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Inverted decoupling: a neglected technique

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Abstract

In a conventional decoupling control scheme, process inputs (signals to valves or set points to lower level flow controllers) are produced as a time-weighted combination of process controller outputs. A lesser known technique, here called "inverted decoupling", produces the process input signals by combining one controller output with the other process input signals. Where this technique is applicable, it has several advantages over conventional decoupling. These advantages as well as potential problems are addressed in this paper. Simulation results, related to distillation column applications, are also presented. © 1997 ISA. Published by Elsevier Science Ltd.

Keywords: Advanced control; Decoupling

1. Introduction

Currently, the leading edge of advanced process control technology is some form of model predictive control. This computationally intensive strategy incorporates feedback, feedforward, decoupling, constraint control and optimization in one comprehensive package. Its use is generally for applications where the process size, complexity and potential economic benefits justify the expenditure and technical support requirements.

There are yet many applications, however, which still rely on more conventional control strategies which can be implemented by configuration of function blocks of contemporary distributed control systems. Many of the benefits of advanced process control can thus be obtained by utilizing advanced regulatory control strategies [1] such as cascade, feedforward and override.

Decoupling is an advanced regulatory control strategy which is applicable to processes with multiple inputs, each of which affects multiple output variables. The process inputs may be signals to the final actuators or set points to lower level flow controllers. If each controller output is connected to a single process input, the control loops will interact with each other. The amount of interaction may be anywhere from mildly annoying to severe, depending upon the process characteristics and the tuning of the controllers themselves.

Several years ago a number of papers [2–4] explored various aspects of decoupling. A similar treatment is given in textbooks [5–10] for undergraduate and graduate level chemical engineering process control courses. In general, these works describe decoupling as a technique in which the process inputs are derived by a time-weighted combination of feedback controller outputs. In this paper, this is called "conventional" decoupling. Two forms of conventional decoupling, described subsequently, have been called "ideal" and "simplified" [2].

An alternative means of decoupling, called in this paper "inverted" decoupling, derives a process input as a time-weighted combination of one feedback controller output and the other process inputs. This achieves the same goal as ideal conventional decoupling while offering additional advantages. Inverted decoupling has rarely been mentioned in the literature; where it has been mentioned, the usual treatment is to emphasize potential problems without a full recounting of its advantages [3,8].

The purpose of this paper is to describe inverted decoupling, set forth its advantages and explore the potential problems. It is hoped that this technique will receive wider recognition and use, particularly by installations that wish to achieve benefits from advanced regulatory control without implementing a large scale model predictive control package.

2. Conventional decoupling

For review, we will first present the general concepts of conventional decoupling, using Fig. 1 as our guide. For simplicity, we limit our discussion to two input, two output processes. While not the most general case, this will cover a significant portion of the applications of interest. The techniques can be extended in a straight forward manner to applications with a greater number of inputs and outputs.

We assume that the process transfer functions, P_{ij}, are known and can be represented by a simple transfer function, such as a first order lag and dead time. This is consistent with actual practice; in fact, many times decoupling is implemented without any dynamic compensation terms.

The following equations relate the process outputs, x_i, to the process inputs, m_j (the symbols represent transforms of the signals):

$$\begin{aligned} x_1 &= P_{11}m_1 + P_{12}m_2 \\ x_2 &= P_{21}m_1 + P_{22}m_2 \end{aligned} \tag{1}$$

while the following equations relate the process inputs m_j to the controller outputs c_k through transforms of decoupling elements D_{jk}:

$$\begin{aligned} m_1 &= D_{11}c_1 + D_{12}c_2 \\ m_2 &= D_{21}c_1 + D_{22}c_2 . \end{aligned} \tag{2}$$

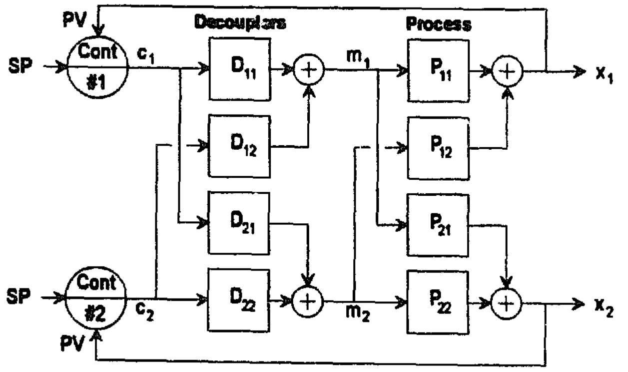

Fig. 1. Ideal conventional decoupling.

Combine Eqs (1) and (2) to eliminate m_1 and m_2:

$$\begin{aligned} x_1 &= (P_{11}D_{11} + P_{12}D_{21})c_1 \\ &\quad + (P_{11}D_{12} + P_{12}D_{22})c_2 \\ x_2 &= (P_{21}D_{11} + P_{22}D_{21})c_1 \\ &\quad + (P_{21}D_{12} + P_{22}D_{22})c_2 . \end{aligned} \tag{3}$$

From here, there are two approaches which are generally used for the determination of the decoupling elements. The first produces complicated transfer functions for the decoupling elements but yields a desirable "apparent" process control loop. The other approach produces simple transfer functions but results in a less-than-desirable apparent process. These decoupling approaches, termed the "ideal" and "simplified", respectively [2], are outlined in the following development.

2.1. Ideal conventional decoupling

We want the apparent process to be the same with decoupling as it would be if there were no decoupling and the other controller were in the manual mode. In other words, we want the apparent process equations to be:

$$\begin{aligned} x_1 &= P_{11}c_1 \\ x_2 &= P_{22}c_2 . \end{aligned} \tag{4}$$

Equating coefficients between Eqs (3) and (4) gives four equations in four unknowns which can be solved simultaneously for the D_{ij}s:

$$\begin{aligned} D_{11} &= \frac{P_{11}P_{22}}{P_{11}P_{22} - P_{12}P_{21}} \\ D_{12} &= \frac{-P_{12}P_{22}}{P_{11}P_{22} - P_{12}P_{21}} \\ D_{21} &= \frac{-P_{11}P_{21}}{P_{11}P_{22} - P_{12}P_{21}} \\ D_{22} &= \frac{P_{11}P_{22}}{P_{11}P_{22} - P_{12}P_{21}} . \end{aligned} \tag{5}$$

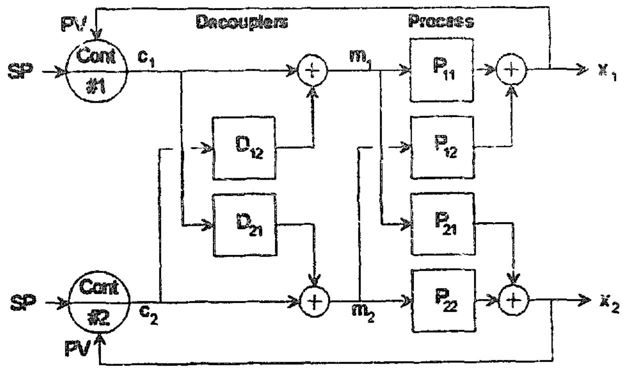

Fig. 2. Simplified conventional decoupling.

Note that while the decoupling elements themselves are relatively complicated, this technique has the advantage of constancy of the apparent process.

2.2. *Simplified conventional decoupling:*

Returning to Eqs (3), if we arbitrarily assign $D_{11} = D_{22} = 1$, we can solve for D_{12} and D_{21} which will make the off-diagonal elements vanish:

$$D_{12} = -\frac{P_{12}}{P_{11}}$$
$$D_{21} = -\frac{P_{21}}{P_{22}}. \tag{6}$$

This yields the struture shown in Fig. 2. Substituting the values for D_{12} and D_{21} into Eqs (3) yields the apparent process equations:

$$x_1 = \frac{P_{11}P_{22} - P_{12}P_{21}}{P_{22}} c_1$$
$$x_2 = \frac{P_{11}P_{22} - P_{12}P_{21}}{P_{11}} c_2 . \tag{7}$$

Here the process loops are decoupled with rather simple-to-implement decoupling elements; if the process is modeled by first order lag plus dead time elements,

$$P_{ij} = \frac{K_{ij} e^{T_{d_{ij}} s}}{T_{ij} s + 1} \tag{8}$$

then each decoupling element is comprised of, at the most, a steady state gain plus lead-lag and dead time elements. In particular cases, either or both of the dynamic terms may not be required.

Notice, however, that the apparent process differs very decidedly from the process when no decoupling is applied and only one controller at a time is in the automatic mode. This implies that the controller tuning would require changing for different modes of operation; furthermore, the response of the loops, with and without decoupling, would differ, presenting a confusing response for the process operator.

Another disadvantage to conventional decoupling is common to both the ideal and the simplified approaches. Conventional decoupling assumes that all computed process input signals retain their integrity, that is, become the actual process input signals. For example, if the computed process input is the set point to a lower level flow controller, it is assumed that the flow rate will actually match the set point. If an abnormality occurs in the flow loop, such as the valve going to a limit or the flow controller itself being switched to manual, then both of the primary loops are affected. These disadvantages motivate us to explore an alternative method of decoupling.

3. Inverted decoupling

If each process input is viewed merely as a disturbance to the other process output, then compensation for that disturbance can be designed using a feedforward approach, as shown in Fig. 3. The required compensation elements are the same as those given by Eqs (6) for the simplified conventional decoupler. Note the signal direction through the decoupling elements, and the location of the summing junction, as compared with the simplified decoupling.

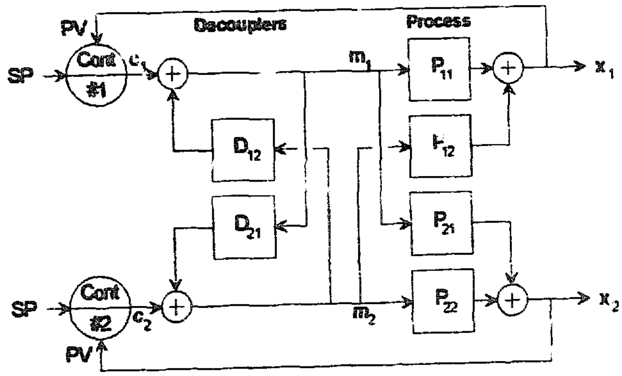

Fig. 3. Inverted decoupling.

Equations representing the decoupling circuit are:

$$\begin{aligned} m_1 &= c_1 + D_{12} m_2 \\ m_2 &= c_2 + D_{21} m_1 . \end{aligned} \tag{9}$$

When these equations are substituted into Eqs (1) and the decoupling element relationships, given by Eqs (6) are used, the resulting equations can be reduced to identically the same apparent process equations as given by Eqs (4). That is, the decoupling circuit was constructed by using the same elements as were used in the simplified conventional decoupling, but the apparent process is the same as was achieved with ideal conventional decoupling. Thus, one advantages of inverted decoupling is:

The apparent process seen by each controller, when decoupling is implemented, is the same as if there were no decoupling and the alternate controller were in the manual mode.

If the process inputs are implemented as cascaded set points to lower level flow controllers, as is often the case, then the input signals to the decoupling elements can be derived directly from the flow transmitter outputs, as shown in Fig. 4. With this implementation, another advantage of inverted decoupling is:

Each decoupled control loop is immune to abnormalities (e.g. valve at a limit or secondary controller in manual) in the secondary of the opposite control loop.

In many distributed control systems, the PID function block often has provisions for an auxiliary input, called the "feedforward" input. This input is summed with the output of the PID portion of the

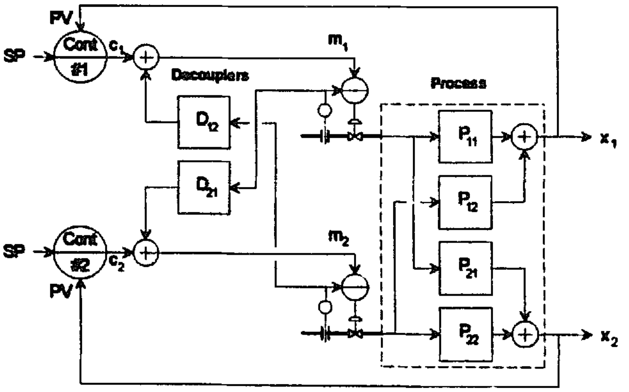

Fig. 4. Implementation of inverted decoupling.

block in the figures above, the feedback controllers and the subsequent summing junction may be incorporated into the same function block. Hence,

Inverted decoupling can often be implemented with in a DCS using a PID function block with feedforward input. This will automatically provide such features as initialization and bumpless transfer between manual and automatic.

With these advantages for inverted decoupling, there are yet some open ended questions to be addressed. These are:

(1) Realizability: are the decoupling elements realizable? (An element is realizable if it does not require future values of its input to determine its output.)

(2) Stability: the inverted decoupling configuration creates an inner loop, c_1 to m_1 to c_2 (through D_{21}) to m_2 to c_1 (through D_{12}). Will this loop be stable?

(3) Robustness: how sensitive is the inverted decoupling technique to model mis-match?

3.1. Realizability

A decoupling element is realizable if it only requires present and past values of its input to compute its output. Mathematically this implies that there cannot be a term of the form $e^{+T_d s}$ in the transfer function, where $T_d > 0$. It will be shown, however, that inverted decoupling can always be configured so that all the decoupling elements are realizable.

(1) If $T_{d11} \le T_{d12}$ and $T_{d22} \le T_{d21}$, then both P_{12}/P_{11} and P_{21}/P_{22} are realizable. In this case, the decoupling configuration shown by Fig. 3 meets the criterion for realizability.

(2) If neither P_{12}/P_{11} nor P_{21}/P_{22} are realizable, then their inverses, P_{11}/P_{12} and P_{22}/P_{21}, are both realizable. Then the alternative decoupling configuration shown in Fig. 5 meets the criterion for realizability. Note that the role of the controllers have been reversed. The apparent process equations now are:

$$\begin{aligned} x_1 &= P_{12} c_2 \\ x_2 &= P_{21} c_1 . \end{aligned} \tag{10}$$

These indicate that the control system is now controlling through the "off-diagonal" elements.

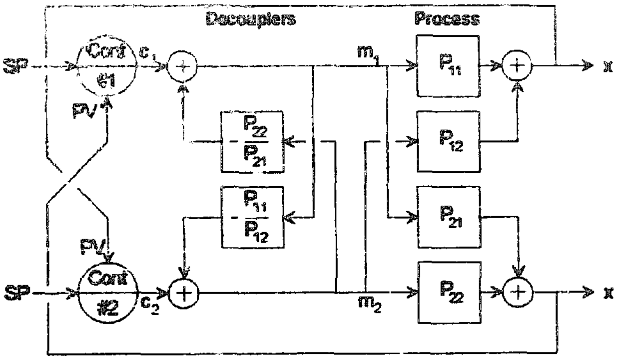

Fig. 5. Alternative inverted decoupling.

(3) If $T_{d11} = T_{d12}$ and $T_{d22} = T_{d21}$, then both elements from each of the pairs $\{(P_{12}/P_{11}), (P_{21}/P_{22})\}$ and $\{(P_{11}/P_{12}), (P_{22}/P_{21})\}$ are realizable, and the choice of decoupling configuration can be made, based on stability considerations described in the next section.

(4) If there is one non-realizable element in each of the pairs $\{(P_{12}/P_{11}), (P_{21}/P_{22})\}$ and $\{(P_{11}/P_{12}), (P_{22}/P_{21})\}$, then an additional time delay can be inserted in the decoupling configuration to force the non-realizable element into realizability. An example will illustrate this technique.

Suppose that $T_{d11} > T_{d12}$ and $T_{d21} > T_{d22}$. Then from the first pair, P_{12}/P_{11} is non-realizable; from the second pair, P_{12}/P_{21} is non-realizable. The time delay to be inserted is the smaller of $T_{d11} - T_{d12}$ and $T_{d21} - T_{d22}$. Suppose further that $T_{d21} - T_{d22} > T_{d11} - T_{d12}$; that is, P_{22}/P_{21} is "more non-realizable" than P_{12}/P_{11}. An additional time delay of the amount $T_x = T_{d11} - T_{d12}$ can be inserted in series with P_{11} as shown in Fig. 6. This effectively increases the time delay through both process elements P_{11} and P_{21}.

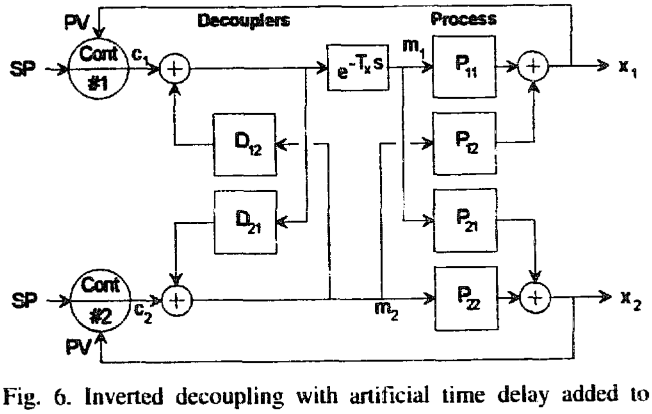

Fig. 6. Inverted decoupling with artificial time delay added to force realizability.

The design of the decoupler elements can now consider the modified process transfer functions:

$$P'_{11} = P_{11} e^{-T_x s}$$
$$P'_{21} = P_{21} e^{-T_x s}. \tag{11}$$

The element P_{12}/P'_{11} contains no dead time, so both it and its inverse, P'_{11}/P_{12}, are realizable. The element P_{21}/P_{22} from the original first pair was realizable; adding the dead time did not make it non-realizable, due to our supposition that

$$T_x = T_{d11} - T_{d12} < T_{d21} - T_{d22}. \tag{12}$$

In other words, P'_{21}/P_{22} is realizable. Thus the proper choice for decoupling components to ensure realizability is:

$$D_{12} = -\frac{P_{12}}{P'_{11}}$$
$$D_{21} = -\frac{P'_{21}}{P_{22}}. \tag{13}$$

Equations describing the apparent processes, as seen by the controllers, are:

$$x_1 = P_{11} e^{-T_x s} c_1$$
$$x_2 = P_{22} c_2. \tag{14}$$

Note that realizability was achieved at the expense of creating an additional time delay in one of the decoupled loops. Furthermore, the advantage of being able to derive the input to decoupling element D_{21} from the secondary loop flow transmitter, as was shown in Fig. 4, was lost. (Decoupling element D_{12} can still be driven from its secondary loop flow transmitter, however.) But because of the requirement for realizability, and the adverse time delay relationships through the process, there is no other choice.

This section has shown that, by inserting an additional time delay, if necessary, it is always possible to choose realizable decoupling elements. It is not claimed that this selection of decoupling elements will be stable, however.

3.2. Stability

For this section, assume that the realizability question has been addressed and the process transfer

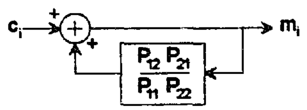

Fig. 7. Equivalent feedback loop created by inverted decoupler.

functions have been modified if necessary or the transfer functions and variables renumbered so that we can consider the inverted decoupling configuration in its standard form as shown by Fig. 3, with the decoupling elements given by Eqs (6). The inner loop, c_1 to m_1 to c_2 (through D_{21}) to m_2 to c_1 (through D_{12}), can be represented by the feedback diagram shown in Fig. 7.

The loop equation is given by

$$\frac{m_i(s)}{c_i(s)} = \frac{1}{1 - \dfrac{P_{12}P_{21}}{P_{11}P_{12}}}$$

for i = either 1 or 2. (15)

If each P_{ij} has been modeled as a first order lag plus dead time, then the loop equation is:

$$\frac{m_i(s)}{c_i(s)} = \frac{1}{1 - \dfrac{K(T_{11}s+1)(T_{22}s+1)}{(T_{12}s+1)(T_{21}s+1)}\mathrm{e}^{-T_\mathrm{d}s}}, \quad (16)$$

where

$$K = \frac{K_{12}K_{21}}{K_{11}K_{22}} \quad (17)$$

$$T_\mathrm{d} = T_{\mathrm{d}12} - T_{\mathrm{d}11} + T_{\mathrm{d}21} - T_{\mathrm{d}22}\,.$$

The question of stability then rests on the location of zeros of the function

$$1 - \frac{K(T_{11}s+1)(T_{22}s+1)}{(T_{12}s+1)(T_{21}s+1)}\mathrm{e}^{-T_\mathrm{d}s}. \quad (18)$$

Thus we seek solutions to the equation

$$\frac{K(T_{11}s+1)(T_{22}s+1)}{(T_{12}s+1)(T_{21}s+1)}\mathrm{e}^{-T_\mathrm{d}s} = 1. \quad (19)$$

Solutions to this equation provide upper and lower limits which the value K, given by process parameters as stated in Eqs (17), must meet in order for the inverted decoupler to be stable.

Note that we are not investigating stability in order to find the setting limits for a tuning parameter; instead, we are trying to determine limits which the parameter K, given strictly by process data, must meet in order for the inverted decoupling to be stable. If K falls outside these limits, then we conclude that inverted decoupling is not stable.

The reason for both upper and lower limits on K is that K may be either positive or negative. If there is an odd number of like signs of the process parameters K_{ij} (Shinskey [10] calls this "negative coupling"), then K will be negative; an even number of like signs will produce a positive value for K. Note the "+" sign by the return signal at the summing junction shown in Fig. 7. If K is negative, then the loop experiences negative feedback, hence the phase lag through the dynamic elements of the transfer function will be 180°. The solution to Eqs (17) will provide a lower (negative) limit for K. If K is positive, the loop experiences positive feedback and the phase lag will be either 0° or 360°. Solving Eqs (17) for both 0° and 360° phase shift will provide two upper limits for K; the real upper limit must be the lower of these.

We proceed by seeking values of ω which satisfy the following the transcendental equation:

$$\operatorname{atan}(\omega T_{12}) + \operatorname{atan}(\omega T_{21}) - \operatorname{atan}(\omega T_{11}) - \operatorname{atan}(\omega T_{22}) + \omega T_\mathrm{d} = n\pi\,. \quad (20)$$

If $K<0$, then use $n=1$. Call the solution ω_1. If $K>0$, then solve this equation twice, both for $n=0$ and $n=2$. This will give two values for ω, ω_0 and ω_2. By the usual consideration that at the solution the magnitude of both sides of Eq. (19) must be equal, we can use these values of ω and solve for the limits to be imposed on K.

If $K<0$, then

$$K_\mathrm{L} = -\sqrt{\frac{(1+\omega_1^2T_{12}^2)(1+\omega_1^2T_{21}^2)}{(1+\omega_1^2T_{11}^2)(1+\omega_1^2T_{22}^2)}}\,. \quad (21)$$

If $K>0$, then

$$K_{\mathrm{U}0} = -\sqrt{\frac{(1+\omega_0^2T_{12}^2)(1+\omega_0^2T_{21}^2)}{(1+\omega_0^2T_{11}^2)(1+\omega_0^2T_{22}^2)}} \quad (22)$$

and

$$K_{U2} = -\sqrt{\frac{\left(1+\omega_2^2 T_{12}^2\right)\left(1+\omega_2^2 T_{21}^2\right)}{\left(1+\omega_2^2 T_{11}^2\right)\left(1+\omega_2^2 T_{22}^2\right)}} . \tag{23}$$

The actual upper limit for K is the lower value of K_{U0} and K_{U2}. In summary, then, the process gains must be such that K meets the following criterion:

$$K_L \le K \le \min(K_{U0}, K_{U2}) . \tag{24}$$

The method outlined above gives rigorous limits of stability and involve solving a transcendental equation. A short cut which yields a good approximation to these limits can be made in many circumstances. If T_d is greater than 0, ω and T_d will be approximately related by $\omega_n \approx n\pi/T_d$, where $n = 1$ for $K < 0$ or $n = 0$ or 2 for $K > 0$. If $n = 0$, then $\omega_0 = 0$, so that by inspection, $K_{U0} = 1$. In fact, K_{U0} is approximately 1 for any value of T_d. If T_d is small relative to the T_{ij}s, then ω is large, and K_L and K_{U2} can be approximated by:

$$\begin{aligned} K_L &\approx -\frac{T_{12}T_{21}}{T_{11}T_{22}} \\ K_{U2} &\approx \frac{T_{12}T_{21}}{T_{11}T_{22}} . \end{aligned} \tag{25}$$

Hence, if T_d is small, or zero, the approximate stability limits are:

$$-\frac{T_{12}T_{21}}{T_{11}T_{22}} < K < \min\left(1, \frac{T_{12}T_{21}}{T_{11}T_{22}}\right) . \tag{26}$$

In this section, we have determined limits that the process conditions must meet in order for inverted decoupling to be feasible, i.e. stable. These limits are stated rigorously by Eqs (20)–(24), or approximately by Eq. (26). Since the decoupling loop transfer function [see Eq. (15)] is the same as for decoupling elements D_{11} and D_{22} for ideal conventional decoupling [see Eqs (5)] then we conclude that identical stability limits would also apply to ideal conventional decoupling.

3.3. Robustness

It has been widely reported in the literature that "ideal decoupling is sensitive to modeling errors" [2–4,8,12,13]. Because of this, and the fact that conventional ideal decoupling requires complicated decoupling elements, ideal decoupling has usually been given no further consideration. Inverted decoupling, a form of ideal decoupling, removes the second of these objections. Because of the advantages offered by inverted decoupling, the first objection warrants further consideration.

The conclusion that ideal decoupling is sensitive to modeling errors has been based on a simulation [2–4] or theoretical study [13] of a dual composition control of a distillation tower, using a conventional reflux-and-boilup control strategy. It is well known that with this strategy, the control loops are closely coupled [11] as indicated by the relative gain number being quite large (say, 20, or larger as the product purity requirements are increased). The relative gain, λ, and K [see Eqs (17)] are related by:

$$K = \frac{\lambda - 1}{\lambda}$$

so that if λ is large, K approaches the upper stability limit given by Eq. (26). Thus there is little margin for error, and in this case, the conclusion is correct.

This conclusion may not necessarily hold true in other cases such as dual composition control of a distillation column using a material balance (distillate-and-boilup) control strategy. Here, it is usually true that $0 < \lambda < 1$, so that $K < 0$. If the dynamics are not adverse, then K will be well within the limits stated by Eq. (26), thus providing tolerance for modeling error.

This is not meant to imply that inverted decoupling is only applicable to dual composition, material balance distillation control strategies. We do claim, however, that the technique should not be discarded based solely upon the observation of its behavior on dual composition, reflux-and-boilup distillation control.

4. Summary

Inverted decoupling has been shown to be a form of ideal decoupling which can be implemented with simple decoupling elements. It has advantages over both ideal and simplified conventional decoupling in that it is tolerant of faults in the lower level control loops. It has an advantage over the traditional simplified decoupling in that the feedback loop behavior, hence the tuning of the controllers, is constant regardless of the mode of the companion control loop. Inverted decoupling shares with conventional decoupling the sensitivity to modeling errors near the stability limits; if there is a comfortable margin of safety to the stability limits, then modeling errors do not appear to be a problem.

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