'Single-term' control of first- and second-order processes with dead time

by J. D. HIGHAM

IN THE APPLICATION of computers to the dynamic control of industrial processes, the next stage of complexity above three-term is to control processes which include a dead time or transport lag. Three-term control becomes ineffective on such a process when the dead time becomes of the same order or greater than the main time constants of the process.

Since these sorts of process are common in industry (e.g. in paper manufacture, cement kiln control, cement cooler control) it is desirable that the algorithm used should be standard and have such ease of commissioning as is available with a three-term controller where, very often, the proportional, integral and derivative terms can be trimmed on-site without resort to expensive process-identification tests before the control system is commissioned. However, since dead-time processes are often the slower processes on a plant, it is not possible to commission them in quite such a free manner as with three-term controllers.

I shall here describe a standard form of control algorithm involving four parameters. Three of them are based on three process parameters: the gain, the time constant and the dead time. The fourth parameter, Q, is used to adjust the closed-loop performance at commissioning time and will depend on the nature of the process disturbances. The process parameters need not be determined very accurately, since Q can be used to reduce the instability otherwise caused by mismeasuring them. Therefore, simple step tests, or calculations, or the experience of the control engineer on the plant, have been found to be sufficient. This is useful since one can have immediate, adequate control of the plant, and then, if required, indulge in more sophisticated process identification (e.g. by chain code techniques) with the plant under control.

Controllers of higher order, involving two independent parameters (in addition to the three process parameters), have been investigated. They involve more accurate determination of the process parameters and the type of process disturbances if one is to achieve even a small improvement over the single-parameter algorithm. Also the complexity of the algorithm itself increases markedly. It is, therefore, difficult to justify the inclusion of more parameters in a general control algorithm.

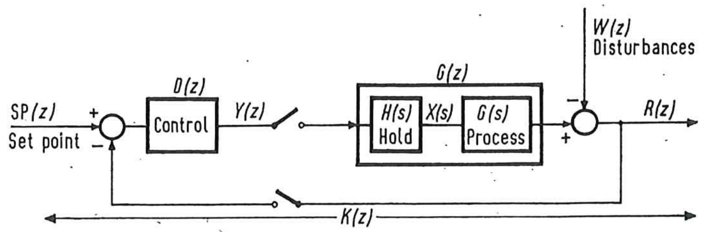

Fig. 1 Control configuration

Control configuration

The control configuration is shown in fig. 1. Note that the signal $Y(z)$ (a number from the computer) passes through a first-order sample-and-hold generating a staircase function $X(s)$ to $G(s)$. The first-order sample-and-hold is an adequate description of the interface between plant and computer where control actuator movement time is less than the sample period. This has always been the case in the variety of process applications that I have dealt with. The controller is designed so that the overall controlled system response to a unit impulse and step change in set point, SP, is as shown in figs 2 and 3.

The response in each case is governed by the parameter Q, so that the closed-loop transfer function of the system is—

$$K(z) = \frac{z^{-(N+1)}Q}{1-(1-Q)z^{-1}} \tag{1}$$

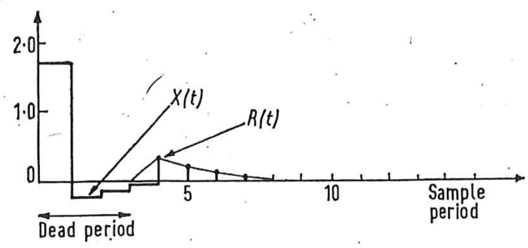

Fig. 2 Controlled response for a unit impulse in SP at $n=0$

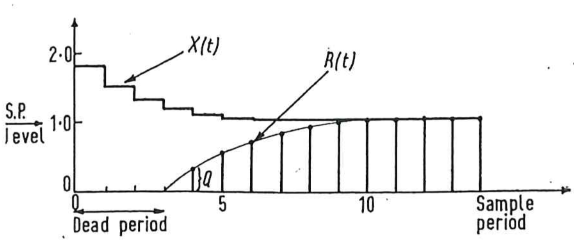

Fig. 3 Controlled response for a unit change in SP at $n=0$

J. D. Higham was awarded a G.E.C. university scholarship to Cambridge in 1960. Later he received a Cambridge open scholarship. He graduated in 1964 with first-class honours in the mechanical sciences tripos, and passed the postgraduate examination in principles of industrial management. That year Higham was awarded a Fulbright travel scholarship to the U.S.A. and a Higgins fellowship at Columbia University, New York. He received a Columbia University master's degree in electrical engineering in 1965. Since then he has been working as a system analyst with G.E.C. Computers and Automation. He is now Software Project Manager on a computer control system being installed in Norway.

The associated step response is shown in fig. 3. Thus the controlled system at the sampling points follows an exponential curve after the dead time, of time constant τ', where $Q = 1-\exp(-T/\tau')$. The transfer function between $W(z)$ and $R(z)$ for $SP(z) = 0$ is merely $1-K(z)$ and the step response can be deduced from fig. 3. Our objective is to find the controller $D(z)$ which will achieve this closed-loop response.

From fig. 1, by block diagram algebra—

$$K(z) = \frac{R(z)}{SP(z)} = \frac{D(z)G(z)}{1+D(z)G(z)} \quad [W(z) = 0] \tag{2}$$

Therefore from (2)—

$$D(z) = \frac{1}{G(z)} \cdot \frac{K(z)}{1-K(z)} \tag{3}$$

Substituting for $K(z)$ from (1)—

$$D(z) = \frac{1}{G(z)} \times \left[\frac{Qz^{-(N+1)}}{1-(1-Q)z^{-1}-Qz^{-(N+1)}}\right] \tag{4}$$

Thus the control algorithm, $D(z)$, contains the parameter Q and the reciprocal of the process transfer function. Obviously the success with which we achieve $K(z)$ will depend on how accurately we know $G(z)$. For a 'tight' system (i.e. where rapid and large changes are made in the control output X), mismeasuring $G(z)$, especially the dead time, can cause poor control and ultimately instability. One overcomes this by making the control less tight (decreasing Q).

Control algorithm for first- or second-order process with dead time an integral number of sample periods

The process transfer function is defined as—

$$G(s) = G \cdot \frac{\exp(-sNT)}{(1+\tau s)} \tag{5}$$

where G = steady-state process gain, T = sampling interval, τ = time constant and NT = dead time (N is an integer).

For second-order systems we assume that—

$$G(s) = \frac{G\exp(-sNT)}{(1+s\tau_1)(1+s\tau_2)} \simeq \frac{G\exp(-sNT)}{(1+\tau s)} \tag{6}$$

where $\tau = \tau_1+\tau_2$. The error in this assumption is small at low frequencies—and we are concerned with the response at low frequencies, since high-frequency closed-loop response is limited by the dead time.

In the z domain $G(s)$ becomes (*1*)—

$$\begin{aligned} G(z) &= Z[H(s).G(s)] \\ &= Z\left\{\frac{[1-\exp(-sT)]G\exp(-sNT)}{s(1+\tau s)}\right\} \\ &= \frac{GLz^{-(N+1)}}{1-(1-L)z^{-1}} \end{aligned} \tag{7}$$

where $L = 1-\exp(-T/\tau)$

A graph of L against τ/T is given in fig. 4. Note $Z[H(s).G(s)] \neq H(z).G(z)$ in general.

Substituting for $G(z)$ in (4) yields—

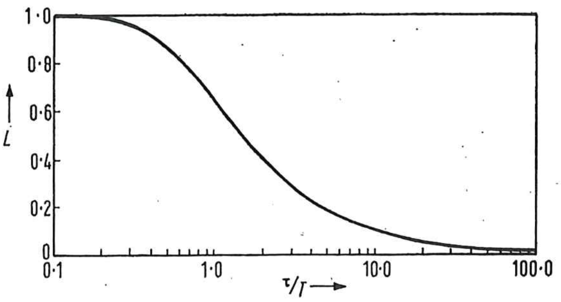

Fig. 4 $L = 1-\exp[-1/(\tau/T)]$

$$\begin{aligned} D(z) &= \left[\frac{1-(1-L)z^{-1}}{G\,Lz^{-(N+1)}}\right] \cdot \left[\frac{Qz^{-(N+1)}}{1-(1-Q)z^{-1}-Qz^{-(N+1)}}\right] \\ &= \frac{1}{G} \cdot \left[\frac{Q/L - z^{-1}Q(1-L)/L}{1-(1-Q)z^{-1}-Qz^{-(N+1)}}\right] \end{aligned} \tag{8}$$

In most cases the control output to be made is a *change* in control output; therefore, we require $\Delta D(z)$ and we multiply (8) by $(1-z^{-1})$. However, there is a factor $(1-z^{-1})$ in the denominator of (8) and this cancels, leaving—

$$\Delta D(z) = \frac{1}{G} \cdot \left[\frac{Q/L - z^{-1}Q(1-L)/L}{1+Qz^{-1}+Qz^{-2} \ldots . + Qz^{-N}}\right] \tag{9}$$

Equ. 9 is the general form of the control algorithm. Its implementation by a standard software scheme (*2*), which enables Q to be a parameter in a single location, is shown in fig. 11, which refers to the example at the end of this article.

Note: the correct form for $N = 0$ (i.e. no dead time) is—

$$\Delta D(z) = \frac{1}{G} \cdot [Q/L - z^{-1}Q(1-L)/L] \tag{10}$$

i.e. the discrete form of a proportional-plus-integral controller with—

$$\text{proportional band} \simeq Q\frac{(1-L)}{L} \times 100\%$$

$$\text{reset time} \simeq \frac{T}{Q} \text{ seconds}$$

Control algorithm for first- or second-order process with dead time a non-integral number of sample periods

Since the sampling time T is often fixed by hardware, it is not always possible to arrange that the dead time is an integral multiple of the sample time. Therefore let—

$$G(s) = \frac{G\exp[-sT(N+m')]}{(1+s\tau)} \tag{11}$$

where $(N+m')T$ = dead time and N = an integer such that $0 \leqslant m' < 1$.

For second-order processes we make the same approximation as in (6). With modified z transforms (*1*) and $m' = 1-m$ in $G(z,m)$, (11) in the z domain becomes—

$$\begin{aligned} G(z) &= Z[H(s).G(s)] \\ &= Gz^{-(N+1)}\left[\frac{(1-J)+(J-1+L)z^{-1}}{1-(1-L)z^{-1}}\right] \end{aligned}$$

where $L = 1-\exp(-T/\tau)$, as before, and $J = \exp[-(1-m')T/\tau] = (1-L)^{1-m'}$. Therefore, we may write—

$$G(z) = \frac{Gz^{-(N+1)}L[(1-M)+Mz^{-1}]}{1-(1-L)z^{-1}} \tag{12}$$

where $$M = \frac{L+J-1}{L} = \frac{(1-L)^{1-m'}-(1-L)}{L}$$

Graphs of M against L for various values of m' are given in fig. 5. We see that (12) is identical with (7), except for the modifying term $(1-M)+Mz^{-1}$ in the numerator. Thus (12) describes the sampled process as shown in fig. 6. For economy of parameters it is convenient (and justifiable) that we ask for $K(z)$ in this case to be similarly modified, i.e.—

$$K(z) = z^{-(N+1)}\left[\frac{Q[(1-M)+Mz^{-1}]}{1-(1-Q)z^{-1}}\right] \tag{13}$$

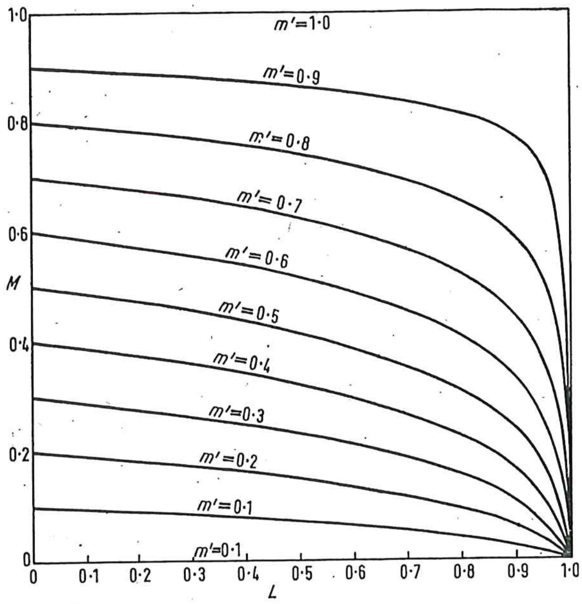

Fig. 5 $M = \{\exp[-(1-m')T/\tau]-(1-L)\}/L$ where $L = 1-\exp(-T/\tau)$

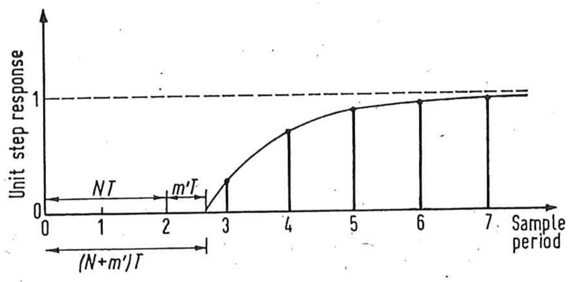

Fig. 6 How equ. 13 follows an exponential at the sampling instants: $n = 2$

This merely means that we are asking the controlled process to behave at the sample periods as if it were a continuous process with a dead time $(N+m')T$ and not NT.

Substituting (13) and (12) in (3) for $D(z)$, we have—

$$D(z) = \left[\frac{1-(1-L)z^{-1}}{Gz^{-(N+1)}L[(1-M)+Mz^{-1}]}\right] \cdot \left[\frac{z^{-(N+1)}Q[(1-M)+Mz^{-1}]}{1-(1-Q)z^{-1}-Qz^{-(N+1)}[(1-M)+Mz^{-1}]}\right]$$

$$= \frac{1}{G}\cdot\left[\frac{Q/L-z^{-1}Q(1-L)/L}{1-(1-Q)z^{-1}-Qz^{-(N+1)}[(1-M)+Mz^{-1}]}\right] \qquad (14)$$

Once again we require $\varDelta D(z)$. Therefore, multiplying (14) by $(1-z^{-1})$, which cancels with a factor $(1-z^{-1})$ in the denominator, we have—

$$\varDelta D(z) = \frac{1}{G}\cdot\left[\frac{Q/L-z^{-1}Q(1-L)/L}{1+Qz^{-1}+Qz^{-2}\ldots\ldots+Qz^{-N}+MQz^{-(N+1)}}\right] \qquad (15)$$

Note that the correct form for $N = 0$ is—

$$\varDelta D(z) = \frac{1}{G}\cdot\left[\frac{Q/L-z^{-1}Q(1-L)/L}{1+MQz^{-1}}\right] \qquad (16)$$

Equ. 15 will be seen to be identical with equ. 9 except for the one term in the denominator involving M. If $m' = 0$ then $M = 0$ and the equations are the same. Note the overall scaling I/G. In practical applications G will include all the input and output constants of the computer-plant interface.

Choice of parameter Q

The most suitable value before commissioning the system may be estimated by a variety of means.

We see in fig. 2 that a unit spike on the set point SP (equivalent to a negative spike on the output with SP = 0) causes the controller to start corrective action, but only by a fraction Q. This is similar but not equivalent to the effect of digital first-order filters on the

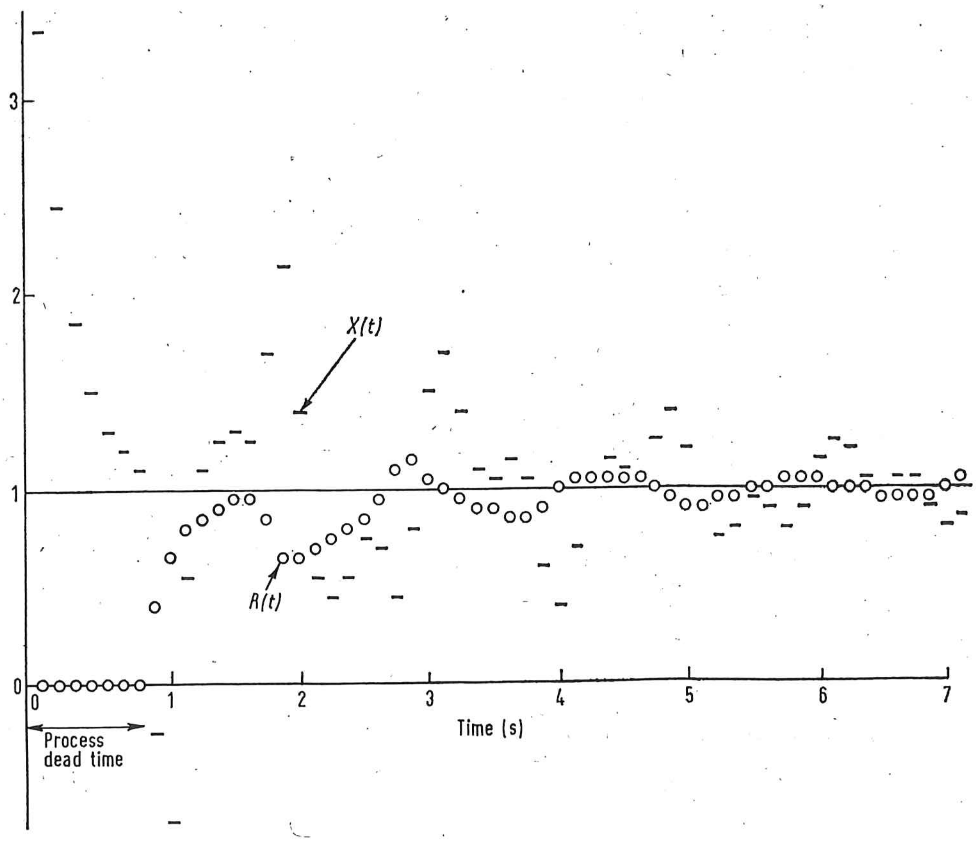

Fig. 7 $T = 0{\cdot}125$s. $Q = 0{\cdot}39$. $L = 0{\cdot}112$. $\tau/\tau' = 4$. Process dead time, −25%

measured variable $R(n)$ as used on a control system without dead time, where the filtered value $R'(n)$ used for control is given by—

$$R'(n) = QR(n)+(1-Q)R'(n-1) \qquad (17)$$

However, we could not achieve the effect shown in fig. 2. by, say, designing an optimal dead-time controller for the process and then adding a filter such as (17). This is because the control algorithm effectively models the process and therefore the filter (17) must be considered as part of the process. Where the measured variable is sampled more frequently than the control action, filtering is useful, and an example is given in the application described at the end of the article.

We see from fig. 3 that the input to the plant $X(t)$ from the control system, is shown as overshooting to correct for a steady change in set point. This is so for $Q>L$, and the percentage overshoot is given simply by $(Q/L-1)$ 100%. In practical applications this will give an upper bound for Q [note also, $Q\leqslant 1$ for stability of (1)].

Where the interest centres on Q/L, and since both Q and L depend on T, it is usually better to consider the ratio—

$$\tau/\tau' = \frac{\log(1-Q)}{\log(1-L)}$$

which is independent of T and relates the time constant of the controlled to the uncontrolled process.

Perhaps the most useful aspect of Q is its use to 'dampen' any instability caused by mismeasuring the process parameters. This has been investigated extensively, but I can give only a brief indication of the effect here. Simulations were performed of a process with $G=1$, $\tau=1$, $NT=1$, and with a control algorithm designed for these parameter values. The step response of the closed-loop system was then investigated for various values of Q in the control algorithm when each of the process parameters was varied by $\pm 25\%$ in turn. From these simulations, the variation in process dead time NT was found to have the most effect on closed-loop performance, and some of the results are given here. Fig. 7 shows the closed-loop step response for $T = 0{\cdot}125$, NT in the process reduced by 25%, $Q = 0{\cdot}39$ and $L = 0{\cdot}112$. The process is indicated by **o**, the set point by a continuous line, and the control output to the process by —. Fig. 7 may be contrasted with fig. 8, where Q is reduced to 0·21, reducing the disturbance caused by incorrect dead-time measurement.

The effect of increasing the sample period by a factor of eight, making it equal to the dead time, was also investigated. The results are shown in figs 9 and 10, which correspond to figs 7 and 8 respectively in that Q has been chosen so that the τ/τ' ratio corresponds. We see that the control is not quite as good, but if one takes into account actuator wear, the slower sampling speed is preferable. Since we can only control disturbance frequencies with period $<2NT$ anyway, this leads to the conclusion that it is not worth controlling more frequently than the dead time of the process, but that fast measurement scanning is useful for

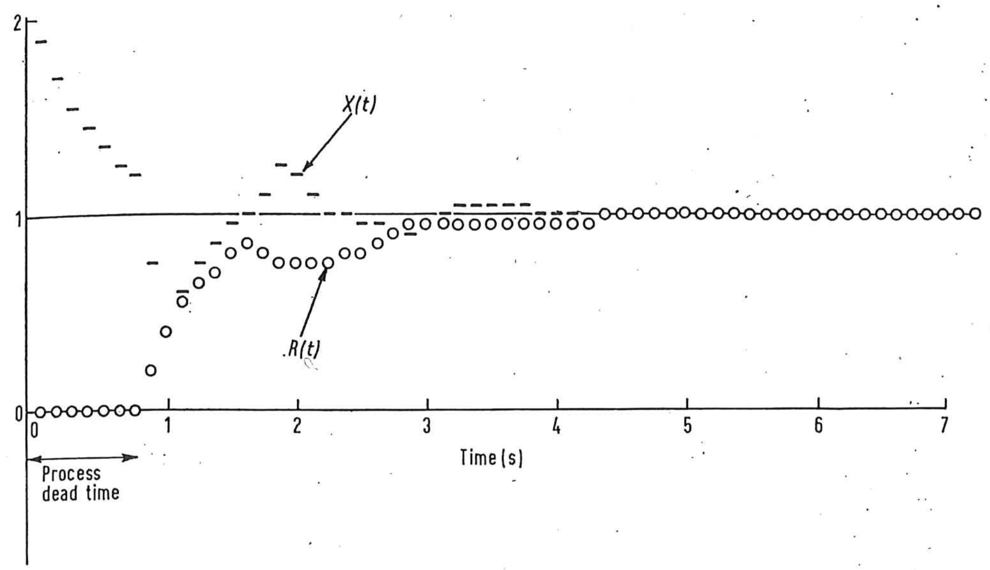

Fig. 8 $T = 0{\cdot}125$s. $Q = 0{\cdot}21$. $L = 0{\cdot}112$. $\tau/\tau' = 2$. Process dead time, −25%

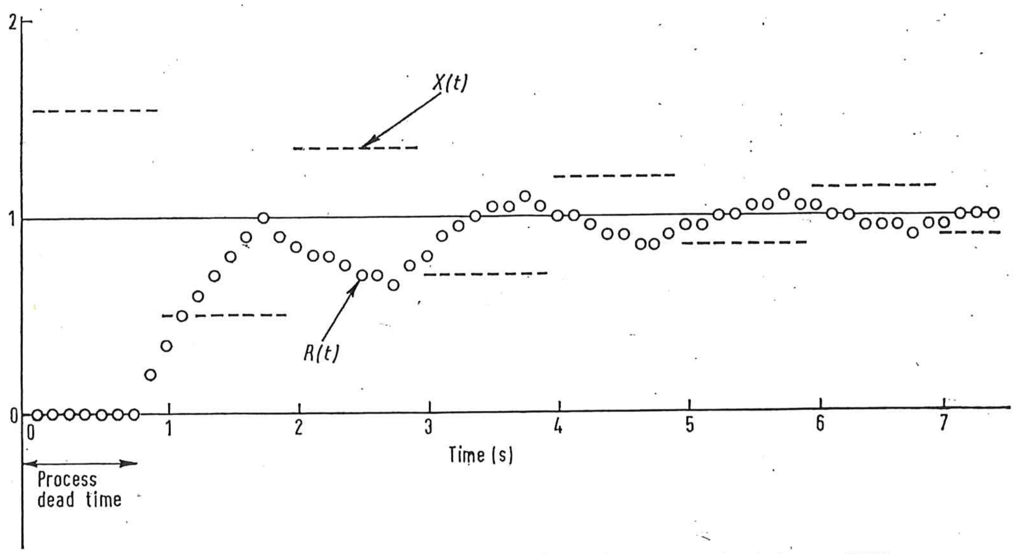

Fig. 9 $T = 1$s. $Q = 0{\cdot}98$. $L = 0{\cdot}632$. $\tau/\tau' = 4$. Process dead time, −25%

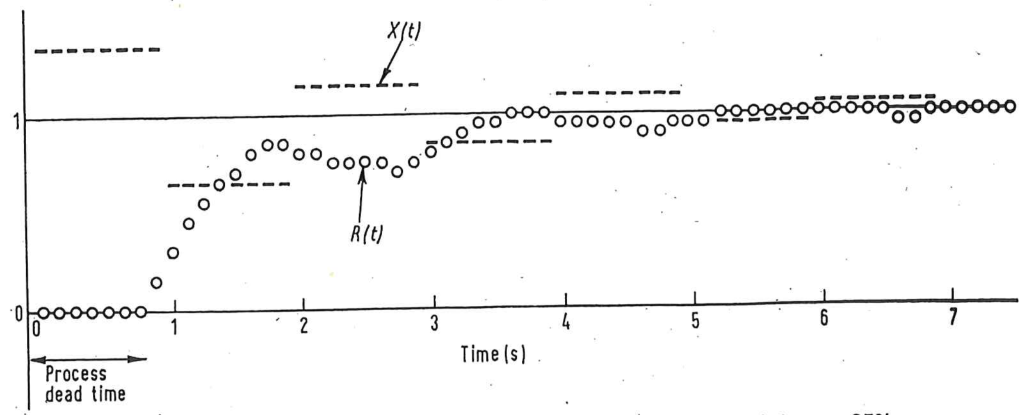

Fig. 10 $T = 1$ s. $Q = 0{\cdot}865$. $L = 0{\cdot}632$. $\tau/\tau' = 2$. Process dead time, −25%

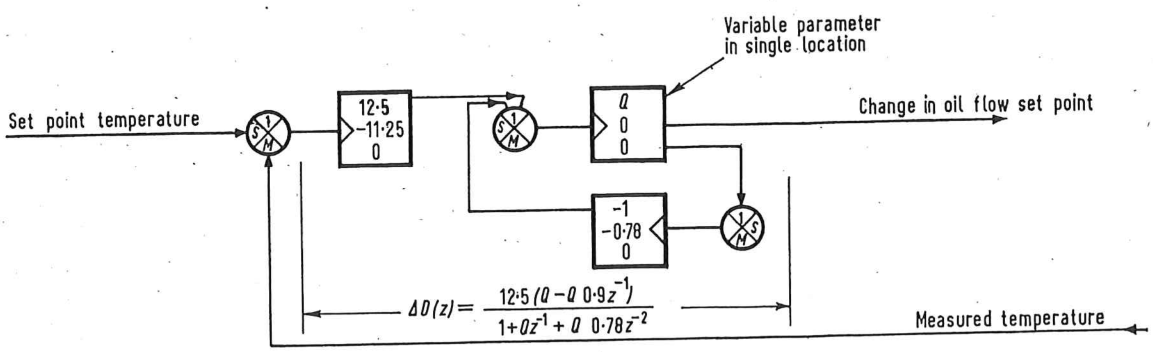

Fig. 11 Example of implementation of algorithm by means of standard software scheme

removing the high-frequency components of the measured variable R. The factors to be taken into consideration when selecting combinations of control sample period, digital filtering sample period, and analogue filtering, are dealt with in detail by Goff (*4*).

Note that for $Q = 1$, algorithm 9 becomes an optimal controller, i.e. the system responds in a dead beat manner in $N+1$ sample periods. This may or may not be useful depending on the sample period T. If T is short compared to τ, L will be small and Q/L will be large and the process very sensitive to process parameter variations.

For $Q = L$, the closed-loop response of $K(z)$ is the same as the open-loop response. This is a useful value of Q to use as an initial value with which to start commissioning, since X, the change in the manipulated process variable, never overshoots the steady-state value required to correct for the error.

Example from cement kiln control

As a practical example of the application, consider the control, *via* the fuel flow (oil), of the burning-zone temperature in a cement kiln. The hardware system gives two sampling frequencies: 1 min and 5 min. There is an inner control loop comparing the flow with the set point and actuating the valve. This is a simple digital equivalent of a proportional-plus-integral controller implemented on the 1 min scan.

The relationship between fuel flow and burning zone temperature is found by step tests to be—

$$G(s) = 0{\cdot}8\,\frac{e^{-9s}}{(1+20s)\,(1+20s)} \qquad (18)$$

The dynamics of the fuel-flow-control loop are fast enough to be ignored in (18). The hardware constrains us to 5 min or 1 min sampling. Therefore $T = 5$ min is chosen as the algorithm sample time, while the measured variable is sampled every minute and filtered in a discrete equivalent of a first-order filter of time constant 5 min. Thus the process and filter is—

$$G(s) = 0{\cdot}8\,\frac{e^{-9s}}{(1+20s)(1+20s)(1+5s)}$$

$$\simeq 0{\cdot}8\,\frac{e^{-9s}}{(1+45s)} \qquad (19)$$

From fig. 4, since $\tau/T = 45/5 = 9$, we have $L = 0{\cdot}1$. Since the dead time is not a discrete number of sample periods we have dead time $= 9 = T(N+m')$. So $N = 1$ and $m' = 0{\cdot}8$. From fig. 5, for $m' = 0{\cdot}8$ and $L = 0{\cdot}1$, $M = 0{\cdot}78$.

From (15) we have—

$$\varDelta D(z) = 12{\cdot}5\,\frac{(Q - Q0{\cdot}9z^{-1})}{(1+Qz^{-1}+0{\cdot}78Qz^{-2})} \qquad (20)$$

It now remains to implement this in the computer software, choosing Q on some basis until commissioning, when the single parameter Q may be trimmed to give the best performance. The implementation of (20) by a standard block diagrammatic software scheme (*2*) is shown in fig. 11. Note that Q is available in a single location for alteration with the computer on-line. The standard procedure for finding the algorithm is summarized as follows:

1 approximate process to $G \cdot \dfrac{\exp[-T(N+m')s]}{1+\tau s}$

2 choose the control sample period T

3 find L for this T and τ from fig. 4

4 find M for this L and m' from fig. 5

5 choose Q and substitute L, M and Q in (15).

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