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EXTREMUM CONTROL SYSTEMS—AN AREA FOR ADAPTIVE CONTROL?

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Abstract

In this paper will first be given a review of older methods and applications of extremum control. It will then be discussed how adaptive control could be applied within this field. This will require the introduction of a parametric model for the system, and the use of system identification to find the best parameter values. One way of modelling such a nonlinear dynamic system is to separate the dynamics and the nonlinearity into two blocks. It is shown how the order between the two blocks will have a large influence on the behaviour of the model.

1. INTRODUCTION

In most control problems, the task of the regulator is to keep some variable at a constant value, or to make it follow a reference signal. In general, the system is then assumed to be linear, and it is possible, in principle, to drive the output to any prescribed value. With such problems, the ordinary PID-regulator can often do a good job. In an extremum control problem on the other hand, the static response curve relating the output to the input has at least one extremum point. It is thus a nonlinear dynamical system. The task of an extremum controller is then to keep the output as close to its extremal value as possible.

There are several examples of practical systems that exhibit this type of behaviour. Control of the air/fuel-ratio for optimal combustion has e.g. been studied on many different plants. Usually, the air flow is then controlled to its optimum setting for the current fuel flow. The optimum may vary e.g. with the fuel quality. Autogeneous ore-grinding is another example, where filling degree in the mill is the input and grinding efficiency is the output. For a water-turbine or a windmill with adjustable blade angles, it is desirable to extract maximum power by a proper setting of the blade angles. This is also an extremal control problem. The paper (1) shows that such problems have been around for a long time. As a matter of fact, Leblanc uses one of the most well-known methods, which is based on adding a perturbation signal to the input and observing its effect on the output.

Extremum control problems started to become more popular after the publication of the famous paper (2). One reason for this was probably improvements in computing technology that made possible the implementation of more and more complicated controllers.

Towards the end of the 50's a couple of commercial optimalizers became available: the Opcon and Quarie controllers. The interest in extremum control seems to have reached a maximum about then and some years thereafter. The number of published papers was higher than ever since, many of them containing optimistic reports of practical applications. Since then the publication rate has decreased, especially in the western countries. Nevertheless, some research has continued, and concepts like system identification and adaptive control have been introduced into this area.

In the past decades, computer technology has developed enormously. This is one reason why it might be rewarding to reconsider extremum control problems. It is now possible to implement rather complex control algorithms in low cost microcomputers, as has already been shown with adaptive control. It should then be possible to benefit from inserting more ideas from adaptive control and identification into the extremum control area. Moreover, with today's competition for market shares and increasing system complexity, even small gains in efficiency may be very valuable.

Several survey papers of different kinds have already been published. General surveys of adaptive and self-optimizing control systems that also include extremum control are e.g. (3), (4), and (5). More specialized surveys of extremum control systems are e.g. (6), (7), and (8). Several basic principles were discussed in detail already in (2).

The rest of this survey will be organized as follows. In Section 2 different models will be discussed. Section 3 is a systematic treatment of proposed schemes for extremum control. A collection of possible practical applications of the theory is discussed in Section 4. Most of these have been tried in practise, and the results are described in the existing literature. Section 5 contains a list of ideas for the use of adaptivity in extremum control problems and finally, a couple of concluding remarks are given.

2. MODELS

As already mentioned, extremum control systems have one major characteristic in common. In the absense of disturbances, the steady-state relation between input and output should be a function with an extremum. The object of control is to stay as close to this extremum as possible despite the influence from dynamics, noise or drifts. In order to use optimal control theory, this desire must be translated into a formal loss function. There are several ways of doing this. One possibility is to use a system model to estimate the slope. The control law can then be designed to keep the slope as close to zero as possible, e.g. with its variance as a measure. It is also possible to use simpler control laws that do not include any system model. But then again it is usually referred to some model for the analysis of performance.

The problem of tuning a regulator for a linear system by minimization of a nonlinear criterion may have the above characteristic. It was the main concern of early extremum control systems, but will not be considered in this survey. There are several reasons for this. For one thing, there are many other methods for tuning regulators, like e.g. stochastic adaptive or model reference methods. It would lead too far to cover all these procedures as well in a single paper. Furthermore, the extremum control problems treated here will be assumed to have unknown nonlinearities, whereas a nonlinear criterion specified by the designer is of course known to him. This knowledge should then be used in the design. Another special feature of the regulator tuning problem for linear systems is that the basic control loop is linear, but an artificial nonlinearity is added in an outer loop. This is in contrast with the extremum systems considered in this survey, where the nonlinearity is assumed to be inherent in the system to be controlled.

Static Systems

A common assumption in the literature is that there is no dynamics in the system. In practice, this condition can be fulfilled by using a sufficiently large sampling interval. But the result may be a slow optimization. In many cases, however, static models may be adequate, and stochastic approximation methods can then be used for optimization to handle noisy measurements. In (9) is given an account for some of the latest developments in the area together with further references, and this survey will be more concerned with dynamic models.

Dynamics

It is not at all clear what is the best and most natural way of modeling a nonlinear dynamic system. To be able to use system identification it is of course desirable to have a model which is linear in its unknown parameters. Any a priori knowledge about the process should then be utilised in the choice of regressors. In this way it may be possible to handle quite complicated, but partially known nonlinear systems. Gallman/Narendra (10) consider general nonlinear systems. Based on approximation theory they discuss some series expansion representations of the output, which are valid in a closed interval of time [0,T]. The presentation includes the Volterra, Wiener and Uryson series.

It is, however, difficult to find model structures that are general enough, and still allow calculations to be done. One attempt is to separate the linear and nonlinear parts into two blocks in series. There are then two possibilities: the nonlinear part can be placed either before or after the linear part. With an input nonlinearity a so called Hammerstein model is obtained, which is a special case of the Uryson series. An output nonlinearity can be viewed as a special case of the Volterra series. This choice will have a large influence on the behaviour of the model as can be seen from the following example.

Example. Consider a first order linear system with white equation noise, and a nonlinearity in the form of a squaring device. Then with the nonlinearity at the input of the linear part according to Fig. 1 the overall system is

$$y(t+1) = ay(t) + bu(t)^2 + e(t)$$

where $e(t)$ is a white noise process.

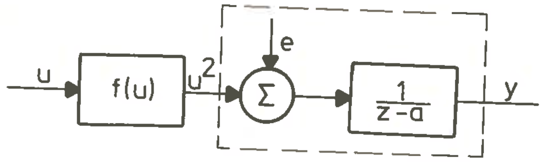

Fig. 1 System with input nonlinearity.

Suppose a stationary solution exists ($|a| < 1$). Expected values then are

$$Ey = \frac{b \cdot Eu^2}{1-a}.$$

If the goal is to minimize Ey (and $b>0$) the best performance is thus achieved by putting $u(t)=0$! Furthermore, if $|a| > 1$ no stationary solution exists.

Now turn to the other case with an output nonlinearity according to Fig. 2. The equations are

$$x(t+1) = ax(t) + bu(t) + e(t)$$
$$y(t) = x(t)^2.$$

For $a = b = 1$ this is the problem considered in (11).

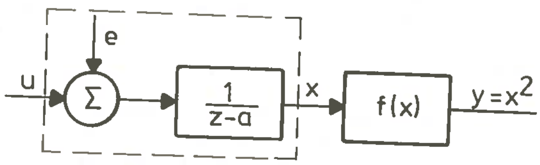

Fig. 2 System with output nonlinearity.

It is shown that because of the nonlinear measurement this is a dual control problem in the sense of Feldbaum (12). The conditional distribution of the state x is discrete, the possible values being $x = \pm|x|$. The conditional mean of x can then be calculated. It is shown that it is not optimal in the long run to have $u(t) = -\hat{x}(t)$. These results would probably not change much if $a = 1-\varepsilon < 1$. Even if a is slightly greater than one, a stationary solution still seems possible. □

There are thus significant differences between the two cases in spite of their identical static response curves. In the first case with the nonlinearity at the input, the optimal control is constant, and thus contains no feedback. The solution to the second problem includes feedback and therefore seems more attractive. It is, however, more difficult to compute, because it has a dual nature even with known parameters.

Maybe an output nonlinearity is in general more important than an input nonlinearity for a good description of a nonlinear system. The only possible effect of a known nonlinearity at the input is to restrict thepossible input values for the linear part. The nonlinear control problem can then be transformed to linear control with positive inputs. If the range of the nonlinearity is the whole of the real axis, then a change of control variable will reduce the problem into a linear one.

Most of the control algorithms described in the literature have been derived for the static case. Much work has been done to analyse the effect of dynamics on such algorithms. Their behaviour can often be improved by slight modifications of the algorithms to compensate for the dynamics. In an absolute majority of these studies the nonlinearity has been applied at the input, giving a so called Hammerstein model. The linear part is frequently of first order with a known time constant.

Only very few papers discuss what happens when there is an output nonlinearity. In some of those papers it is assumed that the intermediate signal is measured. Others assume that it can be reconstructed because no disturbances enter between the input and the intermediate signal. In such cases the problems with an output nonlinearity are circumvented. But more research is needed to find out how to handle systems where the intermediate signal is not available.

Noise and Drift

It may be important in practical systems to take noise and drift into account when designing a regulator. Noise is then usually modelled as white and additive, and is applied at the system output as measurement noise. Other possibilities are to apply it in between the linear and nonlinear parts, or at the input. It is important to note that noise at the input of the nonlinearity is equivalent to a horizontal drift of the nonlinearity. This gives a difficult control problem, which is dual in the sense of Feldbaum. It was shown in (12) that a perturbation signal at the input is required to follow the moving optimum.

Most existing control algorithms are primarily designed for deterministic systems. System noise is then usually handled by analysing its effect on the closed loop system. One way to reduce the effects of noise is of course filtering, which has been found useful and necessary in several schemes.

Time-varying system parameters are usually modelled as first order dynamics driven by white noise. This gives a possibility for tracking the drift. For the nonlinearity such a parameter model can be applied either to the horizontal and vertical positions of the optimum, or to the three parameters of an approximating second order polynomial.

3. CLASSIFICATION OF ALGORITHMS

Surprisingly few new ideas for extremum control have emerged since the 60's. Most of the work has been concerned with analysing the behaviour of known algorithms or slight modifications. Different difficulties are then considered like e.g. measurement noise, input or output dynamics or drift. This is why the old survey paper (8) can still be recommended as a very good introduction to the field. The classification used in this report will follow (8), even though newer modifications will of course also be reviewed.

The first type of systems to be discussed are *perturbation systems*. The effect at the output from a known signal added to the input is then used to derive information about the slope of the nonlinearity. In a so called *switching system* the input is driven at a constant speed until the extremum is passed. The direction of input drift is then reversed according to some fixed rule. *Self-driving systems* use no preset changes in the input. The measurements are used directly to determine the input.

There is also a fourth class of methods that is not described in (8), and seems to have been developed later on. It is based on using a parameterized model in combining parameter identification and extremum control.

A separate classification is given in (6). Rules of thumb are supplied for when to use different methods, and it is shown how to perform certain design calculations.

PERTURBATION METHODS

Already in 1922 Leblanc suggested an application of a perturbation scheme. This may then be the oldest extremum control method, and has also been quite popular. Several applications have been proposed, see e.g. (2), (14), (15), or (16).

The task of an extremum controller is to keep the gradient of the nonlinearity at zero. The problem is thus reduced to an ordinary control problem if the gradient is measured. This can most often not be done directly. A perturbation method may then provide the necessary information. The basic idea is to add a periodic test signal to the control signal, and observe its effect at the output. This is illustrated in Figure 3 for a static nonlinearity. The output and the test signal can e.g. be multiplied and averaged over a number of full periods. The resulting signal is then taken as a substitute for the true gradient, and may e.g. be used in an integral controller as the measured signal that should be kept close to zero.

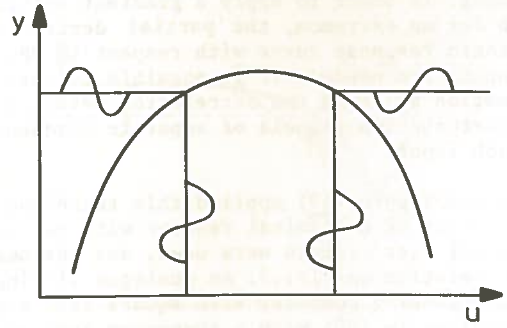

Fig. 3 Effect of an input test signal at the output of a static nonlinearity.

Modifications

Dynamics. The basic perturbation method (based on correlating the test signal and the output) may have to be modified if the system contains dynamics. The dynamics will then introduce a phase lag Θ in the test signal component of the output. The result of correlation will be multiplied by a factor $\cos\Theta$. This gives a sign error in the correlation signal if $\Theta > 90^o$. The overall system may then become unstable. This situation is avoided if a corresponding phase lag is introduced to the test signal before correlation. Such a feature has been found possible and necessary to include in several of the practical applications reported.

Another way to handle the dynamical effects is to use a perturbation signal of sufficiently low frequency. The phase lag Θ will then be small, so that the dynamics can be neglected. This may, however, give a long response time for the overall system.

The control law. In most of the schemes treated in the literature, the input is made proportional to the integral of the correlation signal. A possible improvement would be to use more sophisticated control algorithms based on the same measured signal. One step in this direction was reported in (17). A discrete time model is used with prediction of future disturbances. The correlation signal is taken as the measured error, and minimum variance control is used to keep the process (a gas furnace) at its optimum despite the disturbances.

With the perturbation signal technique, the correlating device must be given a certain amount of time to produce an accurate slope signal. During this time the control signal could be kept constant, so that the total input is varied with the test signal only. The system may then be regarded as a sampled data system where the correlating time is the sampling period.

The test signal. The most commonly used test signal form has been the sinusoid. It is relatively easy to generate using analogue technique, and frequency analysis methods are well suited for examining the effects of such a test signal theoretically. But other test signal forms may also be used, as e.g. a square wave. This is especially easy to generate in a digital computer, and was discussed in e.g. (18).

Several inputs. The perturbation method seems to be well suited for generalization to more than one input. In order to apply a gradient method in the search for an extremum, the partial derivatives of the static response curve with respect to the different inputs are needed. It is possible to obtain this information by using the correlation method above with perturbation signals of separate frequencies for each input.

Price/Rippin (19) applied this technique to the optimization of a chemical reactor with two inputs. Sinusoidal test signals were used, and the best frequency relation was 1:1.5. An analogue six-input extremum-seeking computer with square test signals is described in (20) with a frequency separation of 1:1.05 between each channel. The frequency difference should not be made too small, since the correlation time must be increased in order to separate the effects from different test signals.

For the particular case of only two inputs another method is possible. Two test signals of the same frequency, but with a 90^o phase difference can be used. But then a phase lag in the output due to e.g. dynamics will introduce a cross-coupling in the slope signals, see (8).

Analysis

As with most control systems, theoretical analysis is a valuable complement to practical experiments in finding out how perturbation systems work. Such analysis has been carried out to study e.g. stability questions, possible periodic solutions and the influence of different design parameters.

A thorough experimental investigation of a specific system was reported in (14). The effects of measurement noise and drift of the extremum point were studied in (21). Eveleigh (22) considered the problem of automatic regulator adjustment for a linear system but the analysis applies to more general extremum control systems. The same type of results were also obtained in (16) with good agreements to the results of practical experiments on a gas furnace, maximizing dioxide contents of the flue gas. The stability properties and the loss with different choices of the design parameters was examined in (23) for a special system and a design procedure was given for choosing the parameters.

SWITCHING METHODS

Another basic idea for extremum control is the following. The input is driven at constant speed in the same direction until no further improvement is registered. The drift direction is then reversed. Different algorithms of this type can be described in terms of their specific conditions for altering the direction of input changes. The control law is thus a set of switching conditions. This principle can be mechanized in two ways. The input may be changed continuously or in discrete steps. The second method seems to be quite popular in the Russian literature. Such systems will be called stepping systems.

Continuous Sweep

The paper (24) is a good reference on the continuous sweep method. They consider a static, quadratic nonlinearity with first order dynamics at both input and output. The sweep direction is reversed when the output has decreased from its maximum value by a fixed amount Δ. The design parameters are then the sweep rate and the value of Δ. Tsien/Serdengecti gave design charts and formulae for the input, the output and the so called hunting loss for different values of the design parameters and system time constants. A large portion of the monograph (2) is also devoted to an analysis of the continuous sweep method in the presence of dynamics.

If the output is disturbed by noise the above method may give excessive switching unless the value of Δ is sufficiently increased. This higher Δ-value will on the other hand increase the hunting loss. It is thus necessary to compromize in choosing Δ. Filtering is another possibility for reducing the noise sensitivity. The problem is then that more dynamics is introduced into the system, and the hunting loss will again increase.

Modifications. Unnecessary switching may also be caused by input dynamics. Consider e.g. a maximum-seeking system. After the maximum is passed and the input has been reversed, the input to the nonlinearity will continue to increase for a while due to the input dynamics. The output value at the instant of switching is then taken as the new maximum value, and with large enough dynamic lag this will cause the extra switching. As suggested in (25) this phenomenon is avoided by waiting for a while before starting to find the new maximum value.

The switching conditions may be chosen in many ways. The output may e.g. be measured only at discrete instants. The difference between successive measurements can then be used as an indicator. This was tried in (26) for the control of fuel consumption in a tunnel furnace. Two methods using such differences were analysed in (27). It was found advantageous to keep the input constant for a short while before each reversal of direction.

Several authors have suggested methods relying on differentiation of the output. Naturally, noise will then be a severe problem that has to be handled by proper filtering. Perret /Rouxel (28) consider a static quadratic nonlinearity with a time delay followed by first order dynamics. Phase-plane trajectories are calculated for each of the two directions of input drift. From these, switching conditions are derived which employ the second derivative of the output. This algorithm was applied to the maximization of produced reactive power in an alternator. Hamza (29) describes a very similar method which is claimed to handle arbitrary initial conditions better. It is not clear how these systems can cope with higher order dynamics, non-quadratic nonlinearities or time-variations.

When aiming at an extremum point it seems natural to try making the derivative of the output zero. This leads to using the derivative to determine when to reverse the sweeping direction. Such a method was investigated in (30) for two systems with first order dynamics before and after the nonlinearity respectively. A threshold was introduced, so that switching did not occur until the derivative was less than $-\Delta$ after passage of the maximum. For the case of output dynamics it was found best to put $\Delta = 0$, but with input dynamics Δ should be a small positive number.

A somewhat different technique was described in (31). The problem was to get maximum power from a solar cell on board a satellite. The current/voltage characteristic will change with the distance to the sun, and may look as in Figure 4. The extracted power can then be maximized using a continuous sweep method, where the current and voltage are decreased alternatively so that

$$I_\alpha = k \cdot I_\beta \qquad V_\beta = k \cdot V_\alpha \qquad [1]$$

Then $P_\alpha = P_\beta$. This is a special application, but the same technique could be used for other systems where a product of two related measurable factors is to be optimized, see (68).

Stepping Methods

Consider the static system

$$y = f(u) + e \qquad [2]$$

where $f(\cdot)$ has a single local maximum but is otherwise arbitrary. To begin with, assume that the disturbance $e = 0$. For this system, the input u should be adjusted to give maximal output y. This can be achieved by stepwise changes of u according to the algorithm

$$\Delta u_{n+1} = \Delta u_n \operatorname{sign}(\Delta y_n). \qquad [3]$$

The closed-loop system will then end up with the input oscillating a few steps around the maximum.

There are two design parameters to choose in such a system: the stepping period and the steplength Δu_n. A large steplength is desired in order to find the maximum quickly, but on the other hand this will imply a large loss in the steady state because of large deviations from the optimum. A variable steplength might then be useful. This will, however, complicate the algorithm, and it is not selfevident what criterion to use for the changes of steplength.

At first sight it may seem obvious that the stepping period should be kept as small as possible in order to speed up the system. But when dynamics are included in the model this may no longer be true. The easiest way to handle dynamics is to simply wait for the steady state between each input change. But as this may result in too slow a system, several other methods have been proposed, and some of them will be discussed below.

The influence of noise. Measurement noise will introduce a risk of stepping in the wrong direction when using the control law [3]. The steady state deviations from the optimum will then be increased. The stochastic distribution of the resulting random walk was analysed for different cases in (32), (33), and (34).

Smirnova/Tay (35) suggested a modified method to handle noisy systems better. Several measurements of the output are made for each input value. After each measurement a decision is taken either to stay and continue measuring or to move in either direction. The same basic idea has been used in (36) in a more complicated system that can also handle dynamics.

Dynamics. For a dynamical system the effect of the last input change on the output may be completely

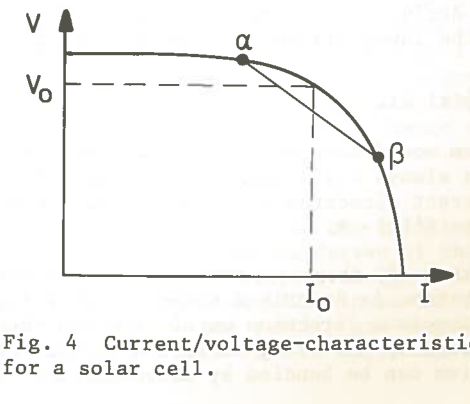

Fig. 4 Current/voltage-characteristic for a solar cell.

hidden in the responses to earlier input changes. Xirokostas and Henderson (37) found that no control at all may be better than using [3], even in the case of a drifting optimum. This basic algorithm thus needs modifying to handle dynamics.

For the case of known all-pole output dynamics Kazakevich (38) suggested that a sufficient number of measurements should be made for each new input value, so that the steady state output could be predicted and used in [3] instead of the current output. This approach was also extended to cover cases with a time delay, measurement noise or drift in the extremum, see e.g. (36), (39), or (40).

A different method was suggested in (37). An unknown nonlinearity with first order output dynamics and measurement noise is considered. The optimum is assumed to drift around both vertically and horizontally. The dynamics are handled by using a weighted sum of old output differences instead of just the last one in the following way:

$$\Delta u_{n+1} = \Delta u_n \ \mathrm{sign}(w_0 \ \Delta y_n + w_1 \ \Delta y_{n-1} + \ldots). \qquad [4]$$

It was shown that the effect of first order dynamics can be completely eliminated using only w_0 and w_1 with $w_k = 0$ for $k > 1$. The vertical drift of the optimum is then also well compensated for by much the same choice of w, whereas the measurement noise will impair control. Simulations were used to show that [4] may give significantly better control than [3]. Further improvements were gained by using a variable steplength.

Galkin (41) analysed the effects of input dynamics in a noise-free system with a constant minimum. The control law used is based essentially on [3], but with a threshold k against switching

$$\Delta u_{n+1} = -\Delta u_n \ \mathrm{sign}(\Delta y_n - k). \qquad [5]$$

It was examined how the design parameters should be chosen to avoid extra switching due to the dynamics.

SELF-DRIVING SYSTEMS

The previously discussed methods employ some form of forced input changes, like a perturbation signal or a predetermined rate of input change. In a self-driving system no such restrictions are imposed on the control signal. Instead, at every instant the available information is used to produce a control signal that will drive the system towards an optimum. Consider once more the static system

$$y = f(u).$$

The first derivative of the output could then be used to drive the input via an integrator so that

$$u(t) = \int^{t} \dot{y}(t) \ dt. \qquad [6]$$

This system would have to be started manually, since $\dot{y} = \dot{u} = 0$ is always a stationary point. But if started in the correct direction with $\dot{u} \neq 0$ it will find a point where $f'(u) = 0$.

Blackman (8) discusses several problems with this type of system. As described above, it will e.g. continue in the same direction until $\dot{y} = 0$ and then stop. So if started in the wrong direction it will continue. This problem can be handled by measuring $\dot{u}$ as well. Then $f'(u) = \dot{y}/\dot{u}$ can be used in the control law instead of just $\dot{y}$. Dynamics will introduce further problems. As explained by Blackman the system may then stick at other points on the curve $y = f(u)$.

Self-driving systems seem to have been paid very little attention to in the literature. Only the paper (42) will be mentioned here. They compensate for the dynamics by taking the measured input through a filter to get the signal u^*. This filter should be a good guess of the system dynamics, and a possible control law is then

$$\dot{u} = k \cdot \dot{y}/\dot{u}^*. \qquad [7]$$

However, to avoid the use of an accurate and therefor expensive divider, a modified control law is suggested. The sign of $\dot{u}$ was taken from [7] according to

$$\mathrm{sign}(\dot{u}) = \mathrm{sign}\ (\dot{y} \cdot \dot{u}^*). \qquad [8]$$

In calculating a proper amplitude of $\dot{u}$, [7] was used with $\dot{u}^* = \dot{u}$ to give

$$\dot{u} = \sqrt{|k \cdot \dot{y}|}\ .$$

This modified algorithm was tested on two simulated examples and was found to work well.

MODEL ORIENTED METHODS

In the schemes discussed so far, little information is collected about the system. Only the output, and maybe the slope of the nonlinearity at the current working point are used. Essentially no information is saved for later use. For the methods treated in this section, the control action is calculated from a model obtained by some kind of system identification. The position of the extremum may e.g. be one parameter in the model. The input may then be chosen as the estimated extremum position. In the simplest case the estimation may reduce to the determination of a single parameter from a couple of noise-free measurements.

There are two main groups within this class of methods. For methods in the first group, each control action is preceded by an identification phase. During this phase, the input must be varied deliberately or by noise to produce good parameter estimates. Based on the estimates a control step is then taken, and the cycle is repeated. With this type of scheme, the parameters identified are often allowed to depend on the current working point, as e.g. slope and curvature. Little information, if any, is therefore exchanged between cycles.

For the second group of methods no separate identification phase exists. The parameters are continuously updated, and control steps are taken based on the current estimates. Since more old data are saved in the estimates, this method will be better only if the model parameters do not change very much with changing working points. To ensure identifyability of the parameters, it may be necessary to superimpose a perturbation on the control signal in this scheme also.

With a low noise level, the first and second derivatives of a static nonlinearity can be determined approximately using only two search steps. This is the essence of the control law suggested in (43), (44).

$$\Delta u = \frac{1}{2} \cdot \frac{[y(u+a) - y(u-a)] \cdot a}{[y(u+a) + y(u-a) - 2y(u)]} \qquad [10]$$

where y(u) is the output with input u. Such a scheme was included in the comparison in (45). The same idea was elaborated further in (46) with known dynamics included before and after the nonlinearity. Linear or exponential drifts of the extremum can also be detected and compensated for.

Higher noise levels can be tolerated if several measurements are made to determine the next control action. Least squares identification is used in (47) to find the parameters of

$$\tilde{y} = \alpha\tilde{u} + \beta\tilde{u}^2 \qquad [11]$$

where $\tilde{y}$ and $\tilde{u}$ denote deviations from the mean values (within one cycle). The input u must then be perturbed, either deliberately or by noise. The optimum is characterized by $\hat{\alpha} = 0$, and is approached by making input changes proportional to $\hat{\alpha}$.

Clarke/Godfrey (48), (49) estimate the slope and curvature by correlating a 3-level test signal u and its square u^2 with the output. Output dynamics with finite memory will then not influence the result, and for a quadratic nonlinearity the optimum can be reached in one step. It is, however, necessary to ascertain that the estimate of the second derivative does not become too small. This can be done e.g. using a fixed limit or a first order filter on the estimate.

Roberts (50) seems to be the first one to suggest a scheme of the second type, where more and more information is gathered about the system. He considers a static system, but includes noise and drift in the model. Several parameters are unknown, including the curvature, position of the optimum, noise level and drift parameters. It is shown that even for known parameters a perturbation signal is needed to follow horizontal drift of the extremum. An optimal perturbation amplitude can be chosen to minimize the mean square deviation from the extremum. When the parameter estimates are correct, a number of signals will have zero mean value. The deviations from zero of these mean values are used to drive the parameter estimates. The input is chosen as the estimated position of the optimum with a superimposed perturbation signal.

Keviczky/Haber exploited the idea of self-tuning extremum control (51). They suggested least squares or stochastic approximation identification to find the parameters of a Hammerstein model. The input was then chosen at each step as if the parameter estimates were correct. With this method parameter drift can be handled by a simple modification of the estimation algorithm.

Bamberger/Isermann (52) developed a program package employing a gradient method for optimizing a Hammerstein model. The parameters of both linear and nonlinear parts can be identified using either the instrumental variable or correlation methods. In the latter case, the final scheme is closely related to that of (48), (49) buth with parameters that are independent of the working point. A successful application to power optimization of a turbine is also reported.

Identification

Model identification is an important part of these model oriented methods. An increasing interest in the identification problem for certain nonlinear systems has been noted in recent years. A survey of this area was given in (53). Some material can also be found in the survey (54) on identification in Russia. The correlation technique has been reviewed in e.g. (55). It seems to be quite useful for nonlinear identification, see e.g. (56),(57).

Most of the work has been done for Hammerstein models, starting with (58), see also (59). Some variants of equation error least squares identification have been discussed in (60), (61), (62), and (63). In all of the papers mentioned above it is assumed that the input is white Gaussian noise, and this is in some cases of importance for the results to hold. This might be a restriction when using the schemes as part of an extremum controller.

COMPARISONS

Many of the papers describing individual methods contain a comparison between the suggested algorithm and some other scheme. In e.g. (64) an improved stepping method is compared to an ordinary perturbation method. But such comparisons do not give an overall picture. With the large number of existing methods for extremum control it would be expected (and wanted) to find several papers comparing different schemes under shifting circumstances. A few algorithms for static optimization with noisy measurements were compared in (65). Also in (45) was used a static, noisy system to evaluate the performance of three extremum-seeking regulators. But no complete comparison of all kinds of methods has been found.

4. APPLICATIONS

Quite a few practical applications of extremum control algorithms have been reported in the literature. Combustion processes seem to have been a major concern in earlier work, but later on several other problem areas have been entered. A selection of tested or suggested applications are listed below in order to give a general feel for the wide range of possible applications.

The most common way to optimize a combustion process is to control the air/fuel-ratio through the air flow. Using different measured variables this has been tried in e.g. (2) for an internal combustion engine, (25) and (64) for a steam generating plant and in (16) on a gas furnace. Draper/Li (2) also varied the ignition timing. The two control variables were alternatively switched to the peakholding regulator. Vasu (14) varied the fuel flow in a flight propulsion system to maximize a certain pressure indicating performance. Several practical experiments were undertaken to find out the influence on the performance of several design parameters in a perturbation scheme.

In certain grinding mills the grinding efficiency will vary with the filling degree of the mill, which can be controlled through the incoming flow of raw material. The optimal point in maximizing efficiency may depend on the quality and composition of this raw material. This type of application was reported in (66) for a cement mill and in (67) for autogeneous ore grinding.

In (15) and (19) the water-gas shift reactor was considered, where hydrogene and carbon dioxide is produced from carbon monoxide and steam. The first authors maximized the amount of carbon monoxide converted, and used the steam as control variable. Their work was extended in (19) to include the temperature as a second control variable.

Further applications from different areas are the previously mentioned solar cell optimization in (31) and the adjustment of a radio telescope antenna to maximize the signal received from a moving object, see (69). Bamberger/Isermann (52) considered the optimization of total power from a steam turbine by controlling the cooling water pump. Table 1 gives an account for what extremum control methods have been used in these applications. As seen from Table 1, no application using self-driving systems has been found. Also, reports of practical work with model-oriented methods are rare, indicating that more research is needed in that area.

Table 1. Extremum control methods in the applications.
P - Perturbation C - Continuous sweep
S - Stepping M - Model oriented

Application Area	Reference	Methods P	C	S	M
Combustion processes	2		x		
	14	x			
	16	x			
	25		x		
	64	x		x	
Chemical proc.	15	x			
- " -	19	x			
Solar cell	31		x		
Turbine power	52				x
Grinding	66	x			x
- " -	67			x	
Antenna adjust.	69			x	

Still more applications have been suggested. Examples are control of blade angles in water turbines or wind mills for power generation, and control of distillation columns to yield maximum production. An interesting environmental problem is the removal of sulphur dioxide from the flue gas of a fluidized bed combustor, see (70). This can be done by feeding certain additive particles into the bed. To keep down the cost of additive particles, it is then desirable to solve the extremum control problem of controlling the combustion temperature to minimize the contents of sulphur dioxide in the flue gas.

5. SOME POSSIBLE ADAPTIVE TECHNIQUES

As shown in the subsection on model-oriented methods, control laws based on an identified system model have already been proposed for extremum control. There are, however, many possible ways to apply adaptive control to this area. Some suggestions are given below, which all make use of a system model with unknown parameters. They should not be regarded as ready to apply methods, but rather as a list of ideas and candidates for further analysis, needing both simulations and theoretical work to be done.

Any a priori knowledge about the system should of course be used in setting up a model. This is especially true for nonlinear systems. It may e.g. provide possibilities for choosing a model structure that allows a good description of the nonlinear phenomena, and still is linear in its unknown parameters to simplify parameter identification. With no such a priori knowledge available, more general nonlinear models have to be used. A possible approximation is then to separate the linear and nonlinear parts. The nonlinearity can be placed either at the input or at the output of the linear block, and the model behaviour will be different in the two cases as discussed in Section 2.

Input Nonlinearity Models

With the nonlinearity at the input, the model is of Hammerstein type. The combined parameter estimation and extremum control problem for this model was discussed in (51) and (52). A basic model with output y and input u is

$$A(q^{-1})\, y = B(q^{-1})\, v + e \qquad [12]$$

where q^{-1} is the backwards shift operator, e is a disturbance and

$$v(t) = \alpha + \beta u(t) + \gamma u(t)^2. \qquad [13]$$

A slight generalization is to use different B-polynomials for the different terms in v(t), but this will not influence the results stated below.

Assume that $\gamma > 0$ and let the object of control be to minimize the expected value of the output. Using dynamic programming it can then be shown that the optimal control law with known model parameters is the constant input

$$u(t) = -\beta/2\gamma. \qquad [14]$$

There is thus no feedback. For unknown parameters Keviczky and Haber (51) suggested an adaptive control law based on least squares identification and certainty equivalence control. It was shown in (71) that this scheme is bound to converge to bad parameter and input values. The reason is that the adaptive control law contains only the estimated parameters and no measurements, so that identifiability is lost when the parameters have converged to any value. To overcome that problem there are several possible ways, which should be tried out and compared.

Estimated optimum as a reference. Keviczky et al (72) suggested that the optimal steady state output value should be calculated from the estimated parameters. Similar to minimum variance control, the expected value of the next output is then equated to the calculated optimum. This gives a control law that is nonlinear in the measurements, and the identification problems are avoided. But some of the optimality is sacrificed, since the output will not be kept at the optimum as much as possible.

Addition of a perturbation signal. A straightforward way to assure parameter identifiability is to add a perturbation signal. Such a signal must be sufficiently rich, as e.g. a pseudo random ternary signal. This method was used in (52). Convergence to the true parameter values can then be achieved by letting the perturbation amplitude tend to zero slowly enough. A disadvantage with this method is that there are two new parameters to choose, the perturbation frequency and amplitude.

Exponential forgetting. The gain in the parameter estimation can be kept away from zero using an exponential forgetting factor. The same thing is achieved if a constant matrix is added each time the covariance matrix is updated. Since the parameter estimates (and thus the input) will then not converge, identifiability is improved, i.e. the estimates can be expected to stay around the true values. If the forgetting factor is allowed to tend to one slowly enough (or the added matrix to zero), there is a hope that the parameters will converge to the true values. However, simulations have indicated that the parameter estimates may behave unsatisfactorily. After periods of staying almost constant they can suddenly make large jumps. Further tests are needed to evaluate this method.

Multistep dynamic programming. The problem of minimizing the expected value of a loss function can in principle be solved using dynamic programming. The practical difficulties however, are often so great that a common approximation is to make a one-step minimization only. In the case discussed here, where the mean output is to be minimized, this approximation leads to the certainty equivalence control law with the identifiability problems mentioned before. By taking the dynamic programming a few steps further, a better control law may result. If the system model contains dynamics, then this improved input will contain feedback from the measurements, which promotes identifiability. This type of control has been discussed and analysed in (73).

Output Nonlinearity Models

With the nonlinearity at the output, the extremum control problem is in general much more difficult to solve. However, a special case is when the intermediate signal between the linear and nonlinear parts can be measured. One way to solve the problem is then to use a self-tuning regulator for the linear part with a reference value calculated from the optimum of the estimated nonlinear part. As in the case with an input nonlinearity, this requires for identifiability that the intermediate signal is sufficiently rich, by noise or deliberate perturbations. There are also other possible solution methods in this case. The minimization of a loss function such as the expected value of the output could e.g. be a useful route to follow, provided that the system equations are written in a suitable form. Again, these are just suggestions that remain to be tested analytically and by simulation.

Known system parameters. As shown in the example of the models section, a dual control problem in the sense of Feldbaum (12) arises when the intermediate signal is not measured. For known system parameters simple examples have been treated in (11), (71), and (74). In (71) a first order integrator system is rewritten so that the least squares method can be used to estimate the state. The system is

$$x(t+1) = x(t) + u(t) + w(t) \qquad [15]$$

$$y(t) = x(t)^2 + e(t) \qquad [16]$$

and the expected value of the output y is to be minimized. Inserting [15] into [16] we get

$$y(t+1) = y(t) + u(t)^2 + 2x(t)\,u(t) + 2v(t)[x(t) + u(t)] + v(t)^2 + \Delta e(t+1) \qquad [17]$$

which can be used for least squares estimation, regarding the last row as a noise term. A certainty equivalence control can be obtained by minimizing $E[y(t+1)|t]$. It was shown by simulation in (71) that the dual control law derived from minimizing $E[y(t+2) + y(t+1)|t]$ gave significantly better results. An obvious alternative that was not tried is to use an extended Kalman filter to estimate x from [15]-[16] directly.

Unknown parameters. When the parameters are unknown it is difficult, if at all possible, to rewrite a system like [15]-[16] to make the least squares method directly applicable. Some kind of approximate nonlinear estimation technique is then needed, as e.g. the extended Kalman filter. It is an open question if it is advantageous to first rewrite the equations (like [17]), or if the original equations should be used. A comparison between these two alternatives for a known parameter case, as e.g. the example above, might give some indications. There is a large number of possible combinations of nonlinear estimation methods, rewriting equations and ways of calculating the control law. It would be desirable to have these possibilities more closely examined, as the output nonlinearity model in some respects seems to give better control laws than the input nonlinearity model. An extension to also include a second linear block at the output of the nonlinearity should only cause minor additional difficulties.

6. CONCLUDING REMARKS

For some reason most of the research on extremum control has been done in Russia and eastern Europe. It can be mentioned, that out of the papers studied for this survey, counting only the ones available in translation, almost 2/3 are from these countries. Most of this work has been published in 'Automation and Remote Control', 'Cybernetics', or the German journal 'Messen, Steuern, Regeln' with a few papers in the IEEE Transactions on Automatic Control. The early IFAC world conferences are also good sources for further references.

Although quite a few practical applications have been reported, in particular with the perturbation method, most of these have concerned pilot plants or laboratory processes. The field of extremum control still needs further development in order to make the technique easy to apply and well suited for routine use in commercial processes. It is believed that the prerequisites for such a development are now at hand. This has been a main reason for carrying out this survey.

First of all, there has been and is a rapid progress in computer technology with powerful microprocessors now appearing at very low cost. It is even becoming economically feasible to replace ordinary analogue PID-controllers by digital versions implemented in microprocessors. This also adds to the possible flexibility of the controller. The increased computing capacity could then instead be used to implement more complicated control algorithms, such as e.g. extremum controllers.

Secondly, the theory development in disciplines like optimization, identification and adaptive control has been substantial. It should then be possible to bring extremum control forward using ideas from these neighbouring areas. A few suggestions based on some specialized models were given in the previous section. Hopefully this survey can help promoting such a progress by presenting the status of extremum control to researchers of these other fields.

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