Controller Tuning from Simple Process Models

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What? Another controller tuning article? Yes, but this time the quarter decay and root locus plots have been shelved in favor of a little algebra. The authors have developed a tuning method which requires only a knowledge of the two dominant poles of a process, reducing the mysteries of controller tuning to a few simple calculations applied to a mathematical model.

CONTROLLER TUNING is still a black art in spite of all the technical articles dealing with the subject that have been published in the past thirty odd years. The pioneer article by Ziegler and Nichols in 1942 (Ref. 1) presented procedures and formulas to tune a controller for a quarter-decay ratio closed-loop response. Lopez, et al (Ref. 2) published correlations to tune a controller for minimum time integrals of selected error functions during step disturbances. Rovira and co-workers (Ref. 3), continuing the work of Lopez, developed similar correlations for step changes in setpoints, and found them to produce more stable responses.

Other cookbook approaches have been presented in numerous textbooks and articles. The fact that none of these seems to have gained widespread acceptance indicates that, as in the case of razor blades, the best solution has not yet been found.

A quarter-decay response is an elegant basis for a treatise, but it is far too underdamped to be accepted by plant operators. Minimum error integral correlations may be easy to develop with search techniques and FORTRAN programs, but no one seems to be convinced that these methods produce the "best" settings for a specific process; in fact, there seems to be little agreement on the definition of best controller settings.

This article presents an approach to controller tuning which was derived from simple algebraic synthesis. The results agree with current industrial practice and have produced acceptable controller parameters in two industrial processes with which the authors have been involved. Settings obtained with the correlations are not intended to be accurate to three significant figures, but as an order of magnitude approximation. Anyone who has ever been involved in the control of an industrial process knows that this is all that could be expected.

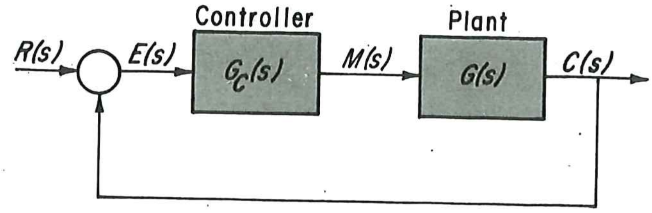

Figure 1. Typical feedback control loop. Given the plant transfer function $G(s)$, the tuning problem is to find the appropriate controller transfer function $G_c(s)$ that results in the desired closed-loop response $C(s)/R(s)$.

Developing process models

A typical feedback control loop is shown in Figure 1. The block labeled "Plant" includes the gains and lags associated with the control valve, the process itself, and the sensor and transmitter. Since the signals between these individual components are not usually available in the control room, the entire process portion of the loop must be combined as a single function for controller design purposes. The controller output signal, $M(s)$, is the input to this combined block; the transmitter output signal, $C(s)$, is the block's output. The controller design problem consists of selecting controller modes and tuning the controller parameters to the specified process.

In order to adapt the controller, the dynamic characteristics of the process must be measured in some manner. All but the crudest trial-and-error tuning methods involve the determination of dynamic plant characteristics, be it with the loop open or closed. Control engineers the world over are familiar with the techniques used to determine deadtimes, process lags and control responses; these methods range from fiddling with controller adjustments and observing plant responses to more sophisticated techniques, such as pulse testing (Ref. 4). Dynamic process values are then used in various tuning algorithms to determine controller settings.

The three techniques previously cited for controller tuning—Ziegler-Nichols, Lopez and Rovira—are based on the open-loop process reaction curve (Ref. 5); this curve, based on measured plant dynamics, represents the time response of a transmitter output to a step change in controller output. Simple linear models have been fitted to these process reaction curves for first-order lag, second-order lag and underdamped processes:

First-order lag plus deadtime (FOPDT)

$$G(s) = \frac{Ke^{-t_0 s}}{\tau s + 1} \qquad (1)$$

Second-order lag plus deadtime (SOPDT)

$$G(s) = \frac{Ke^{-t_0 s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad (2)$$

Underdamped system

$$G(s) = \frac{Ke^{-t_0 s}}{\tau^2 s^2 + 2\tau\zeta s + 1} \qquad (3)$$

To attempt to fit models higher than second-order to the process reaction curve is to try to extract more information than the curve can supply. More sophisticated techniques for determining plant characteristics, such as pulse testing, are required to obtain higher order models. In Equations 1-3, the deadtime value, t_0, includes the effects of all the small time constants that are not accounted for in the model.

Inside the controllers

Over 70 percent of the feedback controllers installed in industry are the standard proportional-integral (PI) or two-mode controller supplied off the shelf by various instrument manufacturers. Practically all PI controllers can be represented by the transfer function:

$$G_s(s) = K_c(1 + \frac{1}{T_i s}) \qquad (4)$$

which can be rearranged in the form of a lead lag with the pole at the origin:

$$G_c(s) = K_c \frac{1 + T_i s}{T_i s} \qquad (5)$$

The integral or reset mode, represented by reset time T_i, cannot in general be turned off.

The balance of the feedback controllers are, for all practical purposes, of the proportional-integral-derivative (PID) type, commonly known as three-mode controllers. These controllers can be represented by the transfer function:

$$G_c(s) = K_c \frac{1 + T_i s}{T_i s} \left[\frac{T_d s + 1}{\alpha T_d s + 1}\right] \qquad (6)$$

The derivative mode of the controller usually consists of a lead lag circuit with the lag set at one-tenth (α=0.1) of the lead. Most PID controllers have a provision for turning the derivative time off (T_d=0), in which case the controller is reduced to a PI or two-mode controller. Both PI and PID controllers have adjustment knobs for setting gains and time constants; these controls are used to tune each individual controller for its specific task.

NOMENCLATURE

C	transmitter signal
G	process transfer function
G_c	controller transfer function
K	steady state gain
K_c	controller gain
M	controller output signal
R	setpoint signal
s	LaPlace transform variable
t	time
t_o	deadtime or transportation lag
T_d	derivative time
T_i	integral or reset time
ζ	damping ratio for second-order process
λ	tuning parameter
τ	time constant of the plant
$\tau_1 \tau_2$	time constants of the factored form of the second order transfer function

Synthesizing the controller

Designing a controller with the synthesis method consists of determining the controller transfer function that is required to produce a specified closed-loop response. Synthesis can be carried out with the aid of such tools as root locus and frequency response plots; but if the plant can be represented by low order models, it is possible to synthesize the controller directly from the closed-loop transfer function. For the system represented in Figure 1, the transfer function becomes:

$$\frac{C(s)}{R(s)} = \frac{G_c(s)\,G(s)}{1 + G_c(s)G(s)} \qquad (7)$$

The synthesis equation is obtained by solving for the controller transfer function $G_c(s)$:

$$G_c(s) = \frac{1}{G(s)} \left[\frac{C(s)/R(s)}{1 - C(s)/R(s)}\right] \qquad (8)$$

Equation 8 is the basic equation from which the controller can be designed. The term $G(s)$ represents the model of the plant dynamics, while the term $C(s)/R(s)$ is the desired closed-loop response. When specifying the desired closed-loop response, care must be taken to insure that the resulting controller is physically realizable. This means that it cannot contain positive deadtime terms or pure differentiation terms (more zeros than poles) and that the gain must be finite. For example, it is not possible for the output to track the setpoint at all times, i.e., $C(s) = R(s)$, because this would require an infinite controller gain.

Dahlin (Ref. 6) and Higham (Ref. 7), working on the synthesis of digital control algorithms, specified the following form of the closed-loop response:

$$\frac{C(s)}{R(s)} = \frac{\lambda e^{-t_0 s}}{s + \lambda} \qquad (9)$$

where t_0 is the deadtime of the plant, and λ is a tuning parameter which specifies the characteristics of the closed-loop response. Note that the steady-state gain of the loop, obtained by setting $s=0$, is unity, insuring the absence of offset.

Substitution of Equation 9 into Equation 8, for a process with no deadtime ($t_0=0$), results in the following synthesis equation:

$$G_c(s) = \frac{1}{G(s)} \frac{\lambda}{s} \qquad (10)$$

By applying Equation 10 to a few typical process loops, we can check its validity against industrial practice.

Flow control—One characteristic of a flow controller is that the response of the flow to a change in valve position is instantaneous; i.e., the lag is essentially zero and the plant can be represented as a pure gain:

$$G(s) = K \qquad (11)$$

Substituting Equation 11 into Equation 10 produces a pure integral controller:

$$G_c(s) = \frac{\lambda}{Ks} \tag{12}$$

In practice, pure integral controllers are not available off the shelf for industrial use. As a consequence, flow controllers are PI controllers with a very small gain and a very short reset time. Equation 5, the PI transfer function, indicates that as the reset time T_i approaches zero, the transfer function approaches that of a pure integral controller: the gain K_c must also be small because of the presence of T_i in the denominator. Other fast loops which use integral controllers include speed control of centrifugal compressers by integrating governors.

First-order lag—When a process can be represented by a pure first-order lag, or when the dominant pole is far from the other poles, the plant transfer function is:

$$G(s) = \frac{K}{\tau s + 1} \tag{13}$$

Substituting Equation 13 into Equation 10:

$$G_c(s) = \frac{\lambda(1 + \tau s)}{Ks} \tag{14}$$

Comparing Equation 14 (plant transfer function) to Equation 5 (PI controller transfer function), and relating the dynamic and steady-state terms to each other, the following tuning parameters result:

$$T_i = \tau \qquad K_c = \frac{\lambda \tau}{K} \tag{15}$$

This is one of the most important results of the controller synthesis method: It says that the reset time T_i should be set equal to the dominant time constant of the plant τ; and that the controller gain K_c can be adjusted to obtain the desired speed of response for the closed loop, since it includes the closed-loop tuning parameter λ. This is the approach used by the authors to tune the temperature controllers on two industrial systems—a stirred tank reactor and a furnace—with excellent results.

The choice of controller modes is consistent with the widespread use of PI controllers in industry. Most level controllers and concentration controllers in stirred tanks with long residence times, as well as most gas pressure controllers, can be represented by first-order lags.

Second-order lag—When the plant can be represented by two dominant poles far from the rest of the poles, the transfer function is:

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{16}$$

Substituting Equation 16 into Equation 10:

$$G_c(s) = \frac{\lambda}{K} \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{s} \tag{17}$$

Comparing the plant transfer function in Equation 17 to the PID controller transfer function in Equation 6 results in the following tuning parameters:

$$T_i = \tau_1 \qquad T_d = \tau_2 \qquad K_c = \frac{\lambda \tau_1}{K} \tag{18}$$

In this case, the reset time T_i is set equal to the longest time constant τ_1, the derivative time T_d is set equal to the next longest time constant τ_2, and the gain is adjusted to obtain the desired speed of response in the closed loop. The lag in the derivative lead-lag term is not predicted, but it is required to keep the controller realizable.

The choice of modes is consistent with industrial use of PID controllers in temperature control loops, where the lag of the temperature bulb and the thermowell is significant when compared to the process lag.

Compensating for deadtime

Some processes have dynamics represented by many interacting lags in series and true transportation lags. To fit a reaction curve to these higher order processes, a deadtime term must be included in the transfer function, as shown in Equations 1-3. When this happens, controller realizability requires a deadtime term in the desired closed-loop response, as shown in Equation 9. Substituting Equation 9 into Equation 8, to synthesize a controller with a deadtime term, results in:

$$G_c(s) = \frac{1}{G(s)} \frac{\lambda e^{-t_0 s}}{s + \lambda(1 - e^{-t_0 s})} \tag{19}$$

The deadtime term in the numerator is cancelled by the corresponding term in $G(s)$ when the equation is simplified, and will disappear from the controller transfer function. The deadtime term $(1-e^{-t_0 s})$ in the denominator, however, represents the deadtime compensator recommended by Smith (Ref. 8); this term, better known as the Smith Predictor, is difficult and expensive to obtain with analog components but is rather simple to implement on a digital computer. The Dahlin controller (Ref. 6), a computerized equivalent of a PI controller with built-in deadtime compensation, has been successfully applied to the digital control of papermachines, known to include significant deadtime.

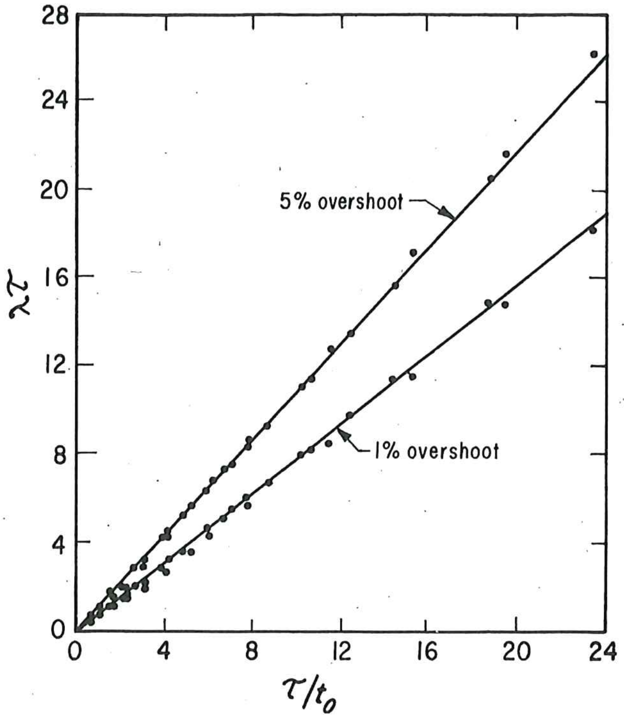

Figure 2: Tuning parameter correlations for a PI controller set for both a one percent and a five percent overshoot.

Because of the difficulty in implementing a deadtime term with analog equipment, the denominator term must be approximated by a first-order Taylor series expansion:

$$e^{-t_0 s} \cong 1 - t_0 s \qquad (20)$$

Substituting the approximation into Equation 19 and simplifying results in:

$$G_c(s) = \frac{1}{G(s)} \frac{\lambda e^{-t_0 s}}{(\lambda t_0 + 1)\, s} \qquad (21)$$

For a first-order lag plus deadtime (FOPDT) approximation of the plant, Equation 1 is substituted into Equation 21 and the resulting expression is compared with Equation 5, the transfer function for a PI controller; this comparison yields the tuning formulas for a PI controller with deadtime compensation:

$$T_i = \tau \qquad K_c = \frac{\lambda \tau}{K(\lambda t_0 + 1)} \qquad (22)$$

For a second-order lag plus deadtime (SOPDT) approximation, Equation 2 is substituted into Equation 21, and the result is compared with Equation 6 to obtain tuning formulas for a PID controller with deadtime compensation:

$$T_i = \tau_1 \quad T_d = \tau_2 \quad K_c = \frac{\lambda \tau_1}{K(\lambda t_0 + 1)} \qquad (23)$$

The effect of deadtime t_0 in Equations 22 and 23 is a reduction in the controller gain K_c for the same value of the tuning parameter λ. These tuning formulas cannot be used when the process is underdamped because complex values for τ_1, τ_2, T_i and T_d would result.

Tuning for 5 percent overshoot

All of the tuning formulas allow for adjustment of the controller gain to meet any specified control loop performance criteria. To illustrate, Figure 2 shows the value of tuning parameter λ used with Equation 22 to tune a PI controller for 5 percent overshoot. A comparative curve for 1 percent overshoot is also shown.

Figure 3 illustrates a graphical correlation of λ with the parameters for the SOPDT model to tune a PID

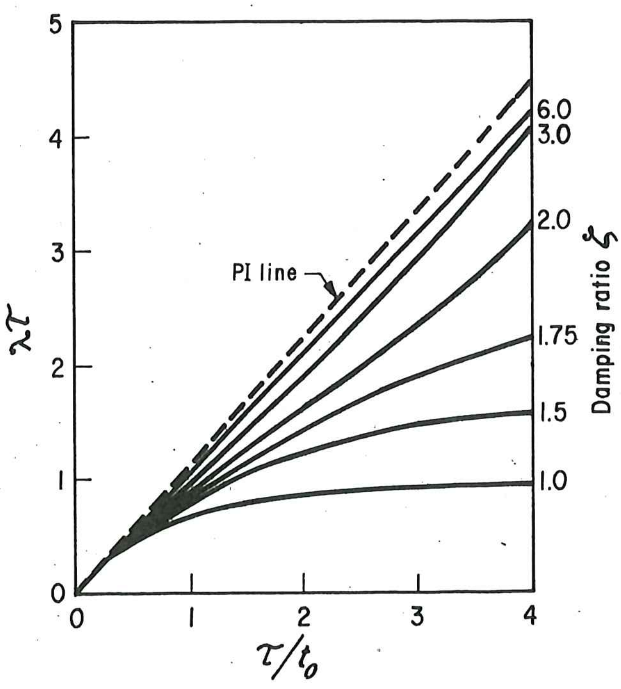

Figure 3. In a PID controller, the value of the tuning parameter λ varies with the damping ratio of the plant model. As the damping ratio increases, the model approaches first-order and the PID controller approaches the PI controller ($T_d \to 0$).

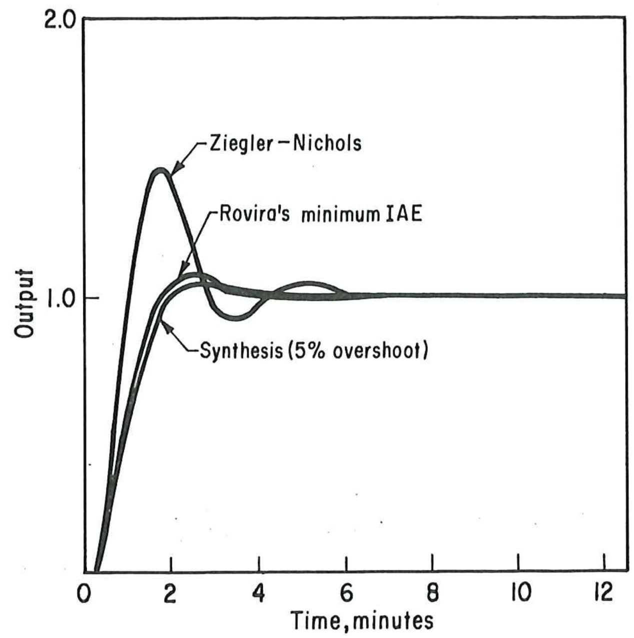

Figure 4. Closed-loop response curves for Ziegler-Nichols, Rovira's Minimum IAE and Synthesis tuning methods indicate that a synthesized controller, set to five percent overshoot, produces a response very close to the IAE method, while the Ziegler-Nichols response is oscillatory. Parameters of this second-order plant model are τ = 3.7 min., t_o = 0.25 min., and K = 1.

controller for 5 percent overshoot. These values are to be used with Equation 23 for tuning the controller. The parameters of the second-order lag are given in terms of the time constant and the damping ratio ζ:

$$\tau = \sqrt{\tau_1 \tau_2}$$

$$\zeta = \frac{\tau_1 + \tau_2}{2\sqrt{\tau_1 \tau_2}} \qquad (24)$$

The correlations shown in Figures 2 and 3 were obtained by digital simulation of a control loop, with the plant represented as a second-order lag plus deadtime. Parameters for the FOPDT model were obtained from the process reaction curve using the two-point method advocated by Smith (Ref. 5).

How well does it work?

A comparison of synthesis tuning of a PI controller with the Ziegler-Nichols technique and with Rovira's minimum IAE (integral of the absolute value of the error) tuning method is shown in Figure 4. Response of the synthesized controller is very close to the minimum IAE response, while the Ziegler-Nichols parameters produce a highly oscillatory response. If the Ziegler-Nichols gain is reduced to obtain 5 percent overshoot, the response is sluggish, with a rise time much longer than for the controller synthesis method. This is because the Ziegler-Nichols reset time is less than half the time constant and does not properly compensate for it.

The controller synthesis technique is more than just another tuning method. It provides a simple guideline for controller tuning which requires only a knowledge of the magnitude of the dominant poles. Controller synthesis views the reset time as a compensator for the dominant pole of the plant and the derivative time as a compensator for the next largest time constant. Synthesizing the reset and derivative time constants reduces controller tuning at the process site to proportional gain adjustments. The gain is adjusted to balance overshoot against rise time according to the needs of each specific control problem; it could not be simpler.

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