Theoretical Consideration of Retarded Control

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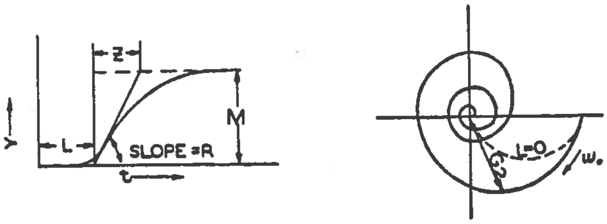
This paper is concerned with a theoretical study of the control of a single-capacity process with dead-period lag. Characteristic equations corresponding to the application of proportional, proportional-plus-derivative, proportional-plus-reset, and proportional-plus-reset-plus-derivative responses are used to graph the controller parameters necessary to obtain a desired degree of stability. The degree of stability is taken to be associated with the amplitude ratio of the lowest-frequency harmonic mode. Effects of the various controller parameters are shown and a method is suggested to determine the adjustable parameters for a desired degree of stability.

EVER since the publication by Callender, Hartree, and Porter (1)[3] considerable attention has been directed to the study of the dynamics of control of retarded systems. Some interest has been shown in the "optimum adjustment" of the control parameters for particular types of control functions and process characteristics (2–7). It is the purpose of this paper to study the control of a single-capacity process with dead-period lag. The controller will be assumed to be conventional; i.e., it will have available proportional, integral or reset, and derivative responses.

The two principal components of the control loop are the process and the controller. The process is considered to include all parts of the installation exclusive of the controller. For this discussion the final control element or valve will be included with the process.

The process can be characterized by its reaction curve which is the chart record obtained when the valve is given a sudden sustained disturbance with the controller disconnected. Such a record is shown in Fig. 1(a) for a unit change in pressure. There appears to be a period of time during which the pen moves but little and this dead time or lag L may be of some magnitude in comparison with the transfer lag (the lag due to the lumped capacity of the process). The dead time is due to the fact that the process is really a continuum where the parameters which describe the process are distributed. The lag due to the finite time of transport of the signal (for example, a long tube which carries a compressible fluid) is called a distance-velocity lag. If the continuum contains no inertia, it may be represented by a number of cascaded lumped resistance-capacity networks. Increasing the number of the cascaded elements gives a better approximation to the continuum since the order of contact with the time axis increases with the number of elements in the lumped circuit approximation. However, the complexity of the problem increases with the number of elements.



(a) Process-reaction curve

(b) Process Nyquist plot

Fig. 1

A good approximation which has the advantage of simplicity may be obtained by introducing a certain amount of dead time along with one or two resistance-capacity elements. In this paper we approximate the reaction curve by using dead time and a single-capacity lag.

The following differential equation can be used as the first approximation to the process

$$\frac{dY}{dt} + \frac{R}{M} Y = R\,\Delta F(t - L) \quad \text{[1]}$$

where
Y = pen deviation from set point, in.
R = unit reaction rate, in/psi min
t = time, min
M = process sensitivity, in/psi
L = dead-period lag, min
ΔF = controller output change, psi
Z = process time constant, min

The frequency response G_2 is

$$G_2 = \frac{Y(i\omega_0)}{\Delta F(i\omega_0)} = \frac{R\,e^{-i\omega_0 L}}{\frac{R}{M} + i\omega_0}, \quad i = \sqrt{-1}$$

as shown in Fig. 1(b) where ω_0 is the applied angular frequency.

We will consider a controller to regulate the process which has proportional, integral and derivative response functions. This controller may be represented by the following differential equation

$$-\Delta F(t) = S\left[U\int_0^t Y(\sigma)d\sigma + Y(t) + T\frac{dY(t)}{dt}\right] \ldots [2]$$

where
S = proportional sensitivity, psi/in
U = reset rate, min^{-1}
T = derivative time, min

The controller response to a unit step in pen deviation and frequency response G_1 are shown in Figs. 2(a) and 2(b). It is more interesting to make a phase-magnitude plot for a sinusoidal variation in $Y(t)$ as shown in Fig. 3. This shows that the conventional controller can be considered as a band-rejection filter and amplifier, the low-frequency corner being determined by the reset rate and the high-frequency corner by the derivative time. Proportional sensitivity sets the amount of gain in the rejection band.

Since we are concerned with "regulators" we will consider

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[3] Numbers in parentheses refer to Bibliography at end of paper.

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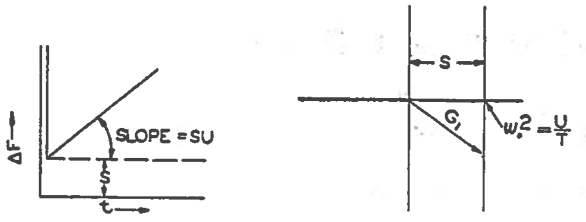


(a) Controller transient response (b) Controller Nyquist plot

FIG. 2

only those disturbances which can be represented by a change in load and not by a change in set point. One can easily go from one to the other. In this investigation the load change occurs at the valve end of the process at zero time and produces the same reaction curve as a step change in pressure at the valve. For the approximation to a single-capacity process with dead-period lag we can write

$$\frac{dY(t)}{dt} + \frac{R}{M} Y(t) = R\,\Delta F(t - L) + R\,\Delta D(t - L) \ldots [3]$$

where ΔD is the load change, psi. In order to get the control-loop equation in nondimensional form, we introduce the following notation:

$\mu = \frac{RL}{M}$ = self-regulation index of process ($\mu \to 0$ when $M \to \infty$)

$\tau = \frac{t}{L}$ = dimensionless "time"

$\theta(\tau) = \frac{Y(L\tau)}{RL\Delta D_0}$ = dimensionless "pen deviation"

SRL = dimensionless proportional sensitivity setting

UL = dimensionless reset "rate" setting

$\frac{T}{L}$ = dimensionless derivative time setting

$\nu_1 = SRL(UL)$ = integral parameter

$\nu_2 = SRL$ = sensitivity parameter

$\nu_3 = SRL\left(\frac{T}{L}\right)$ = derivative parameter

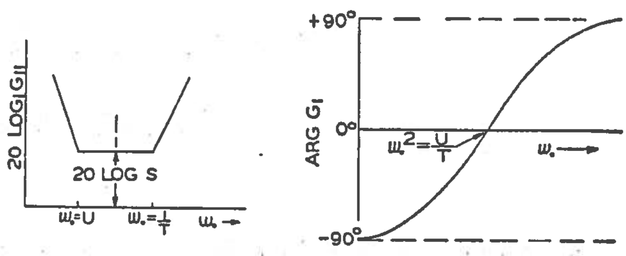
r a constant disturbance ΔD_0 the control loop may be represented by the differential equation

$$\frac{d\theta}{d\tau} + \mu\theta(\tau) = -\nu_1 \int_0^{\tau-1} \theta(\sigma)d\sigma - \nu_2\theta(\tau - 1) - \nu_3 \frac{d\theta(\tau - 1)}{d\tau} + 1 \ldots\ldots [4]$$

The ultimate aim in the adjustment of controllers is to obtain a response curve which will satisfy the user's requirement for good control. The quality of control is therefore relative to the application. The user usually wants minimum area under the response curve, minimum deviation, and minimum cycling. Ziegler and Nichols (2) suggest that the amplitude ratio of the response curve be about 0.25, and this is a commonly accepted rule of thumb in the process industry.

Since retarded action implies that the response curve consists of an infinite number of harmonic modes, it would be fruitless to prescribe the amplitude ratio for each mode. We shall, therefore, designate the "degree of stability" to be associated with the amplitude ratio of the fundamental (lowest-frequency) harmonic mode.

Adjustment of the controller will be based on information ob-



(a) Log-magnitude versus frequency (b) Phase versus frequency

FIG. 3

tained from the control region. We define the control region to be the graphic relationship between the adjustable control parameters necessary to obtain a prescribed degree of stability of the response curve. The characteristic equation of the control loop is used to plot the control region.

The control region may also be obtained by means of the well-known methods of frequency analysis. Instead of using the amplitude ratio of the fundamental harmonic mode as a measure of degree of stability, the loci of "constant magnitude" are commonly used. Since this method has received considerable attention in the past years, we will not consider it as a basis of analysis. Moreover, there is at times serious error in estimating stability and response from these diagrams (8).

We will now consider the special cases of proportional control, proportional-plus-derivative control, proportional-plus-reset control, and proportional-plus-reset-plus-derivative control.

PROPORTIONAL CONTROL ($\nu_1 = \nu_3 = 0$)

The control relationships are obtained from the characteristic equation which for this case becomes

$$p + \mu + \nu_2 e^{-p} = 0 \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

There is an infinite number of roots corresponding to Equation [5], the real roots ($p = -\delta_n$) being shown in Fig. 4. We are interested primarily in the pair of complex roots with the lowest-frequency component which we denote by

$$p = \omega(-r \pm i) \ldots\ldots\ldots\ldots\ldots\ldots [6]$$

where a = amplitude ratio of fundamental mode

$$r = -\frac{1}{2\pi} \ln a$$

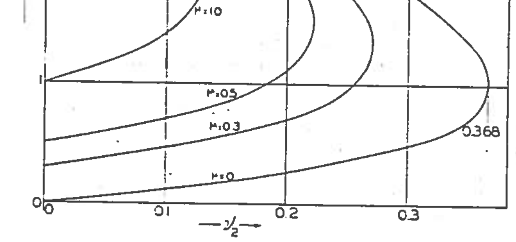


FIG. 4 REAL ROOTS OF CHARACTERISTIC EQUATION [5]

$\omega = 2\pi \frac{L}{P}$ dimensionless angular frequency of fundamental mode

P = period, min

Substitution of Equation [6] into [5] and separation of the real and imaginary parts yields

$$\frac{\tan \omega}{\omega} = \frac{1}{\omega r - \mu}$$

and

$$\nu_2 = \frac{\omega}{\sin \omega} e^{-r\omega}$$

The foregoing equations define the control region for proportional control as shown in Fig. 5. The sensitivity parameter is plotted against the self-regulation index μ. The solid lines are contours of constant amplitude ratio a of the fundamental while the broken lines are loci of constant dimensionless period P/L.

If the disturbance is a Heaviside step of height ΔD_0

$$\theta(\tau) = \frac{1}{\nu_2 + \mu} + \sum_{n=0}^{\infty} A_n e^{-\omega_n r_n \tau} \cos (\omega_n \tau - \phi_n) + \sum_{n} B_n e^{-\delta_n \tau} \ldots\ldots [7]$$

provided there are no repeated roots of the characteristic equation and where

A_n = harmonic amplitudes

ω_n = harmonic frequencies

ϕ_n = associated phase angles

$r_n \omega_n$ = damping constants for each harmonic mode

δ_n = damping constants corresponding to real roots

The amplitudes of the first three harmonics are shown in Table 1.

TABLE 1 APPROXIMATE AMPLITUDES OF HARMONICS, PROPORTIONAL CONTROL

(Amplitude ratio of fundamental = 0.25)

μ	A_0	A_1	A_2
0	0.858	0.0332	0.010
0.1	0.836	0.0332	0.010
0.3	0.696	0.0332	0.010
0.5	0.588	0.0332	0.010
0.7	0.502	0.0332	0.010
1.0	0.406	0.0332	0.010

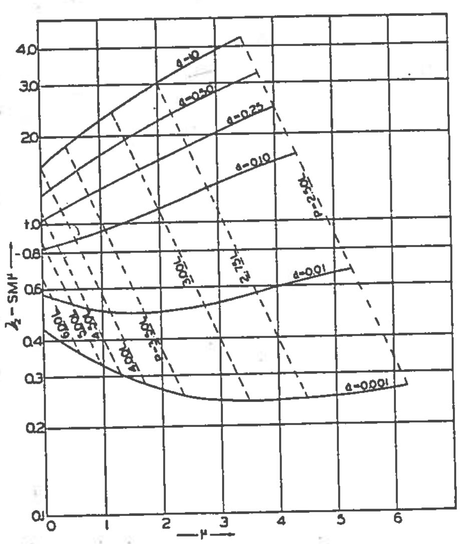


FIG. 5 CONTROL REGION FOR PROPORTIONAL CONTROL

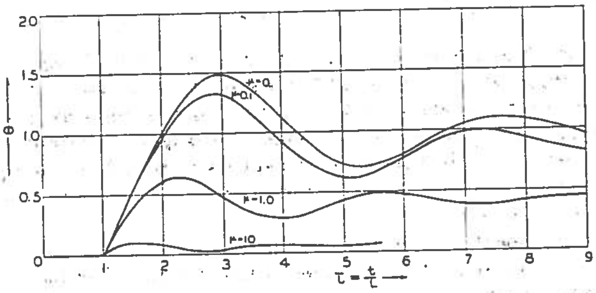


FIG. 6 RESPONSE CURVES FOR PROPORTIONAL CONTROL, a = 0.25

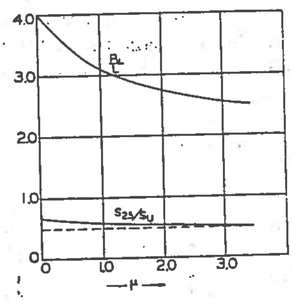


FIG. 7 COMPARISON OF ULTIMATE AND 0.25 AMPLITUDE RATIO SENSITIVITIES

TABLE 2 OFFSET, INITIAL DEVIATION, AND PERIOD, PROPORTIONAL CONTROL

(Amplitude ratio of fundamental = 0.25)

τ_I = time of initial deviation

$\theta(\tau_I)$ = magnitude of initial deviation

θ_f = offset

μ	SRL	θ_f	P/L	τ_I	$\theta(\tau_I)$
0	1.03	0.970	4.7	2.97	1.49
0.10	1.05	0.870	4.4	2.86	1.33
1.0	1.30	0.435	3.4	2.28	0.68
10.0	5.19	0.066	2.2	2.00	0.10

It is evident that for $0 \leq \mu \leq 1$ all of the higher harmonics in Equation [7] are negligible in comparison with the fundamental. Thus, if one chooses the amplitude ratio of the fundamental to be 0.25, the response curve will have approximately this degree of stability.

The offset $1/(\nu_2 + \mu)$ and the time and magnitude of the initial deviation (height of the first peak) of the response curve are shown in Table 2 for various values of self-regulation and an amplitude ratio of 0.25. It is seen that the initial deviation is about one and one-half times the offset.

A few representative response curves are shown in Fig. 6 for the case of proportional control. These curves are easily sketched by obtaining from Fig. 5 the sensitivity setting ν_2 and period for the process characteristic and desired degree of stability. For an amplitude ratio of 0.25, the time and amount of initial deviation as well as the offset may be determined from Table 2.

Fig. 7 shows the ratio of the sensitivity settings S_{25} necessary to obtain an amplitude ratio of 0.25 to the settings necessary to obtain an amplitude ratio of unity (the ultimate sensitivity S_u) as well as the period corresponding to this ultimate sensitivity.

PROPORTIONAL-PLUS-DERIVATIVE CONTROL ($\nu_1 = 0$)

The characteristic equation corresponding to this case is

$$p + \mu + e^{-p}(\nu_2 + \nu_3 p) = 0$$

As before, we obtain the equations for the control region

$$\nu_3 = (r \sin \omega - \cos \omega)e^{-\omega r} - \mu e^{-\omega r} \frac{\sin \omega}{\omega}$$

$$\nu_2 = (1 + r^2)e^{-r\omega}\,\omega \sin \omega - \mu\, e^{-r\omega}(r \sin \omega + \cos \omega)$$

Figs. 8 and 9 show the μ-contours in the control region for amplitude ratios of 0.25 and 1.0 (the stability-limit case). As one chooses settings along a 0.25-amplitude contour for a fixed μ, it is found that the controlled response changes considerably in character. Table 3 shows what happens to the offset, the time of initial deviation, the magnitude of the initial deviation, and period for $\mu = 0$.

It is evident that for the offset to be a minimum ν_3 should be between 0.3 and 0.4. For larger values of ν_3 the offset increases at a rapid rate. The corresponding values are shown in Table 4 for a self-regulation index μ of 0.3.

If 0.25-amplitude ratio is desirable and if minimum offset is required, then the controller can be adjusted according to Table 5. The period, initial deviation, magnitude of initial deviation, offset, and linear approximations for the settings also are shown for various values of μ.

TABLE 3 CHARACTERISTICS OF RESPONSE CURVES, PROPORTIONAL-PLUS-DERIVATIVE CONTROL

$\mu = 0$ $\qquad a = 0.25$

ν_3	ν_2	θ_f	P/L	τl	$\theta(\tau l)$
0.1	1.120	0.89	4.21	2.80	1.36
0.2	1.195	0.84	3.84	2.67	1.27
0.3	1.235	0.81	3.49	2.57	1.20
0.4	1.225	0.82	3.13	2.49	1.15
0.5	1.081	0.93	2.74	2.45	1.12

TABLE 4 CHARACTERISTICS OF RESPONSE CURVES, PROPORTIONAL-PLUS-DERIVATIVE CONTROL

$\mu = 0.3$ $\qquad a = 0.25$

ν_3	ν_2	θ_f	P/L	τl	$\theta(\tau l)$
0.1	1.182	0.68	3.76	2.55	1.02
0.2	1.245	0.65	3.48	2.45	0.98
0.3	1.267	0.64	3.17	2.38	0.94
0.4	1.228	0.65	2.86	2.31	0.91

TABLE 5 CHARACTERISTICS FOR PROPORTIONAL-PLUS-DERIVATIVE CONTROL WITH MINIMUM OFFSET AND 0.25 AMPLITUDE RATIO

$$\nu_2 \approx 0.161\,\mu + 1.240$$
$$\nu_3 \approx -0.111\,\mu + 0.335$$

μ	θ_f	P/L	τ_l	$\theta(\tau_l)$
0	0.81	3.40	2.55	1.17
0.1	0.75	3.31	2.48	1.09
0.3	0.64	3.16	2.38	0.94
0.5	0.55	3.05	2.28	0.83
0.7	0.49	2.96	2.20	0.74
1.0	0.42	2.88	2.12	0.64

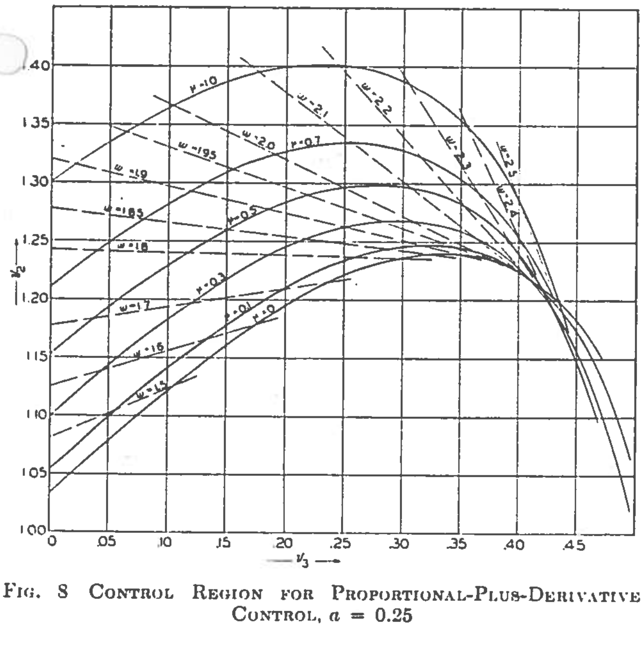


FIG. 8 CONTROL REGION FOR PROPORTIONAL-PLUS-DERIVATIVE CONTROL, $a = 0.25$

The response curves for $\mu = 0$ and $\mu = 0.3$ are shown in Fig. 10.

PROPORTIONAL-PLUS-RESET CONTROL ($\nu_3 = 0$)

The addition of reset response removes offset, but tends to make the system more unstable. The characteristic equation for this case may be written

$$p^2 + p\mu + e^{-p}(\nu_1 + \nu_2 p) = 0$$

and the control region is defined by

$$\nu_2 = \omega e^{-r\omega}\,[2r \cos \omega + (1 - r^2) \sin \omega] + \mu e^{-r\omega}\,[r \sin \omega - \cos \omega]$$

$$\nu_1 = \nu_2\, r\omega + \omega^2\, e^{-r\omega}\,[(1 - r^2) \cos \omega - 2r \sin \omega] + \mu\, \omega e^{-r\omega}\,[r \cos \omega + \sin \omega]$$

These equations allow one to plot the control region shown in Fig. 11. The control parameter ν_2 is plotted against ν_1 for various μ and for an amplitude ratio of 0.25. The contours for the stability-limit case and for critical damping are shown in Figs. 12 and 13, respectively.

It is also known that the control area is

$$\int_0^\infty \theta(\sigma)d\sigma = \frac{1}{\nu_1}$$

As one progresses along a particular contour of 0.25 amplitude ratio, it is found that the control area does not change very much in the neighborhood of maximum ν_1 but the frequency does. It is desirable to keep the frequency as large as possible and retain minimum control area.

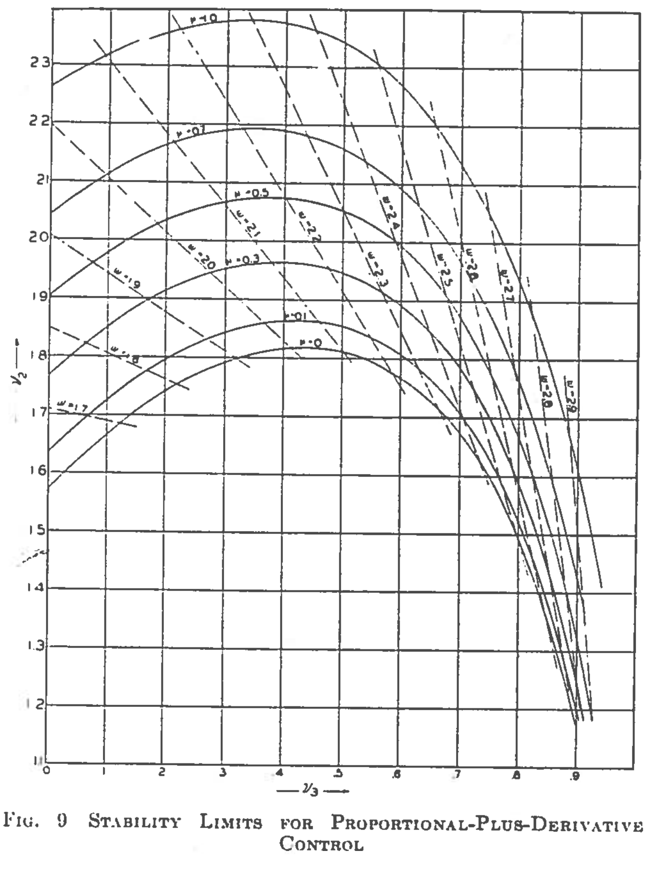


FIG. 9 STABILITY LIMITS FOR PROPORTIONAL-PLUS-DERIVATIVE CONTROL

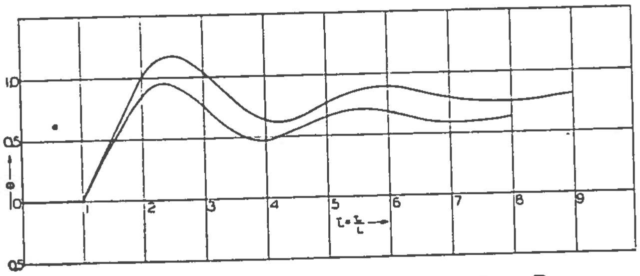


FIG. 10 RESPONSE CURVES FOR PROPORTIONAL-PLUS-DERIVATIVE CONTROL, $a = 0.25$, MINIMUM OFFSET
(Upper curve $\mu = 0$, lower curve $\mu = 0.3$)

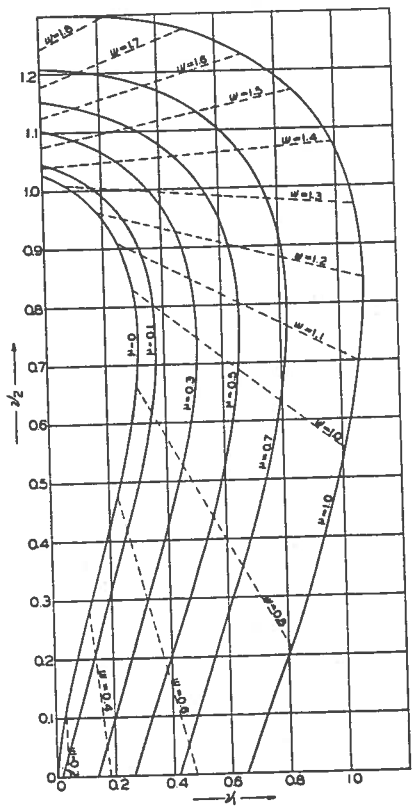


FIG. 11 CONTROL REGION FOR PROPORTIONAL-PLUS-RESET CONTROL, $a = 0.25$

Fig. 14 shows response curves for settings chosen along the 0.25 amplitude contour in the control region.

If it is desirable that the response curve be critically damped so that most of the area of the curve is above the control point, then Fig. 13 is used. For this case the criterion of minimum control area is the selection of the maximum value of ν_1 on the contour of critical damping.

PROPORTIONAL-PLUS-RESET-PLUS-DERIVATIVE CONTROL

If derivative action is added to a proportional-plus-reset controller, the characteristic equation is

$$p^2 + p\mu + e^{-p}(\nu_1 + \nu_2 p + \nu_3 p^2) = 0$$

Again we find

$$\nu_2 = e^{-\omega r}\left\{(2r\omega - \mu)\cos\omega + [\mu r + \omega(1 - r^2)] \times \sin\omega\right\} + 2r\omega\nu_3$$

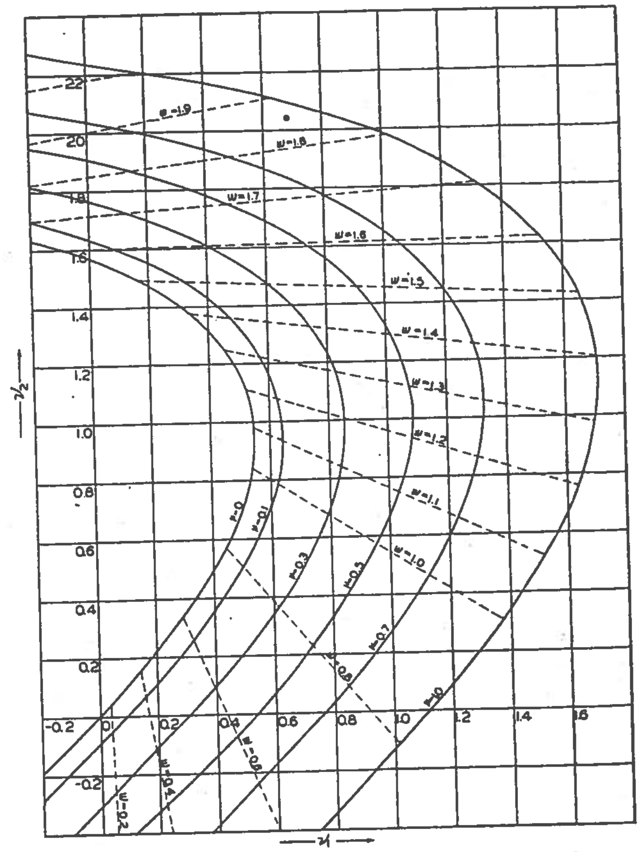


FIG. 12 STABILITY LIMITS FOR PROPORTIONAL-PLUS-RESET CONTROL

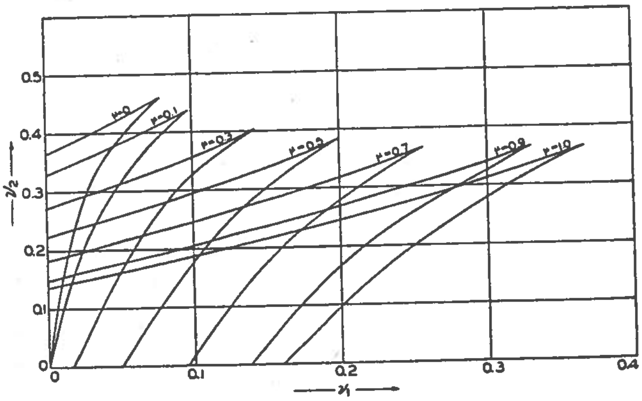


FIG. 13 CRITICAL DAMPING CONTOURS FOR PROPORTIONAL-PLUS-RESET CONTROL

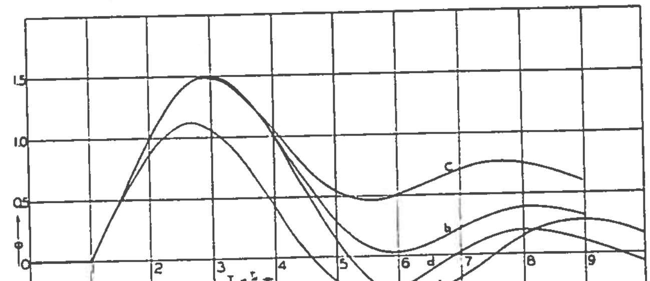


FIG. 14 RESPONSE CURVES FOR PROPORTIONAL-PLUS-RESET CONTROL

(a) $\mu = 0$, $\nu_1 = 0.27$, $\nu_2 = 0.90$; (b) $\mu = 0$, $\nu_1 = 0.18$, $\nu_2 = 0.96$; (c) $\mu = 0$, $\nu_1 = 0.073$, $\nu_2 = 1.01$; (d) $\mu = 0.3$, $\nu_1 = 0.45$, $\nu_2 = 0.93$

$$\nu_1 = \omega r \nu_2 + \omega e^{-\omega r} \left\{ [\mu r + \omega(1 - r^2)] \cos \omega + (\mu - 2r\omega) \times \sin \omega \right\} + \omega^2(1 - r^2)\nu_3$$

The control regions obtained from the foregoing equations are shown in Figs. 15 and 16 when the fundamental harmonic component of the response curve has an amplitude ratio of 0.25.

There is an infinite number of modes which add up to the actual response. The introduction of derivative makes it possible to have a set of values of ν_3, ν_2, and ν_1 yield not only a mode having 0.25 amplitude ratio but also a critically damped mode. The contours for the stability limit are shown in Figs. 17 and 18.

Representative response curves for parameters chosen on a 0.25 amplitude ratio contour for $\mu = 0$ and $\mu = 0.3$ are shown in Figs. 19 and 20. The values of the parameters in Fig. 20 were taken so as to make the control area a minimum when $\nu_3 = 0.5$ (curve a), to make the parameter ν_2 a maximum when $\nu_3 = 0.5$ (curve b), and to satisfy both 0.25 amplitude ratio and critical damping when $\nu_3 = 0.5$ (curve c). The parameter $\nu_3 = 0.5$ gives approximately the largest possible value of ν_2 satisfying simultaneously the condition of the critical damping and 0.25 amplitude ratio. Curve c may be considered optimum.

Conclusions

We have shown how the control regions are used to determine the control-parameter settings for a prescribed degree of stability of the response curve. One must be able to obtain the process constants μ, R, and L from the process-reaction curve and to write the linearized approximate equations for the controller.

Since there is a degree of latitude in the actual controller settings, this method suffices for most practical cases. This is evident by the fact that the settings prescribed by Ziegler and Nichols have been commonly accepted. However, the Ziegler-Nichols settings do not take into consideration the self-regulation of the process.

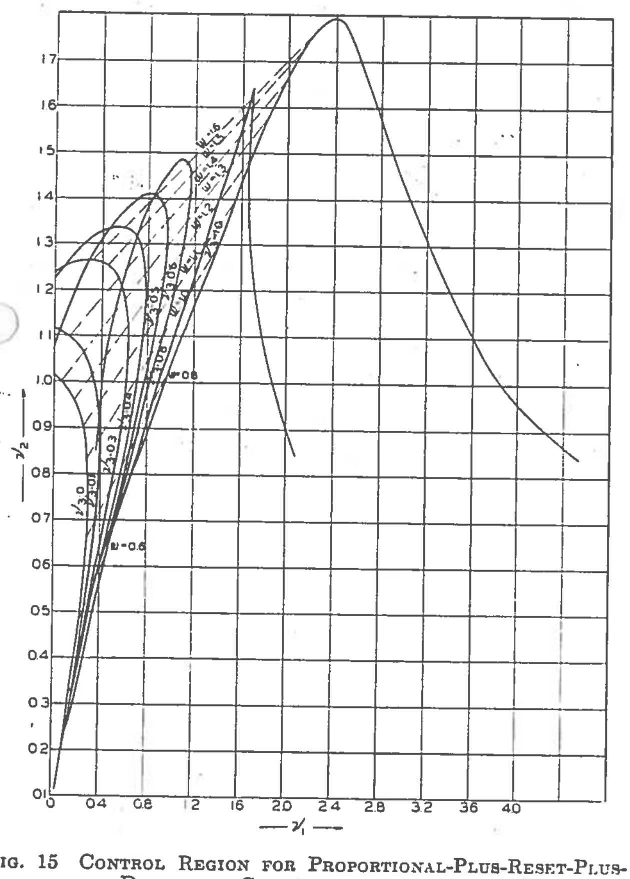


Fig. 15 Control Region for Proportional-Plus-Reset-Plus-Derivative Control, $\mu = 0$, $a = 0.25$

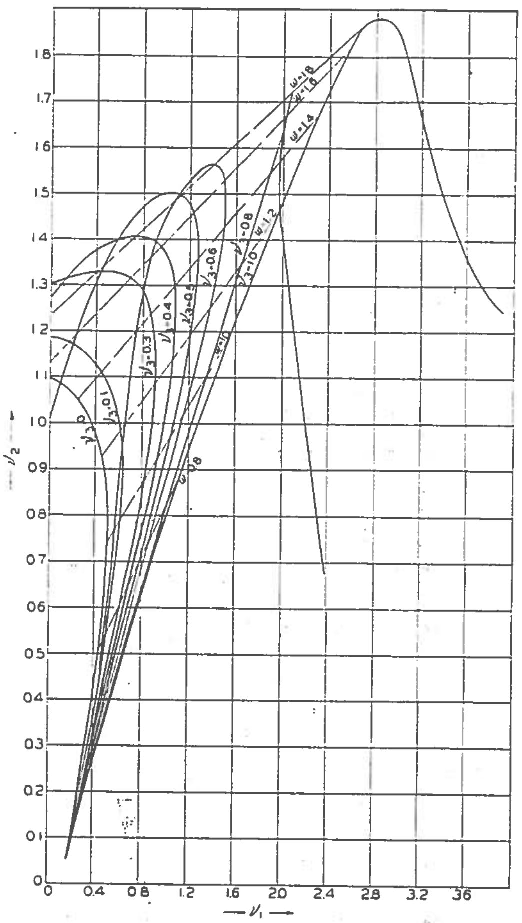


Fig. 16 Control Region for Proportional-Plus-Reset-Plus-Derivative Control, $\mu = 0.3$, $a = 0.25$

We therefore suggest the following settings, if the degree of stability specified by 0.25 amplitude ratio for the fundamental mode is desirable:

Proportional control (criterion, 0.25 amplitude ratio)

$$\nu_2 = 1.03 + 0.35\,\mu \quad [8]$$

Proportional-plus-derivative (criteria, 0.25 amplitude ratio and minimum offset)

$$\left.\begin{array}{l}\nu_2 = 1.24 + 0.16\,\mu \\ \nu_3 = 0.34 - 0.11\,\mu\end{array}\right\} \quad [9]$$

Proportional-plus-reset (criteria, 0.25 amplitude ratio and compromise between minimum area and period)

$$\nu_2 = 0.9 + 0.083\,\mu \qquad \nu_1 = 0.27 + 0.6\,\mu \quad [10]$$

Proportional-plus-reset-plus-derivative (criteria, 0.25 amplitude ratio and critical damping modes dominant, maximum ν_2)

$$\nu_2 = 1.35 + 0.25\,\mu \qquad \nu_1 = 0.54 + 0.33\,\mu \qquad \nu_3 = 0.5 \quad [11]$$

If there is no interaction between controller adjustments then one can obtain the actual controller adjustments as follows

$$S = \frac{\nu_2}{RL} = \text{sensitivity, adjustment knob setting}$$

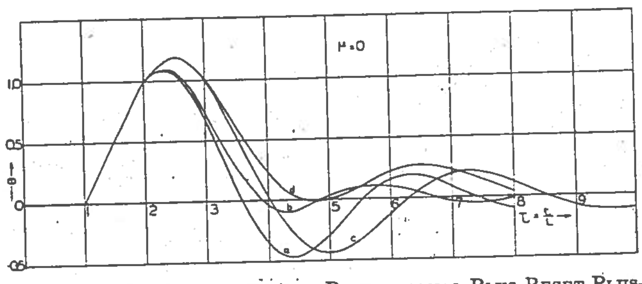


FIG. 19 RESPONSE CURVES FOR PROPORTIONAL-PLUS-RESET-PLUS-DERIVATIVE CONTROL

(a) $\nu_1 = 0.8$, $\nu_2 = 1.41$, $\nu_3 = 0.5$; (b) $\nu_1 = 0.54$, $\nu_2 = 1.35$, $\nu_3 = 0.5$; (c) $\nu_1 = 0.57$, $\nu_2 = 1.21$, $\nu_3 = 0.3$; (d) $\nu_1 = 0.34$, $\nu_2 = 1.27$, $\nu_3 = 0.3$

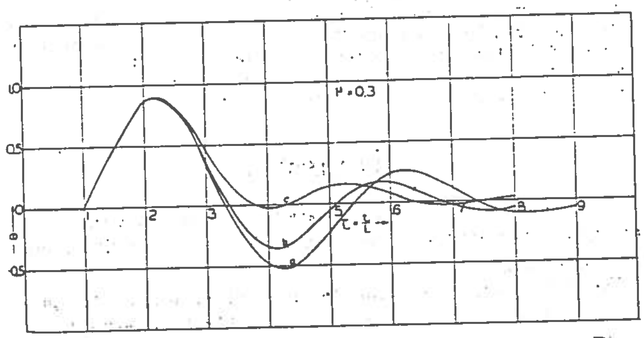


FIG. 20 RESPONSE CURVES FOR PROPORTIONAL-PLUS-RESET-PLUS-DERIVATIVE CONTROL

(a) $\nu_1 = 1.24$, $\nu_2 = 1.45$, $\nu_3 = 0.5$
(b) $\nu_1 = 1.02$, $\nu_2 = 1.5$, $\nu_3 = 0.5$
(c) $\nu_1 = 0.64$, $\nu_2 = 1.43$, $\nu_3 = 0.5$

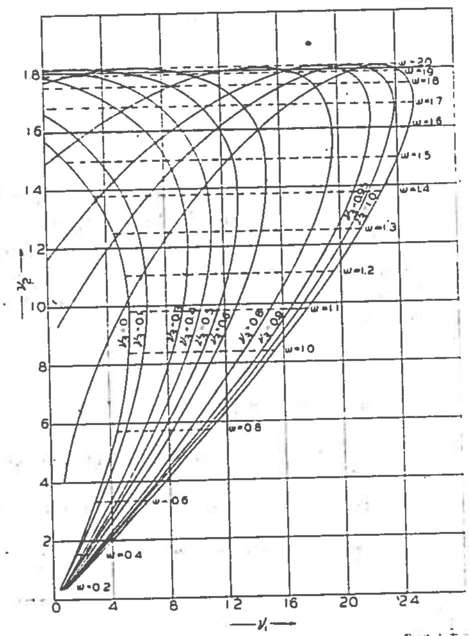


FIG. 17 STABILITY LIMITS FOR PROPORTIONAL-PLUS-RESET-PLUS-DERIVATIVE CONTROL, $\mu = 0$

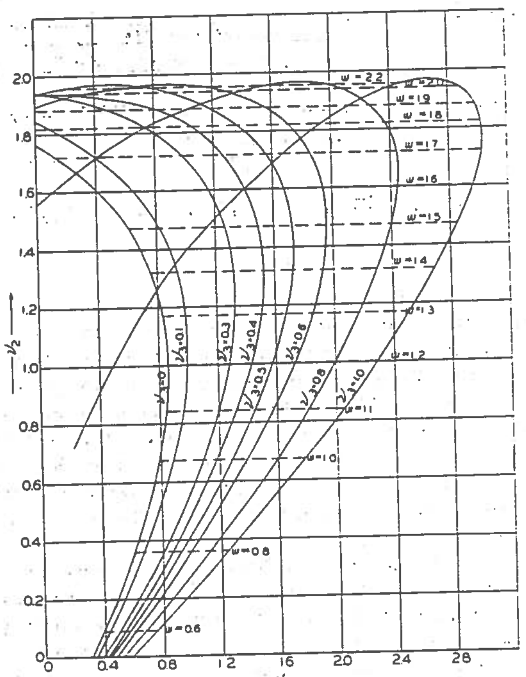


FIG. 18 STABILITY LIMITS FOR PROPORTIONAL-PLUS-RESET-PLUS-DERIVATIVE CONTROL, $\mu = 0.3$

$$U = \frac{\nu_1}{\nu_2 L} = \text{reset rate, adjustment knob setting}$$

$$T = \frac{\nu_3}{\nu_2} L = \text{derivative time, adjustment knob setting}$$

If there is interaction between controller settings, then the parameters ν_1, ν_2, and ν_3 are first obtained from the linearized controller equation. For example, a cascade controller (9) has the following relation between adjustable parameters and the control constants

$$\nu_2 = S'(1 + U'T')RL \qquad \nu_1 = S'U'RL^2 \qquad \nu_3 = S'T'R \quad [12]$$

where S' = adjustable knob labeled "Sensitivity"
U' = adjustable knob labeled "Reset Rate"
T' = adjustable knob labeled "Derivative Time"

Relationships [11] yield ν_1, ν_2, and ν_3, and then Equations [12] are used to obtain S', U', T', the actual knob settings of the cascade controller.

The control regions show that increasing the self-regulation index μ makes the loop gain SM smaller so that for a fixed process sensitivity, M, the controller sensitivity S decreases. Also, the control area and period decrease as μ increases.

The use of reset to remove offset makes the control less stable since a smaller value of sensitivity must be used in comparison with proportional alone. Moreover, the control-region graphs show that the period increases with the addition of reset to proportional control.

The addition of derivative action allows one to use increased sensitivity and reset rate. Hence the period is decreased further and the control area is decreased, giving much better control. It is apparent that the addition of derivative action decidedly improves control for values of self-regulation $0 \leq \mu \leq 1$. For processes having $\mu > 1$ derivative offers little advantage from the standpoint of load-change disturbances. However, as pointed out in reference (9) start-up of a batch process requires the use of derivative to prevent overpeaking. Hence derivative is almost always a desirable response.

ACKNOWLEDGMENT

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Discussion

Y. TAKAHASHI.[4] The authors are to be congratulated on presenting a paper with very clear conclusions and very useful control-region diagrams.

The authors' statements on the complexities of industrial processes and the presence of continuum are highly important, considering such complexities were occasionally ignored by some control mathematicians, leading to utopian conclusions such as infinite gain as an optimum.

Theoretically the horizontal asymptote in Fig. 3 (*a*) should be defined by S' of the following equation

$$\Delta F/Y = S'(1 + U'/p)(1 + T'p)$$

Putting this identical to the corresponding form of Equation [2] of the paper we get

$$S' = \frac{S}{2}\left(1 \pm \sqrt{1 - 4UT}\right)$$

But, in most cases, especially near the optimum settings, the difference between S and S' is very slight; moreover, the authors' horizontal line of the height "20 log S" comes nearer to the exact attenuation curve than that of S'.

Compared with the results of other papers,[5] the values from the authors' equation are among the higher. Generally, the conclusions depend upon the definition of "optimum," so if the transient response itself is considered instead of the fundamental mode of oscillation, they are influenced by the nature of disturbance, defined statistically or as a time function. For the latter case the writer tried an analysis. Assuming the disturbance of the form

$$\Delta D = \Delta D_0(1 - e^{-t/T_d})$$

and defining the optimum to be the minimum control area of $\int_0^\infty |\theta(\sigma)|d\sigma$, it was seen that the longer the T_d, the stronger the optimum settings; for example, the optimum settings of proportional controls are

	$\mu = 0$	$\mu = 1$	
$SRL =$	0.89	0.91	for $T_d = 0$
$SRL =$	1.07	1.40	for $T_d = 2L$

Finally, the writer would like to point out Dr. Oppelt's comments[6] on the importance of Ziegler-Nichols' ultimate sensitivity method. This enables us to take into account the self-regulation of the process, and according to Oppelt, its results approximately coincide with Hazebroek-Waerden's optimum values for proportional-plus-reset controls.

[4] Professor, University of Tokyo, Chiba-City, Japan.

[5] Authors' Bibliography (2, 3, 5, 6, 7), and "On the Automatic Control of Generalized Passive Systems," by Kun Li Chien, J. A. Hrones, J. B. Reswick, Trans. ASME, vol. 74, 1952, p. 175.

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Authors' Closure

For the process $\mu = 0$ Fig. 21 below shows the integral $\int_0^\infty \left|\theta(\sigma) - \frac{1}{\nu_2 + \mu}\right| d\sigma$ as well as the offset and amplitude ratio for various values of ν_2. It is evident that the absolute area defined by the integral has a very broad minimum. It appears that changing ν_2 from 0.58 to 1.25 does not change the absolute area more than 25 per cent whereas the offset changes about 50 per cent. Hence, 0.25 amplitude ratio appears to give a good compromise between offset and minimum absolute area in the case of proportional control. It can also be shown that this criterion of absolute area agrees with our results for a three-term controller. The authors wish to point out that Professor Takahashi has done considerable work along these lines.[7]

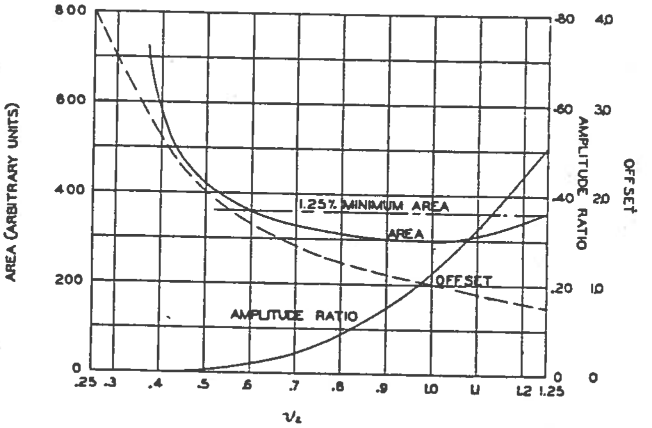

Fig. 21

Professor Takahashi's discussion brings out some interesting aspects concerning the interaction of knob settings in the cascade type of controller. As long as $U' < 1/T'$, the knob settings correctly describe the controller functions. However, when $U' > 1/T'$, the straight line approximations to the frequency response of the controller indicate that the knob labeled reset actually controls the derivative response and vice versa. Also, the sensitivity depends more strongly on the settings of the reset and derivative knobs.

It may be noted that the optimum settings given by Hazebroek and van der Waerden are based on a change in set point.

[7] "Automatic Control," by Y. Takahashi, The Science and Technique Book Co., Kanazawa, Japan, 1949 (in Japanese.)