keyword: controllability

21E
1943

NB: Very similar to RGA by McAvoy - at least use of time response.

Process Lags in Automatic-Control Circuits

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Methods are given for quantitative determination of time lags in automatically controlled processes. The area under recovery curves is taken as a direct measure of process difficulty, and this area is shown to vary as the second power of the time lag. A "recovery-factor" term, part of a complete expression for controllability, is introduced which makes possible a classification of processes in dimensions of the process itself, regardless of controller or valve mechanism used. Values of this recovery factor from various industrial applications are given in tabular form. Several processes are examined for the time lag, and means of reducing this unfavorable characteristic are demonstrated. It is felt that this paper will be useful to engineers who are interested in improving the controllability of the processes which they design.

THE importance of automatic controllers in the operation of modern plants is increasing yearly if the number of controllers used is any indication. Knowledge of instrument characteristics is also increasing; the theoretical action of each control effect has been expressed as a mathematical equation, and the newer instruments follow the equations very closely. Adjustment dials are even calibrated in terms of the constants appearing in the equations which describe the responses. Industry's demands for closer and closer control have forced the development of the refined control effects which the instrument manufacturer has supplied. Now it appears that the picture has become top-heavy. The science of instrument design has exceeded the study of process design for controllability.

In the application of automatic controllers, it is important to realize that controller and process form a unit; credit or discredit for results obtained are attributable to one as much as the other. A poor controller is often able to perform acceptably on a process which is easily controlled. The finest controller made, when applied to a miserably designed process, may not deliver the desired performance. True, on badly designed processes, advanced controllers are able to eke out better results than older models, but on these processes, there is a definite end point which can be approached by instrumentation and it falls short of perfection.

The chronology in process design is evidently wrong. Nowadays an engineer first designs his equipment so that it will be capable of performing its intended function at the normal throughput rate plus a factor of safety. The control engineer or instrumentman is then told to put on a controller capable of maintaining the static equilibrium for which the apparatus was designed. The control engineer faced with this do-or-die ultimatum recommends a type of controller basing his judgment on experience with similar jobs. If his analogy is good, the correct controller is selected. When the plant is started, however, it may be belatedly discovered that, in spite of the correct equipment design for steady-state conditions and the correct instrument selection, control results are not within the desired tolerance. A long expensive process of "cut and try" is then begun in order to make the equipment work. Both engineers realize that some factor in equipment design was neglected but generally they can neither identify the missing ingredient nor correct it in future design.

The missing characteristic can be called "controllability," the ability of the process to achieve and maintain the desired equilibrium value. Design for steady-state conditions is not enough if exact maintenance of variables is necessary. Control action consists of continuous correction of process changes, tending to destroy equilibrium at the desired value and, as such, its study involves not steady-state but transient characteristics of the process and controller.

A tubular heater for raising milk to the pasteurizing temperature may be designed with ample heating surface, and the steam supply may be adequate, but the maintenance of a constant milk outlet temperature by steam-valve manipulation is very difficult if milk flow or incoming temperature vary suddenly. A good controller will be able to bring the temperature back to the correct value following one of these disturbances but only at the expense of some deviation for a certain length of time. During the recovery period a loss results, since any increase in temperature spoils the "cream line" of the product and any drop requires reprocessing. These deviations are so important in milk pasteurization that most of the equipment now used has been designed to make excellent control results possible.

The problem of equipment design for controllability involves transient conditions and transients usually involve exponential curves and an order of mathematics not at the finger-tips of the average engineer. Even if he were able to deal with transient phenomena, he would not know where to start, since to the authors' knowledge no complete formula for controllability has ever been published. A great many of the factors affecting controllability have been identified and investigated in the numerous papers sponsored by the A.S.M.E. Committee on Industrial Instruments and Regulators. All of these factors affect controllability; no one is a complete solution to the problem, and each factor uncovered increases the certainty that the problem is a complex one, not to be solved in a day. As it now stands the plant designer is almost justified in disregarding the entire matter, hoping that the magic quantity, controllability, is included in his apparatus but turning the burden over to his instrument engineer.

Sooner or later, however, these factors affecting process controllability will have to be smoked out and reduced to definite "good-practice" rules which will be as much a part of equipment design as safety factors. Furthermore, establishment of rules is not enough; simple methods of applying the rules must be developed at the same time so that the complex mathematics involved will not be the stumbling block. It was possible to calculate the equilibrium conditions existing in a fractionating column before the McCabe-Thiele method of graphical analysis was developed but that method reduced the time required to a reasonable figure.

Unfortunately, the authors are not able to give a formula for controllability. It appears that when such a formula is devised it will consist of several factors. One might be called the "recovery factor," the ability of the process to recover from the maximum change in demand or load. Another, a "load factor," must take into account the point in the process at which the disturbance occurs. That expression will cover the thing called

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Contributed by the Committee on Industrial Instruments and Regulators, and presented at the Fall Meeting, Rochester, N. Y., October 12–14, 1942, of The American Society of Mechanical Engineers. Re-presented at 1942 Annual Meeting for discussion only.

Trans. of the A.S.M.E, 65, 433-444 (1943) "supply and demand side capacity" (1).[3] The third might be called the "mobility factor," the ability of the process to follow demands for different values of the variable. This factor would be important when controller set points had to be changed suddenly or changed gradually as on a "time-schedule" control problem. The failure of some time-schedule temperature-control applications has been due to lack of mobility, not lack of the recovery factor. Added to these three may be other secondary factors as yet unexplored.

In this paper, a tentative formula is set up for the first or "recovery factor" involving three process characteristics. One of these, time lag, is shown to be of primary importance and simple methods are given for approximating the time lag on actual examples of industrial control installations. This paper then attempts to deal with *only one* term in *only one* of the factors affecting controllability—the time-lag term in the recovery factor.

A Control Circuit

Illustrated in Fig. 1 is a control circuit consisting of a controller and a process (2). Note that no control valve is shown, it being considered a portion of the process. Between pen and output lies the controller which transforms pen behavior into appropriate output behavior. The output effects the process, changing some variable which is translated into a pen movement through the measuring portion of the circuit at the right. If for every output there were a definite pen position no controller would be necessary and manual control would suffice. The purpose of the controller is to keep the pen at the desired point in spite of the load changes which are shown entering the process at several points. It is these load changes which require altered output in order that the same pen position be maintained.

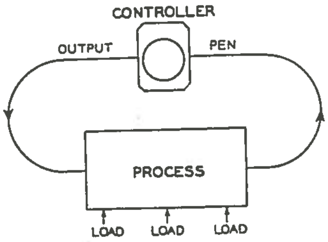

Fig. 1

Between output and pen lies the relatively uncharted portion of the control circuit, the process. It is with the latter that this paper is chiefly concerned, though a brief résumé of the control effects present in modern controllers and their characteristics must be included if only to establish a terminology. Air-operated instruments will be considered simply because they are the type most familiar to the authors, so output will be given as an air pressure in pounds per square inch. Pen movement will be given in inches in most cases.

In the process examples to follow it will be assumed that one set point is to be maintained regardless of process load conditions, so a controller with proportional and automatic reset (proportional-speed-floating) responses will be used. The first of these two control effects, proportional response, gives an output change proportional to pen movement; the magnitude of this response will be called "sensitivity," and the unit of sensitivity will be the output-pressure change per inch of pen movement. Automatic-reset response detects the deviation of the pen from the desired set point and gives a rate of output-pressure change proportional to the discrepancy. The magnitude of automatic-reset response will be called "reset rate," the number of times per minute which it duplicates the proportional-response output change caused by the discrepancy between pen and set point. A third control effect called "pre-act" or "derivative" response is often used on processes with long time lags. This response in its pure form gives an output-pressure change proportional to the rate of pen movement and its unit has been called the "pre-act time" in minutes. This response will be considered in this paper only to the extent of pointing out the processes on which it could be used effectively.

Process Reaction Curve

The magnitude of controller responses can be determined by impressing various pen movements and noting the resulting output-pressure behavior. It would appear reasonable then that some process characteristics could be identified by impressing an output-pressure change and noting the resulting pen behavior. The authors (5) have found that the "reaction curve" drawn by the pen in response to a sustained change in output pressure can be analyzed to give a fair picture of the process from the standpoint of optimum controller settings.

In order to visualize a process-reaction curve, consider the control circuit of Fig. 2 in which an actual process replaces the blank box of Fig. 1. Cold water flowing to a tank is heated by steam injection and flows at constant rate through a pipe line to the bulb of a temperature controller located some distance away. The pen moves in response to temperature changes at the bulb, and the output pressure alters valve opening and the corresponding steam flow to the tank. The principal load change on the process comes from variations in the temperature of incoming cold water. This process is chosen because it exhibits the least complex type of time lag, notably a "distance velocity" (3), or dead-period lag. A definite length of time is required for water from the tank to flow to the bulb; consequently, the temperature of water at the bulb will lag the tank temperature by this interval.

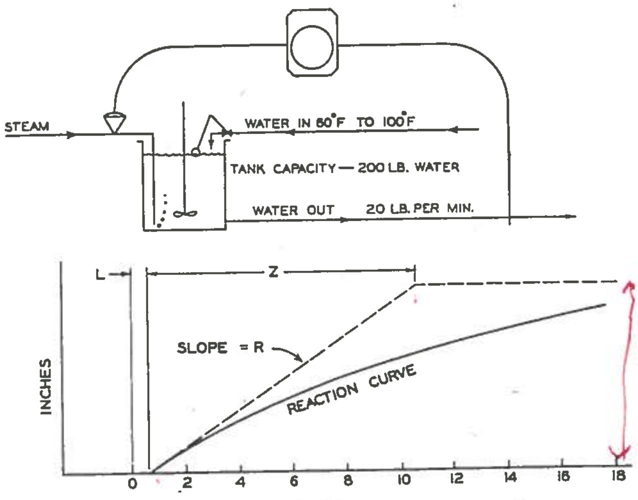

Fig. 2

$\tau = Z =$

Actually there are other lags, in the control circuit of Fig. 2, such as the lag of the bulb to changes in water temperature, the lag of moving the valve from one position to another, and small lags in the controller itself. In addition, the heat content of tank walls and of the pipe leading to the bulb would have an effect but it can be assumed that these factors are negligible in the example.

If the system Fig. 2 were in balance at a constant temperature and a small sustained change in output pressure of F pounds per square inch were made which opened the steam valve slightly, the tank temperature would immediately start to increase to-

$u = F$

$y = k \cdot \frac{e^{-Ls}}{1+Zs}$

largest slope: $\frac{k}{Z} = R$

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[3] Numbers in parentheses refer to the Bibliography at the end of the paper.

ward a new balance point. After the interval necessary for water to flow from tank to bulb, the pen would move accordingly. This process-reaction curve is shown in Fig. 2. Two characteristics of the reaction curve are used to determine the optimum controller settings, the "lag" L (minutes) and the maximum rate of pen movement caused by the impressed output change which is called the reaction rate R (inches per minute). Experimental work has shown that optimum settings for a controller with proportional-and-automatic-reset responses are approximately

$\frac{1}{K}$

$$K_c = \text{Sensitivity} = \frac{0.9\,F}{RL}\ \text{psi per in.} = 0.9\,\frac{\tau}{\theta}\,\frac{\Delta u}{K'}$$

$1/K$

$$\tau_I = \text{Reset rate} = \frac{0.3}{L}\ \text{per min} \Rightarrow \tau_I = 333 \cdot \theta$$

These settings appear to be very nearly correct on the processes tested for wide variations in the values of R and L. On those infrequent processes in which L becomes greater than Z_0, Fig. 2, the settings given are too conservative.

There are two drawbacks to the use of experimental reaction curves for process analysis. In the first place the disturbance caused by running a reaction curve, can seldom be tolerated on a continuous process. In the second place, it is necessary to have a process on which to run the test, and this the designer does not have since pilot-plant and full-scale units will usually have widely different control characteristics. Nevertheless, reaction curves are very practical because they give a simple pictorial representation of a process and an explanation of process difficulties which is almost impossible to reach by chasing air, steam, and temperatures around the control circuit. In addition, process-reaction curves can often be calculated quite easily as will be shown. In fact, it is often easier to calculate a reaction curve than to believe that so simple a picture actually gives an indication of controller settings.

CALCULATED REACTION CURVE

In order to calculate the controller settings required for the process of Fig. 2, it is only necessary to find values of F, R, and L. Assume the following data:

Water in tank, lb	200
Water in line between tank and bulb, lb	12
Water flow lb per min	20
Steam flow (maximum) lb per min	6
Incoming-water temperature (minimum), F	60
Incoming-water temperature (maximum), F	100
Outlet-water temperature, F	160

Diaphragm-operated valves normally require a pressure change of about 12 psi to give full stroke. Each pound per square inch change in output will make $^1/_{12}$ of the total steam flow of 6 lb per min or $^1/_2$ lb per min. This assumes that the valve has the linear characteristics which are correct for this process (4, 5). If the tank temperature were constant at 160 F and a 2-psi change in output were made, increasing the heat flow by 1000 Btu per min the tank temperature would start to rise $^{1000}/_{200}$ or 5 F per min. Assuming 1 in. on the instrument chart or scale equivalent to 25 F, the reaction rate R, resulting from a 2-psi change in output, would be 0.2 in. per min, and the unit reaction rate R_1 for a 1-psi change would be

$$R_1 = \frac{R}{F} = \frac{0.2}{2} = 0.1 \text{ in. per min per psi}$$

The time lag of the process will be the time necessary for the water flow of 20 lb per min to displace the 12 lb of water between tank and bulb

$$L = \frac{12}{20} = 0.6 \text{ min}$$

The controller settings for proportional-and-automatic-reset responses will then be

$$\text{Sensitivity} = \frac{0.9\,F}{RL} = \frac{0.9}{R_1L} = 15 \text{ psi per in.}$$

$$\text{Reset rate} = \frac{0.3}{L} = 0.5 \text{ per min}$$

In terms of output pressure, the maximum load change, ΔF, which can occur in the process would be the difference in heat input required to raise 20 lb per min of 60 F water to 160 F and that required for 100 F inlet water divided by the valve constant of 500 Btu per min per psi

$$\begin{aligned}(20)(160 - 60) &= 2000 \text{ Btu per min}\\ (20)(160 - 100) &= \underline{1200}\\ \text{Maximum load change} &= 800/500 = 1.6 \text{ psi} = \Delta F\end{aligned}$$

Now let us see how this calculated reaction curve can be translated into one measure of process controllability.

RECOVERY FACTOR

On this process, a controller adjusted to the foregoing values of sensitivity and reset rate would correct the maximum load change of 1.6 psi (incoming-water change from 60 to 100 F) at the expense of a recovery curve similar to that shown in Fig. 3. The shaded area under the curve can be taken as a measure of maximum process difficulty inasmuch as recovery curves for load changes smaller than 1.6 psi would enclose less area. In order to determine the area, it is only necessary to remember that automatic-reset response changes controller output at a rate proportional to the distance between pen and set point, the latter converted by proportional response into an output change. It can be shown that the "net area" under the recovery curve of Fig. 3 will then be equal to

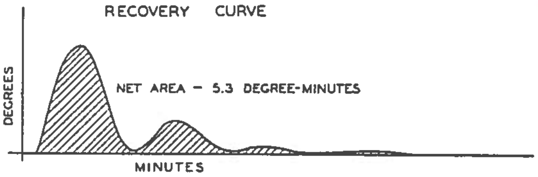

FIG. 3

$$\text{Net area} = \frac{\Delta F}{(S)\,(RR)} \text{ in-min} \qquad [1]$$

where ΔF = load change, psi
S = sensitivity, psi per in.
RR = reset rate, per min

The worst load change of 1.6 psi which can afflict the process considered will then produce an area of 0.21 in-min or 5.3 deg-min.

It is obvious from Equation [1] that lowering either controller setting will increase the net area. Likewise, increasing the sensitivity will increase the amplitude ratio of oscillations in the recovery curve and also increase the area. Raising the sensitivity from $0.9/R_1L$ to $2/R_1L$ would give an oscillation which would never die out and the area would become infinite. An increase

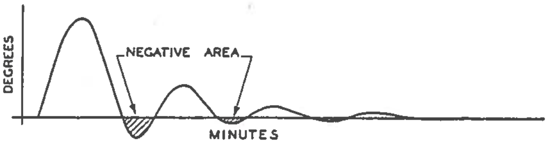

FIG. 4

in reset rate would allow the recovery curve to swing on both sides of the set point adding a negative area as shown in Fig. 4. This reversed swing would undo an equal portion of the work done by automatic reset while the pen was above the set point and the total area would become

$$\text{Total area} = (\text{net area}) + (2)\ (\text{negative area}) \ldots\ldots\ldots\ldots [2]$$

In the authors' experience the settings given reduce the total area to about the minimum possible without introducing an additional control effect such as pre-act response.

If the optimum controller settings are then

$$\text{Sensitivity} = \frac{0.9}{R_1 L} \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

$$\text{Reset rate} = \frac{0.3}{L} \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

These values may be substituted in Equation [1] to give

$$\text{Net area} = 3.7\,\Delta F R_1 L^2 \text{ in-min} \ldots\ldots [5]$$

In the recovery curve of Fig. 3 where the net area is equal to the total area, the latter is also equal to $3.7\,\Delta F R_1 L^2$. This holds true for processes similar to that shown in Fig. 2 where the load change occurs at essentially the same point in the circuit as that at which the output-pressure effect takes place. This is the same as saying that processes in which the reaction curve, caused by a sustained change in load, and the reaction curve caused by an equivalent valve change are identical, the net area is equal to the total area, and the total area is equal to $3.7\,\Delta F R_1 L^2$. Quite often, however, load changes occur at other points in the circuit, and the recovery curve swings on both sides of the set point, even at the optimum controller settings. This would be the case in the process of Fig. 2, if the principal load change were not the temperature of incoming water but a heat gain or loss in the pipe line between tank and bulb. In that event the total area would be greater and a "load factor" would replace 3.7 in Equation [5] when solved for the total area under a recovery curve from maximum load change. The total area can be expressed as

$$\text{Total Area} = (\text{Load Factor})\ (\text{Recovery Factor}) \ldots\ldots [6]$$

where both the load factor and recovery factor are characteristics of the process. The effect of load changes at various points in the process has not been completely investigated by the authors so they cannot quantitatively fix the load factor except for the limited case noted in which it is equal to 3.7. This factor has been qualitatively investigated by others as the relation between supply and demand side capacities (1). The recovery factor, $\Delta F R_1 L^2$, equal to 0.0575 in-min or 1.44 deg-min in the given example, has been used by the authors as a means of process classification and appears to be a good yardstick for evaluating this phase of process controllability.

In a previous paper (5), it has been pointed out that values of R_1 and L can be determined during adjustment of a controller on an application. The proportional-response sensitivity, which just gives sustained oscillation, is called the "ultimate sensitivity" S_u, and the period of oscillation at this sensitivity is called the "ultimate period" P_u. If S_u is taken in pounds per square inch per inch, and P_u in minutes, R_1 and L are determined by the formulas

$$R_1 = \frac{8}{(P_u)\ (S_u)} \text{ in. per min per psi} \ldots\ldots\ldots\ldots [7]$$

$$L = \frac{P_u}{4} \text{ min} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [8]$$

The difference between the controller-output-pressure readings at minimum and maximum loads is equal to ΔF pounds per square inch. The recovery factor is then

$$\boxed{\text{Recovery factor} = \overset{\text{dist.}}{\Delta F} R_1 L^2 =} \frac{(\Delta F)(P_u)}{2S_u} \text{ in-min} \ldots\ldots [9]$$

Miscellaneous values of ΔF, R_1, L, and the recovery factor taken from various applications are given in Table 1 only to show the range of recovery factors. Note that on the temperature-control applications the recovery factor is also converted to "degree-Fahrenheit-minutes," and the pressure-control applications given as "psi-min." Temperature and pressure applications can only be compared on the "inch-minute" basis. Most of these values are calculated from ultimate sensitivity, period, and ΔF readings, taken during instrument adjustment. Some are taken also from experimental reaction curves.

TABLE 1 FACTORS FOR VARIOUS APPLICATIONS

	ΔF	R_1	L	$\Delta F R_1 L^2$, in-min	$\Delta F R_1 L^2$	
Ammonia absorber..........	10	0.07	8.7	53	1100	Deg F-min
Fractionating column........	5	0.06	7.5	17	410	Deg F-min
Superheater.................	3	0.38	2.1	5	240	Deg F-min
Oil-tube still................	3	0.13	3.1	3.7	180	Deg F-min
Wet bulb...................	6	0.02	4.5	2.4	45	Deg F-min
Dry bulb...................	2	0.22	0.77	0.26	5	Deg F-min
Milk heater................	0.5	0.60	0.67	0.13	3	Deg F-min
Canners' retort.............	2	0.17	0.03	0.0003	0.008	Deg F-min
Column vent................	0.5	0.2	0.08	0.0006	0.036	Psi-min
Air pressure................	0.2	37.5	0.002	0.00003	0.0006	Psi-min
Water flow..................	4	0.56	0.12	0.032	0.55	Gal

Determination of ultimate sensitivities and periods during controller adjustment and subsequent notation of maximum and minimum output-gage readings provide a ready means of arriving at process characteristics in terms of the recovery factor. It is hoped that industrial plants will tabulate these data for all control applications so that a rational classification of processes will someday result.

Process Design to Reduce Lag

The recovery factor has been identified as one of the important characteristics determining the controllability of industrial processes. Let us now turn to the question of process redesign to reduce this factor. In the process of Fig. 2 a reduction in size of the maximum load change would reduce the recovery factor although generally load changes are a "death and taxes" sort of quantity and cannot be avoided. Assuming this is the case in the process of Fig. 2, our efforts will have to be directed at R_1 and L. An increase in tank size will reduce R_1 since the heat storage will be increased. The effect of doubling the tank size would be to halve the unit reaction rate and consequently double the proportional-response sensitivity. Fig. 5 shows that each wave in the new recovery curve would have just one half the amplitude as before so the area would be halved. Obviously a reduction in valve size

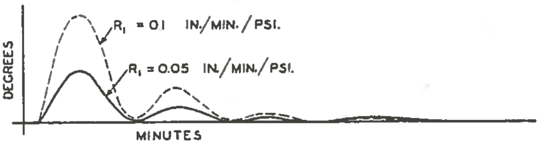

FIG. 5

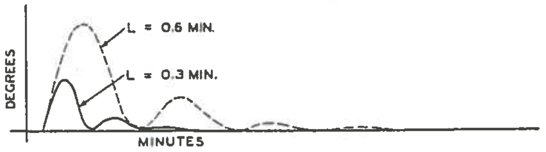

FIG. 6

would reduce R_1 but increase ΔF proportionately. In this case, and generally, reduction of $R_1\Delta F$ in a process entails increase in the physical size and cost of the apparatus.

Moving the bulb to a position one half as far from the tank would halve the lag of our process and allow both sensitivity and reset rate to be doubled. The new recovery curve. Fig. 6 would have one half the amplitude and one half the period of the former process and consequently only one fourth the net area. This is shown by the recovery factor which varies as the second power of the lag. Reduction of process lag usually means only a process rearrangement and has a greater proportionate effect than comparable change in R_1. The remainder of this paper will therefore consider only means of reducing process lags, leaving a study of reaction rate for a future paper.

LAG OF MULTIPLE-CAPACITY CIRCUITS

It will be relatively easy for the process designer to identify the simple distance-velocity lags and take steps to reduce them to a minimum. Faced with the process of Fig. 2, he would move the temperature bulb as close as possible to the tank. Unfortunately, however, most processes are made up of a series of resistances and capacities and the effective lag is a complex function of the number and size of these *RC* (resistance-capacity) units (3). Exact determination of lags is not usually necessary and it is believed that the following method of approximation is sufficiently accurate for practical purposes:

In Fig. 7 another process is shown in which a constant flow of water is heated in one tank and overflows to a second tank. The reaction curve for this process would be S-shaped. It has been found by experiment that this reaction curve can be approximated by the two dotted lines in Fig. 7. The slanting line is drawn tangent to the point of inflection and intersects the original temperature a time *L* after the output change was made; this time of *L* min being considered the effective time lag of the circuit.

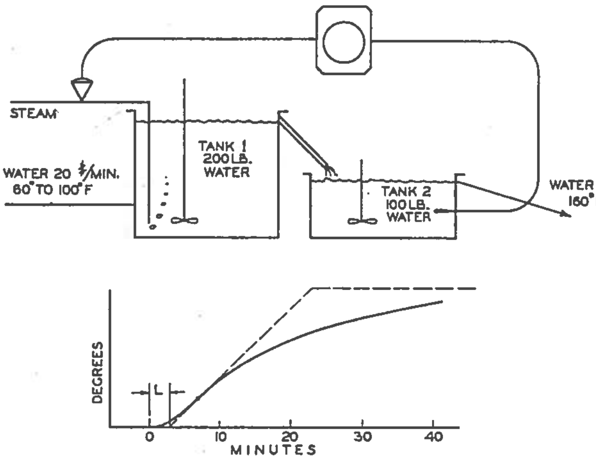

FIG. 7

In order to determine the lag in this circuit, it is first necessary to evaluate the time constant of the two principal capacities separately. A sudden change in steam flow of say 100 Btu per min to the first tank would cause its temperature to rise as shown in Fig. 8, rapidly at first and then slower and slower as it approached the new equilibrium point. This curve is an exponential curve and has one characteristic in which we are interested. At any instant, the temperature is rising at a rate proportional to the remaining temperature difference, so it would always reach equilibrium in a definite length of time if it continued at that rate. Note that the two tangents to the curve of Fig. 6 reach the final temperature in the same interval. This time has been called the "time constant," "characteristic time," "lag," etc. (2, 6) of the exponential curve. Here it will be referred to as the "impedence," *Z*, of the *RC* unit. The sudden introduction of 100 Btu per min to tank 1 over and above the amount of heat necessary to maintain 160 F will first cause the tank to rise at a rate of 100/200 or 1/2 F per min. Eventually the 20 lb per min of incoming water will be heated 100/20, or 5 deg to 165 F and the system will again be at balance. The impedence, *Z*, of the curve is then 5 deg divided by $^1/_2$ deg per min or 10 min. Any other change in heat flow could have been used still giving the same value. In like manner the second tank alone would respond to a sudden change in its incoming-water temperature by giving a similar curve with an impedence *z* of 5 min. The response of both tanks together to a sudden change in heat input of 100 Btu to tank 1 is shown in Fig. 9. Tank 1 rises on an exponential curve with a 10-min time; the temperature of tank 2 rises on the S-shaped curve. At any time *A* it is rising at such a rate that it would reach the corresponding incoming temperature *B*, 5 min later at *C*. This curve is complex mathematically but the lag *L* can be readily determined from Fig. 10. The ratio of z/Z is 0.5, showing a factor of 0.39. The smaller *z* multiplied by this factor gives *L* for the two impedences of 5 and 10 min as (0.39) (5) or 1.95 min. Actually *L* for the entire circuit would include an impedence due to the valve motor and one due to the bulb but in this case they are small by comparison and can be neglected.

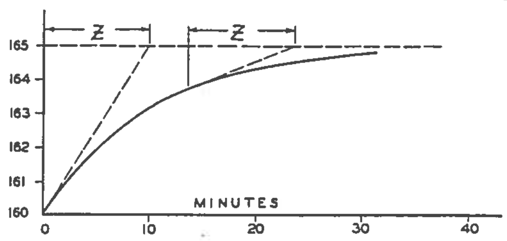

FIG. 8

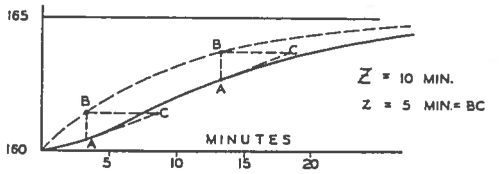

FIG. 9

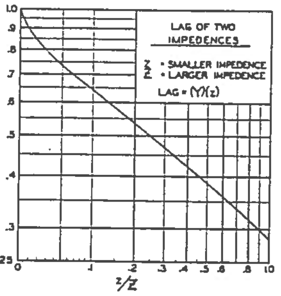

FIG. 10

Reduction of *L* in the circuit can be accomplished by reducing the impedence of either unit, preferably the smaller. Making tank 2 one half as large would cut its time to $2^1/_2$ min, and *L* would become 1.25 min, while halving the size of tank 1 so that both tanks had 5-min impedences would only reduce *L* to 1.41 min. Thus tank 2 should be considered first and the greatest possible reduction made in its capacity.

If tank 2 could be completely removed and the bulb placed in tank 1, the system would apparently have only one capacity and consequently no lag. A great deal of work has been done on these single-capacity systems (7), although they are only approached in industrial circuits since other impedences become appreciable. In this case the impedence of the valve would enter the picture as small z.

The response of diaphragm valves to a sudden output change at the controller is not a true exponential, although an approximate impedence can be used which varies with motor size, controller-relay-valve capacity, and the friction in the line between controller and valve. Figures between 0.05 min and 0.5 min are found on different sizes of valves with a multiplying factor of 2 or 3 for long connecting lines. A normal figure might be 0.15 min. Eliminating tank 2 would then leave two major impedence units again, tank 1 with 10-min impedence, and the diaphragm valve with 0.15-min impedence. The lag from Fig. 10 would be 0.13 min. This lag represents about the lowest limit for considering the use of the derivative (pre-act) responses now available. Some reduction in lag could be accomplished by reducing the size of valve motor or length of connecting line though this is not always possible.

Further improvement then would consist of complete elimination of tank 1, making the process simply a steam-water mixer, at which time another impedence would become appreciable, that of the bulb. A definite amount of heat is required to raise the temperature of a bulb and there is only so much area through which the heat can flows, so the controller bulb in the flowing water would have a definite impedence. This time for bulbs has been investigated quite thoroughly and data are available (6, 8). With water flowing at good velocity past a bulb its time is roughly 0.05 min. The remaining combination of $Z = 0.15$ for the valve and $z = 0.05$ for the bulb would give, from Fig. 10, a lag of 0.022 min or 1.3 sec. If the bulb were located a short distance downstream of the mixing point, a small distance-velocity lag might exist which could be added to the 0.022-min figure. Successive reduction in the number and size of impedences in the circuit of Fig. 7 has made possible a 100 to 1 reduction in L. Note that only the time-lag term in the recovery factor is being considered. In certain cases the increased unit reaction rate which may attend reduction in lag can overbalance the good results although in general any reduction in lag will improve controllability.

The complete reaction curve for the circuit of Fig. 7 is not quite the same as that of Fig. 9 since the former has the actual valve and bulb impedences included. With small error the lag shown in Fig. 7 can be calculated by adding the two small times of 0.15 and 0.05 to the 1.95-min lag of the two principal impedences giving a total circuit lag of 2.15 min. This problem of approximating the circuit lag when several impedences are present divides itself into three groups:

1 The lag of a circuit consisting of one very large and several very small impedences will be very nearly equal to the sum of the small ones. Example: One 10-min and three 0.05-min impedences would give a lag of approximately 0.15 min.

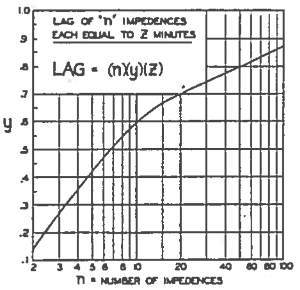

FIG. 11

2 The lag of a circuit consisting of two large and several very small impedences is approximately equal to the lag of the two major times determined from Fig. 10 plus the smaller times. Example: The lag resulting from one 8-, one 3-, and two 0.1-min impedences would be the lag of the two larger ones, 1.3 min, plus 0.2 min, or 1.5 min.

3 The lag of a circuit consisting of several equal impedences is found from Fig. 11. Example: Five impedences each with a time Z of 0.8 min would give a lag equal to $5 \times 0.42 \times 0.8$ or 1.68 min. This case is found in fractionating columns, absorbers, etc., where each plate from the point of measurement up to a regulated liquid reflux flow constitutes an impedence equal to the volume of liquid held on each plate divided by the reflux volume. Several impedences much smaller than the value of the large equal ones can be added directly to the lag found from Fig. 11, without much error, e.g., two 0.1-min impedences would increase the foregoing lag to 1.88 min.

PRESSURE CONTROL

The simple pressure-control circuit shown in Fig. 12 can have three appreciable impedences, namely, valve, tank, and measurement. The valve impedence can be taken as 0.15 min. The measurement impedence varies widely and depends upon the length and size of connecting tubing, upon the material in the line, and upon the volume displacement of the measuring element in the controller. For air pressures measured by Bourdon springs with $^3/_{16}$-in-ID connecting tubing the time will be about 1 sec per 100 ft of tubing. In this problem we will assume a 0.01-min measuring impedence. The tank impedence will not be constant at all loads but will be a maximum at the no-load condition. With no outflow, the slightest valve opening will cause the tank pressure to rise toward the supply pressure of 100 psi, so under these conditions the impedence will be essentially infinite. At any rate it will be a great deal longer than either of the other two impedences in the circuit, so rule 1 applies and the circuit lag will be equal to 0.15 + 0.01 or 0.16 min.

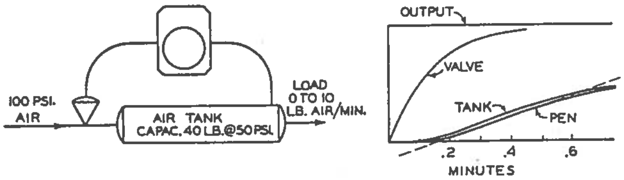

FIG. 12

A pressure-control application can often be improved by reducing the length of tubing between tank and instrument or filling the tubing with a less viscous medium. Using a smaller valve motor and shortening the air connection between instrument and valve can reduce this impedence. The "booster relays" offered by some manufacturers are designed to give faster valve action. As a general rule, damping introduced in either the measurement or output connections will increase the lag since these applications follow rule 1.

FLOW CONTROL

In a liquid-flow-control circuit such as that of Fig. 13 there are

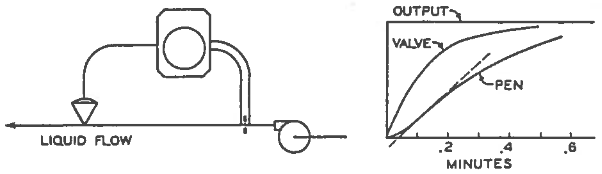

FIG. 13

only two appreciable impedences, those of valve and measurement. Except for small inertia effects in the flowing liquid its flow rate follows valve movement. Measurement time varies with manometer displacement, length of connections, and material moved in the connecting lines. The response of the mercury manometers used in industry while not a true exponential can be given an effective impedence which is generally not less than 0.15 min, even with short connecting lines filled with material of low viscosity. Combining this with a normal valve impedence of 0.15 min the lag shown in Fig. 10 is 0.04 min. This checks well with actual practice since the 0.04-min lag would give an ultimate period (5) of 4 times this value or 10 sec. Industrial flow controls rarely show periods less than this figure. Reduction of either valve or measurement impedence improves a flow-control application; the former has been covered under the pressure-control example. Manometer impedence can be lessened by opening the mercury-damping valve with which most are equipped and locating the manometer as close to the orifice as possible using remote transmission systems if it is necessary to carry the indication of flow to a central panel. Recent development of so-called "aneroid" manometers is a step in the right direction, as they have less inherent resistance to change and less displacement than the corresponding mercury type.

AIR HEATER

Control of conventional air-heating apparatus represents a rather difficult control application when air temperatures must be held within close limits and must recover quickly from changes in load. A good example of current interest is the problem of controlling carburetor air temperature in aircraft-engine testing where the multitude of readings cannot be taken until carburetor air temperature is correct. The wide change in heating load occurring when engines are "gunned" must be corrected quickly as time is an important factor.

Conventional design of air-heating equipment usually neglects all consideration of the priceless ingredient, controllability, and as a result adequate control is often not attained. Fig. 14 shows the system normally used and its reaction curve. A tubular steam-heated surface is placed in the air duct, a bulb installed downstream, and a control valve located on the steam line to the heat exchanger. Three appreciable impedences are present, the valve, the heating surface, and the bulb. The data are given in Fig. 14; an air flow of 10 to 40 lb per hr at some temperature between 0 and 70 F is to be heated to 80 F with steam at 50 psi.

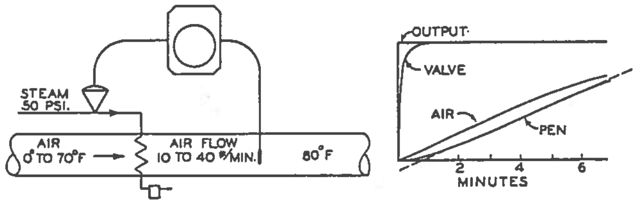

FIG. 14

The heating surface necessary for maximum load of 40 lb per min of zero-deg air would be approximately 75 sq ft, after including a factor of safety. The weight of metal in the heating surface would be in the neighborhood of 150 lb which would have a heat capacity of about 15 Btu per deg.

The impedence of the heating surface will vary with load but will be greatest under minimum load. In this case it can be most easily estimated by calculating the equilibrium existing at minimum load and finding the rate of fall toward incoming-air temperature with no heat inflow. To heat 10 lb per min of 70-deg air to 80 requires 23.7 Btu per min. The temperature of the 75-sq ft heating surface will only be a few degrees above the air temperature, say 83 F. If, from this equilibrium, the steam were suddenly shut off, the heating-tube temperature would fall toward 70 F, the incoming air temperature. The initial heat flow would be 23.7 Btu per min which if continued would dissipate the 15 × (83—70) or 195-Btu content of the metal in 195 /23.7 or 8.2 min. The heat-content figure of 195 Btu assumes that all the metal in the coil is at the 83-deg skin temperature, which is essentially correct inasmuch as the air film constitutes the largest resistance to heat flow. It also disregards the heat content of the steam in the tubes but the 8.2-min figure for surface impedence is sufficiently exact to show the weakness of the system.

Impedences of bulbs in air are quite large and depend upon bulb diameter and air velocity. The time in min for a $^1/_2$-in-diam bulb is about $100/U^{0.5}$ where U is the air velocity in feet per minute. The constant of 100 for $^1/_2$-in. bulbs becomes 72 for $^3/_8$-in. bulbs, 23 for $^3/_{16}$-in. bulbs, and 12 for $^1/_8$-in. capillary bulbs. If 1000 fpm is taken as maximum duct velocity in this problem, minimum velocity would be 250 fpm and the impedence of a $^3/_{16}$-in-diam bulb would be 1.4 min. The small steam valve would have an impedence of about 0.1 min.

The circuit lag can then be evaluated by combining the three impedences of 8.2, 1.4, and 0.1 min according to rule 2 which gives a lag of 0.88 min. Sudden changes of air flow through the duct of Fig. 14 will cause disturbances from which the system can recover only after a considerable time has elapsed. The period of oscillation (5) will be about $5.7\,L$, or 5 min, and if two appreciable waves are required in the recovery curve before the correction is essentially complete, at least 10 min will have elapsed. This delay is generally intolerable in aircraft-engine testing.

The process of Fig. 14 could be improved somewhat by installing a bulb of smaller diameter although even the $^1/_8$-in. bulb would still leave a process lag of 0.64 min. Pre-act response included in the controller would reduce the effective lag by perhaps 40 per cent to 0.4 min. Even so the period of oscillation would be 2.3 min. So instead of patching up this poor process let us consider a complete redesign keeping our eyes on controllability.

In Fig. 15 a rearranged process is sketched which has only two appreciable impedences instead of three. Air flows through two ducts in parallel; one contains the heating coil, the other is a by-pass. Temperature is controlled by operating a damper to mix cold and hot air. Since this system is designed for controllability, the smallest-diameter bulb available will be used, even though it is no longer the controlling (smaller) impedence. The motor for damper operation will be somewhat larger than before so its time can be estimated at 0.15 min. The bulb will have an impedence of 0.76 min, which combined with the 0.15-min valve impedence gives a circuit lag of only 0.08 min. This compares favorably with the 0.88-min lag of the first system. Further reduction in lag could be accomplished by using a "booster relay" to lower the valve impedence. Decreased bulb impedence would have only a small effect on the lag.

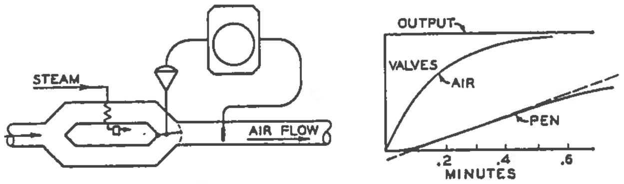

FIG. 15

The increased controllability of this process over that shown in Fig. 14, if measured in terms of the recovery factor, would be as the ratio of the lags squared or 120 to 1. This is not an insignificant improvement. It should also be noted that ΔF in the recovery factor would also be reduced by the redesign because very little repositioning of the damper would be required to compensate for a large increase in the air flow. The ratio of hot and cold air would remain essentially constant as long as incoming temperature remained the same.

CONCLUSION

In this paper the authors have attempted to show the quantitative effect of process time lag on control. One yardstick for the measurement of controllability, the recovery factor, has been introduced which seems to show that controllability varies as the square of the lag. Equations have been given for determination of the recovery factor on control circuits during instrument adjustment, so that this characteristic of processes may be easily found and tabulated. Rapid solution for the effective lag of multiple-capacity circuits is made possible by Figs. 10 and 11, and illustrative problems have been solved.

The few examples, illustrating lag reduction by process redesign, should point the way toward a better understanding of this important step in control improvement. Methods must also be made available for evaluating the other factors affecting controllability, as well as the terms other than lag in the recovery factor. Analysis of tubular liquid heaters for lag is possible, but an example was omitted from this paper, as the authors feel that their present method can be simplified considerably. It is hoped that the picture of controllability, and the methods developed in this paper for lag determination, will be of some immediate assistance to the conscientious process engineer who is anxious to include greater controllability in the equipment he designs.

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Discussion

W. F. HICKES.[4] Ever since the instrument industry learned to make controllers with two and three types of response, the on-off controller has been regarded as a "poor relation" not to be mentioned in the same breath with its betters. Yet every practical instrument man knows that on-off control is the best control for the many processes which are essentially single capacity and substantially free from distance-velocity lag. It is interesting to see the result obtained by applying the authors' methods of calculating controller settings to such a process. It was stated that for optimum performance

$$\text{Sensitivity} = \frac{0.9}{RL}$$

However, for a single-capacity process without distance-velocity lag, the reaction curve has its maximum slope at the origin and $L = 0$. Substituting this value in the previous equation, we find that the optimum sensitivity is infinite. In like manner, we may calculate reset rate with a similar result; the optimum reset rate is infinite. Since a controller having infinite sensitivity or infinite reset rate is an on-off controller, we have confirmed the conclusion reached through practical experience that, for the case under consideration, the on-off controller is the best controller. Thus, the lowly on-off controller is not a cheap substitute that will "get by" but is actually supreme in its proper field.

It should be added in passing that this applies only to a true on-off controller, one without appreciable dead space or throttling action. Also, even the most perfect on-off mechanism will be worthless if a sluggish thermal system or inadequate control-relay capacity makes a single-capacity process effectively multicapacity.

P. W. KEPPLER.[5] The fact is brought out in this paper that the recorder (or indicator) type of flow controller in many cases suffers excessively from the inertia accompanying the metering element. The authors have made no mention of the nonrecording (and nonindicating) type of flow controller in which the motion of the measuring elements is made negligible, thereby eliminating fluid displacement (as well as other bad effects due to the motion of the sensitive measuring elements). Nonrecording and nonindicating flow controllers are of course used very extensively.

The authors recommend placing the metering element of the indicator close to the orifice and using a device for remote indicating. This, however, only eliminates fluid displacement in connecting tubing, not that in the meter, nor the inertia of the meter.

A much better solution would probably be to make the instrument servo-operated and thereby derive an inertia-free flow measure for controlling. For example, the flow differential could be converted into air pressure by a suitable pilot-valve arrangement, essentially without fluid displacement. This air pressure could in turn be used for controlling as well as for recording or indicating. This would probably cost no more than a device for remote indicating, and would appear to offer a much better solution.

Besides eliminating inertia, an essentially motionless sensitive element should also be much easier to make accurate and durable.

These thoughts would seem to apply to a greater or lesser extent to all controllers and instruments where improvements can be economically justified.

A. A. MARKSON.[6] This paper must be rated as a substantial addition to the Society's literature on automatic control. The method of attack is empirical and experimental and will probably find a certain disfavor among "control mathematicians." What is not generally realized is that the differential equations of highly irreversible processes are well handled by systematic empiricism. This is a platitude to workers in the fields of heat transfer, hydraulics, and aerodynamics, to name several. From this point of view, the writer considers the authors' work as a valuable step in this direction. However, empiricism, which is not carefully founded on the fundamental equations of a science, should be handled cautiously. While the authors' formulas for "reset rate" and "sensitivity" are undoubtedly very useful, the writer believes it worth their trouble to set these formulations up in dimensionally consistent form. For example, reset rate is given the dimension "per minute," which is at the least somewhat confusing.

The examples given to show how controllability may be improved are worth study. The interesting case of where control

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may be improved by a change in the controlled variable or by the use of auxiliary variables has not been considered, doubtless as being outside of the scope of the paper. One illustration of the former will serve to show how, in certain very practical applications, the choice of the proper controlled variable radically alters the controllability of the process. Let us suppose it is desired to control the temperature in a deaerating water heater by bleeding live steam to the heater. This control may be effected either from the heater water temperature or from the heater pressure, since the two are uniquely related. Both practice and the principles of the paper show that the pressure control is usually preferable.

The matter of terminology is a troublesome one in the control field. Use of coined words such as "pre-act" and "impedence" will naturally encounter the purist's scorn. Yet they have much to recommend them because, like a good trade-mark, they identify some very definite phenomena with their proper field of technology. The use of the term "sensitivity" to denote proportional response is unfortunate. To bring this point out clearly, consider the use of the term as applied to galvanometers, the sensitivity of which is often referred to as deflection per microvolt. If we take a relatively crude galvanometer and add a relay or magnifier to it, we can give it the same deflection per microvolt as a better instrument. Thus, the two instruments have now the same "sensitivity," as defined by the paper. Yet changes can occur which will cause absolutely no response whatever in the poorer instrument.

It does not seem right to appropriate a term long used in metrology as a figure of instrument merit merely to denote relay action, especially since there is still a definite need for reserving it as a figure of merit even in the control field. The use of this term for proportional response goes back, without doubt, to the days when controllers "hunted" because they were too "sensitive." The term we employ to denote proportional response is "gradient." When used in a resetting controller, it is denoted as "temporary gradient." This but adds one more expression to the terminology of control which is badly in need of standardization.

J. B. McMahon.[7] This paper discusses how process lags affect process "controllability," in an attempt to make it possible for process designers to incorporate "controllability" in their designs, as well as the other necessary factors. However, the basic assumption is made in the paper that "controllability" is a matter of degree, or difficulty, and that all processes are controllable to some extent.

This is frequently not the case. Before the difficulty of controlling a process can be checked or calculated, it is necessary to determine whether it is controllable at all. This question has never been widely discussed in the literature on automatic control but has been mentioned by the writer.[8]

Briefly, the factors to be considered are as follows:

(*a*) Susceptibility to measurement.
(*b*) Significance of measurement.
(*c*) Susceptibility to automatic control.
(*d*) Magnitude of process lags.

(*a*) Many factors, such as chemical composition, which may be measured in the laboratory, are not susceptible to continuous measurement by an industrial instrument.

(*b*) Many factors capable of being measured are not satisfactory criteria of changes in process conditions, e.g., the temperature of vapors, leaving a column fractionating a binary mixture, is not a satisfactory criterion for heat supply to the column.

(*c*) The obvious way of satisfying (*b*) may be very uneconomic. The response to corrective action must be consistent. The significance of the measurement must be consistent, e.g., both above and below ebullition temperature is a significant measure of the heat content of water or steam, but not at the point of change of state.

(*d*) The final result, such as vapor pressure of the end product of a fractionating column, may be capable of being measured so as to satisfy requirements (*a*), (*b*), and (*c*), but the process lags introduced by the method of measurement may be so great as to preclude all hope of compensating successfully for any variations in operating conditions.

It may be thought that the foregoing considerations are only rarely of importance, and, considering the great bulk of all automatic-control applications, that is true. However, this is because most applications are repetitions of jobs which have already been handled successfully. However, when new processes are developed, or old ones are redesigned, such considerations become vital. Attempting to redesign a process so that it becomes more controllable may readily result in its becoming entirely unmanageable, due to neglect of these factors.

Another factor which is important is that of the self-regulation of the process itself. All of the process examples cited in this paper show definite positive self-regulation. However, numerous applications exist in which there is no tendency for the process to balance out, after an upset, within reasonable limits; and in many cases, an upset tends to accentuate itself. Exothermic chemical reactions may be very violent in their unbalancing tendencies.

The writer feels that the art of automatic control has not yet reached the point of progress where it is possible to lay down very definite rules with respect to preadjustment of automatic controllers, or with respect to process design, except on the basis of experience. Experience may lead to rearrangement of apparatus so that better control results are possible, but in practically all process designs, efficiencies, heat exchange, recoveries, etc. will continue to be the dominating factors.

It may be thought that this is an argument for doing nothing with respect to process controllability. It is actually a plea to go slowly. Twenty years ago, before automatic controllers reached their present state of development, many elaborate automatic-control installations were made, which in a short time proved to be more decorative than useful, and many of them were far from beautiful. The results proved very harmful to the industry, and the effects have not yet died out in many places. If process designers become too sold on the possibility of design for controllability, and the results are disappointing, the art can very readily receive another such setback.

J. C. Peters.[9] From the standpoint of exact quantitative solutions, most practical process-control problems fall into one of two classes; one class in which the solution is fairly easy but scarcely worth while, and another in which the solution would be of considerable interest but which involves great practical difficulty.

The authors have directed their attention to what might be considered as short-cut methods for practical use. The value of such methods is determined by the extent to which they fit actual cases. While the authors have presented but little evidence that their methods have wide application, they are probably awaiting the reports of others before making too definite statements on this point. The writer is glad to report that he has applied the equation relating "reset rate" and "lag" with encouraging results.

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[7] Application Engineer, Republic Flow Meters Company, Chicago, Ill. Mem. A.S.M.E.

[8] "Mechanical Engineers' Handbook," by Lionel S. Marks, fourth edition, McGraw-Hill Book Company, Inc., New York, N. Y., section on "Automatic Control," by J. B. McMahon, pp. 2116–2123.

[9] Research Engineer, Leeds & Northrup Company, Philadelphia, Pa. Mem. A.S.M.E.

In the present paper, a rather unusual case of temperature control is taken as the basis for calculating the area under the control curve. The authors indicate that they fully realize that the "load factor" may be considerably different for different types of processes. The writer has determined this factor for a particular case of temperature control and obtained a value of 15, as compared with the value of 3.7 applying to the process of Fig. 2 of the paper.

It is to be pointed out that, if a "measure of process difficulty" is to be considered as a measure of control difficulty, the equation for it should include a factor dependent upon the nature of the disturbances to which the process is subjected. When disturbances always take place very slowly, excellent control may be obtainable, whereas the same process may be practically uncontrollable if subjected to sudden changes.

Process designers may well pay particular attention to the emphasis placed upon the importance of the elimination of what the authors refer to as "lag." As is pointed out, lag may result from the fact that a material must be transported over a certain distance before its effect is felt, or it may result from the combined effect of capacity and resistance to flow. In the case of a thermal system, this lag may be reduced to a minimum by seeing to it that the adjusted heat supply is given as favorable a thermal relation to the point of measurement as circumstances permit.

While, in general, control terms are not sufficiently standardized to justify quibbling about them, it seems proper to call the authors to task for the use of the term "impedence" while speaking of the time-constant of an RC circuit. If electrical analogies are to be used, certainly the well-established term, "time-constant" is a natural one to employ. As an electrical engineer, the writer usually thinks of impedences measured in ohms, and to him, to express "impedance" in minutes, seems intolerable.

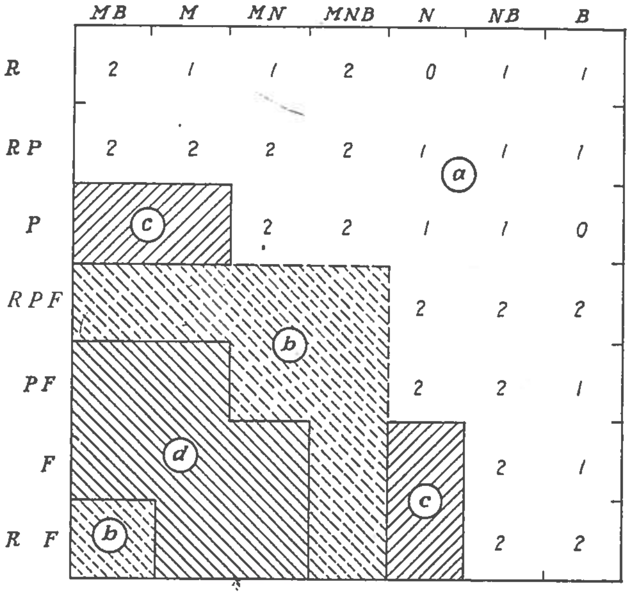
In conclusion, the writer wishes to express his appreciation to the authors for the great amount of thought which they have put into this paper. Its fresh and practical approach may well lead the way to a more rational analysis of control-application problems.

Ed S. Smith.[10] Instead of being the single entity urged by the authors, "controllability" seems to this writer to consist of different elements in different cases and not to be properly a blanket concept at all.

A stable regulated system, comprising a plant and its regulator, may be aptly considered as forming a chain whose length increases with the number of lags (or "capacities") in series, the chain having to extend the whole distance between its two ends. As long as a regulator either supplies missing links or strengthens too-weak ones, it controls equally regardless of the location of the links. The shorter chain with a missing link is no more controllable than a longer one also having a missing link. The correct link must be supplied in each individual case, and its location is lost by the use of RL or any other blanket controllability factor.

The diagram, Fig. 16 of this discussion, shows the effect on stability of missing elements, or links, in a plant or process having inertia (mass) M, damping N, and/or self-regulation B, when controlled by a regulator having rate R ("pre-act" in the paper), proportional P, and/or floating F ("reset" in the paper) components, the rate component alone being incomplete as a regulator.

From Fig. 16, it appears that there can be no correlation between any controllability factor such as RL of the paper and the performance of regulated systems generally. In other words, there is a fatal lack of correspondence between the mathematics of the paper and the physical system, which keeps its method from being generally applicable.



Symbol	No. Cap System	Performance
a	0-2	Always Stable
b	3	Stability Depends on RPF Values
c	2	Unstable, Steady Hunting
d	3	Unstable, Increasing Hunting

Fig. 16 Stability Diagram Showing Complex Nature of Controllability

The authors' method is not suitable, without extensive modification, to floating regulators and plants. Its use in such fields is questioned as being outside of the field of proportional control in which its use has been tested.

The authors should test the limits of usefulness of their method on plants, respectively having the following relations

$$\ddot{X} + \dot{X} + X = 0 \text{ and } \ddot{X} + X = 0$$

The first plant would not need any regulator at all, while the second would hunt steadily without one.

A rational use of either reaction curves or known plant coefficients with reasonably representative differential equations is believed by the writer to furnish a sounder basis for control engineering than the rules proposed by the authors. The differential equations are handier than their solutions because of the fact they are "always true," while the solutions of course depend upon initial conditions and take many forms. However, families of typical curves are available which enable the differential equations to be evaluated and used conveniently for control purposes.

Two incidental notes are as follows:

1 The method of Fig. 9 of the paper appears to be limited to the case of real time constants and thus not to apply to the most interesting case of damp oscillations with their complex time constants.

2 Offhand, there appears to be something wrong with Equation [2] of the paper for Fig. 4, since areas below the set point should, as a matter of physical common sense, have equal weight with those above it, an improper use of signs possibly being involved.

The authors are requested to include, preferably as an appendix, a copy of their mathematical developments so that each reader will not have to supply them himself and to increase the use of the method suggested within its limits.

This paper has performed a service in directing attention to-

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[10] Eclipse Aviation, Bendix, N. J. Mem. A.S.M.E.

ward the usefulness of reaction curves as diagnostic symptoms in prescribing the remedial regulator.

AUTHORS' CLOSURE

As Mr. Hickes points out, an "on-off" controller, one having very high proportional-response sensitivity, is theoretically and actually ideal for processes which are essentially single capacity. Since no thermal systems are infinitely fast in their response, and no relays have infinite capacity, some time lag will exist in all temperature-control circuits, nevertheless, they can approach the desirable characteristics of $R_1L = 0$, when R_1 is very small. Even though a finite R_1L intercept is present on a process, an "on-off" controller often gives stable control, simply because the controller sensitivity is not infinitely high as the name implies but lower than $1/(R_1L)$.

The authors accept and appreciate Mr. Keppler's discussion regarding the use of pressure elements with small displacement. It is hoped that the paper will help point the way toward such useful improvements in instrument design. Servo-operation of measuring devices does not necessarily eliminate measurement lag inasmuch as some lags are always introduced in each stage of amplification of the servo-mechanism.

Mr. Markson's confusion over the "per minute" units of reset rate might be eliminated if he considered it as being something dimensionless per minute. Actually, it is the number of times per minute at which automatic reset response duplicates the proportional-response output change caused by the pen deviation. Instead of expressing the magnitude of reset response by saying that it produces so many pounds per square inch per minute per inch deviation, it is given as pounds per square inch per minute per pounds per square inch change in proportional-response output. This leaves the unit as 1/min or "repeats per minute." All equations are dimensionally consistent if F is expressed in pounds per square inch, R in inches per minute, and L in minutes. Term R_1 is then inches per minute per pound per square inch.

Terminology is troublesome. The authors feel that the magnitude of proportional response is correctly expressed by "sensitivity;" also, that the deflection of a galvanometer per microvolt is correctly called its sensitivity. It is thought that the threshold of potential necessary to overcome friction in a galvanometer is something else which might be called "sensibility," defined as "that quality of an instrument which makes it indicate very slight changes of condition." "Gradient" is a word worthy of consideration as an alternate for sensitivity.

In the application of automatic-control instruments to a process, consideration must certainly be given to the points brought out by Mr. McMahon. In this paper, the authors chose to bypass the first two, which only involve measurement and not control. Generally, the object of automatic control is to hold a pen at one set point which represents an optimum condition in the process, whether it is indicated as temperature, pressure, or some other variable. Generally also, under point (c), response to output change is reasonably consistent, at least to the extent that a positive output-pressure change from equilibrium causes a pen to move in a direction opposite to that caused by a negative-output change. There are cases, for example, in azeotropic distillation, where increased reflux flow may cause a temperature deviation in either direction, depending upon the equilibrium conditions in the column. If composition is the desired quantity in this case, temperature is really a nonsignificant measurement of composition.

Mr. McMahon questions whether controllability is a matter of degree, and rightly, since it entails a definition of terms. The authors, in this paper, arbitrarily took the area under a recovery curve as one measure of controllability. This is only one of many possible bases for comparison of control results. For example, the maximum amplitude of an oscillation resulting from a load change might be taken as the sole criterion. This is possibly the basis of Mr. McMahon's contention that some processes are not controllable or synonymously "unmanageable." Probably even the processes to which he refers are controllable but not within the required tolerance.

It might be well to explain the reason for choosing this basis and the significance. On most control applications, the set point simply represents optimum conditions in a process. Deviations in either direction increase processing cost either by increasing steam cost or producing an inferior product, which must be reprocessed or sold at a lower price. If process thru-put is uniform, a plot of processing cost per minute against deviation from the set point might be a probability curve for the average process. This curve of the form $y = (1 - ae^{-bx^2} + c)$ with minimum cost at the set point is shown in Fig. 17 of this closure. It can be seen that, around the set point, this curve is fairly well approximated by the two straight lines which represent the process evaluation used in this paper, i. e., that processing cost increases at a uniform rate with deviation.

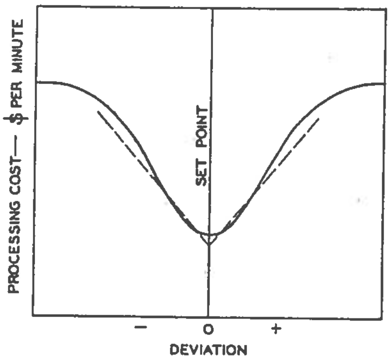

FIG. 17

Obviously, a probability curve does not represent all processes. Many would be discontinuous at some point. For example, an increase in temperature beyond a certain point might increase processing cost very suddenly, if at that point a reaction started which blew a wing off the plant. Again, considerable level variations about a set point might cause no trouble at all until a tank overflowed or a pump lost suction.

Self-regulation of a process appears to be one of the very minor considerations on processes, such as those discussed in the paper, those in which L is less than Z_0 (Fig. 2). If the 8.2-min "impedence" of Fig. 14 were made infinitely large, the process would not be self-controlling but the lag would only be increased from 0.88 to the sum of the remaining impedences or 1.5 min.

Mr. Peters brings out a very important point which cannot be overlooked in future work on the "load factor," and that is the type of load change, or rather, the rate at which the load change occurs. The authors carefully side-stepped this in their analysis of the process shown in Fig. 2. In this process, the load change was located so that it affected the process in exactly the same manner as a change in controller output. It should be apparent that, under these conditions, the rate of load change makes absolutely no difference in the area under a recovery curve. The recovery curve for a gradual load change would deviate a small amount and remain away until the load ceased to change; the pen distance from the set point being just sufficient to make the rate of output change from automatic reset response correspond to the rate of load change. When the load change was complete, the pen would return exponentially to the set point. In this case as well as that of an equal sudden load change, the area under the recovery curve would be equal to $\Delta F/(S)(RR)$.

On the other hand, a sudden load change, such as a flow of water entering just upstream of the bulb, Fig. 2, would cause a very rapid initial deviation and give a recovery curve with a large negative area. The load factor would be in the neighborhood of 65. But a gradual load change, even at this unfavorable location, would shrink the load factor back toward 3.7. Thus the load factor, as Mr. Peters indicates, depends upon both location and rate of load change.

The authors do not choose to defend their use of the term "impedence" in place of the more bulky "time constant of a resistance capacity unit." The "ence" ending was used to distinguish it from electrical "impedance." It is felt that a happier word might be chosen to describe this concept which is so necessary in process evaluation.

The exact mathematical solution of automatic-control problems, as championed by Mr. Smith, is certainly a desirable goal. However, the first paragraph of Mr. Peters' discussion expresses the authors' opinion in the matter.

Negative areas do have equal weight with positive areas so the signs of Equation [2] are correct.

The authors are glad to append equations for Figs. 10 and 11 of the paper, even though it is thought that the approximation which they express can be determined graphically more easily and with sufficient accuracy.

Equation for Fig. 10

$$Y = \frac{L}{z} = 1 + \frac{1}{X} - \frac{1}{X^{\frac{1}{1-X}}} - \frac{\log_e X}{1 - X}$$

where $X = \frac{z}{Z}$

Equation for Fig. 11:

$$y = \frac{L}{nZ} = 1 + \frac{1}{n} + \frac{n-2}{n(n-1)} + \frac{(n-2)(n-3)}{n(n-1)^2} + \frac{(n-2)(n-3)(n-4)}{n(n-1)^3} + \dots + \frac{(n-2)!}{n(n-1)^{n-2}} - \frac{(n-2)!}{n(n-1)^{n-1}} e^{n-1}$$