or by any motive power other than steam which did not involve combustion in the motive units themselves. This act required that the change of motive power be made on or before July 1, 1908, and provided that a penalty of $500 per day be exacted on and after that date for failure to comply with its terms.

Prior to this, the Westinghouse Electric and Manufacturing Company had developed the high-tension single-phase system. The New York Central plans called for a relatively short electrified zone and they decided to adopt the 650-volt d-c third-rail system. The New Haven Railroad planned on an initial electrification to Stamford and New Haven with possible future extensions eastward and decided to adopt the high-voltage single-phase system. These three companies were also partners in a pioneering rectifier motive power application in 1913, 1914, and 1915.

A Pennsylvania Railroad combination baggage and passenger car (car no. 4692) was equipped with four type-*308* 225-hp 600-volt traction motors and d-c control equipment borrowed from the Long Island Railroad and with transformers, multi-anode rectifiers, and accessory a-c control equipment supplied by Westinghouse. This car was tested on the Westinghouse test track at East Pittsburgh and then placed in revenue service on the New York, New Haven and Hartford Railroad. It hauled two trail cars and performed 8,757 miles in revenue service on the Harlem River branch and 13,588 miles on the New Canaan branch for a total revenue service of 22,345 miles.

The Pennsylvania Railroad and Westinghouse again pioneered rectifier car no. 4561 in 1949. Here again, the Pennsylvania Railroad supplied a combination passenger and baggage car with Long Island Railroad d-c motor and control equipment (two 559 *DR3* motors and associated control) while Westinghouse supplied the transformer and rectifier equipment. This car started operation July 14, 1949, and has been in continuous service since then.

The Pennsylvania then purchased two 2-cab rectifier locomotives which have been in revenue service for approximately 3 years. The New Haven Railroad then purchased 100 new rectifier multiple-unit cars and ten rectifier locomotives.

**E. W. Ames** and **V. F. Dowden:** Table I gives the kilovolt-amperes (kva) required for all auxiliaries of the rectifier car from test readings of amperes for each circuit. Table II is tabulated on the same basis as Table I for the stand-by kva at 11-kv trolley for the 409 motor car and two trail cars. The rectifier cars accelerate at a 1-mile-per-hour-per-second rate while the older cars accelerate between the 0.5- and 0.75-mile-per-hour-per-second rate.

**Table II. Stand-by Kva at 11-Kv Trolley for 409 Motor Car and Two Trail Cars**

| | Kva |
|---|---|
| Transformer and traction motor blower $37\times420=$ | 15.6 |
| Main transformer (805 kva) excitation kva $3.4\times1{,}100=$ | 37.4 |
| Air-brake compressor motor (1/3 time) $(36\times347)/3=$ | 4.2 |
| Car heaters* including two trail cars = M-g set 1-kw output | 150.0 |

* Car heating is maximum available.

The new cars, with a higher acceleration rate, have enabled the railroad to reduce the scheduled time for a run. The new cars offer many passenger comforts, such as automatically controlled heating or air conditioning, fluorescent lighting, forced-air circulation, and many other comforts not available on the older-type equipment. Power consumption for the 409 motor car with two trailers is not available on the same basis as that stated in the paper.

# Conditional Feedback Systems—A New Approach to Feedback Control

G. LANG
NONMEMBER AIEE

J. M. HAM
MEMBER AIEE

**Synopsis:** In the classical single-loop feedback system, feedback acts not only to modify the influence of disturbances but also to determine the basic character of the input-output response. The inherently close association of these two effects has been a constant trial for designers and students since the inception of classical feedback, and has usually required that design requirements be compromised. Basically new configurations for feedback systems are introduced in which these effects of feedback are separated. In the new systems, feedback acts solely to reduce the influence of disturbances and thus to determine the response of the system to external loads and internal parameter variations. The character of the input-output response is independent of transmission around the feedback loop. The term "conditional feedback" has been introduced to distinguish the limited role that feedback plays in these new systems as compared with the classical system. Conditional feedback systems permit requirements on input-output response and on disturbance-output response to be met independently and offer a broad new range of performance characteristics for both linear and nonlinear systems in which there is substantial energy storage.

THE classes of closed-loop systems introduced in this paper were evolved after an unsophisticated review of the problems encountered and the solutions accepted in present-day engineering practice in control and regulation. In the design of these new systems, the need for compromise in performance specifications as a result of conflicts between input-output response and disturbance-output response has largely been eliminated. Such conflicts are inherent to the conventional feedback control system because of the intimate relationship existing between the input-output transmission characteristic and the loop transmission characteristic. Since the latter is dictated by requirements for loop stability and for the suppression of disturbances, little variation is allowed the former, and at that not without considerable compromise. A useful analogy for the foregoing condition is found in the intimate connection that exists between the frequency response and the phase characteristic of minimum phase networks. These restrictions can be overcome by the use of networks having nonminimum phase characteristics. In a like manner, a basic change in topology can be used to extend the present boundaries of feedback control systems.[1–4]

## Classification of Control Problems

The classification of control problems adhered to in this paper distinguishes two basic categories defined as follows:

1. The servo class: A servo problem requires the generation of an output signal that bears a prescribed functional relationship to an input signal. This problem is commonly met in all types of signal transmission. A servo system is applied to the solution of a servo problem.

2. The regulator class: A regulator problem requires the elimination or reduction of the effects on a controlled quantity of extraneous and generally poorly defined disturbances. A regulator system is applied to the solution of a regulator problem.

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G. Lang is with the Ferranti Electric Limited, Toronto, Ont., Canada, and J. M. Ham is with the University of Toronto, Toronto, Ont., Canada.

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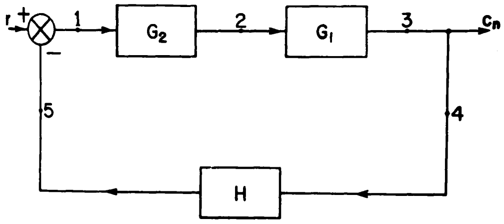



Fig. 1 (left). A representative feedback system

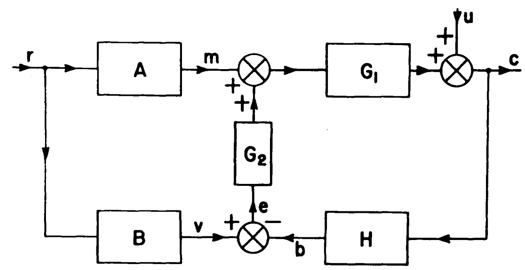



Fig. 2 (right). Configuration of a conditional feedback system

Feedback control systems are applied to both the aforementioned problems because they can act simultaneously as a servo and as a regulator. This property has been found to be of the utmost value in the solution of the many problems in which both servo and regulator requirements exist. Purely servo problems do not necessarily require feedback for their solution, nor do purely regulator problems, e.g., the glow-discharge tube is used to provide voltage regulation without feedback.

## Conventional Feedback Control Systems

The dual role played by the conventional feedback control system is easily exposed by considering the Laplace transform $C$ of the output signal $c$ that results when an additive signal $u_n$ having the transform $U_n$ is applied successively to the nodes $n=1, 2, 3, 4$, and 5, in the representative feedback system of Fig. 1. For an input signal $r$ with the transform $R$ and successive additive signals $u_n$, the successive outputs $c_n$ have the transforms

$$
\begin{aligned}
C_1 &= \frac{G_1G_2U_1}{1+HG_1G_2}+\frac{G_1G_2R}{1+HG_1G_2}=F_1U_1+FR \\
C_2 &= \frac{G_1U_2}{1+HG_1G_2}+\frac{G_1G_2R}{1+HG_1G_2}=F_2U_2+FR \\
C_3 &= \frac{U_3}{1+HG_1G_2}+\frac{G_1G_2R}{1+HG_1G_2}=F_3U_3+FR \\
C_4 &= \frac{-G_1G_2U_4H}{1+HG_1G_2}+\frac{G_1G_2R}{1+HG_1G_2}=F_4U_4+FR \\
C_5 &= \frac{-G_1G_2U_5}{1+HG_1G_2}+\frac{G_1G_2R}{1+HG_1G_2}=F_5U_5+FR
\end{aligned}
\tag{1}
$$

The response terms on the right of equation 1 express the principle of superposition for the signals $u_n$ and $r$. It follows from equation 1 that

$$
\begin{aligned}
F_1 &= F \\
F_2 &= \frac{F}{G_2} \\
F_3 &= \frac{F}{G_1G_2} \\
F_4 &= -HF \\
F_5 &= -F
\end{aligned}
\tag{2}
$$

From equation 2 it is clear that disturbances at nodes 2 and 3 are quelled by concentrating gain in $G_2$ and $G_1$, whereas disturbances at the input to $H$ (considered as an output-sensing device) are of a particularly troublesome nature and cannot be reduced without lowering the gain or bandwidth of $H$. This latter view points out the necessity for care in the choice of components for output sensing.

For a linear system, a disturbance at node 2 can be considered as a disturbance of different amplitude and frequency distribution entering at node 3. Herein all external disturbances will be represented in terms of an equivalent load disturbance $u$.

## Disturbances

The importance of knowing the disturbance problems associated with a particular system design cannot be too heavily stressed. If there are no external disturbances and available system components are linear and not subject to parameter variations, an open-loop system is ideally suited to most servo problems provided a suitable open-loop transfer function can be obtained.

Open-loop systems have the particular advantage that real time delays are often unimportant. It is the character of the delayed response that is usually of major interest. Further, in such systems, adequate control of the influence of certain nonlinearities and load disturbances can be gained by providing stable compensating nonlinear elements and an output with a suitably low driving-point impedance. The influence of disturbances entering the system between input and output can often be reduced by isolating the system from known sources of disturbance, e.g., transmission lines may be transposed to reduce the effect of induced signals.

There will remain, however, many classes of parameter variation, nonlinearity, and internal disturbance that are not amenable to treatment by such techniques. For these a closed-loop regulating system is required. A closed loop will modify the influence of those system variations for which a closed loop is not necessary, but improved system performance will usually be obtained when all variations susceptible to open-loop compensation are so treated. Closed-loop regulation is required in nearly all machine- and process-control problems. Another important role for closed-loop control is to modify or synthesize transfer functions which, in many cases, would be most intractable to synthesis in a simple manner by open-loop techniques.[5]

## Configuration and Basic Properties of Conditional Feedback Systems

Two distinct control problems have been identified: the servo or signal transmission problem and the regulator or disturbance suppression problem. The performance requirements for signal transmission and disturbance suppression in practical applications are usually distinct, but the signal and disturbance behavior characteristics of classical feedback systems have been shown to be inherently interdependent. Hence, the design requirements for input-output response and disturbance-output response have often had to be compromised. Since feedback in many control problems is essential solely to effect a suitable reduction in the influence of disturbances, the question arises whether there are systems in which the use of feedback can be so restricted. Systems possessing this property are called conditional feedback systems. A basic configuration for a linear conditional feedback system is shown in Fig. 2. In Fig. 2, $G_1$ is the Laplace transfer function describing the main transducer, $r$ is the input signal, $u$ is a disturbance, and $c$ is the output. For the system shown, the transform $C$ of the output $c$ is

$$C=\frac{1}{1+G_1G_2H}U+\frac{AG_1\left(1+\frac{B}{A}G_2\right)R}{1+G_1G_2H} \tag{3}$$

In equation 3, let $B$ be defined by the equality

$$G_2\frac{B}{A}=G_1HG_2$$

or

$$B=AG_1H \tag{4}$$

Then equation 3 becomes

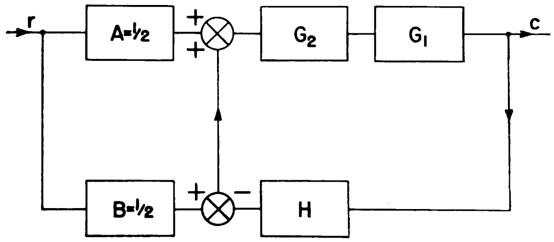
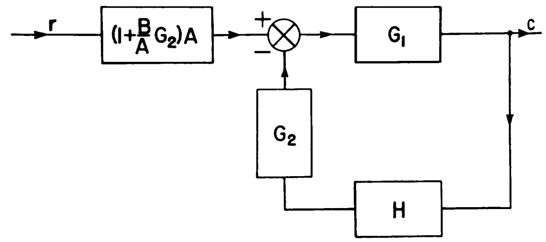



Fig. 3 (left). The conditional configuration equivalent to the classical system

Fig. 4 (right). Equivalence of conditional system to a classical system with a prefilter




$$C=\frac{1}{1+G_1G_2H}U+AG_1R \tag{5}$$

from which

$$\left(\frac{C}{R}\right)_{u=0}=AG_1 \tag{6}$$

and

$$\left(\frac{C}{U}\right)_{r=0}=\frac{1}{1+G_1G_2H} \tag{7}$$

Equation 6 shows that when $B$ is defined by equation 4 the input-output response of the system is unaffected by the feedback loop and can be of a basically different character from the disturbance-output response in equation 7, the form of which is completely determined by the feedback loop. The significance of these equations is immense for they show that a conditional feedback configuration permits design requirements on input-output response and on disturbance-output response to be considered independently. This fact implies that a broad new range of performance characteristics is available from conditional feedback systems.

Before giving a design procedure and illustrative design for the conditional system of Fig. 2, some discussion of its internal behavior is in order. If the transfer function $B$ satisfies equation 4, the feedback signal $b$ in the absence of disturbances is exactly equal to the signal $v$ produced at the output of $B$ by the input signal $r$. Hence the signal $e$ at the output of the lower comparator is identically zero and there is no feedback. The transfer ratio $C/R$ for the signal transmission is then simply the product of the transfer functions in the direct path from input to output; see upper branch of the block diagram in Fig. 2.

These facts point to a special significance for the system component described by the transfer function $B$. The function of $B$ is most readily suggested by considering the system when $H=1$. In this case, the feedback signal $b=c$. Hence when $B$ is defined by equation 4, the output signal from $B$, namely $v$, is equal to the output signal $c$. Clearly then $B$ represents the desired input-output transfer function and the signal $v$ represents the desired output response. For this reason, the system component described by the transfer function $B$ in Fig. 2 will be called a "reference model." The reference model is an undisturbed representation of the transfer characteristics of the main transducer as defined in equation 4, and will usually be realized with simple $R$, $L$, and $C$ elements. When $H$ is other than unity, $v$ is the desired output signal as modified by $H$. $H$ commonly describes the output-sensing device.

An interesting comparison between the classical feedback system of Fig. 1 and the conditional system of Fig. 2 is obtained by developing a conditional configuration that is equivalent to the classical system. This system is shown in Fig. 3. From equation 3 it follows that, if $A=B=1/2$, the system in Fig. 3 behaves as the classical system of Fig. 1. Clearly the classical system calls for an ideal reference model. Since ideal behavior is not to be expected from practical equipment, it is not surprising that the use of an ideal reference model entails special system performance limitations.

W. K. Linvill pointed out to the authors that the conditional feedback system of Fig. 2 is equivalent to a classical system with a prefilter, as shown in Fig. 4. In Fig. 4, the response $C$ to $R$ and $U$ is that given by equation 3. When the transfer function $B$ in Fig. 4 is defined by equation 4, the prefilter has the particular transfer function $A\ (1+G_1G_2H)$, the response ratio $C/R=AG_1$, and there is no signal in the feedback path; hence, the combination of this particular prefilter and the classical system has the properties of a conditional system. It should be observed that both of the foregoing equivalences are valid only when the system components are linear.

Since feedback in a conditional feedback system is used solely to reduce the influence of disturbances, it is important to examine carefully the action of disturbances on this new configuration. In a conditional feedback system such as that of Fig. 2, there is no loop feedback if there are no disturbances. The error signal $\epsilon$ obtained by comparing the desired signal $v$ with the fed back signal $b$ serves as a measure for the effect of disturbances. In developing conditional systems, it has been observed that in the comparison of $v$ with $b$ either a subtraction or a ratio operation may be used. If a subtractive comparison is used, an additive correction is made at the input to $G_1$, as in Fig. 2. If a ratio comparison is used, the signal input to $G_1$ is corrected by multiplying the signal output from $A$, namely $m$, by some function of the signal $\epsilon$ from the ratio comparator.

The action of two types of disturbances will be examined. First consider the steady-state response of the system in Fig. 2 to a constant additive disturbance $u$. The output signal $c$ will be in error by a constant and the error signal $\epsilon$ will be constant whether or not there is an input signal $v$. Next consider the system of Fig. 2 with the subtractive comparator replaced by a ratio comparator and the additive corrector replaced by a multiplicative corrector. Let there be a constant multiplicative disturbance such as may be caused by a decrease in the static gain factor of $G_1$. The output signal $c$ will be in error by a constant factor and the error signal $\epsilon$ will be constant. However, if additive and multiplicative disturbances occur together the error signal $\epsilon$ at the output of both a subtractive and a ratio comparator will contain frequency components of the input signal. These observations suggest that the nature of disturbances has an important bearing on the design of conditional feedback systems; this fact is discussed later in the paper.

The response of the linear conditional feedback system in Fig. 2 to an additive disturbance of general form is described by equation 7. The form of this response is controlled by an appropriate choice for the transfer function $G_2$. All of conventional feedback theory and design technique is relevant to making this selection.

It is important to observe that disturbances influencing the system component, described by $A$ in Fig. 1, without affecting the reference model $B$, are compensated by feedback through $G_2$ to the output of $A$. Parameter variations in the components of a conditional feedback system such as that shown in Fig. 2 have essentially the same influence on the output as the corresponding variations in the classical feedback system of Fig. 1. Hence most of the literature on parameter sensitives is relevant. To illustrate this fact consider a variation $dG_1$ in the transfer function of the main transducer. The corresponding variation in the transfer function of the output signal is $dC$. For the classical system of Fig. 1

$$dC=\frac{1}{1+G_1G_2H}\,\frac{C}{G_1}\,dG_1 \tag{8}$$

For the conditional system, the variation $dC$ has exactly the same form.

The foregoing discussion of a linear conditional feedback system has shown that this new configuration permits the independent control of the input-output response and of the disturbance-output response. The disturbance-output response can always be made as good as that of the corresponding classical system while the input-output response can have forms hitherto unrealizable. A design procedure for conditional feedback systems based on the foregoing development is described in the following.

## Design Procedure and Illustrative Example

The special properties of conditional feedback systems as developed in the foregoing lead to a design procedure that is unusually direct and simple. A suggested procedure is outlined in the following. The particular significance of the steps will be illustrated in a sample design.

1. From a careful study of the system problem determine the required input-output response characteristics and the required disturbance-output response characteristics; special care should be given in determining the basic nature of disturbances.

2. Select a main transducer and associate with it compensating elements to realize the desired input-output response. This step involves the choice of the transfer functions $G_1$ and $A$ in Fig. 2.

3. Select a suitable output-sensing device and determine its transfer function $H$.

4. Design an undisturbed reference model described by the transfer function $B=AG_1H$.

5. Design a loop-compensating network described by the transfer function $G_2$ to realize the desired disturbance-output response to the extent that the fundamental requirement of loop stability allows.

These steps in design form a direct sequence in which there is no need for compromise among the steps. To illustrate the design procedure for conditional feedback systems a sample design will be given for a conditional system and a corresponding classical system that employs the same main transducer. The feedback loop transmission function is to be the same in both systems so that a direct comparison of performance characteristics is justified.

### Example

It is desired to construct a positional servomechanism using a main transducer described by the transfer function

$$G_1=\frac{K_1\epsilon^{-T_1s}}{s};\ K_1=10,\ T_1=0.2 \text{ second} \tag{9}$$

$G_1$ corresponds to a time-delayed integration. Pure time delays in the loop of a classical feedback system impose a definite limit on the bandwidth that can be realized in input-output response. Such is not the case when a conditional feedback system is employed. The configuration of the conditional system is shown in Fig. 2. The corresponding classical system is shown in Fig. 1. The steps in designing the conditional system are now given.

From equation 6 it follows that the transfer function $A$ must be synthesized to make the product $AG_1$ represent the desired form of the input-output response. Suppose that the desired response to a unit step has the form

$$c=\left(1-\epsilon^{\frac{-(t-T_1)}{T}}\right);\ T_1=0.2,\quad T=0.01 \text{ second} \tag{10}$$

From equation 10 the input-output response transform is

$$\frac{C}{R}=\frac{\epsilon^{-T_1s}}{1+Ts} \tag{11}$$

From equation 11 it follows that a suitable form for $A$ is

$$A=\frac{K_0T_0s}{1+T_0s} \tag{12}$$

From equations 9 and 11, the values of $K_0$ and $T_0$ are given as

$$T_0=T=0.01,$$

$$K_0=\frac{1}{T_0K_1}=10 \tag{13}$$

The transfer function $A$ is readily realized with gain and a simple resistance-capacitance network.

It is important to note that it is not necessary to effect the complete compensation of the main transducer with elements placed in the box marked $A$. The synthesis of the desired input-output response transform may include tandem compensation of $G_1$ placed in the upper branch of the feedback loop, classical feedback around $G_1$, and the like. However, if the feedback loop is required to have nonzero transmission at zero frequency to fulfill its regulating function, compensating elements which do not transmit at zero frequency must be placed in $A$.

Now an output-sensing device may be selected and its transfer function $H$ determined. To simplify the details of this design example, $H$ is considered to be unity. Design of the undisturbed reference model is now carried out. From equation 4 and the condition $H=1$, it follows that the transfer function $B$ for this model is

$$B=\frac{C}{R}=\frac{\epsilon^{-T_1s}}{1+Ts};\ T_1=0.2,\quad T=0.01 \text{ second} \tag{14}$$

If the time delay $T_1$ of the main transducer is subject to parameter variation, the value used in equation 14 for designing the reference model is some mean value. In practice $B$ will be realized with miniature elements having tolerances on their characteristics established by the allowable tolerances in the response ratio $C/R$.

To complete the design of the conditional feedback system, a loop-compensating transfer function $G_2$ has to be designed to realize the desired disturbance-output response, as defined by equation 7. Since it is desired to compare the performance of the conditional system with that of the corresponding classical system, the selection of $G_2$ will be based on the following considerations. The classical system in Fig. 1 corresponds to the conditional system in Fig. 2 when the function $G_1$, $G_2$, and $H$ are the same. The disturbance-output response of these systems is identical and is described by equation 7. Since the input-output response of the conditional system is independent of $G_2$, the classical system will compare most favorably with the conditional system if the form of $G_2$ is selected to obtain the best input-output response from the classical system.

The response ratio $C/R$ for the classical system in Fig. 1 is

$$\frac{C}{R}=\frac{G_1G_2}{1+HG_1G_2};\ H=1 \tag{15}$$

In equation 15, $G_2$ is to be designed to obtain the best response ratio. The classical cut-and-try procedure is followed. The basic form for $G_2$ is taken to be

$$G_2=K_2\frac{\frac{1}{a_d}T_ds+1}{T_ds+1}\,\frac{T_is+1}{\frac{1}{a_i}T_is+1} \tag{16}$$

$G_2$ can be realized with gain, a lead network, and a lag network. The parameters $K_2$, $a_d$, $T_d$, $a_i$, and $T_i$ are to be selected.

Suppose that a static gain factor

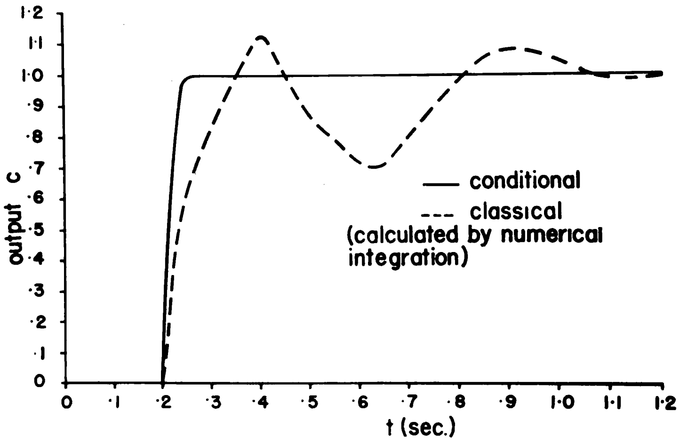



**Fig. 5. Step responses of the conditional and classical design**

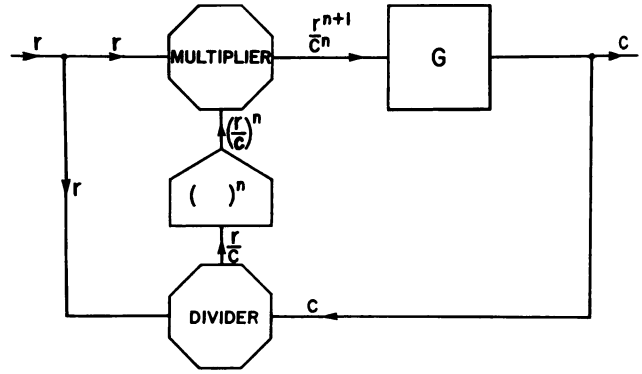



**Fig. 7. A simplified multiplicative system**

$$K=\lim_{s\to 0} sHG_1G_2=100 \tag{17}$$

is required in the feedback loop. From equations 9 and 17 it follows that

$$K=K_1K_3$$

or

$$K_3=\frac{K}{K_1}=10 \tag{18}$$

In equation 16, the gain factor $a_d$ of the lead network is arbitrarily assigned the value 0.1. From a polar plot of $G_1$, the value $T_d=0.025$ is selected for the time constant of the lead network. On the assumption that the lag network acts as a gain factor $a_t$ at frequencies in the neighborhood of the critical point in the $G_1G_2$ plane, $a_t$ is assigned a value $a_t=1/40$, which results in a response peak of about 1.3 in ratio $C/R$. The time constant $T_t$ of the lag network is assigned a value $T_t=2.5$ seconds, which results in a phase shift through the lag network of less than 2 degrees at frequencies in the neighborhood of the critical point in the $G_1G_2$ plane. The open-loop response of the classical system is thus defined as

$$G_1G_2=\frac{100\epsilon^{-0.2s}}{s}\;\frac{0.25s+1}{0.025s+1}\;\frac{2.5s+1}{100s+1} \tag{19}$$

The input-output response of the conditional system as defined by equation 11 is now to be compared with the input-output response of the corresponding classical system as defined by equations 9, 19, and 15. In Fig. 5, the responses of the two systems to a unit step are compared and in Fig. 6 the magnitude and phase characteristics of the response ratios $C/R$ are compared.

In evaluating the response comparisons in Figs. 5 and 6, it is important to remember that the disturbance-output responses of the two systems are identical. The conditional system clearly has much superior input-output response characteristics. It is important again to emphasize that there is freedom of choice for the character of the input-output response in a conditional feedback system.

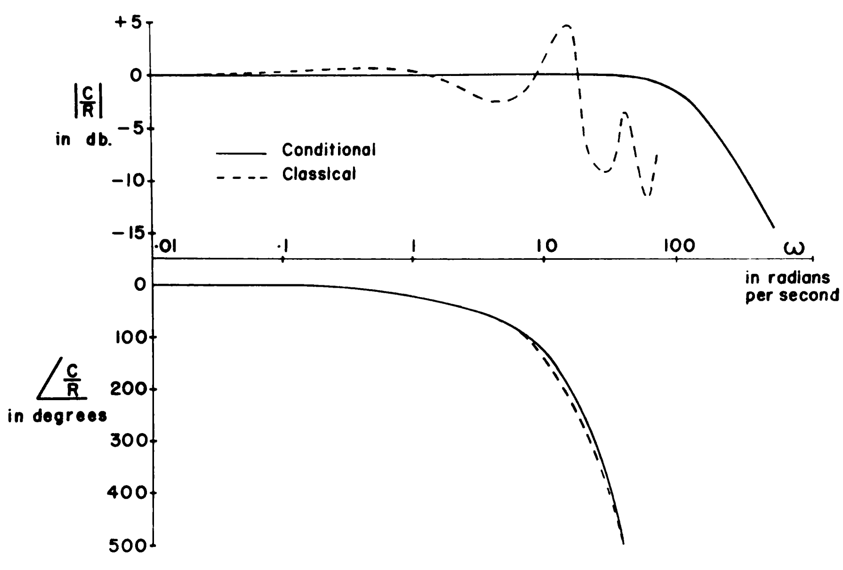



**Fig. 6. Frequency responses of the conditional and classical designs**

In a classical system, the effort to realize maximum bandwidth in the input-output response almost always leads to underdamped oscillatory characteristics. This fact is particularly true when pure time delays are present in the feedback loop.

The foregoing example is suggestive of the extended range of performance characteristic available to feedback system designers* when conditional configurations are used. It is also important to note that the input-output bandwidth may be reduced while the disturbance output bandwidth is maintained.

Apart from having basic significance in the domain of linear systems, conditional feedback configurations have certain special merits for systems containing nonlinearities and, indeed, a new class of nonlinear servomechanism has been evolved. The extension of the concept of conditional feedback to nonlinear systems is discussed in the following.

## Extension of the Principle to Nonlinear Systems

### Disturbances

The casual observation that an error could be properly measured as a ratio rather than as a difference has led to a rather novel embodiment of the conditional topology and to some interesting conclusions concerning the nature of disturbances. It has been remarked previously that a steady-state multiplicative disturbance results in a constant error signal when the error signal is measured as the ratio of the undisturbed signal to the disturbed signal in a conditional system. In a like manner, a steady-state additive disturbance will result in a constant error signal when the error is measured as a difference. Hence, it is useful to classify arbitrary disturbances as being dominantly multiplicative or dominantly additive in character accordingly as the

* The systems described here are the subject of patent action by Ferranti Electric Limited.

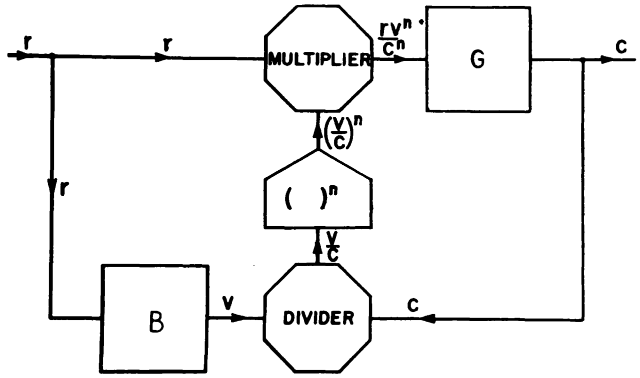
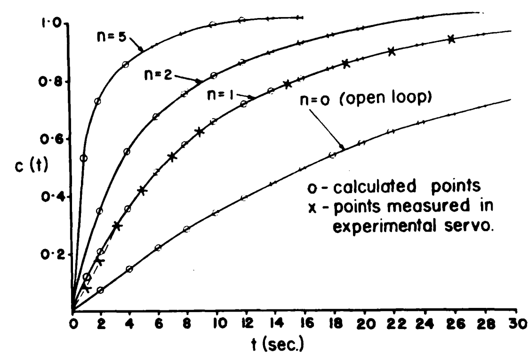



**Fig. 8 (left). A multiplicative conditional servo**

**Fig. 9 (right). The incremental response of a simple multiplicative servo**




ratio measured error or the difference measured error has the lower bandwidth. A significant difference in bandwidth in these measurements, when interpreted in terms of basic bandwidth restrictions on the feedback loop, will suggest the adoption of a particular type of conditional system for dealing with a disturbance.

In view of these observations, conditional feedback systems have been classified as being either multiplicative or additive in character. Multiplicative conditional feedback systems are believed to represent an entirely new class of systems.

## A Heuristic Description of the Multiplicative System

A simplified multiplicative conditional system is shown in Fig. 7. Three computing devices have been introduced in this configuration: a divider, a power-raising device, and a multiplier. Consider the system as a regulator when $r$ is a positive constant and the low frequency gain of $G$ is unity. In the absence of a disturbance, the following signal relations exist

$$c=r,\ \frac{r}{c}=1,\ \frac{r^{n+1}}{c^n}=r \tag{20}$$

If the low-frequency gain of $G$ changes from unity to $k$, where $k$ represents a new steady-state gain such as would result from a change in load, a new system steady state will arise in which the new signal relations are

$$c=r\,{}^{n+1}\sqrt{k},\ \text{and}\ \frac{r}{c}=\frac{1}{{}^{n+1}\sqrt{k}} \tag{21}$$

Hence, multiplicative feedback has effectively altered the gain to a value ${}^{n+1}\sqrt{k}$. This behavior is analogous to the reduction of an additive disturbance in the conventional feedback loop by a factor $1/(1+k)$, where $k$ is the low-frequency loop gain.

When the system is used as a servo, it is necessary to introduce a reference model $B$. Such a system is shown in Fig. 8, where $B(s)=G(s)$ as in the additive system. Further, let the low frequency gain of $B$ and of $G$ be unity. In the absence of a disturbance, the following relations hold provided $r$ and $c$ are positive

$$\frac{v}{c}=1,\ r\frac{v^n}{c^n}=r,\ C=GR,\ \text{and}\ V=BR \tag{22}$$

Since the over-all system is linear in the absence of disturbances, the Laplace transform may be used to describe the input-ouput relationship.

Assume now that a disturbance occurs changing the low frequency gain of $G$ to $k$. In the ensuing steady state, $v/c$ will have a constant value because signal variations in $v$ are duplicated by signal variations in $c$. Since $v/c$ is a constant, the system again has a linear input-output response relationship, and

$$C=R\,{}^{n+1}\sqrt{k}G\ \text{and}\ \frac{v}{c}=\frac{1}{{}^{n+1}\sqrt{k}} \tag{23}$$

The question now arises as to the behavior of the system during the transient interval. At the present time, it may be said that, for all signals and disturbances having frequency components that are confined to a frequency range over which $G$ is essentially a constant, the resultant signal $c$ can be represented as the product of two functions: the output $c$ caused by the disturbance if $r$ is maintained at unity, and the output $c$ caused by the signal alone. All questions pertaining to the stability of the multiplicative loop cannot be answered at this time. The following section contains an example of typical modes of behavior for simple multiplicative feedback systems.

## Response of a Simple Multiplicative Servo

Consider the servo system shown in Fig. 7. In the light of the equivalence shown in Fig. 3, the configuration in Fig. 7 is the equivalent of a classical system employing a ratio comparator. This system has interesting modes of behavior. Suppose $G$ has the form

$$G=\frac{K}{1+Ts};\ T=24\ \text{seconds},\ K=1 \tag{24}$$

Let $r$ and $c$ initially have the steady value $r=c=1$. The incremental response in $c$ to an additive step in $r$ is shown in Fig. 9 for various values of the exponent $n$.[6,7] For comparison, the step response of $G$ is shown. From the system of Fig. 7 it is readily shown that the differential equation defining the response $c$ is

$$\frac{T}{n+1}\frac{d}{dt}(c^{n+1})+c^{n+1}=r^{n+1}\ \text{for}\ c>0 \tag{25}$$

An equation of the same form governs the response of the system to a multiplicative disturbance such as a sudden decrease in the gain factor $K$ of $G$. Equation 25 shows that response in the variable $c^{n+1}$ has the time constant

$$T_{n+1}=\frac{T}{n+1} \tag{26}$$

The exponent $n$ in a multiplicative servo is equivalent to the loop gain of a classical feedback system employing a subtractive comparison. This result as well as that given in equation 21 provides a simple example of the use of multiplicative feedback to alter and calibrate the transmission characteristics of a transducer.

The nonlinear behavior illustrated in the foregoing is produced intentionally by employing essentially nonlinear system components. However, almost all practical control systems contain nonlinearities that are natural to the components and generally undesired. The concept of conditional feedback alters the significance of many of these residual nonlinearities.

## Natural Nonlinearities in Additive Conditional Feedback Systems

Such nonlinearities as torque saturation in motors, backlash in gears, stiction on shafts, clipping in amplifiers, and nonlinear gain in synchros cause the designer great trouble. Most of these nonlinearities contribute to the instability of a feedback loop and especially to the instability of the classical feedback system. The reason for this condition may be expressed in an interesting manner with the help of Fig. 3. Fig. 3 shows that the classical feedback system is equivalent to a conditional configuration in which the reference model is ideal. Hence the signal $\epsilon$ in the feedback loop contains components caused by all of the nonlinearities in the loop, the influence of each of which feedback is endeavoring to suppress. The classical feedback system does not permit any form of nonlinearity or any form of imperfect linearity to be fully tolerated; it strives for the ideal and commonly becomes unstable in so doing.

However, it is clear that in many practical applications certain residual nonlinearities in the relation of output to the input are not undesirable. It is an unfortunate limitation of the classical feedback system that such admissible nonlinearities contribute to the instability of the feedback loop. The conditional feedback system, on the other hand, provides a direct means for accepting tolerable nonlinearities in the input-output response and at the same time largely prevents these nonlinearities from influencing the stability of the feedback loop. This remarkable behavior is realized by building into the reference model, $B$ in Fig. 2, the tolerable nonlinearities in the components $A$, $G_1$, and $H$. The desired response $v$ at the output of $B$ then contains components caused by these nonlinearities. These components of $v$ cancel the components of the fed-back signal $b$ caused by the nonlinearities in $A$, $G_1$, and $H$. Hence the stability of the feedback loop, in so far as it is excited by the input signal alone, is unaffected. The conditional configuration does not remove the influence of any nonlinearities on the stability of the feedback loop when excited by an additive output disturbance. Hence some care must be exercised in exploiting the aforementioned input-output characteristics.

An excellent example of the use of a conditional feedback system to overcome a serious instability in a classical system is the following. It is well known that a classical system that is conditionally stable for small input signals may be seriously unstable for large signals which cause an element such as a motor to torque saturate. The instability is caused by a reduction in effective loop gain. If the torque-saturating characteristic is included in the reference model of a conditional system, this instability cannot be excited by input signals.

The basic significance of the foregoing is that, for purposes of input-output response, the conditional feedback system permits the regulating action of the feedback loop to act in a manner best suited to the particular application. Perhaps no examples of this fact are more illuminating than those that spring from the problem of the human operator in a feedback system.

### Application of Conditional Configurations to the Human Operator Problem

Whether it be in an automobile, an aeroplane, a steel mill, or an economic system, the importance of system response characteristics in the presence of human operators is profound.[8] It is recognized that the human operator is described by neither a well-defined nor a linear transmission function. Certain basic characteristics can, however, be distinguished. There is what approximates to a pure time delay in motor response to a sudden, say, visual stimulus. There is a blocking action against stimuli received in too rapid succession. The pattern of response changes as experience in a given environment is accumulated. Conditional feedback systems provide the means for exploiting the basic characteristics of the particular human operator by achieving optimum input-output response with adequate loop stability. The design example given is suggestive of the improvement that can be realized in the presence of pure time delays. By providing a linear or nonlinear representation of the human operator in the reference model, the operator is called upon to behave as this model rather than to behave ideally as in the classical system. If the reference model $B$ in Fig. 2 is considered to have a variable structure, it is apparent that the conditional feedback configuration provides an interesting technique for the determination of approximate describing functions for the human operator. The determination is carried out by adjusting the structure of $B$ until the loop signal $\epsilon$ falls to some rms value.

## Conclusions

This paper has introduced the concept and elaborated the basic theoretical and practical implications of conditional feedback. Systems are said to be conditional feedback systems when feedback in these systems is used solely in a regulating role to reduce the influence of disturbances of all types. It is distinctive of linear additive conditional feedback systems that the forms of the input-output responses and of the disturbance-output responses are independent. This fundamental fact permits the realization of a broad new range of response characteristics, especially in the presence of such classically difficult factors as pure time delay. The design procedure for such systems is unusually direct and simple.

A classification of system disturbances as being basically additive or multiplicative in character has led to the concept and practical implementation of multiplicative feedback systems employing ratio comparisons and multiplicative corrections. A new range of response characteristics is available from these systems.

Conditional feedback systems provide means for accepting certain component nonlinearities in the relation of system output to system input and, at the same time, for removing the influence of these nonlinearities on the stability of the feedback loop as excited by the input signal. This fact alters the significance for the designer of many residual nonlinearities and permits certain modes of instability to be suppressed. Conditional feedback systems, except through the new class of multiplicative systems, make no new contribution to the regulator problem but they provide a basis for improvement for all classes of servo problems and not least for the class involving human operators.

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# Discussion

**George C. Newton, Jr.** (Massachusetts Institute of Technology, Cambridge, Mass.): The authors are to be commended for this interesting approach to the problem of suppressing disturbances without imposing

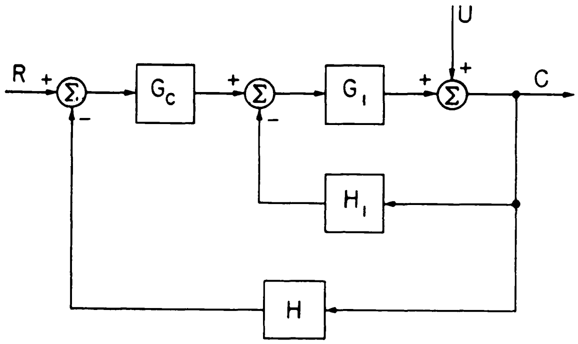
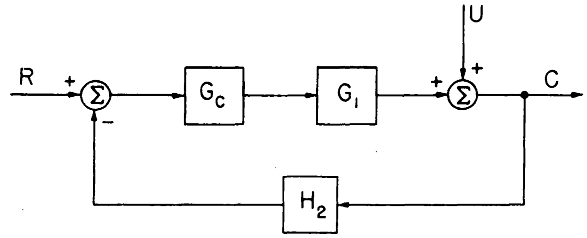



**Fig. 10 (left). Parallel feedback configuration**

**Fig. 11 (right). Single-loop equivalent to parallel feedback configuration**




constraints on the input-output transfer characteristic. This is an important problem and can be approached in a number of different ways. The authors' approach is not an unreasonable one in those instances where the input signal is available. Frequently, however, only the difference between the input signal and the primary feedback signal is available with any degree of precision; e.g., a radar tracking system and a position follow-up using synchros for error-sensing. It may be concluded from this paper that the classical single-loop feedback system is unable to do such as can be done with a conditional feedback system configuration. Thus, one may feel at a loss when meeting a problem in which the input signal is not available for filtering prior to summing with the primary feedback signal. It is one purpose of this discussion to show that conventional configurations may be used to achieve results identical with those achieved by the authors. In these configurations there is no need for operating on the input signal.

Consider first the configuration shown in Fig. 10. In this figure, an auxiliary feedback path is used to suppress the effects of disturbances. After the auxiliary feedback transfer function $H_1$ has been adjusted, the other adjustable transfer functions (usually $G_c$ only) can be adjusted to achieve the desired input-output transmission characteristic. To observe this consider the equation for the output signal

$$C=\frac{G_cG_1R+U}{1+G_1H_1+G_1G_cH} \tag{27}$$

This equation compares with equation 5. The behavior of this parallel feedback configuration is identical with the authors' system providing

$$AG_1=\frac{G_cG_1}{1+G_1H_1+G_1G_cH} \tag{28}$$

$$1+G_1G_2H=1+G_1H_1+G_1G_cH \tag{29}$$

These equations may be solved for $G_c$ and $H_1$, assuming that $A$, $G_1$, $G_2$, and $H$ are known. The solutions are

$$G_c=A(1+G_1G_2H) \tag{30}$$

$$H_1=[G_2-A(1+G_1G_2H)]H \tag{31}$$

Thus, in principle, the authors' system can be realized without need for operations on the input signal. In actual practice, the auxiliary feedback in the case of a positional servomechanism might come from a tachometer attached to the output shaft and $H_1$ would be adjusted to limit the effect of disturbances. There is no need to use equation 31 since $G_2$ is arbitrary within wide limits. $G_c$ is then adjusted to achieve the desired input-output transfer function.

It has been shown that a parallel feedback configuration can be used to suppress disturbances without constraining the input-output transfer functions. Theoretically, it is perfectly possible to realize identical system behavior using a single-loop equivalent to the parallel feedback configuration. Fig. 11 shows such a single loop. If, in this figure, the transmission of the feedback elements is set in accordance with the equation

$$H_2=H+\frac{H_1}{G_c} \tag{32}$$

the behavior of the single loop will be identical with the behavior of the parallel feedback configuration. Thus, it should not be concluded that single-loop feedback systems are unable to achieve independent adjustment of the disturbance-suppression characteristic and the input-output transmission characteristic. However, in practice, realization of the required transfer functions is usually simpler if a parallel feedback configuration or the configuration suggested by the authors is used.

The authors' application of their conditional principle to multiplicative systems appears to be novel. In connection with process control problems there may be considerable merit in the use of conditional multiplicative feedback configurations. However, a number of questions are left unanswered. For example, in the case of complex $G$ functions, is not the stability sensitive to the output signal level since the loop gain for incremental signals depends on this quantity? Also, are the advantages of the multiplicative system sufficient to justify the use of multiplier, divider, and power-raising devices which are not as easily realized as summing and operational amplifying devices? I look forward to the future publications on the multiplicative systems promised by the authors.

**H. C. Ratz** (Ferranti Electric Limited, Toronto, Ont., Canada): Consideration of the conditional feedback approach to control problems described by the authors leads naturally to discussion of its relationship to conventional feedback systems. The relationship of conditional systems to classical systems is topologically equivalent to the relationship of bridge circuits to series-parallel circuits. In the conditional system, when the model is chosen for balance, as given by equation 4, there is no signal in the feedback path through $G_2$ and, hence, the choice of signal input-output response is independent of the stability of the feedback loop. In Fig. 4, the authors have drawn a classical system with a prefilter which has the same transfer function as the conditional system of Fig. 2. However, in Fig. 4, there is a signal in the feedback path through $G_2$, the effect of which must be exactly cancelled by the prefilter.

It is possible to draw a conventional feedback system without a prefilter which is linearly equivalent to a conditional system. Fig. 12 shows such a system which is topologically equivalent to Fig. 2. The Laplace transform of the output in Fig. 12 is given by equation 3 with respect to both signal and disturbance. Furthermore, if the model is defined by equation 4, then equations 5, 6, and 7 apply in this case also. Thus, the input-output response is unaffected by any adjustment of $G_2$ which may be made to alter the character of the disturbance-output response.

There are, however, at least two fundamental differences between this conventional configuration and a conditional system. In Fig. 12, there is always signal in the feedback path so that system stability depends upon the linearity of the components; and the realizability of the transfer functions in the feedback loop is subject to more severe restrictions than in the conditional case. Thus, in special cases, linear equivalents can be formulated, but the contribution of conditional servos is in the simple treatment of nonlinear systems and in the new wide range of feedback configurations which are realizable. Models are already in use for process control problems but the new approach of the authors is to use the model in such a way as to eliminate feedback signals from the error-detecting element. In the conditional system, the feedback loop is employed only to reduce the effect of deviations from model behavior and other disturbances.

**H. Tyler Marcy** (International Business Machines Corporation, Endicott, N. Y.): It is very appropriate that the authors

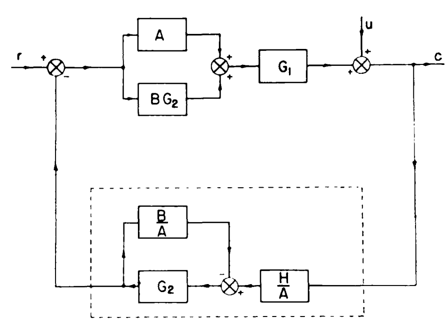



**Fig. 12. The conditional feedback system drawn in the configuration of a conventional system**

should present this interesting paper at this time. The concept of a control system making use of information contained in the input signal to decrease dynamic errors caused by changes in the signal may be a key technique in many of the more comprehensive process control problems which are receiving much attention today.

There is pertinent literature applying to this subject which makes the word "new" in the paper title subject to question. The concepts of feed-forward have been discussed previously.[1,2,3] It has been widely known that, should it be possible to operate on the input signal, dynamic errors in the control system can be reduced. An important consideration is that feed-forward techniques of error reduction do not change the degree of stability or the natural frequencies of the control system, i.e., the roots of a characteristic equation remain the same. Harris[1] also considered feed-forward as a means for compensating nonlinear factors such as dry friction.

Perhaps the dominant limitation with the use of feed-forward is the nature of the input signal. It must first be possible to measure it directly and then to operate on the signal in a predictive sense. In the presence of noise, this can become impractical. We have, however, learned much about statistical treatment of this sort of problem, since feed-forward was widely discussed some years ago. Regardless of where the disturbance to a control system is applied, predictive knowledge of that disturbance can be employed to decrease dynamic control errors resulting from the disturbance.

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**Rufus Oldenburger** (Woodward Governor Company, Rockford, Ill.): This valuable contribution to the science of automatic control emphasizes the importance of designing for both servo and regulator performance in many automatic control problems. In the design of speed governors we study the response to speed-setting changes and load rejections, as well as other disturbances. We have found that the design of our governors on the basis of characteristic roots so as to give fast closed-loop response to sudden load changes has also given good response to speed-setting changes.

Unfortunately, our most troublesome disturbance is what I like to call the "noise" in the speed signal. This signal is put out by the speed-measuring element. This element corresponds to the authors' feedback element $H$. As the authors point out, unfortunately this disturbance cannot be reduced without compromising $H$. The noise can come from gear drive irregularities, prime-mover shaft runout, or other causes. It is a part of the signal one does not wish to respond to, and severely limits the mathematical operations that can be performed on the speed measurement in the computer part of the control.

Prof. James Reswick[1] introduced an auxiliary feedback loop in which the transfer function $AG_1$ of the forward part is taken to be the transfer function of the extra feedback path, except for a constant of proportionality. If $H=1$ the authors also introduce an element with transfer function $AG_1$ into an extra loop, but in a different manner by placing it in a forward branch.

The inclusion by the authors of a discussion of nonlinear components is most timely. The American Society of Mechanical Engineers will devote its April 1956 Instruments and Regulators Division conference to nonlinear control. The Russians specialize in this area, as well as in the automatic control field in general; they have a journal devoted entirely to the science of automatic control. The Macmillan Company will soon publish an American Society of Mechanical Engineers book entitled "Frequency Response," which I am editing. This book is to contain leading contributions from all over the world on the various phases of this subject.

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**H. P. Birmingham** (Naval Research Laboratories, Washington, D. C.): This important paper is of particular interest to the human engineer concerned with the man as an element in a man-machine control system. A problem under attack is the matter of human performance in pursuit tracking and in compensatory tracking. In compensatory tracking, the human operator is presented only the error (difference between input and output) and manipulates his control on the basis of this information. In the pursuit case, the operator sees the input and his output, or terms proportional thereto, on the same display and is required to keep the difference (error) at a minimum. It has been shown that under some conditions, the human is able to maintain a smaller average error in the pursuit case. It is expected that the authors' analysis can be used to show how the man can turn in this superior performance where the course as well as the error is shown to him, by acting analogously to a conditional system.

**G. Lang** and **J. M. Ham:** We greatly appreciate the interest shown by the discussers. Dr. Newton's observation that input signals are not always available with precision is well made but we consider the occasions when input signals cannot be measured with useful accuracy to be rare.

Dr. Newton proceeds to show that in a linear world a conventional feedback configuration, such as shown in Figs. 10 and 11, can be made to have transfer functions identical with those of the basic conditional configuration in Fig. 2. This fact is implied by the prefilter equivalent shown in Fig. 4. As Mr. Ratz has shown in Fig. 12, there are many such linear equivalents. However, we wish to emphasize that our conditional configuration leads to direct realization techniques not readily implemented for general linear equivalents. For example, Dr. Newton's equations 30 and 31 show how to pick conventional compensating functions $G_c$ and $H_1$ to achieve equivalence with a conditional configuration. Given the conditional configuration and the freedom to pick $A$ and $G_2$ independently, equations 30 and 31 are readily used to define $G_c$ and $H_1$, but we wonder how Dr. Newton would pick $G_c$ and $H_1$ to give independent input-output and disturbance-output responses without prior reference to a conditional configuration.

In the literature on feedback control systems we are not aware of any collected treatment on the problem of selecting compensating transfer functions to achieve independent conditions on input-output and disturbance-output responses, particularly when nonminimum phase components are present. A practical reason for the difficulty in achieving such conditions is readily discerned from equations 30 and 31. These equations show that any changes in $G_c$ or in $H_1$ affect both $C/R$ and $C/U$ so that the cut-and-try procedure is commonly resorted to. In this connection, our sample design is exemplifying the directness with which a conditional system design can be realized even under the difficult condition of a pure time delay in the main actuator.

A block diagram for Fig. 10 showing how to realize $G_c$ and $H_1$ as specified in equations 30 and 31 is given in Fig. 13. If the system is synthesized as shown, it is clear that many more elements are required than for the equivalent conditional system. While it is true that in a linear world $G_c$ and $H_1$ can be realized with gain and general passive filters, it is not clear how such filters are to be synthesized in practice.

Dr. Newton's remarks about the equivalence of Fig. 11 to the conditional configuration are quite correct but are subject to the qualifications outlined in the foregoing. It should be observed that all of the equivalences discussed by Dr. Newton depend for their validity on linearity. In this connection the remarks of Mr. Ratz are particularly relevant.

With regard to multiplicative systems, Dr. Newton's observation that for complex $G$ functions the loop stability depends on the output signal level is quite correct. To the question of the justification for using multiplier, divider, and power-raising devices in these systems, it may be remarked that multiplicative systems appear to have other than physical worth, namely as model elements for economic systems where per-unit changes are significant. Further, a generalization of the concept of signal comparison leads to insight into the nature of disturbances as suggested in the section entitled "Extension of the Principles to Nonlinear Systems."

Mr. Ratz's comments may be regarded

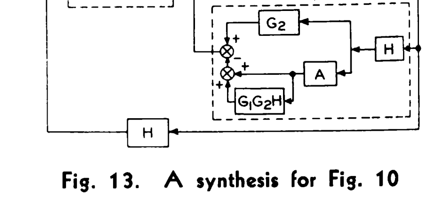


**Fig. 13. A synthesis for Fig. 10**

as an amplification of the paper and of the remarks of Dr. Newton, and deserve careful reading. Fig. 12 represents the conventional single-loop feedback system equivalent to the conditional configuration. Again it must be noted that, if this system is synthesized as shown in Fig. 12, more elements are required than for the equivalent conditional system.

Mr. Marcy emphasizes the relationship of the conditional feedback systems introduced by us to so-called feed-forward systems to which considerable attention has certainly been given. Perhaps the work of J. R. Moore[1] most closely resembles ours. To the extent that any linear system may be regarded as a combination of feed-forward and feedback elements, the conditional topology introduced by us can be so interpreted. However, the concept and topology of conditional feedback systems employing a reference model has not been described previously in the literature, as far as we know. As Mr. Marcy suggests, the concept of conditional feedback may be particularly illuminating for problems concerning transient process control.

The remarks of R. Oldenburger are interesting. We have heard of Professor Reswick's recent work and are looking forward to seeing his paper.

We appreciate the interest of Dr. Birmingham in the application of conditional feedback systems to the human operator problem. Dr. Birmingham rightly distinguishes the tasks of compensatory and pursuit tracking. These correspond to the regulator and servo definitions as used in the paper. We believe that the conditional topology is significant for the engineering of man-machine systems. A research program on this topic is in progress at the University of Toronto.

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# Design of Control Systems for Minimum Bandwidth

GEORGE C. NEWTON, JR.
MEMBER AIEE

**Synopsis:** This paper presents a method of designing feedback control systems which minimizes bandwidth for a specified transient error. Reduction of bandwidth is desired in order to attenuate noise, simplify the compensation, and ease the requirements on components operating at high power levels.

## Design Specifications

### BANDWIDTH TEST

Minimum bandwidth is obtained by adjusting the system weighting function to produce minimum output noise under specially contrived circumstances termed the "bandwidth test." In this paper the bandwidth test is a conceptual scheme for defining bandwidth. During the bandwidth test a stochastic noise signal is used for the control system input. The output of the control system is passed through a filter and its rms value measured. The rms value of the filtered output is correlated with the bandwidth of the control system by comparison with the filtered output of a standard system of any prescribed form but having adjustable bandwidth. The bandwidth of the standard system is adjusted to produce an rms value of its filtered output equal to that of the control system. Both the standard system and the control system are driven from a common noise source during the bandwidth test. Fig. 1 is a block diagram showing this scheme for relating control system bandwidth to its filtered output during a bandwidth test. By definition, the bandwidth of the control system is equal to the bandwidth of the standard system when the rms values of the filtered outputs are equal. In order that minimizing the filtered output of the control system shall be equivalent to minimizing its bandwidth, the standard system together with the noise source and filter must produce a monotonically increasing rms output with increasing bandwidth.

### PERFORMANCE INDEX

The purpose of any control system is to constrain its output to match a desired output (ideal value) within an acceptable tolerance when acted upon by an input (command) and disturbances. The measure used to assess the agreement between the desired and actual outputs is usually called a performance index. For the design method of this paper the input and desired output are arbitrary transient signals. The integral square of the error between the desired output and the actual output is used as the performance index. In the process of adjusting the control system to minimize its bandwidth, only those weighting functions are used which make the integral-square error equal to or less than a specified value.

### SOLUTION FOR WEIGHTING FUNCTION

In the section entitled "A Variational Approach" a general solution is obtained for the weighting function which the control system should have in order to possess minimum bandwidth for a specified integral-square error. This solution is obtained by variational methods. The data necessary to obtain a particular solution are as follows:

In connection with the bandwidth test:

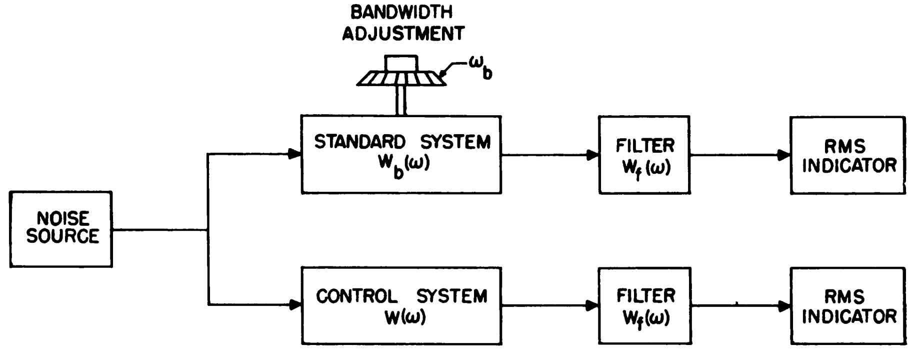


Fig. 1 Scheme for defining of system bandwidth

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GEORGE C. NEWTON, JR., is with Massachusetts Institute of Technology, Cambridge, Mass.

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