non-convex (Sourlas and Manousiousthakis 1995). Nonetheless, several design strategies have been proposed. For example, in Sourlas and Manousiousthakis (1995), an ℓ_1 model matching approach is used, and a numerical procedure is considered to obtain the associated optimal controller. The complexity of this approach, motivates several alternatives including the introduction of a weighting function to make the problem convex (Goodwin *et al*. 1999). However this approach does not guarantee a stabilizing solution, since the weighting factor depends explicitly upon an unknown parameter. Moreover, this feature also makes it unclear how one can achieve a good fit in some given frequency range.

Other approximations to the problem of decentralized synthesis include time-varying controllers that minimize a quadratic design criterion (Savkin and Petersen 1998), and sequential loop closure (Mayne 1973, Güçlü and Özgüler 1986, Bryant and Yeung 1994, Hovd and Skogestad 1994).

One issue of importance is that of integrity (Campo and Morari 1994), i.e., whether a system retains stability when one or more controllers are taken out of service. Necessary and sufficient conditions for integrity have been given in Gündes and Kabuli (2001). This, and other properties, have also been explored in a recent PhD thesis (Kariwala 2004).

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The topic of performance bounds and benchmarks has been extensively studied in the case of centralized designs (see Chen *et al.* (2000), Toker *et al.* (2002), Chen *et al.* (2003), Su *et al.* (2003), Silva and Salgado (2005) and the references therein). However, there has been little prior work on extending these results to the decentralized case. Some advances in the case of a minimum variance benchmark have been reported in Kariwala (2004), but these lead to a loose performance bound due to simplifying assumptions.

In Yuz and Goodwin (2003), an optimal Youla parameter is found for a diagonal model of the process and, based on an associated linear approximation, a convex minimization problem is formulated yielding a *correcting* term that minimizes a quadratic measure of the achieved (*real*) sensitivity. This approach was used to develop a means of numerically assessing decentralized performance. However, this approach does not show how the associated measure is related to intrinsic system properties such as open loop poles, zeros, delays, etc. More recent work in Salgado and Conley (2004) uses a, so called, participation matrix to asses decentralized performance. The result is linked to intrinsic system properties, but is not directly related to measurable performance attributes such as overshoot, rise time and others.

Our goal in the current paper is to gain insight into the extra performance loss that results from the use of a decentralized architecture. Thus, our goal is to study fundamental performance limitations for decentralized control systems. Our results extend known results on fundamental performance limitations for feedback systems (see for example Bode (1945), Freudenberg and Looze (1985, 1988), Åström (1991), Chen (1995, 2000), Åström (1997), Seron *et al.* (1997), Sung and Hara (1998)), allowing the explicit consideration of the restricted structure in a decentralized design.

Specifically, we develop a particular measure of decentralized performance which has the dual features of (a) being related to intrinsic system properties such as delays, non-minimum phase zeros, etc. and (b) being directly related to measurable performance attributes such as rise time, etc.

An interesting observation is that our result depends, *inter alia*, on the well known RGA (Bristol 1966). Thus, the result gives further insight into and credibility for, this commonly used tool for evaluating input–output pairings in decentralized control systems. Not surprisingly, the results presented here show that, whilst the RGA certainly plays a key role, it is not the only issue of importance.

Here we restrict attention to open-loop stable, square multivariable linear systems. We develop the results in a continuous-time context but anticipate that similar results hold, *mutatis mutandis*, in the discrete-time case.

2. Preliminaries

Consider the $p \times p$ linear feedback control loop shown in figure 1. In this figure, $\mathbf{G}(s)$ and $\mathbf{C_d}(s)$ denote a $p \times p$ stable plant and a $p \times p$ **diagonal** controller, respectively.

Since our ultimate goal is to design a decentralized controller, we also define a nominal diagonal plant model; i.e.

$$\mathbf{G_o}(s) = \operatorname{diag}\{G_{11}(s), G_{22}(s), \ldots, G_{pp}(s)\}, \tag{1}$$

where

$$[\mathbf{G}(s)]_{ij} = G_{ij}(s) \tag{2}$$

and we also define the additive error transfer function as

$$\mathbf{G}_\epsilon(s) = \mathbf{G}(s) - \mathbf{G_o}(s). \tag{3}$$

The following technical assumptions will be used throughout the paper.

Assumption 1: $\mathbf{G}(0)$ *and* $\mathbf{G_o}(0)$ *are non-singular.*

Note that this assumption is quite standard and is necessary to be able to track arbitrary step references with no steady state error.

Typically, the design of $\mathbf{C_d}(s)$ will be based entirely on $\mathbf{G_o}(s)$. We thus add the following restriction.

Assumption 2: *The controller* $\mathbf{C_d}(s)$ *belongs to the class* $\mathcal{C}$ *of controllers, which is defined as the class of all diagonal, proper and stabilizing controllers for* $\mathbf{G_o}(s)$.

We then have the following.

Lemma 1 (Decentralized closed loop sensitivity)**:** *For any stable plant* $\mathbf{G}(s)$ *we have the following*:

(i) *The class* $\mathcal{C}$ *of controllers can be parameterized as*

$$\mathbf{C_d}(s) = [\mathbf{I} - \mathbf{Q_d}(s)\mathbf{G_o}(s)]^{-1}\mathbf{Q_d}(s), \tag{4}$$

where $\mathbf{Q_d}(s)$ *is any stable proper diagonal transfer function.*

(ii) *The achieved sensitivity function* $\mathbf{S}(s)$ *when* $\mathbf{C_d}(s)$, *as in* (4), *is utilized in the feedback loop of figure* 1 *is*

$$\mathbf{S}(s) = \mathbf{S_o}(s)\mathbf{S_\Delta}(s) \tag{5}$$

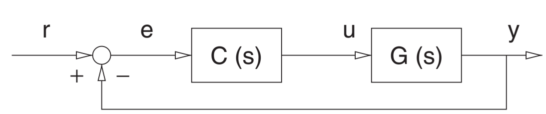

Figure 1. Multivariable control loop.

where

$$\mathbf{S_o}(s) = \mathbf{I} - \mathbf{G_o}(s)\mathbf{Q_d}(s) \tag{6}$$

$$\mathbf{S_\Delta}(s) = [\mathbf{I} + \mathbf{G}_\epsilon(s)\mathbf{Q_d}(s)]^{-1}. \tag{7}$$

Proof:

(i) This is the standard Youla parametrization for the nominal system (Goodwin *et al.* 2001).
(ii) This results from using the definition of the sensitivity for the control loop of figure 1, i.e.

$$\mathbf{S}(s) = [\mathbf{I} + \mathbf{G}(s)\mathbf{C_d}(s)]^{-1},$$

followed by the substitution of $\mathbf{C_d}(s)$ as given in (4). □

Without loosing too much generality, we will further assume the following:

Assumption 3 : $\mathbf{Q_d}(s)$ *is rational, i.e. it has no pure time delays in any of its elements.*

This supposition implies that the delay in each element of the nominal complementary sensitivity, $\mathbf{T_o}(s)$, is simply the delay of the corresponding diagonal element in $\mathbf{G_o}(s)$.

We next observe that all of the well known performance limitations for linear MIMO systems apply to the current situation, since these results typically hold irrespective of the nature of the controller (subject only to the closed loop stability constraint). These performance limits can be expressed either in the time or frequency domains. As a prelude, we state the following result which holds for a SISO loop:

Lemma 2 (SISO cheap control performance bound and a fundamental limitation)**:** *Consider any open loop stable SISO plant without zeros on the imaginary axis written in the form*

$$G_{\mathbf{o}}(s) = B_p(s)\tilde{G}_{\mathbf{o}}(s)e^{-s\tau}, \tag{8}$$

where τ is the pure time delay of the plant, $\tilde{G}_{\mathbf{o}}(s)$ is a proper, stable and minimum-phase (MP) transfer function, and $B_p(s)$ is a Blashke product of the form

$$B_p(s) = \prod_{\ell=1}^{n_z} \frac{-s + z_\ell}{s + z_\ell}, \tag{9}$$

where $\{z_\ell\}_{\ell=1,\ldots,n_z}$ denotes the set of non-minimum phase (NMP) zeros of $G_o(s)$.

Assume that this plant is under feedback control achieving zero steady state error for step references. Denote the closed loop error for unit step references by $e(t)$.

(i) *The energy of $e(t)$ is bounded below by*

$$\inf \int_0^\infty e^2(t)\, dt = \tau + 2\sum_{\ell=1}^{n_z} \frac{1}{z_\ell}. \tag{10}$$

(ii) *Denote by $\{p_j\}_{j=1,\ldots,n_p}$ and by $\{z_k\}_{k=1,\ldots,n_c}$, $n_c \geq n_z$, the sets of poles and zeros of the complementary sensitivity function $T_o(s)$, respectively. Then*

$$\int_0^\infty e(t)\, dt = \lim_{s\to 0}\left[\frac{S_o(s)}{s}\right] = \tau - \sum_{j=1}^{n_p}\frac{1}{p_j} + \sum_{k=1}^{n_c}\frac{1}{z_k}. \tag{11}$$

(iii) *The (idealized) controller that achieves the minimum bound (10) is such that*

$$\int_0^\infty e(t)\, dt = \tau + 2\sum_{\ell=1}^{n_z}\frac{1}{z_\ell}. \tag{12}$$

Proof:

(i) See Goodwin *et al.* (2003).
(ii) The closed loop error satisfies

$$E(s) = \frac{S_o(s)}{s} = \int_0^\infty e(t)e^{-st}\, dt \tag{13}$$

and since the closed loop has zero steady state error for step references, it follows that

$$\int_0^\infty e(t)\, dt = \lim_{s\to 0}\left[\frac{S_o(s)}{s}\right]. \tag{14}$$

Also, since $G_o(s)$ is stable, all admissible complementary sensitivity functions that achieve zero steady state error for step references can be written in the form (recall Assumption 3)

$$T_o(s) = B_p(s)D(s)e^{-s\tau}, \tag{15}$$

where

$$D(s) = \frac{\prod_{\ell=1}^{n_z}(s+z_\ell)\prod_{k=n_z+1}^{n_c}(s-z_k)}{\prod_{j=1}^{n_p}(s-p_j)} \times \frac{\prod_{j=1}^{n_p}(-p_j)}{\prod_{\ell=1}^{n_z} z_\ell \prod_{k=n_z+1}^{n_c}(-z_k)} \tag{16}$$

and $n_p \geq n_c + rd\{G_o(s)\}$ ($rd\{\}$ denotes relative degree). Therefore,

$$\lim_{s\to 0}\left[\frac{S_o(s)}{s}\right] = \lim_{s\to 0}\left[\frac{1 - B_p(s)D(s)e^{-s\tau}}{s}\right]. \tag{17}$$

Using L'Hôpital's rule and the definitions of $B_p(s)$ and $D(s)$, it is straightforward to establish that

$$\lim_{s\to 0}\left[\frac{S_o(s)}{s}\right] = \tau - \sum_{j=1}^{n_p}\frac{1}{p_j} + \sum_{k=1}^{n_c}\frac{1}{z_k} \quad (18)$$

which proves the result.

(iii) It is a standard result (Goodwin *et al.* 2003) that the closed loop whose error satisfies (10) is such that the associated Youla parameter is rational. In addition, the zeros of the optimal complementary sensitivity are the non-minimum phase zeros of the plant (then $n_c = n_z$) and has n_p poles, where $n_p = n_z + rd\{G_o(s)\}$. The location of those poles is as follows: $rd\{G_o(s)\}$ of them are at $s \to -\infty$ and the other n_z, at $s = -z_\ell$ for all ℓ. This implies, according to (11), that

$$\int_0^\infty e(t)dt = \tau - \sum_{\ell=1}^{n_z}\frac{1}{(-z_\ell)} + \sum_{\ell=1}^{n_z}\frac{1}{z_\ell} = \tau + 2\sum_{\ell=1}^{n_z}\frac{1}{z_\ell}. \quad (19)$$

□

Remark 1: Note that, if in a general control loop, $T_o(s)$ has as zeros only the non-minimum phase zeros of the plant, then $n_c = n_z$ and part (ii) of the previous lemma reduces to

$$\int_0^\infty e(t)dt = \tau - \sum_{j=1}^{n_p}\frac{1}{p_j} + \sum_{\ell=1}^{n_z}\frac{1}{z_\ell}. \quad (20)$$

This case is consistent with most nominal design procedures, e.g. $\mathcal{H}_2$ optimization problems.

3. Main result: time domain limits in decentralized control

In this section inescapable time-domain performance limitations that arise from the use of a decentralized architecture, are examined. These constraints are evaluated with reference to the equivalent measures when a centralized architecture is used. A common factor shared by both architectures is the RGA.

We first review some ideas in decentralized control. The RGA of a transfer matrix $\mathbf{G}(s)$ is defined as (Skogestad and Postlethwaite 1996, Albertos and Sala 2004)

$$\Lambda = \mathbf{G}(s) \otimes [\mathbf{G}^{-1}(s)]^T \Leftrightarrow \Lambda_{ij} = G_{ij}(s)\cdot[\mathbf{G}^{-1}(s)]_{ji}, \quad (21)$$

where $\otimes$ denote element by element multiplication.

When $s = 0$ is considered, we have the original definition of the RGA introduced in Bristol (1966). The more general case, i.e. when $s = j\omega$, $\omega \in \mathbb{R}$, leads to the idea of Dynamic Relative Gain Array (DRGA). In this paper, only the RGA for $s = 0$ will be considered.

The RGA is a useful tool for assessing input–output pairings when a complete decentralized control strategy has to be designed. Standard rules state that input–output pairs associated with negative, or large RGA terms, should be avoided and those with RGA terms near 1 should be preferred.

Negative elements in the RGA arise when the d.c. gain of the elements $G_{ij}(s)$ are significantly larger than $\det\{\mathbf{G}(s)\}$, evaluated at d.c. In turn, negative RGA elements are usually accompanied by positive RGA elements larger than 1. This is due to a property of the RGA matrix, namely that the elements in every row (and in every column) add to 1. We refer to this type of plant as *poorly conditioned*.

The reasons to avoid input–output pairings associated with negative RGA terms are stated in the following lemma adapted from Morari and Zafiriou (1989), Campo and Morari (1994), Skogestad and Postlethwaite (1996), Albertos and Sala (2004).

Lemma 3 (Negative RGA elements and closed loop stability)**:** *Suppose that* $\mathbf{G}(s)$ *is a stable and proper transfer function that has non-singular DC gain. Define* $\mathbf{G_o}(s) = \mathrm{diag}\{G_{ii}(s)\}$. *If* $\Lambda_{ii} < 0$ *for some i and a stabilizing decentralized controller* $\mathbf{C_d}(s) = \mathrm{diag}\{C_{ii}(s)\}$, *with integration in each channel, is designed for* $\mathbf{G_o}(s)$, *then*

(i) *The real loop* (*i.e. the loop that considers* $\mathbf{G}(s)$ *and* $\mathbf{C_d}(s)$) *will either be unstable or, if in the real loop the ith loop is opened* (*i.e. controller* $C_{ii}(s)$ *is taken out of service and the plant input* u_i *is kept bounded*), *then the resulting loop will be unstable.*

(ii) *If* $\mathbf{G}(s)$ *has dimension* 2×2, *then the real loop will always be unstable.*

Proof: The proof follows along the same lines as in the proof of Theorem 10.4 in Skogestad and Postlethwaite (1996). □

Lemma 3 states that it is unadvisable to consider a nominal diagonal model whose entries are associated with one or more negative RGA elements. If one insists on considering such nominal models, then in the best case, the loop may be made stable, but if, due to maintenance or saturation, one loop associated with negative RGA elements goes out of service, then the resulting loop will be unstable. This means that the real loop will lack integrity (Campo and Morari 1994).

For the reasons given above, in the sequel it will be assumed that the nominal diagonal model has been selected avoiding input–output pairings associated with negative RGA elements.

We next note that the following result holds for both centralized and decentralized designs:

Theorem 1 (Fundamental MIMO limitations for every control architecture)**:** *Consider a MIMO feedback control loop having stable closed-loop poles located to the left of $-\alpha$ for some $\alpha > 0$. Also assume that zero steady state error occurs for reference step inputs in all channels. Then, for a positive unit reference on the rth channel, the loop (vector) error and (vector) output, denoted respectively by $\mathbf{e^r}(t)$ and $\mathbf{y^r}(t)$, satisfy the following:*

(i) *For any plant zero z_o with left directions $\mathbf{h}_1{}^T, \ldots, \mathbf{h}_{\mu_z}{}^T$, satisfying $\Re\{z_o\} > -\alpha$, we have that*

$$\int_0^\infty \mathbf{h}_i{}^T \mathbf{e^r}(t) e^{-z_o t}\, dt = \frac{h_{ir}}{z_o}, \quad i = 1, 2, \ldots, \mu_z, \tag{22}$$

where h_{ir} is the rth element in $\mathbf{h_i}$.

(ii) *Also, for $\Re\{z_o\} > 0$,*

$$\int_0^\infty \mathbf{h}_i{}^T \mathbf{y^r}(t) e^{-z_o t}\, dt = 0, \quad i = 1, 2, \ldots, \mu_z. \tag{23}$$

Proof: This is a standard result for any stable MIMO design, see for example Goodwin *et al.* (2001). □

Note, in particular, that if there are no MIMO delays or non-minimum phase zeros, then no significant design limitations apply to centralized designs. The next result holds for decentralized designs:

Theorem 2 (Fundamental MIMO limitations in a decentralized architecture)**:** *Consider a stable MIMO feedback control loop based on a decentralized architecture and subject to Assumptions 1, 2, 3. Assume that zero steady state error occurs for step references in all channels, and that a positive unit step reference is applied on the rth channel. Then, the jth component of the loop error, $e_j^r(t)$, satisfies*

$$\int_0^\infty e_j^r(t)\, dt = \frac{G_{jj}(0)}{G_{rj}(0)} \Lambda_{rj} \left\{ \tau_j - \sum_{k=1}^{n_j} \frac{1}{p_k^j} + \sum_{\ell=1}^{m_j} \frac{1}{z_\ell^j} \right\}, \tag{24}$$

where $\{p_k^j\}_{k=1,\ldots,n_j}$ and $\{z_\ell^j\}_{\ell=l,\ldots,m_j}$ denotes the sets of poles and zeros, respectively, of the jth diagonal entry of the nominal (diagonal) complementary sensibility $\mathbf{T_o}(s)$. Also, Λ_{rj} is the (r, j)th element of the RGA matrix, and τ_j is the pure delay in $G_{jj}(s)$.

Proof: We note that, to achieve zero steady state error, we require that

$$\mathbf{Q_d}(0) = [\mathbf{G_o}(0)]^{-1}. \tag{25}$$

This result implies, upon using (7), that

$$\mathbf{S}_\Delta(0) = [\mathbf{I} + (\mathbf{G}(0) - \mathbf{G_o}(0))\mathbf{Q_d}(0)]^{-1} = \mathbf{G_o}(0)\mathbf{G}^{-1}(0). \tag{26}$$

For a unit step in the rth channel, we have that the Laplace transform of the control error in that channel satisfies

$$\mathbf{E^r}(s) = \mathbf{S}(s)\mathbf{v_r}\frac{1}{s}, \tag{27}$$

where $\mathbf{v_r}$ is the null column vector save for a unit element in the rth row.

Since (25) holds, we have that $\mathbf{E^r}(s)$ converges for all $\Re\{s\} > -\alpha$, and hence

$$\int_0^\infty e_j^r(t)\, dt = \lim_{s\to 0} \mathbf{v_j}{}^T \mathbf{E^r}(s) = \lim_{s\to 0} \mathbf{v_j}{}^T \frac{\mathbf{S}(s)}{s}\mathbf{v_r} \tag{28}$$

$$= \lim_{s\to 0} \mathbf{v_j}{}^T \frac{\mathbf{S_o}(s)\mathbf{S}_\Delta(s)}{s}\mathbf{v_r}. \tag{29}$$

Now we have that

$$\mathbf{v_j}{}^T \frac{\mathbf{S_o}(s)}{s} = \begin{bmatrix} 0 & \ldots & 0 & \frac{S_{o_{jj}}(s)}{s} & 0 & \ldots & 0 \end{bmatrix} \tag{30}$$

which implies, jointly with (26), that

$$\int_0^\infty e_j^r(t)\, dt = \lim_{s\to 0}\left[\frac{S_{o_{jj}}(s)}{s}\right] G_{jj}(0)[\mathbf{G}^{-1}(0)]_{jr}. \tag{31}$$

The result follows using part (ii) of Lemma 2 and the definition of the RGA. □

Remark 2: If all open loop stable zeros are cancelled in the nominal design and the controller is itself minimum phase then the previous theorem reduces to (see Remark 1)

$$\int_0^\infty e_j^r(t)\, dt = \frac{G_{jj}(0)}{G_{rj}(0)} \Lambda_{rj} \left\{ \tau_j - \sum_{k=1}^{n'_j} \frac{1}{p_k^j} + \sum_{\ell=1}^{m'_j} \frac{1}{z_\ell^j} \right\}, \tag{32}$$

where $\{z_\ell^j\}_{\ell=1,\ldots,m'_j}$ is the set of non-minimum phase zeros of $G_{jj}(s)$ and $\{p_k^j\}_{k=1,\ldots,n'_j}$, the set of closed loop poles for the jth nominal loop.

We see that the RGA plays a key role in the result of Theorem 2. However, (24) gives additional insight since it shows that the RGA is only part of the story in the quantification of the time domain limitations for decentralized control architectures. Other factors which influence the result are the nominal closed loop bandwidths (i.e. the inverse of the poles $p_1^j, p_2^j, \ldots, p_{n_j}^j$), the nominal closed loop zeros and the time delays in the diagonal transfer function elements $G_{jj}(s)$. Non-minimum phase zeros in the diagonal elements of the nominal model (obviously not cancelled) play central roles in the third term of (24).

It is interesting to compare of Theorem 2 with the following result for all MIMO designs.

Theorem 3 (Fundamental MIMO limitation in the genreal MIMO case)**:** *Consider a stable full-MIMO control loop, based on a $p \times p$ plant $\mathbf{G}(s)$ with non singular DC gain. The controller is assumed to be parameterized in Youla form, with a rational $p \times p$ full MIMO parameter $\mathbf{Q}(s) = [Q_{ij}(s)]$.*

(i) *Suppose that there is zero steady state error for step references in all channels. Then, for a step reference in the rth channel, the jth component of the control error, denoted by $e_j^r(t)$, satisfies*

$$\int_0^\infty e_j^r(t)\,dt = \sum_{i=1}^{p} \frac{G_{ji}(0)}{G_{ri}(0)} \Lambda_{ri} \left(\tau_i^{jr} - \sum_{\ell=1}^{n_{p_i}^{jr}} \frac{1}{p_{i\ell}^{jr}} + \sum_{k=1}^{n_{z_i}^{jr}} \frac{1}{z_{ik}^{jr}} \right), \tag{33}$$

where Λ_{ri} denotes the (r,i)th element of the RGA matrix for $\mathbf{G}(s)$, τ_i^{rj} is the time delay in $G_{ji}(s)Q_{ir}(s)$, $\{z_{ik}^{jr}\}_{k=1\cdots,n_{z_i}^{jr}}$ denote the set of zeros of $G_{ji}(s)Q_{ir}(s)$ and $\{p_{i\ell}^{jr}\}_{\ell=1\ldots,n_{p_i}^{jr}}$, the set of poles of $G_{ji}(s)Q_{ir}(s)$.

In particular, if $j=r$, (33) becomes

$$\int_0^\infty e_j^j(t)\,dt = \sum_{i=1}^{p} \Lambda_{ji} \left(\tau_i^{jj} - \sum_{\ell=1}^{n_{p_i}^{jj}} \frac{1}{p_{i\ell}^{jj}} + \sum_{k=1}^{n_{z_i}^{jj}} \frac{1}{z_{ik}^{jj}} \right) \tag{34}$$

(ii) *Also, we have*

$$\sum_{i=1}^{p} \frac{G_{ji}(0)}{G_{ri}(0)} \Lambda_{ri} = \begin{cases} 0 & \text{if } j \neq r \\ 1 & \text{if } j = r. \end{cases} \tag{35}$$

Proof:

(i) Suppose that $j \neq r$. Since the control loop is stable, $E(s)$ converges for $s=0$. Therefore,

$$\mathbf{E}^{\mathbf{r}}(s) = \mathbf{S}(s)\frac{\mathbf{v_r}}{s} \Rightarrow E_j^r(0) = \int_0^\infty e_j^r(t)\,dt = \lim_{s\to 0} S_{jr}(s)\frac{1}{s}. \tag{36}$$

Note that in order to have integration in the loop, $S_{jr}(0) = 0$. On the other hand,

$$\begin{aligned} S_{jr}(s) &= -T_{jr}(s) \\ &= -\sum_{i=1}^{p} G_{ji}(s)Q_{ir}(s) \\ &= -\sum_{i=1}^{p} \frac{\prod_{k=1}^{n_{z_i}^{jr}}(\beta_{ik}^{jr}s+1)}{\prod_{\ell=1}^{n_{p_i}^{jr}}(\alpha_{il}^{jr}s+1)} K_i^{jr} e^{-\tau_i^{jr}s}, \end{aligned} \tag{37}$$

where $\alpha_{il}^{jr} = (-p_{il}^{jr})^{-1}$, $\beta_{ik}^{jr} = (-z_{ik}^{jr})^{-1}$ and $K_i^{jr} = G_{ji}(0)Q_{ir}(0)$. Note that

$$S_{jr}(0) = 0 \Leftrightarrow \sum_{i=1}^{p} K_i^{jr} = 0. \tag{38}$$

Therefore, using L'Hôpital's rule it follows that

$$\begin{aligned} E_j^r(0) &= -\sum_{i=1}^{p} K_i^{jr} \lim_{s\to 0} \frac{d}{ds}\left(\frac{\prod_{k=1}^{n_{z_i}^{jr}}(\beta_{ik}^{j}s+1)}{\prod_{\ell=1}^{n_{p_i}^{jr}}(\alpha_{il}^{jr}s+1)} e^{-\tau_i^{jr}s} \right) \\ &= \sum_{i=1}^{p} K_i^{jr} \left(\tau_i^{jr} + \sum_{l=1}^{n_{p_i}^{jr}} \alpha_{il}^{jr} - \sum_{k=1}^{n_{z_i}^{jr}} \beta_{ik}^{jr} \right). \end{aligned} \tag{39}$$

The result follows considering the definitions of β_{ik}^{jr}, α_{il}^{jr} and noting that, in order to have zero steady state error for step references

$$\begin{aligned} \mathbf{Q}(0) = \mathbf{G}^{-1}(0) &\Leftrightarrow Q_{ij}(0) = [\mathbf{G}^{-1}(0)]_{ij} \\ &\Rightarrow K_i^{jr} = G_{ji}(0)Q_{ir}(0) = \frac{G_{ji}(0)}{G_{ri}(0)}\Lambda_{ri}. \end{aligned} \tag{40}$$

If $j=r$, the proof is analogous to the previous one and therefore, omitted. Note that in this case $S_{jj}(s) = 1 - T_{jj}(s)$ and $S_{jj}(0) = 0 \Leftrightarrow \sum_{i=1}^{p} K_i^{jj} = 1$.

(ii) This result is proven as follows:

$$\begin{aligned} \sum_{i=1}^{p} \frac{G_{ji}(0)}{G_{ri}(0)}\Lambda_{ri} &= \sum_{i=1}^{p} G_{ji}(0)\big[\mathbf{G}^{-1}(0)\big]_{ir} \\ &= \mathbf{G}_{j*}(0)\big[\mathbf{G}^{-1}(0)\big]_{*r} = \begin{cases} 0 & \text{if } j \neq r \\ 1 & \text{if } j = r \end{cases} \end{aligned} \tag{41}$$

where $\mathbf{A}_{j*}$ denotes the jth row of $\mathbf{A}$ and $\mathbf{A}_{*j}$, the jth column of $\mathbf{A}$. □

Remark 3: Theorem 3 applies to all MIMO designs which stabilize the full MIMO plant $\mathbf{G}(s)$. Thus Theorem 3 also applies to decentralized designs, as treated in Theorem 2. To use Theorem 3 for the evaluation of the accumulated errors in a decentralized design, it is necessary to interpret the Youla parameter $\mathbf{Q}(s)$ appropriately. Indeed, let $\mathbf{T_o}(s)$ be a nominal (diagonal) complementary sensitivity resulting from a decentralized design. Then the corresponding diagonal Youla parameter is

$$\mathbf{Q_d}(s) = \mathbf{G_o}^{-1}(s)\mathbf{T_o}(s) = \mathbf{G_o}^{-1}(s)(\mathbf{I} - \mathbf{S_o}(s)). \tag{42}$$

We can then evaluate the corresponding Youla parameter for use with $\mathbf{G}(s)$ as follows:

$$\mathbf{Q}(s) = \mathbf{G}^{-1}(s)\mathbf{T}(\mathbf{s}) = \mathbf{G}^{-1}(s)(\mathbf{I} - \mathbf{S_\Delta}(s) + \mathbf{G_o}(s)\mathbf{Q_d}(s)\mathbf{S}(s)), \tag{43}$$

where $\mathbf{T}(s)$ is the achieved complementary sensitivity in the real loop. If $\mathbf{Q}(s)$ as in equation (43) is utilized in Theorem 3 then the result reduces to the expression given in Theorem 2 for the decentralized case.

In view of the above discussion, we will use Theorem 3 only for centralized architectures. When a decentralized architecture is taken into account, we will use the more explicit result given in Theorem 2 for this case.

4. Interpretation of the results

This section presents some implications of the results in decentralized performance evaluation.

4.1. *Centralized versus decentralized performance*

While Theorems 1 and 3 hold for every linear MIMO controller, Theorem 2 holds only for decentralized architectures. The main difference between Theorem 3 and Theorem 2 is that equation (33) implies that, in a not necessarily decentralized architecture, the accumulated error depends on a linear combination of effects; in particular, the coefficients of that linear combination satisfy (35). Hence, a centralized architecture can yield a lower accumulated error by using the MIMO interaction, implicit is this linear combination, in a beneficial fashion in the design. This is not possible in the decentralized case due to the restricted architecture.

Further insight into the comparison between centralized and decentralized designs is provided in the following corollary to Theorems 2 and 3.

Corollary 1 (Comparison between centralized and decentralized designs)**:** *Consider a rational MP plant* $\mathbf{G}(s)$ (*with no delays*) *having a NMP nominal model* $\mathbf{G_o}(s)$ *satisfying the conditions of Theorem* 2. *Then,*

(i) *If a centralized architecture is used, then the accumulated errors can be made arbitrarily small.*

(ii) *If a decentralized architecture is used and* $[\mathbf{T_o}(s)]_{jj}$ *is such that it has as zeros only the NMP zeros of the diagonal elements* G_{jj}, *then the* (*absolute value of the*) *accumulated errors are bounded from below by*

$$\left|\int_0^\infty e_j^r(t)\,dt\right| \geq \left|\frac{G_{jj}(0)}{G_{rj}(0)}\Lambda_{rj}\left\{\tau_j + \tau_{\text{dom}} + \sum_{\ell=1}^{m'_j}\frac{1}{z_\ell^j}\right\}\right|, \tag{44}$$

where the same notation as in Remark 2 has been used and τ_{dom} *denotes the dominant time constant of the jth loop* (*a real dominant pole is assumed*).

Proof:

(i) Since the full MIMO plant $\mathbf{G}(s)$ is assumed MP, one can choose the corresponding Youla parameter as

$$\mathbf{Q}(s) = \mathbf{G}^{-1}(s)\ \mathrm{diag}\left\{\frac{1}{(\alpha_i s + 1)^{n_i}}\right\}_{i=1\cdots p}, \tag{45}$$

where $\alpha_i > 0$ and n_i are appropriate integers such that $\mathbf{Q}(s)$ proper. In this case,

$$\mathbf{T}(s) = \mathrm{diag}\left\{\frac{1}{(\alpha_i s + 1)^{n_i}}\right\}_{i=1\cdots p}, \tag{46}$$

which implies that $e_j^r \equiv 0$ for $j \neq r$. For $j = r$ it suffices to use part (ii) of Lemma 2. Letting $\alpha_i \to 0$ then yields the result.

(ii) Using equation (32) it follows that

$$\left|\int_0^\infty e_j^r(t)\,dt\right| = \left|\frac{G_{jj}(0)}{G_{rj}(0)}\Lambda_{rj}\left\{\tau_j - \sum_{k=1}^{n'_j}\frac{1}{p_k^j} + \sum_{\ell=1}^{m'_j}\frac{1}{z_\ell^j}\right\}\right|. \tag{47}$$

Also, the poles of the nominal jth loop cannot be made arbitrarily fast, due to the robust stability requirement. It follows that there must be, at least, one relatively slow dominant pole, which we denote p_{dom}. The result then follows if one lets $\tau_{\text{dom}} = -(1/p_{\text{dom}})$. □

Remark 4: The last corollary states that it is always possible, in the centralized control of MP MIMO plants, to achieve lower accumulated errors than when using a decentralized design. This is as expected since $\mathbf{G}(s)$ is assumed here to have no MIMO zeros and therefore there are no (important) limitations on the achievable performance for centralized designs. On the other hand, decentralized designs need to deal with the possibility that the nominal model has NMP zeros and that the nominal closed loop poles cannot be made arbitarily fast, due to robust stability requirements.

Next, consider a more general situation when the full MIMO plant is NMP. Assume first that the RGA has diagonal elements larger than 1 (which implies that negative elements exist in the same row and in the same column). Say that $\Lambda_{jj} > 1$. Assume that a decentralized controller is designed for the given plant. Then, the error in channel j, when a unit step reference is applied to that channel, is given by (24) with $j = r$, that is

$$\int_0^\infty e_j^j(t)\Big|_{\text{decent}} dt = \Lambda_{jj}\left\{\tau_j - \sum_{k=1}^{n_j}\frac{1}{p_k^j} + \sum_{\ell=1}^{m_j}\frac{1}{z_\ell^j}\right\} > \left\{\tau_j - \sum_{k=1}^{n_j}\frac{1}{p_k^j} + \sum_{\ell=1}^{m_j}\frac{1}{z_\ell^j}\right\}. \quad (48)$$

We next consider the same plant, but under centralized control. Also assume for simplicity that all products $G_{ji}(s)Q_{ij}(s)$, $i = 1, 2, \ldots, p$, have the same delays, and the same poles and zeros as $[\mathbf{T}_o(s)]_{jj}$. Then equation (33) together with (35), for $j = r$, implies that

$$\int_0^\infty e_j^j(t)\Big|_{\text{cent}} dt = \left(\tau^{jj} - \sum_{\ell=1}^{n_p^{jj}}\frac{1}{p_\ell^{jj}} + \sum_{k=1}^{n_z^{jj}}\frac{1}{z_k^{jj}}\right) = \left\{\tau_j - \sum_{k=1}^{n_j}\frac{1}{p_k^j} + \sum_{\ell=1}^{m_j}\frac{1}{z_\ell^j}\right\}. \quad (49)$$

This result establishes that centralized control yields a lower accumulated error than the decentralized one. This result relies on the assumption that $\Lambda_{jj} > 1$. If that is not the case, the centralized design can still deliver smaller accumulated errors in the remaining channels. To see this assume that all products $G_{ji}(s)Q_{ij}(s)$, $i = 1, 2, \ldots, p$, have the same delays, and the same poles and zeros. Then using (33) and (35), with $j \neq r$, yields

$$\int_0^\infty e_j^r(t)\Big|_{\text{cent}} dt = 0. \quad (50)$$

The comparison is further highlighted when NMP zeros are present. Consider a poorly conditioned 2×2 plant, with $\Lambda_{11} = \Lambda_{22} > 1$. It will be assumed that this pairing defines the nominal diagonal model $\mathbf{G_o}(s)$ used for decentralized design.

Suppose that the plant has only one NMP MIMO zero at $s = z$, associated with the canonical left direction $\mathbf{v}_j{}^T$. Note that this implies that $G_{jj}(s)$ has a NMP zero at $s = z$.

Consider a nominal (decentralized) design that cancels all stable zeros of the nominal model and has (very) fast closed loop poles. Under these conditions, Theorem 2 implies that

$$\int_0^\infty e_j^j(t)\, dt = \Lambda_{jj}\frac{1}{z}. \quad (51)$$

On the other hand, consider a centralized design based on the same criteria as in the decentralized case, i.e.: fast uncancelled closed loop poles and no uncancelled stable zeros. Also, assume that $Q_{ij}(s)$ cancels all of the stable zeros of $G_{ji}(s)$. Under these conditions, according to Theorem 3, we have that

$$\int_0^\infty e_j^j(t)\, dt = \sum_{i=1}^{p}\Lambda_{ji}\frac{1}{z} = \frac{1}{z}. \quad (52)$$

Note that in the above we have used the fact that each row sum of the RGA equals 1.

Since the plant is assumed to be poorly conditioned, $\Lambda_{jj} > 1$. Then, (51) implies that the long term average of $e_j^j(t)$, in the decentralized case, will be large relative to the centralized case. Note that this average will be even greater if z is a slow NMP zero. On the other hand, (52) shows that even for poorly conditioned plants, it is possible to achieve good performance with a centralized design, and that this is easier if z is a fast NMP zero. In conclusion, having $\Lambda_{jj} > 1$ may seriously deteriorate the achievable performance in the decentralized case, as compared to the centralized case.

If $0 < \Lambda_{jj} < 1$, then the accumulated error in the jth channel, when using a decentralized architecture, may be smaller than for the case of centralized control. However, the effect in the rest of the channels can be always made smaller in the centralized architecture due to the structure of (33) and the constraint (35) for $j \neq r$.

4.2. *Bounds on transient times*

Accumulated errors, as discussed so far, generate insight into control loop performance. However, accumulated errors are not a norm, and a small value does not necessarily mean that the error itself is small. In this section, lower bounds for the settling time in decentralized control loops are derived. Also the interdependence of these bounds with other time-domain performance indicators is explored. Several cases of interest are discussed and compared with the centralized case.

To generate a unified treatment of the subject, first note that the results in Theorems 2 and 3, all take the generic form

$$\int_0^\infty e(t)\, dt = \Omega. \quad (53)$$

We thus define a feedback control scalar error function, $e(t) = r(t) - y(t)$, with Laplace transform $E(s)$. We assume that this error is driven by a unit step input, i.e. $r(t) = \mu(t)$, and that $E(s)$ can be described by

$$E(s) = (1 - F(s))\frac{1}{s} \tag{54}$$

with

$$F(s) = \sum_{i=1}^{M} F_i(s) e^{-s\tau_i}, \tag{55}$$

where $F_i(s)$ is stable and strictly proper $\forall i \in (1, 2, \dots, M)$. We also assume that $F(0) = 1$.

With this error definition we consider the accumulated error as in (53).

We next analyse several specific cases.

Case 1: $\Omega < 0$ In this case, the error must be predominantly negative, that is, the negative accumulation must be larger than the positive accumulation. This implies that $y(t)$ overshoots the reference $r(t)$. We can then build a lower bound, $e_a(t)$, for the error, as shown in figure 2. This bound is defined by

$$e_a(t) = \begin{cases} 1 & 0 < t < \tau_{\min} \\ -\varepsilon & \tau_{\min} \le t < t_2 \\ -\beta\varepsilon e^{-(t-t_2)p_d} & t \ge t_2, \end{cases} \tag{56}$$

where $0 < \beta < 1$, $\varepsilon = -\min_t e(t)$, $\tau_{\min} = \min\{\tau_1, \tau_2, \dots, \tau_M\}$ and $p_d = |\Re\{\text{dominant poles of } F(s)\}|$.

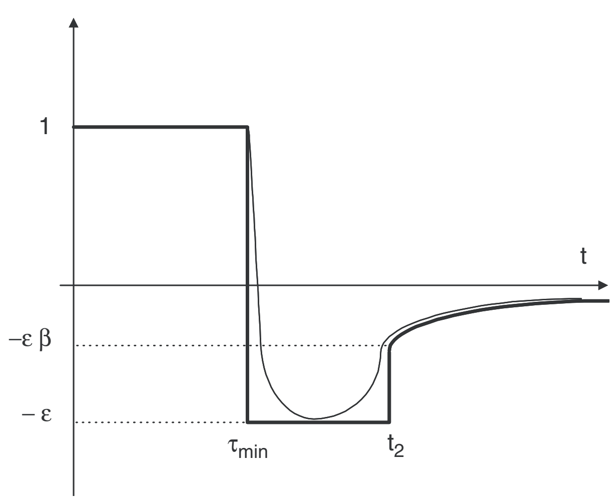

Figure 2. Example of admissible error function (solid-thin) and lower bound (solid-thick) for case 1.

Therefore

$$\begin{aligned} \Omega &\ge \tau_{\min} - \varepsilon(t_2 - \tau_{\min}) - \beta\varepsilon \int_{t_2}^{\infty} e^{-p_d(t-t_2)}\, dt \\ &= \tau_{\min} - \varepsilon(t_2 - \tau_{\min}) - \frac{\beta\varepsilon}{p_d}. \end{aligned} \tag{57}$$

If we recall that, in this case, $\Omega < 0$ and making the assumption that $p_d(t_2 - \tau_{\min}) \ge 1$, we then have

$$t_2 - \tau_{\min} \ge \frac{|-\Omega + \tau_{\min}|}{\varepsilon(1+\beta)} = \mathcal{B}_1. \tag{58}$$

The following observations apply

- $t_\Delta = t_2 - \tau_{\min}$ is the fraction of the settling time that can be reduced by the designer.
- Inequality (58) provides a lower bound for t_Δ, which has a direct relation with $|\Omega - \tau_{\min}|$, i.e. with the error accumulated in $[\tau_{\min}, \infty)$, and an inverse relation with ε and β, i.e. with the size of the overshoot and the speed of response.
- An alternative formulation for (58) is

$$\varepsilon \ge \frac{|-\Omega + \tau_{\min}|}{(t_2 - \tau_{\min})(1+\beta)}, \tag{59}$$

where now a lower bound for the overshoot is obtained.
- The results above are valid not only for $\Omega < 0$, but also for the more general case $\Omega < \tau_{\min}$.

Assume now that $F(s)$ has a zero located at $s = -c$, $c > 0$, with $c < |p_d|$, then an additional constraint on the error function arises, namely

$$\int_0^{\infty} e(t) e^{ct}\, dt = -\frac{1}{c} < 0. \tag{60}$$

This constraint expresses the fundamental limitation in Theorem 1, which is independent of the controller structure.

When the lower bound for the error defined in figure 2 is used in (60) we obtain

$$-\frac{1}{c} \ge \int_0^{\tau_{\min}} e^{ct} dt - \varepsilon \int_{\tau_{\min}}^{t_2} e^{ct} dt - \beta\varepsilon \int_{t_2}^{\infty} e^{-(t-t_2)p_d} e^{ct} dt \tag{61}$$

which leads to

$$t_2 - \tau_{\min} \ge \frac{1}{c}\left(\ln\left(\frac{1}{\varepsilon} + 1\right) + \ln\left(\frac{x}{\beta + x}\right)\right) = \mathcal{B}_2, \tag{62}$$

where $x = (p_d/c) - 1$. Note that, if $p_d \gg c$, then

$$t_2 - \tau_{\min} \geq \frac{1}{c}\ln\left(\frac{1}{\varepsilon} + 1\right). \tag{63}$$

Since (58) and (62) must be simultaneously satisfied, we finally have that

$$\begin{aligned} t_2 - \tau_{\min} &\geq \max\{\mathcal{B}_1; \mathcal{B}_2\} \\ &= \max\left\{\frac{|-\Omega + \tau_{\min}|}{\varepsilon(1+\beta)};\ \frac{1}{c}\left(\ln\left(\frac{1}{\varepsilon}+1\right) + \ln\left(\frac{x}{\beta + x}\right)\right)\right\}. \end{aligned} \tag{64}$$

Case 2: $\Omega > 0$ We next consider the case when the accumulated error is positive due to a NMP zero in $F(s)$, located at $s = c$, $c > 0$. Then,

$$\int_0^\infty e(t)e^{-ct}\,dt = \frac{1}{c} > 0 \tag{65}$$

$$\int_0^\infty y(t)e^{-ct}\,dt = 0. \tag{66}$$

Equation (66) implies that $y(t)$ must be negative in a nonzero time interval, that is, there exists undershoot. In other words, the error $e(t)$ must be larger than 1 in some nonzero time interval.

In this case, an upper bound $e_b(t)$ for the error can be built, as shown in figure 3. Here,

$$e_b(t) = \begin{cases} 1 & 0 < t < \tau_{\min} \\ 1 + \delta & \tau_{\min} \leq t < t_2 \\ \gamma(1+\delta)e^{-(t-t_2)p_d} & t \geq t_2, \end{cases} \tag{67}$$

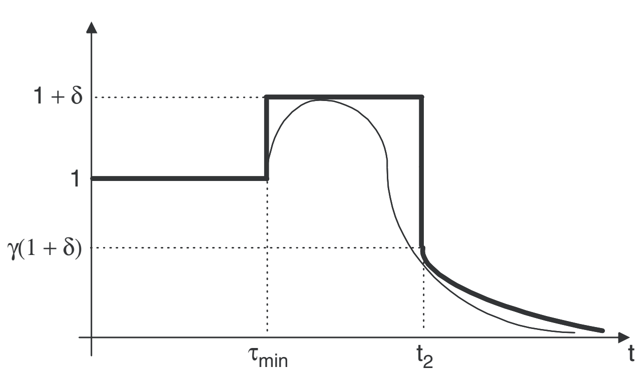

Figure 3. Example of admissible error function (solid-thin) and upper bound (solid-thick) for case 2.

where $0 < \gamma < 1$, $1 + \delta = \max_t e(t)$ and $p_d = |\Re\{\text{dominant poles of } F(s)\}|$.

Substituting these bounds into (65), and proceeding as in the derivation of $\mathcal{B}_2$ in (62), we obtain

$$t_2 - \tau_{\min} \geq \frac{1}{c}\left(\ln\left(\frac{1+\delta}{\delta}\right) + \ln\left(\frac{q-\gamma}{q}\right)\right) = \mathcal{B}_3, \tag{68}$$

where $q = 1 + p_d/c$. Note that if $p_d \gg c$, then the settling time satisfies

$$t_2 - \tau_{\min} \geq \frac{1}{c}\ln\left(\frac{1+\delta}{\delta}\right). \tag{69}$$

As in the case when $\Omega < 0$, the accumulated error leads to an additional constraint for the settling time. This constraint can be obtained in the same way as for case 1, that is

$$\begin{aligned} \int_0^\infty e(t)\,dt = \Omega &\leq \tau_{\min} + (1+\delta)(t_2 - \tau_{\min}) \\ &+ \gamma(1+\delta)\int_{t_2}^\infty e^{-(t-t_2)p_d}\,dt. \end{aligned} \tag{70}$$

Hence, assuming that $p_d(t_2 - \tau_{\min}) \geq 1$, we have that

$$t_2 - \tau_{\min} \geq \frac{\Omega - \tau_{\min}}{(1+\delta)(1+\gamma)} = \mathcal{B}_4. \tag{71}$$

The above results show that the settling time, $t_2 - \tau_{\min}$, must satisfy the following constraint

$$\begin{aligned} t_2 - \tau_{\min} &\geq \max\{\mathcal{B}_3; \mathcal{B}_4\} \\ &= \max\left\{\frac{1}{c}\left(\ln\left(\frac{1+\delta}{\delta}\right) + \ln\left(\frac{q-\gamma}{q}\right)\right);\ \frac{\Omega - \tau_{\min}}{(1+\gamma)(1+\delta)}\right\}. \end{aligned} \tag{72}$$

Note that if small overshoot is specified, i.e. $\delta \to 0$, then $\mathcal{B}_4$ remains bounded, while $\mathcal{B}_3$ diverges. This suggests that, in this case, pairing has no significant influence on the achievable performance of a decentralized loop.

5. Examples

This section presents several examples which illustrate the ideas presented in this work.

5.1. *Example 1*

Consider the following MP plant $\mathbf{G}(s)$ with RGA given by $\boldsymbol{\Lambda}$, where

$$\mathbf{G}(s) = \begin{bmatrix} \frac{4}{(s+1)(s+4)} & \frac{3}{2(s+1)(s+3)} \\ \frac{2}{(s+1)(s+4)} & \frac{3}{(s+1)(s+3)} \end{bmatrix};$$

$$\boldsymbol{\Lambda} = \frac{1}{3}\begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}. \tag{73}$$

5.1.1. Decentralized control. With the nominal model $\mathbf{G_o}(s)$ given by (1), a decentralized controller $\mathbf{C_d}(s)$ is designed to achieve a nominal complementary sensitivity given by

$$\mathbf{T_o}(s) = \begin{bmatrix} \frac{16}{(s+4)^2} & 0 \\ 0 & \frac{16}{(s+4)^2} \end{bmatrix}. \tag{74}$$

If we now compute the accumulated errors for a unit step reference at channel 1, we have based on Theorem 2

$$\int_0^\infty e_1^1(t)\big|_{\text{decent}}\, dt = \frac{2}{3} \tag{75}$$

$$\int_0^\infty e_2^1(t)\big|_{\text{decent}}\, dt = -\frac{1}{3}. \tag{76}$$

5.1.2. Centralized control. We next consider a centralized design, based on $\mathbf{G}(s)$, to achieve a true complementary sensitivity equal to the nominal one above, i.e., $\mathbf{T}(s) = \mathbf{T_o}(s)$. This latter design uses the Youla-parameter:

$$\mathbf{Q}(s) = \begin{bmatrix} \frac{48(s+1)}{9(s+4)} & -\frac{24(s+1)}{9(s+4)} \\ -\frac{32(s+1)(s+3)}{9(s+4)^2} & \frac{64(s+1)(s+3)}{9(s+4)^2} \end{bmatrix} \tag{77}$$

If we now compute the accumulated errors for unit step reference at channel 1, we have based on Theorem 3 that

$$\int_0^\infty e_1^1(t)\big|_{\text{cent}}\, dt = \frac{1}{2} \tag{78}$$

$$\int_0^\infty e_2^1(t)\big|_{\text{cent}}\, dt = 0. \tag{79}$$

Comparing (75), (76) with (78), (79) we see that, the centralized design has the potential for better performance, since the accumulated errors are smaller than are achievable in the decentralized case.

5.2. *Example 2*

Consider the NMP plant

$$\mathbf{G}(s) = \begin{bmatrix} \frac{12s-1}{(12s+1)(4500s+1)} & \frac{12s-1}{(12s+1)(4500s+1)} \\ \frac{-2}{3(4500s+1)} & \frac{12s-1}{(12s+1)(4500s+1)} \end{bmatrix} \tag{80}$$

whose RGA is given by

$$\boldsymbol{\Lambda} = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}. \tag{81}$$

Note that $\mathbf{G}(s)$ has two NMP zeros: one at $s = 1/60$ with left direction $\mathbf{h_1} = 2^{-1/2}[-1 \quad 1]^T$ and the other, at $s = 1/12$ with left direction $\tilde{\mathbf{h}}_\mathbf{1} = [1 \quad 0]^T$.

We will consider two design strategies: a decentralized one and an $\mathcal{H}_2$ (sub)optimal centralized one. We use the term "real loop" to describe the situation when the controller is applied to the true (non diagonal) plant model.

5.2.1. Decentralized control. Suppose that the nominal decentralized design is characterized by

$$\mathbf{T_o}(s) = \begin{bmatrix} \frac{-0.15(s-0.08333)}{(s+0.08333)(s+0.15)} & 0 \\ 0 & \frac{-0.15(s-0.08333)}{(s+0.08333)(s+0.15)} \end{bmatrix} \tag{82}$$

and that a step reference of unit magnitude is applied to the first channel in the real loop. Note that the NMP zero at $s = 1/60$ is a non canonical one and does not appear in any of the diagonal elements of the plant.

Figure 4 shows the accumulated error in each channel, $J_1(t)$ and $J_2(t)$, where

$$J_1(t) = \int_0^t e_1^1(\eta)\, d\eta, \quad J_2(t) = \int_0^t e_2^1(\eta)\, d\eta. \tag{83}$$

To apply Theorem 2 observe that in the nominal designs associated with both channels, there is an uncancelled NMP zero at $s = 0.08333$ and two uncancelled stable

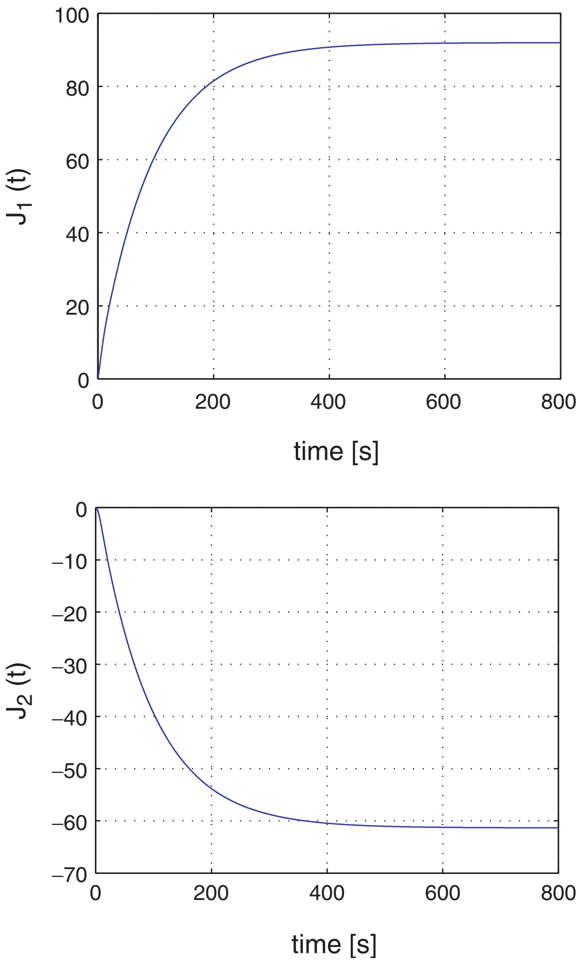

Figure 4. Accumulated integral of the errors for $\mathbf{r}(t) = [1\,0]\mu(t)$ (decentralized case) – Example 2.

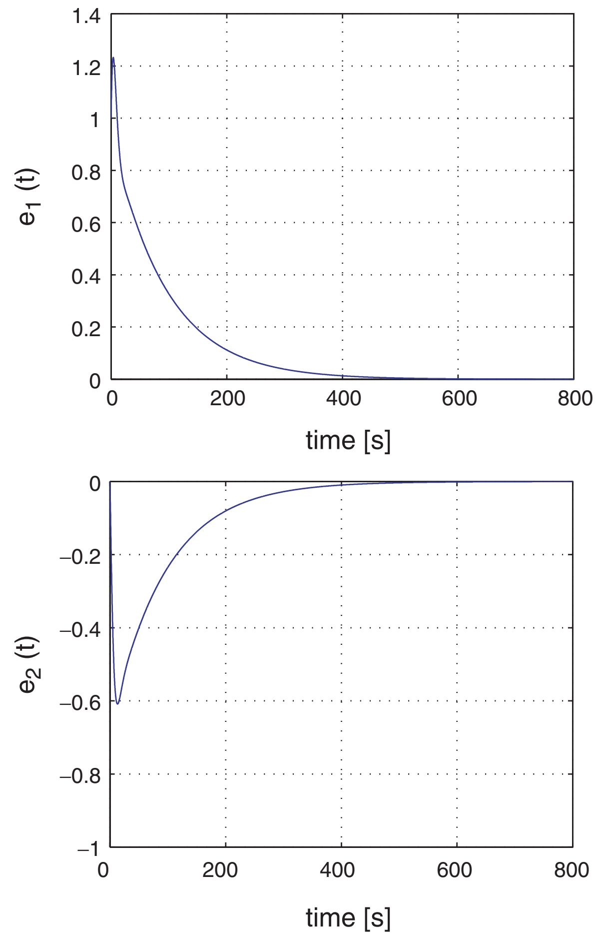

Figure 5. Loop errors for $\mathbf{r}(t) = [1\,0]\mu(t)$ (decentralized case) – Example 2.

closed loop poles at $s = -0.15$ and $s = -0.08333$. Therefore, Theorem 2 predicts

$$
\begin{aligned}
J_1(\infty) &= \int_0^\infty e_1^1(t)\,dt \\
&= \Lambda_{11}\left\{0 + \frac{1}{0.08333} + \frac{1}{0.15} + \frac{1}{0.08333}\right\} = 92 \\
J_2(\infty) &= \int_0^\infty e_2^1(t)\,dt = \frac{G_{22}(0)}{G_{12}(0)}\Lambda_{12} \\
&\quad \times \left\{0 + \frac{1}{0.08333} + \frac{1}{0.15} + \frac{1}{0.08333}\right\} = -61.33.
\end{aligned} \tag{84}
$$

Note that these results are verified by the steady state values shown in figure 4 based on the simulated behaviour.

The corresponding closed loop error response is shown in figure 5, under the same conditions as used to generate figure 4. We next interpret figure 5 in the light of the results in §4.2. Using the notation of figure 3, we choose $\delta \approx 0.23$ and $\gamma \approx 0.0813$ (this leads to $|e_1(t)| < 0.1$ for $t \geq t_2$), and from figure 5 we see that $t_2 \approx 210$. We next compare this experimental result with the bound given in (72). Specifically, we note that the dominant pole of $\mathbf{S}(s)$ is located at $s = -p_d = -0.0107$. The bounds $\mathcal{B}_3$ and $\mathcal{B}_4$ are then given by

$$
\begin{aligned}
\mathcal{B}_3 &= \frac{1}{0.08333}\left(\ln\left(\left[1 - \frac{0.08333\gamma}{p_d + 0.08333}\right]\frac{1+\delta}{\delta}\right)\right) = 19.22 \\
\mathcal{B}_4 &= \frac{1}{(1+\gamma)(1+\delta)}(92) = 60.17
\end{aligned} \tag{85}
$$

where we have used (24) to evaluate the appropriate value of Ω. Hence we see that (72) is indeed satisfied. Moreover, we see that the bound $\mathcal{B}_4$ is the more restrictive.

5.2.2. Centralized control. Using the results in Silva and Salgado (2005) it is possible to show that the stable (and not necessarily proper) Youla parameter $\mathbf{Q_{opt}}(s)$ that minimizes the 2-norm of the loop error for any given reference direction is given by

$$\mathbf{Q_{opt}}(s) = \arg \min_{\mathbf{Q}(s)\ \text{stable}} \|\mathbf{E}(s)\|_2^2 = (\xi_c(s)\mathbf{G_o}(s))^{-1}, \tag{86}$$

where $\xi_c(s)$ is an unitary (and with identity DC gain) zero interactor for $\mathbf{G_o}(s)$ (Silva and Salgado 2005), given in this case by

$$\xi_{\mathbf{c}}(s) = \begin{bmatrix} \dfrac{-0.38462(s+0.08333)(s-0.04333)}{(s-0.08333)(s-0.01667)} & \dfrac{0.92308s}{(s-0.01667)} \\ \dfrac{-0.92308s(s+0.08333)}{(s-0.08333)(s-0.01667)} & \dfrac{-0.38462(s+0.04333)}{(s-0.01667)} \end{bmatrix}. \tag{87}$$

In order to obtain a proper controller, $\mathbf{Q_{opt}}(s)$ must be detuned to obtain a stable and proper parameter $\mathbf{Q_{subopt}}(s)$. This can be done considering, for example,

$$\mathbf{Q_{subopt}}(s) = \mathbf{Q_{opt}}(s) \begin{bmatrix} \dfrac{1}{\alpha_1 s+1} & 0 \\ 0 & \dfrac{1}{\alpha_2 s+1} \end{bmatrix}, \tag{88}$$

where $\alpha_1, \alpha_2 > 0$ for stability. Note that if $(\alpha_1, \alpha_2) \to (0, 0)$, $\mathbf{Q_{opt}}(s)$ is recovered.

To compare the responses in this case with the decentralized one, $\alpha_1 = 10$ and $\alpha_2 = 2$ were selected. With this choice $\mathbf{Q_{subopt}}(s)$ is given by

$$\mathbf{Q_{subopt}}(s) = \begin{bmatrix} \dfrac{-353.0769(s+0.06373)(s+0.0002222)}{(s+0.1)(s+0.01667)} & \dfrac{-726.9231(s-0.1548)(s+0.0002222)}{(s+0.5)(s+0.01667)} \\ \dfrac{180(s+0.08333)(s+0.0002222)}{(s+0.1)(s+0.01667)} & \dfrac{-1350(s+0.0002222)(s+0.08333)}{(s+0.5)(s+0.01667)} \end{bmatrix}. \tag{89}$$

Figure 6 shows the loop errors for a step reference in the first channel. It can be seen that the centralized case clearly shows better settling time and smaller amount of overshoot in the error. Additional simulations showed that this conclusion also applies for other reference directions.

Figure 7 shows the accumulated errors $J_1(t)$ and $J_2(t)$, defined as above, for the same reference as used to generate figure 6. The steady state values achieved by these functionals are smaller than their decentralized counterparts. (Compare figure 7 with figure 4.)

To verify the result in Theorem 3, $J_1(\infty)$ and $J_2(\infty)$ can be calculated explicitly. To that end, note that here we have

$$\begin{aligned} G_{11}(s)Q_{11}(s) &= \frac{-0.078462(s+0.06373)(s-0.08333)}{(s+0.1)(s+0.08333)(s+0.01667)} \\ G_{12}(s)Q_{21}(s) &= \frac{0.04(s-0.08333)}{(s+0.1)(s+0.01667)} \\ G_{21}(s)Q_{11}(s) &= \frac{0.052308(s+0.06373)}{(s+0.1)(s+0.01667)} \\ G_{22}(s)Q_{21}(s) &= \frac{0.04(s-0.08333)}{(s+0.1)(s+0.01667)}. \end{aligned} \tag{90}$$

Therefore, Theorem 3 predicts

$$\begin{aligned} J_1(\infty) &= \Lambda_{11}\left(0 + \frac{1}{0.1} + \frac{1}{0.08333} + \frac{1}{0.01667} - \frac{1}{0.06373} + \frac{1}{0.08333}\right) \\ &\quad + \Lambda_{12}\left(0 + \frac{1}{0.1} + \frac{1}{0.01667} + \frac{1}{0.08333}\right) \approx 70.9 \\ J_2(\infty) &= \frac{G_{21}(0)}{G_{11}(0)}\Lambda_{11}\left(0 + \frac{1}{0.1} + \frac{1}{0.01667} - \frac{1}{0.06373}\right) \\ &\quad + \frac{G_{22}(0)}{G_{12}(0)}\Lambda_{12}\left(0 + \frac{1}{0.01667} + \frac{1}{0.1} + \frac{1}{0.08333}\right) \\ &\approx -55.38, \end{aligned}$$

which are in agreement with the asymptotic values shown in figure 7.

5.3. *Example 3*

The next example illustrates a case where the RGA (when considered in isolation) does not give the full picture of the achievable decentralized performance. (Of course this is known in terms of other considerations but is further exemplified by the accumulated error results presented in this paper.)

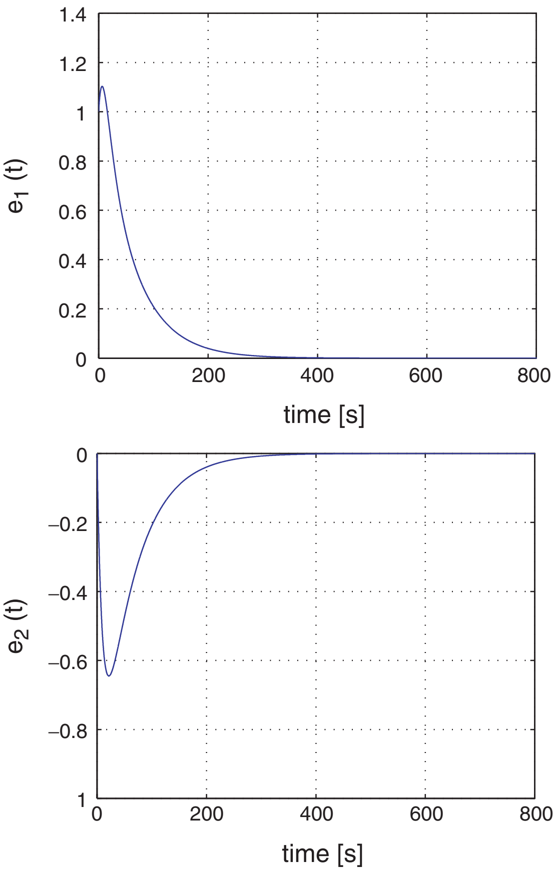

Figure 6. Loop error for $\mathbf{r}(t) = [1\,0]\mu(t)$ (centralized case) – Example 2.

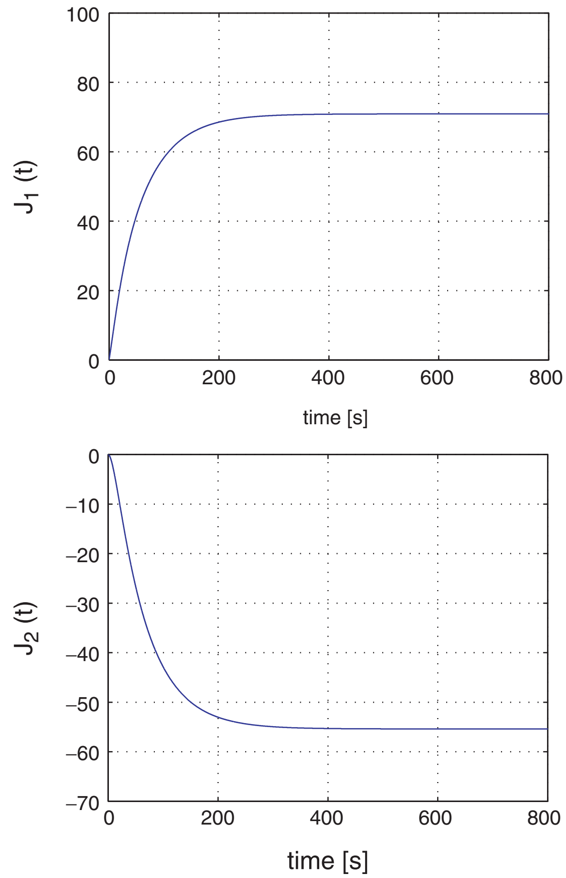

Figure 7. Accumulated integral of the errors for $\mathbf{r}(t) = [1\,0]\mu(t)$ (centralized, non optimal case) – Example 2.

Consider the plant

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{-10\,s+1}{(5\,s+1)(6\,s+1)} & \dfrac{30\,s+1/2}{(5\,s+1)(6\,s+1)} \\ -\dfrac{1}{2}\dfrac{1}{(5\,s+1)(6\,s+1)} & \dfrac{-2\,s+1}{(5\,s+1)(6\,s+1)} \end{bmatrix} \tag{91}$$

whose MIMO zeros are located at $s = -0.0750 \pm 0.2385j$ (i.e. $\mathbf{G}(s)$ is a minimum phase plant). The RGA for this plant is given by

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}. \tag{92}$$

5.3.1. Decentralized control. The nominal model is given by

$$\mathbf{G_o}(s) = \begin{bmatrix} \dfrac{-10\,s+1}{(5\,s+1)(6\,s+1)} & 0 \\ 0 & \dfrac{-2\,s+1}{(5\,s+1)(6\,s+1)} \end{bmatrix} \tag{93}$$

which implies that any nominal stabilizing decentralized design must have a complementary sensitivity of the form

$$\mathbf{T_o}(s) = \begin{bmatrix} (-10s+1)\bar{T}_{11}(s) & 0 \\ 0 & (-2s+1)\bar{T}_{22}(s) \end{bmatrix}, \tag{94}$$

where $\bar{T}_{11}(s), \bar{T}_{22}(s)$ are stable transfer functions with an appropriate relative degree. Suppose that $\bar{T}_{ii}(s)$ has no zeros. From Theorem 3.2, and assuming that fast closed loop poles are chosen, we have

$$\int_0^\infty e_1^1(t)\,dt \geq 10 \cdot \Lambda_{11} = 8,$$

$$\int_0^\infty e_1^2(t)\,dt \leq 10 \cdot \frac{G_{11}(0)}{G_{21}(0)} \Lambda_{21} = -4 \tag{95}$$

$$\int_0^\infty e_2^1(t)\,dt \geq 2 \cdot \frac{G_{22}(0)}{G_{12}(0)} \Lambda_{12} = 0.8,$$

$$\int_0^\infty e_2^2(t)\,dt \geq 2 \cdot \Lambda_{22} = 1.6. \tag{96}$$

Actually, the results in (95) and (96) represent loose bounds on the accumulated errors, since these bounds were calculated assuming that the nominal design can be made arbitrarily fast. This is clearly unrealistic, due to the modelling errors that arise when considering the diagonal nominal model. By way of illustration, note that in this case, the left multiplicative modelling error is given by

$$\mathbf{G}_{\boldsymbol{\Delta}}(s) = \mathbf{G}(s)\mathbf{G}_{\mathbf{o}}^{-1}(s) - \mathbf{I} = \begin{bmatrix} 0 & -\dfrac{1}{2}\dfrac{60\,s+1}{2\,s-1} \\ \dfrac{1}{2}\dfrac{1}{10\,s-1} & 0 \end{bmatrix}. \tag{97}$$

The corresponding singular values are sketched in figure 8. In order to assure robust stability of the real loop, it is sufficient that

$$\bar{\sigma}\{\mathbf{G}_{\boldsymbol{\Delta}}(j\omega)\mathbf{T}_{\mathbf{o}}(j\omega)\} = \bar{\sigma}\left\{\begin{bmatrix} 0 & \dfrac{1}{2}\,\bar{T}_{22}(s)\,(60\,s+1) \\ -\dfrac{1}{2}\,\bar{T}_{11}(s) & 0 \end{bmatrix}\right\} < 1, \quad \forall\omega, \tag{98}$$

where $\bar{\sigma}\{\cdot\}$ denotes the largest singular value. (See also Remark 4.) Therefore, the bandwidth of $\bar{T}_{22}(s)$ should be selected smaller than about 0.03[rad/s]. Note that there is no restriction on the bandwidth of $\bar{T}_{11}(s)$. Using the last considerations, it is possible to refine the bounds given in (95), (96) to

$$\int_0^\infty e_1^1(t) \geq 10 \cdot \Lambda_{11} = 8$$

$$\int_0^\infty e_1^2(t) \leq 10 \cdot \frac{G_{11}(0)}{G_{21}(0)}\Lambda_{21} = -4 \tag{99}$$

$$\int_0^\infty e_2^1(t) \geq (2+33) \cdot \frac{G_{22}(0)}{G_{12}(0)}\Lambda_{12} = 14,$$

$$\int_0^\infty e_2^2(t) \geq (2+33) \cdot \Lambda_{22} = 28. \tag{100}$$

We thus see that there are severe limitations on the integral of errors for a decentralized design for this example. These limitations arise from the non-minimum phase behaviour of the diagonal elements $G_{11}(s)$ and $G_{22}(s)$.

5.3.2. Centralized control. Note that, since $\mathbf{G}(s)$ is minimum phase, there are no significant performance limitations based on the use of a centralized MIMO design. This can be achieved, for example, by choosing

$$\mathbf{Q}(s) = \mathbf{G}^{-1}(s)\begin{bmatrix} \dfrac{1}{\alpha_1 s+1} & 0 \\ 0 & \dfrac{1}{\alpha_2 s+1} \end{bmatrix}, \tag{101}$$

where $\alpha_1, \alpha_2 > 0$. (See Corollary 1, part (i).) This choice for $\mathbf{Q}(s)$ implies that

$$\int_0^\infty e_1^1(t)\,dt = \alpha_1, \quad \int_0^\infty e_1^2(t)\,dt = 0 \tag{102}$$

$$\int_0^\infty e_2^1(t)\,dt = 0, \quad \int_0^\infty e_2^2(t)\,dt = \alpha_2. \tag{103}$$

Indeed, $e_1^2(t) \equiv e_2^1(t) \equiv 0$ for this centralized design.

Comparing the results in (102), (103) with those in (99), (100) we see that it is always possible to choose values for α_1, α_2, in the centralized design, that achieve lower accumulated errors than in the proposed decentralized design. This is as expected since $\mathbf{G}(s)$ has no MIMO NMP zeros and therefore there are no (important) limitations on the achievable performance of the centralized control of this plant. On the other hand, in the decentralized case, the fact that the nominal model has NMP zeros imposes restrictions on the design and performance, which are revealed using the results presented in this paper.

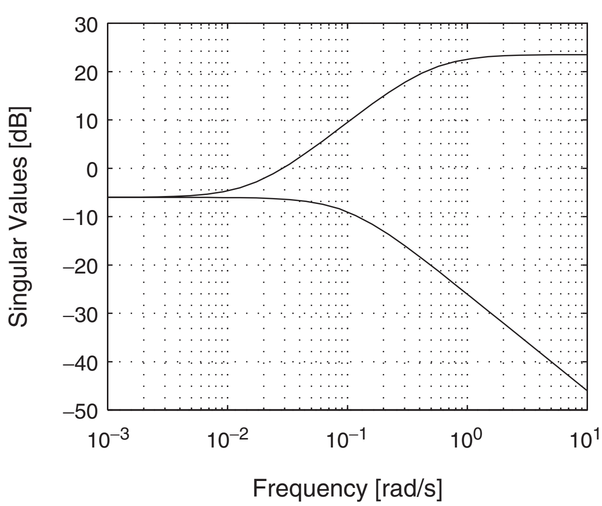

Figure 8. Singular values of $\mathbf{G}_{\boldsymbol{\Delta}}(s)$ – Example 3.

5.4. *Example 4*

The aim of this example is to show how one can use accumulated errors, as an indicator of performance of decentralized control loops. In particular, we will consider the conditions under which it is possible to make the accumulated errors as small as possible with a decentralized control structure.

Consider the NMP 3×3 plant model given by

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-10(s+0.4)}{(s+4)(s+1)} & \frac{0.5}{(s+1)} & \frac{-1}{(s+1)} \\ \frac{2}{(s+2)} & \frac{20(s-0.4)}{(s+4)(s+2)} & \frac{1}{(s+2)} \\ \frac{-2.1}{(s+3)} & \frac{3}{(s+3)} & \frac{30(s+0.4)}{(s+4)(s+3)} \end{bmatrix}. \tag{104}$$

This model has a non-canonical NMP zero at $s = 0.2295$ and a RGA given by

$$\mathbf{\Lambda} = \begin{bmatrix} 2.8571 & -1.2857 & -0.57143 \\ -2.8571 & 3.2381 & 0.61905 \\ 1 & -0.95238 & 0.95238 \end{bmatrix}. \tag{105}$$

The above array suggests the pairing of the ith input with the corresponding ith output, if a decentralized control structure is to be considered. Therefore, a suitable nominal model is given by the NMP transfer function

$$\mathbf{G_o}(s) = \operatorname{diag}\left\{\frac{-10(s+0.4)}{(s+4)(s+1)}, \frac{20(s-0.4)}{(s+4)(s+2)}, \frac{30(s+0.4)}{(s+4)(s+3)}\right\}. \tag{106}$$

Since $\mathbf{C_d}(s)$ is assumed to belong to the class of all stabilizing controller for $\mathbf{G_o}(s)$, then it is clear that all nominally admissible complementary sensitivities are given by

$$\mathbf{T_o}(s) = \begin{bmatrix} T_{11}(s) & 0 & 0 \\ 0 & (-s+0.4)\bar{T}_{22}(s) & 0 \\ 0 & 0 & T_{33}(s) \end{bmatrix}. \tag{107}$$

In the nominal diagonal case, and without paying attention to the nominal NMP zero at $s = 0.4$ or to the magnitude of the control action, there is no need to consider any zeros in $T_{11}(s)$, $\bar{T}_{22}(s)$ or $T_{33}(s)$, and the poles of $\mathbf{T_o}(s)$ may be chosen arbitrarily. Therefore, Theorem 2 allows one to establish the following lower bounds for the accumulated error, due to a unit step change

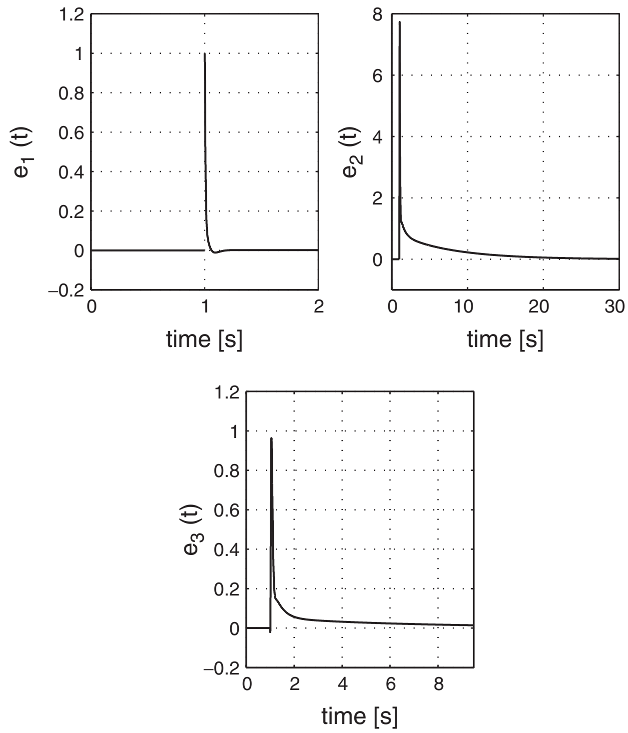

Figure 9. Loop errors of the proposed decentralized design for $\mathbf{r}(t) = [1\,0\,0]\mu(t-1)$ – Example 4.

in the first channel:

$$\int_0^\infty e_1^1(t)\,dt \geq \Lambda_{11}\{0 - 0 + 0\} = 0 \tag{108}$$

$$\int_0^\infty e_2^1(t)\,dt \geq \frac{G_{22}(0)}{G_{12}(0)}\Lambda_{12}\left\{0 - 0 + \frac{1}{0.4}\right\} = 6.43 \tag{109}$$

$$\int_0^\infty e_3^1(t)\,dt \geq \frac{G_{33}(0)}{G_{13}(0)}\Lambda_{13}\{0 - 0 + 0\} = 0. \tag{110}$$

To illustrate (108)–(110), figure 9 shows the error of the true loop for a step change in the first channel, for the following choice for $\mathbf{T_o}(s)$:

$$\mathbf{T_o}(s) = \operatorname{diag}\left\{\frac{60000}{(s+300)(s+200)}, \frac{-3000(s-0.4)}{(s+40)(s+30)}, \frac{6}{(s+3)(s+2)}\right\}. \tag{111}$$

We have chosen a narrow bandwidth for $T_{33}(s)$ to avoid stability problems in the real loop. The simulated accumulated errors are 0.024, 6.51 and 0.47 for channels 1, 2 and 3, respectively. These results can be seen to be consistent with the bounds given above.

It is clear that the choice made above for $\mathbf{T_o}(s)$ is not advisable, since no consideration has been made of robustness issues, nor consideration of the fundamental limitations imposed by the nominal NMP zero. As a matter of fact, it is illustrative to note that the response of the proposed design, although very fast in channels 1 and 3, exhibits unacceptable undershoot in the output of channel 2. It is therefore reasonable to conjecture that any suitable norm used to evaluate the performance of the real loop based on the proposed controller, would show this performance degradation. As an example, the two norm of the error, J, in the case of figure 9 is found to be $J_{\text{decent}} = 6.76$, which is considerably greater then the centralized minimum $J_{\text{opt cent}} = 2.82$, which can be evaluated using the results in Chen *et al.* (2000).

6. Conclusions

This paper has presented a time domain constraint on the integral of the error in a MIMO system subject to a decentralized control architecture constraint. This time domain result can be considered as an additional cost arising from the use of decentralized control. Interestingly, the result is related to the well known RGA, but shows that the RGA is not the only issue of importance in decentralized performance evaluation. Thus, the result gives further insight into the use of this measure to determine input–output pairings in decentralized systems and its interplay with other key plant features such as time delays, NMP zeros, etc.

In addition, the results allow one to obtain explicit bounds for the settling time and undershoot in decentralized architecture control schemes. These bounds can be used to quantify the performance loss arising from decentralized control, and to compare it with centralized designs.

An important conclusion is that, in the centralized case, the design can take advantage of the plant interactions to achieve better performance than in a decentralized one. Usually, decentralized designs simply ignore the plant interactions and therefore, do not attempt to use the interactions in a beneficial fashion.

Future work in this area could include the extension to the case of open loop unstable systems. Also, further research is desirable on the issue of evaluation of decentralized performance bounds, considering the structure restricted minimization of suitable performance indices, such as the 2-norm of the loop error for step references. (The centralized case is treated in Chen *et al.* (2000), Toker *et al.* (2002), Su *et al.* (2003), Silva and Salgado (2005). A solution to this problem would lead to insightfully comparisons between decentralized and centralized control performance, allowing one to explicitly characterise the cases in which decentralized controllers are guaranteed to perform poorly, etc. In Sourlas and Manousiousthakis (1995) some progress has been made in this direction, but the numerical procedure proposed, does not provide insight into the nature of the solution. Another extension of interest would be the consideration of other control strategies, such as triangular or block diagonal designs. This could provide insight into how the structure enrichment of the controller may (or may not) lead to better performing control loops. Preliminary results in this direction have been reported in Salgado and Conley (2004).

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