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Brief Paper

Automatic Tuning of PID Controllers Based on Transfer Function Estimation*

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Key Words—Adaptive control; closed-loop systems; describing functions; identification; industrial control; limit cycles; PID control; process control.

Abstract—A method for the automatic tuning of PID controllers in a closed loop, based on the estimation of a parametric 'black-box' transfer function model, is proposed. The system is excited by generating limit cycle oscillations at two different frequencies, which are approximately the crossover frequency and the critical frequency for the feedback loop. A discrete parametric transfer function model is estimated from the experimental data. Important parameters concerning the estimation, such as the prefilter cut-off frequency and the sampling interval, are determined automatically from the experimental data. The PID parameters are determined from a constrained optimization in the frequency domain. The constraints are classical control system properties, such as the maximum amplitudes of the sensitivity and the complementary sensitivity functions. Given these constraints, the PID parameters are determined such that the low frequency amplitude characteristic for the controller is maximized. Simulation experiments show that the tuning procedure has low sensitivity to disturbances and noise during the tuning experiment.

1. *Introduction*

METHODS FOR the automatic tuning of PID controllers, based on knowledge of the process transfer function at one or more frequencies, have been proposed by several authors. Some of these methods determine the process transfer function at a single frequency only (Åström and Hägglund, 1984a, b; Schei, 1991, 1992). Other methods are based on the estimation of parametric transfer function models.

The PID parameters can be determined from the transfer function estimates in a number of ways. Analytical methods for the computation of the PID parameters from the estimated transfer function models are, however, limited to very simple models, typically first-order models for PI control and second-order models for PID control (Gawthrop and Nomikos, 1990). For general transfer function models the computation of the PID parameters involves some numerical search routine. Methods for determination of the control parameters based on the minimization of quadratic performance objectives in the time domain are presented by Nishikawa *et al.* (1984), Radke and Isermann (1987), Isermann (1989), and by Vega *et al.* (1991). A drawback with these methods is that it is not trivial to relate the choice of parameters in such performance objectives to classical control loop characteristics, such as the gain and phase margins, etc. Hence, the determination of the robustness of these controllers is not straightforward.

The robustness can be specified more explicitly by performing the optimization in the frequency domain. A method for designing H_∞ optimal PID controllers is presented by Grimble (1990). The optimization problem is based on the minimization of the H_∞ norm of the sensitivity and complementary sensitivity functions multiplied by frequency dependent weights. However, a serious difficulty with such methods is to specify the weights automatically. An alternative frequency domain approach is presented by Prada *et al.* (1990). In this method the PID parameters are determined such that the system obtains specified values for the gain and phase margins. The relationship between the derivative and integral times is fixed in order to obtain a unique solution.

The proposed procedure below for automatic tuning of PID controllers has the following properties.

- A perturbation signal is generated automatically such that the system is excited mainly in the frequency range that is most important for the determination of control system performance. Input and output data from this experiment are stored. The rest of the computations are performed off-line.
- A discrete parametric transfer function model is estimated from the experimental data. Appropriate values for the prefilter parameters and the sampling interval during parameter estimation are determined automatically. It is assumed that the experimental data are stored with a relatively high sampling frequency. These data are then filtered and decimated prior to the transfer function estimation.
- The PID parameters are determined from a constrained optimization in the frequency domain. The constraints are maximum amplitudes of the sensitivity and the complementary sensitivity functions for the feedback loop. Given these constraints, the PID parameters are determined such that the low frequency controller amplitude characteristic is maximized.

The paper is organized as follows. Section 2, which contains the main contribution in this paper, presents excitation methods for closed-loop systems based on the generation of limit cycle oscillations at approximately the crossover frequency and the critical frequency for the feedback loop. These excitation methods have not been published before. The choice of the various parameters concerning the transfer function estimation is presented in Section 3. These choices are based on the average oscillation frequencies during the excitation experiment. The optimization method for determination of the PID parameters is presented in Section 4. This method also seems to be new. Simulation experiments are shown in Section 5.

2. *Generation of the perturbation signal*

It is assumed that a stable (and conservative) controller is established prior to the tuning, and that the purpose of the tuning is to improve the performance of the control system. For processes that are open-loop stable, conservative controllers might often be established by simple transient response experiments.

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The most important frequency range for control system performance is usually between the crossover frequency (gain equal one) and the critical frequency (phase equal $-180°$) for the loop transfer function. Methods for generating limit cycle oscillations at these two frequencies are described below. It is assumed that the process and the controller are continuous time systems.

In the first method the system is excited by connecting a relay function and a linear filter in a feedback path from the measurement to the reference signal for the controller as shown in Fig. 1. $M_r(s)$ is the closed-loop transfer function from the controller reference, r, to the measurement, y. For most systems a limit cycle will be generated due to the nonlinear characteristic of the relay. The linear filter $d(s)$ is used to influence the frequency of the limit cycle oscillation. The reference signal will vary in steps between $r_0 - \Delta r$ and $r_0 + \Delta r$, where Δr is the amplitude of the relay function.

We assume that there is generated a stable limit cycle in the system. The oscillation frequency, ω_{lc}, is approximately determined by

$$\angle d(j\omega_{lc})M_r(j\omega_{lc}) \approx -180°. \quad (1)$$

This follows from a describing function approximation of the relay characteristic.

Firstly, we assume that the controller has one degree of freedom. The process input is then

$$u(s) = C(s)(r(s) - y(s)), \quad (2)$$

where $C(s)$ is the transfer function for the controller. The closed-loop transfer function $M_r(s)$ is then equal to the complementary sensitivity function, $M(s)$.

$$M_r(s) = M(s) = \frac{G(s)C(s)}{1 + G(s)C(s)}, \quad (3)$$

where $G(s)$ is the transfer function for the process.

With $d(s) = 1$ we see from equation (1) that, if a stable limit cycle exists in the system, the phase of $M_r(j\omega)$ is approximately $-180°$ at the oscillation frequency. Hence, this frequency is approximately equal to the critical frequency for the loop transfer function.

With $d(s) = 1/s$ we see from equation (1) that, if a stable limit cycle exists, the phase of $M_r(j\omega)$ is approximately $-90°$ at this oscillation frequency. It can be seen from a Nichols diagram (Schei, 1992) that this frequency is between the crossover frequency and the critical frequency for the loop transfer function.

The second excitation scheme is shown in Fig. 2. We assume that a stable limit cycle is generated in this system. From the describing function approximation of the relay characteristic we now find that the oscillation frequency, ω_{lc}, is approximately determined by

$$\angle(2M_r(j\omega_{lc}) - 1) \approx -90°. \quad (4)$$

The transfer function on the left-hand side of equation (4) is

$$2M_r(j\omega_{lc}) - 1 = \frac{2G(j\omega_{lc})C(j\omega_{lc})}{1 + G(j\omega_{lc})C(j\omega_{lc})} - 1 = \frac{G(j\omega_{lc})C(j\omega_{lc}) - 1}{G(j\omega_{lc})C(j\omega_{lc}) + 1}. \quad (5)$$

From equations (4) and (5) we have that

$$\frac{G(j\omega_{lc})C(j\omega_{lc}) - 1}{G(j\omega_{lc})C(j\omega_{lc}) + 1} \approx -kj \Rightarrow G(j\omega_{lc})C(j\omega_{lc}) \approx \frac{1 - kj}{1 + kj}, \quad (6)$$

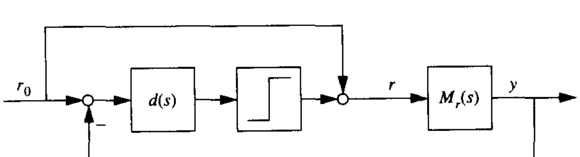

FIG. 1. Block diagram showing the relay feedback path from the process measurement, y, to the reference signal for the controller, r. A limit cycle oscillation with frequency equal to the critical frequency is approximately generated if $d(s) = 1$.

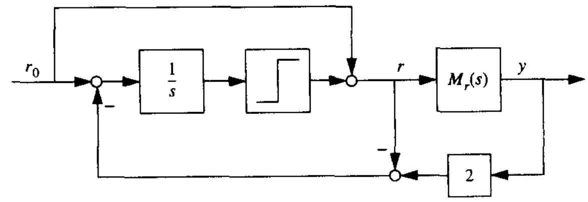

FIG. 2. Block diagram for generation of a limit cycle oscillation at the crossover frequency.

where k is some real positive number. From equation (6) we see that

$$|G(j\omega_{lc})C(j\omega_{lc})| \approx 1. \quad (7)$$

The frequency of the limit cycle oscillation, ω_{lc}, generated by the scheme in Fig. 2, is approximately equal to the crossover frequency.

A suitable perturbation signal can now be generated by letting the system oscillate for a predetermined number of periods at the various oscillation frequencies as described above. The major part of the excitation energy is then distributed in the frequency range between the crossover frequency and the critical frequency for the loop transfer function. Also, this will usually be so after the control parameters are adjusted to the new values. It should be noticed that the critical frequency is independent of variations in the proportional gain for the controller.

Most processes encountered in the process industry have smooth transfer functions that can be determined quite accurately in a narrow frequency range from knowledge of the transfer function at two distinct frequencies in this range. Hence, it is assumed to be sufficient that the system is excited by generating limit cycle oscillations at two different frequencies, the critical frequency and the crossover frequency are chosen.

It is assumed above that the controller has one degree of freedom. However, in a PID controller the derivative action is normally applied to the measurement signal only, not to the reference signal. The PID controller has then two degrees of freedom, and the control law can be expressed as

$$u(s) = C_r(s)r(s) - C_y(s)y(s). \quad (8)$$

The closed-loop transfer function from r to y is now

$$M_r(s) = \frac{G(s)C_r(s)}{1 + G(s)C_y(s)} \quad (9)$$

and the complementary sensitivity is

$$M(s) = M_r(s)\frac{C_y(s)}{C_r(s)}. \quad (10)$$

Hence, we can generate limit cycle oscillations at the critical frequency and the crossover frequency by including the filter $C_y(s)/C_r(s)$ at the output from $M_r(s)$ in Figs 1 and 2.

Some theoretical questions regarding the existence and uniqueness of relay oscillations and the accuracy of the describing function approximation are analysed by Åström and Hägglund (1984a) and Tsypkin (1984).

3. *Transfer function estimation*

The proposed tuning method is based on the identification of a parametric black-box transfer function model. Important results concerning the variance and the bias distributions of transfer function estimates are presented by Ljung (1985) and by Wahlberg and Ljung (1986). It is of prime importance to know the frequency range where we want the transfer function model to resemble the real system. In the previous section it was shown how this frequency range could be approximately determined and excited by the nonlinear feedback paths shown in Figs 1 and 2. The sampling frequency and the prefilter characteristics can then be chosen automatically based on average oscillation periods. It is assumed that the raw experimental data are stored with a relatively high sampling frequency. These data are then prefiltered and decimated prior to the transfer function estimation.

The simple ARX model structure is used for estimation of the process transfer function. Wahlberg and Ljung (1986) have found that this model structure can lead to approximately the same results as more advanced structures, if the data are properly filtered prior to the parameter estimation. The ARX model structure tends to emphasize high frequencies in the fit between the transfer function model and the real system. Hence, it is important to combine this model structure with a low-pass filter. The filter is chosen to be a fourth-order Butterworth filter with a cut-off frequency twice the estimated critical frequency. This estimate is based on the average oscillation period for the scheme in Fig. 1 with $d(s) = 1$. The sampling frequency is set to 15 times the estimated critical frequency.

The choice of model order is a compromise between variance and bias considerations. The variance of the estimated transfer function model increases with increasing model order. On the other hand, the bias in the transfer function estimate will decrease with increasing model order, since more parameters give more flexibility to adjust the model to the true system. A second-order model seems to be a reasonable compromise between these conflicting objectives. If we approximately regard the excitation signal to consist of two single frequencies, we know that the data are sufficiently informative with respect to a second-order model (Ljung, 1987). The model is also flexible enough to fit exactly to the real system at these two frequencies. Hence, we can assume that the bias will be low around the two oscillation frequencies when the model is of second-order and most of the excitation energy is concentrated around these two frequencies.

A second-order ARX model with input u and output y is stated as

$$y(t) = -a_1 y(t-T) - a_2 y(t-2T) + b_1 u(t-kT) + b_2 u(t-(k+1)T) + e(t). \tag{11}$$

A time delay, kT, where T is the sampling interval, is included in the model. k will be a small integer if the sampling interval is chosen as above. The prediction error parameter estimate of $\theta = [a_1, a_2, b_1, b_2]^T$, which minimizes the mean square of the prediction errors, is computed with the well-known *least squares* method. The parameter k is determined by computing θ with k varying from one up to a maximum possible value. The set of k and θ that give the smallest value of *Akaike's Information Criterion* is chosen as the estimated model.

The process transfer function estimate is determined from the model in equation (11) and the estimates of k and θ.

$$\hat{G}(e^{j\omega T}, \hat{\theta}, \hat{k}) = \frac{\hat{b}_1 + \hat{b}_2 e^{-j\omega T}}{1 + \hat{a}_1 e^{-j\omega T} + \hat{a}_2 e^{-2j\omega T}} e^{-\hat{k}j\omega T}, \qquad \hat{\theta} = [\hat{a}_1 \hat{a}_2 \hat{b}_1 \hat{b}_2]^T. \tag{12}$$

The PID parameters are determined under the assumption that the controller and the process are continuous time systems. $\hat{G}(e^{j\omega T}, \hat{\theta}, \hat{k})$ is then used as an estimate of the continuous time frequency response for the process.

4. *Determination of the PID parameters*

The transfer function for the feedback part of the controller is expressed as

$$C_y(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + (T_d/N_f)s}\right), \tag{13}$$

where K_p is the proportional gain, T_i is the integral time, and T_d is the derivative time. It is assumed that the derivative part is filtered by a first-order filter with time constant T_d/N_f. The parameters K_p, T_i and T_d are determined from a constrained optimization in the frequency domain. The constraints define certain stability margins that we want to impose on the system.

In process control applications we often want the highest possible attenuation of low-frequency disturbances, while maintaining certain stabilty margins. Hence, we choose to maximize the low-frequency amplitude characteristic for the controller. Since the integral term is dominating at low frequencies, the objective function is chosen to be $f(K_p, T_i) = T_i/K_p$. By minimizing f we maximize the asymptotic amplitude characteristic of $C_y(j\omega)$ at frequencies below $1/T_i$.

The performance and robustness of a control system is often stated in terms of the sensitivity function, $N(j\omega)$, and the complementary sensitivity function, $M(j\omega)$. By using the estimated model $\hat{G}(e^{j\omega T})$ we get

$$N(j\omega) = \frac{1}{1 + \hat{G}(e^{j\omega T})C_y(j\omega)}, \qquad M(j\omega) = \frac{\hat{G}(e^{j\omega T})C_y(j\omega)}{1 + \hat{G}(e^{j\omega T})C_y(j\omega)}. \tag{14}$$

The peak values of $|N(j\omega)|$ and $|M(j\omega)|$ are chosen as constraints in the optimization. Hence, we require that $|N(j\omega)| - n_p \le 0$, and $|M(j\omega)| - m_p \le 0$ for all ω. The value of n_p is usually chosen to be in the range 1.3–3.0. m_p is usually between 1.0 and 1.6. It has to be at least 1.0 in order to impose a reasonble constraint at low frequencies.

If we prefer to specify the constraints in terms of minimum values of gain and phase margins for the feedback loop, we can easily find the corresponding values of n_p and m_p from equation (14).

The total optimization problem is stated as

$$\begin{aligned} \text{minimize} \quad & f(K_p, T_i) = T_i/K_p \\ \text{subject to} \quad & g_1(\omega_i) = |M(j\omega_i)| - m_p \le 0 \\ & g_2(\omega_i) = |N(j\omega_i)| - n_p \le 0 \\ & K_p \le K_{p,\max}, \end{aligned} \tag{15}$$

where the frequencies ω_i are chosen as a set of frequencies in the relevant frequency range. A constraint on the proportional gain is included in order to limit the high-frequency amplification in the controller. Without this constraint, first or second-order minimum phase processes would lead to virtually unlimited bandwidth.

The controller design method above is quite natural from a simple loop-shaping argument. Usually we want the sensitivity to be as small as possible at frequencies below the bandwidth of the system, while limiting the peak amplitude values of the sensitivity and the complementary sensitivity functions. This is approximately obtained by equation (15). $1/T_i$ is usually close to the bandwidth of the system, and we see from equation (14) that the sensitivity is approximately minimized at frequencies below $1/T_i$.

The parameters n_p and m_p determine the trade-off between robustness and performance in different frequency domains. It is well known that the amplitude of the complementary sensitivity function determines the stability robustness of the system against multiplicative norm-bounded perturbations (Doyle and Stein, 1981). Small values of n_p and m_p give large stability margins. It can be seen from a Nichols diagram that the gain margin will be determined by n_p and the phase margin by m_p, if these parameters are given reasonable values. However, small values of n_p and m_p generally increase the sensitivity at low frequencies. This is mathematically expressed by Bode's Integral Theorem (Bode, 1945), which states that the integral $\int_0^\infty \ln(N(j\omega))\,d\omega$ is a constant.

The solution of equation (15) is based on a sequential quadratic programming method. In this method, a quadratic programming subproblem is solved at each iteration. Extensive simulation experiments indicate that the optimization procedure will usually converge to the optimal solution if we are able to specify an initial feasible solution. It is seen easily from a Nichols diagram that, for stable processes with at most one integration, we can always find a feasible proportional controller by choosing the proportional constant sufficiently small. It is assumed that $n_p > 1$ and $m_p > 1$. The

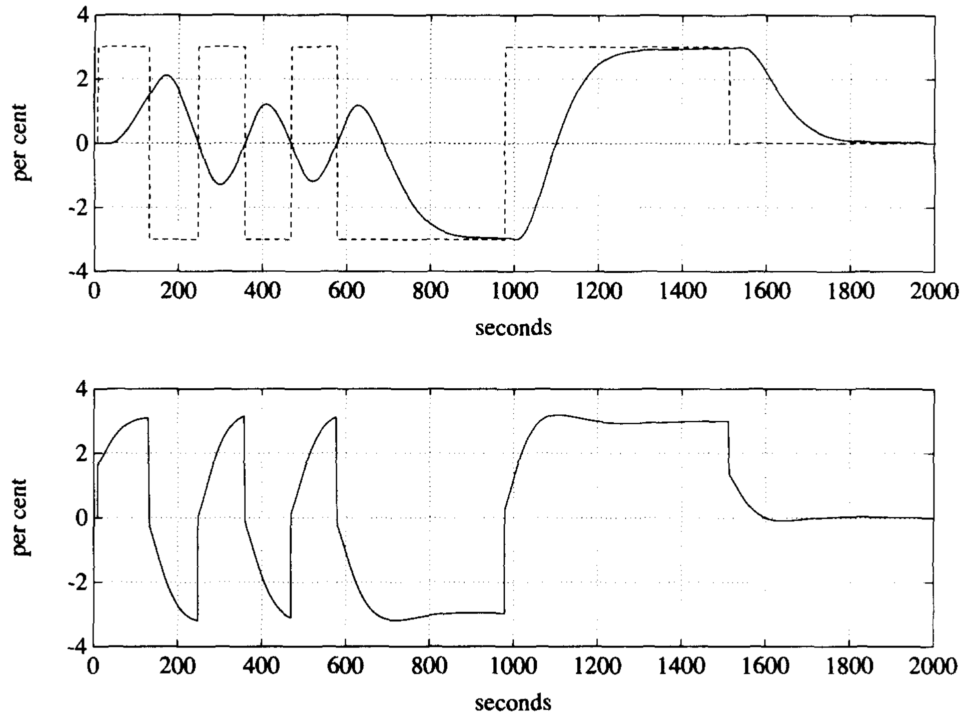

FIG. 3. Above: output measurement (——) and reference signal (--) during closed-loop tuning. Below: process input.

same is true for stable processes with at most two integrations, if we include a sufficiently long derivative time in the controller.

An important advantage with this frequency domain approach is that it is easy to choose reasonable values for the parameters that have to be specified by the user. The parameters that have to be specified are n_p and m_p, or alternatively, minimum allowable gain margin and phase margin.

5. *Simulation experiments*

The auto-tuning method will be demonstrated with the following simulation model:

$$G(s) = \frac{(1 - 10s)}{(1 + 60s)(1 + 20s)(1 + 20s)} e^{-10s}. \tag{16}$$

Conservative control parameters were first obtained from a simple pulse response experiment. The constraints were set

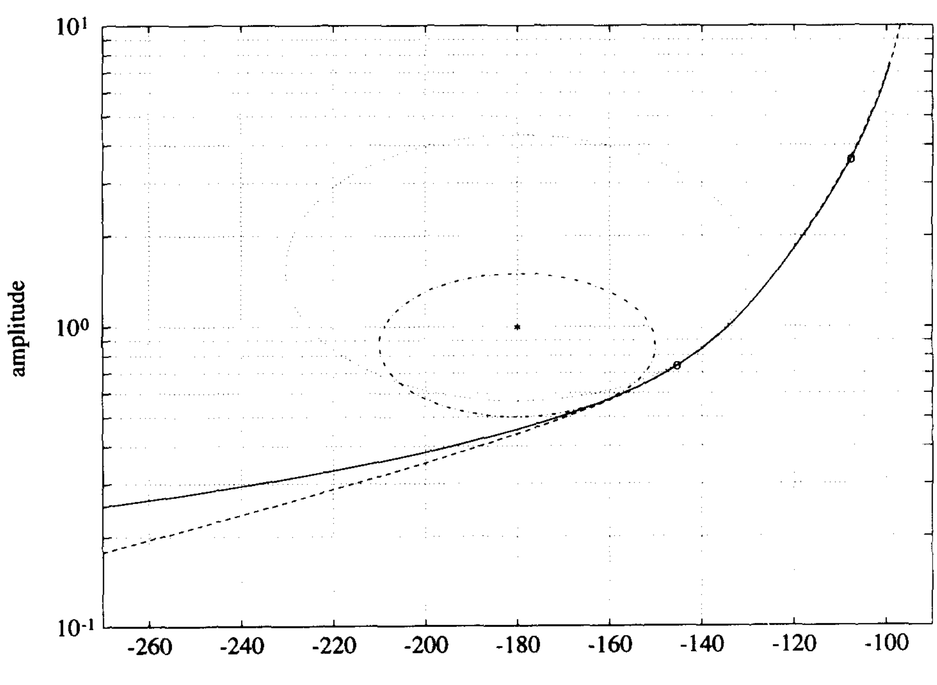

FIG. 4. Nichols plot for the estimated loop transfer function (——) and for the true system (--). The constraints $g_1 = 0$ (··) and $g_2 = 0$ (-·) are also shown. The two oscillation frequencies are marked with circles in the diagram.

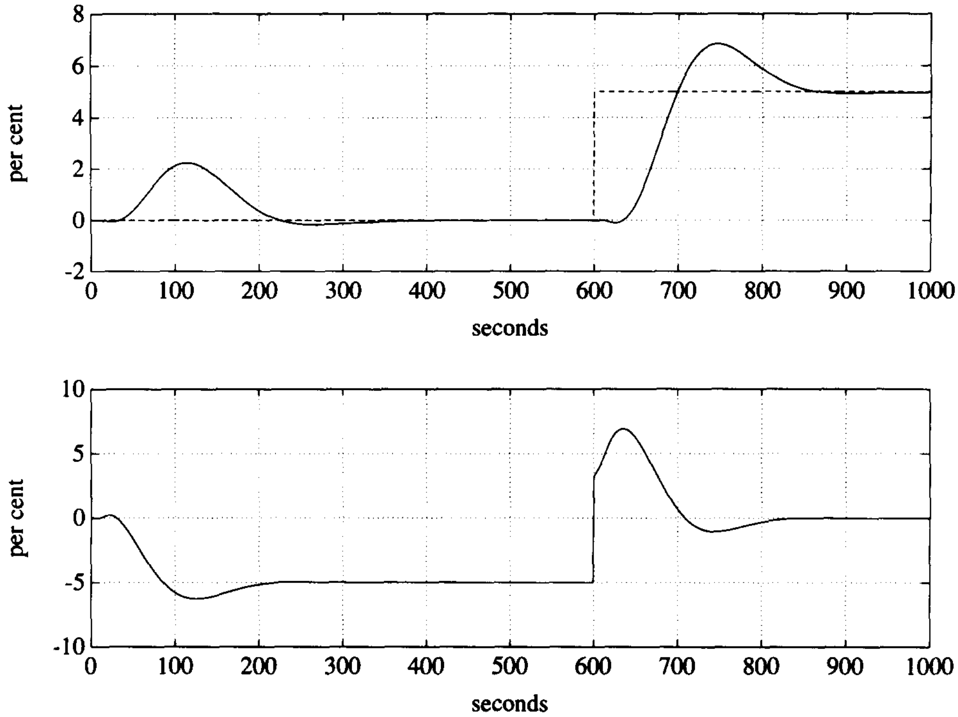

FIG. 5. Above: output measurement (——) and reference signal (--). The simulation starts with a load disturbance, and there is a step in the reference signal after 600 s. Below: process input.

to $n_p = 1.4$ and $m_p = 1.1$, and the parameters for a PI controller were determined to $K_p = 0.46$ and $T_i = 65.4$ s.

The system is now excited in closed loop. The reference signal and the process input and output responses during the tuning are shown in Fig. 3. The reference signal perturbation is generated by the schemes in Figs 1 and 2, with two oscillation periods at approximately the critical frequency and one oscillation period at approximately the crossover frequency. The first relay switch occurs when the measurement has reached 50 per cent of the first step in the reference signal.

The experimental data are prefiltered and decimated, and the transfer function estimate is based on the ARX model in equation (11):

$$\hat{G}(e^{j\omega 15}) = \frac{0.0830 + 0.0048e^{-j\omega 15}}{1 - 1.3895e^{-j\omega 15} + 0.4773e^{-2j\omega 15}} e^{-3j\omega 15}. \quad (17)$$

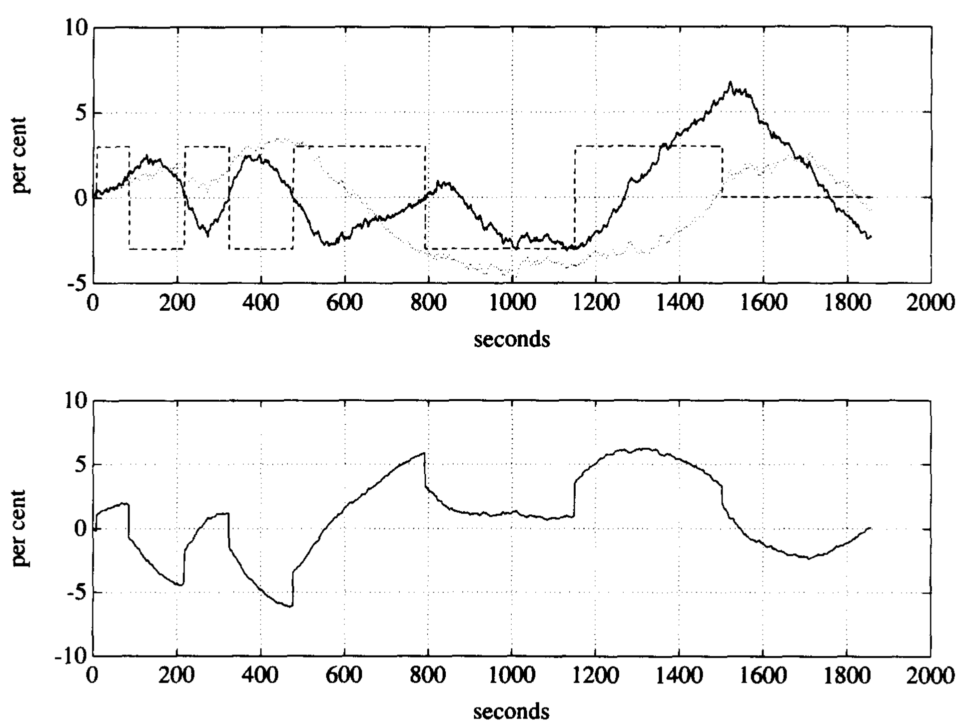

FIG. 6. Above: output measurement (——) and reference signal (--) during closed-loop tuning. The noise and disturbance are shown with the dotted line. Below: process input.

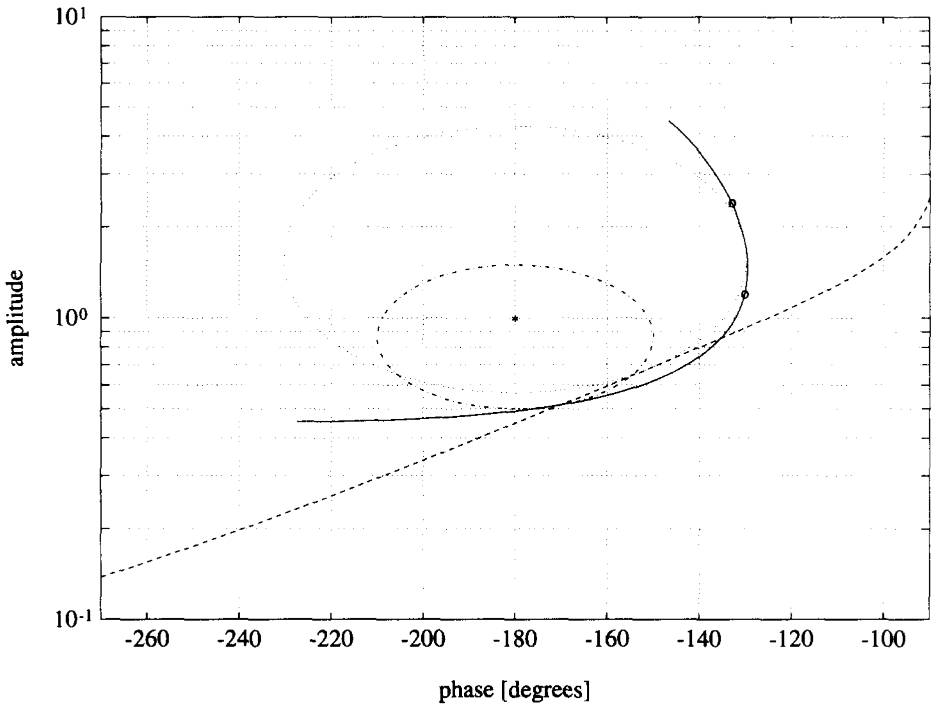

FIG. 7. Nichols plot for the estimated loop transfer function (——) and for the true system (– –). The two oscillation frequencies are marked with circles in the diagram.

The parameters for a PID controller are determined under the constraints $n_p = 2.0$ and $m_p = 1.3$. The parameter N_f in equation (13) is set to 5.0. The new PID parameters are $K_p = 1.64$, $T_i = 68.9$ s and $T_d = 20.5$ s.

Nichols plots for the estimated and the true system are shown in Fig. 4. We see that the estimated model fits very well to the true system for all important frequencies.

The system is simulated in Fig. 5. The derivative action is not applied to the reference signal:

$$C_r(s) = K_p\left(1 + \frac{1}{T_i s}\right). \tag{18}$$

The same tuning experiment is repeated with process

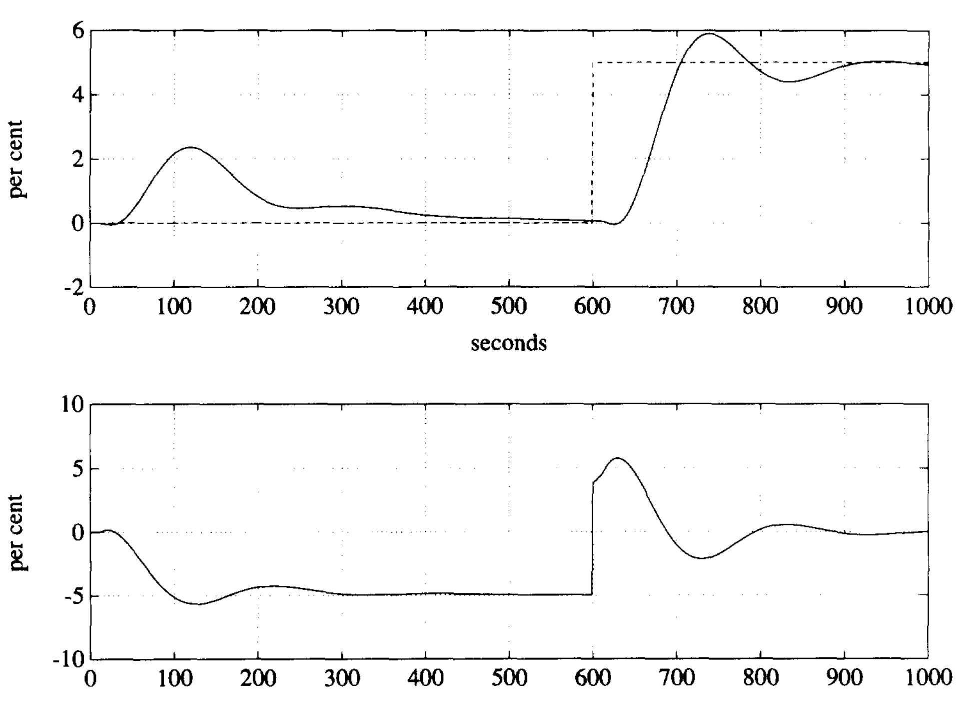

FIG. 8. Above: output measurement (——) and reference signal (– –). The simulation starts with a load disturbance and there is a step in the reference signal after 600 s. Below: process input.

disturbance and measurement noise acting on the system. The process disturbance is generated by filtering discrete white noise with sampling interval 1 s and standard deviation 50% through a second-order Butterworth low-pass filter with a cut-off frequency of 0.010 rad/s. The disturbance sequence is added to the process output. The measurement noise is white with standard deviation 1% filtered through a first order low-pass filter with a cut-off frequency of 0.10 rad/s.

The tuning experiment is shown in Fig. 6. The oscillation periods are highly irregular due to the process disturbance. The transfer function estimate is now

$$\hat{G}(e^{j\omega 23}) = \frac{0.2831 - 0.2797e^{-j\omega 23}}{1 - 1.9469e^{-j\omega 23} + 0.9364e^{-2j\omega 23}} e^{-2j\omega 23}. \tag{19}$$

The parameters for a PID controller are computed under the same constraints as above, that is, $n_p = 2.0$ and $m_p = 1.3$. The new PID parameters are $K_p = 1.75$, $T_i = 134$ s and $T_d = 11.8$ s.

Nichols plots for the estimated and the true system are shown in Fig. 7. We see that the estimated model fits reasonably well to the true system in the most important frequency range, but the low-frequency part of the model is very inaccurate.

The system is simulated in Fig. 8. The longer integral time compared with Fig. 5 makes the response to the load disturbance slightly more sluggish. We see that the performance of the control system is reasonably good, even if the identified parametric model is very different from the true system. The reason is that the amplitude and the phase of the estimated model is relatively accurate in the most important frequency range between the crossover frequency and the critical frequency for the feedback system.

6. *Conclusions*

Simulation experiments show that the presented tuning method has low sensitivity to disturbances and noise during the tuning experiment. The main reason for the good performance is that the identification experiment and the parameters concerning the transfer function estimation are chosen with the intention to obtain an accurate transfer function estimate in a narrow frequency range. This frequency range is determined automatically from the closed-loop excitation experiment.

Extensive simulation experiments also indicate that the constrained optimization method for determining the PID parameters gives very good performance and robustness. It is an important advantage with this method that the trade-off between robustness and performance is determined explicitly through the choice of constraints. The trade-off between performance in different frequency domains is also determined through the choice of sensitivity constraint, n_p. A small value of n_p will lead to low sensitivity in the frequency domain around the bandwidth of the feedback loop. A higher value of n_p increases the amplification of disturbances around the bandwidth, but the sensitivity at low frequencies will decrease.

A disadvantage with auto-tuning methods based on off-line identification is that they are not suitable for adaption to continuous changes in the process. Hence, these process variations should be considered when choosing the constraints n_p and m_p.

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