and frequency of one or two normal modes of lower frequency, and a closer examination of the behaviour of the control system with any set of parameters can then be made by the solution of the control equations on the differential analyser. A large number of such solutions has been carried out and an optimum set of control parameters is stated.

The modified control law appears to have some advantage over that previously investigated, from the points of view both of the kind of control obtainable and of the design and operation of the control gear.

A method of obtaining in practice a control law of the form studied is briefly indicated.

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$$g = \frac{Ke^{-\theta s}}{\tau s+1}$$

$\theta = 1$

$\tau = 10$

change to 2θ

$K_u = 10$

$\angle g(j\omega_u) = -\theta\omega - \mathrm{arctg}(10\omega) = -\pi$

ω	$\angle g - \pi$
1	−0.67
2	0.379
1.6	−0.033
1.62	−0.0012
1.63	−0.002

$$|g(j\omega_u)| = \frac{k}{\sqrt{16.3^2+1}} = 0.061 \cdot k$$

$$K_u = \frac{1}{0.061 \cdot k} = 10 \qquad K = 1.633$$

10

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Optimum Settings for Automatic Controllers

By J. G. ZIEGLER[1] and N. B. NICHOLS,[2] ROCHESTER, N. Y.

In this paper, the three principal control effects found in present controllers are examined and practical names and units of measurement are proposed for each effect. Corresponding units are proposed for a classification of industrial processes in terms of the two principal characteristics affecting their controllability. Formulas are given which enable the controller settings to be determined from the experimental or calculated values of the lag and unit reaction rate of the process to be controlled. These units form the basis of a quick method for adjusting a controller on the job. The effect of varying each controller setting is shown in a series of chart records. It is believed that the conceptions of control presented in this paper will be of assistance in the adjustment of existing controller applications and in the design of new installations.

A PURELY mathematical approach to the study of automatic control is certainly the most desirable course from a standpoint of accuracy and brevity. Unfortunately, however, the mathematics of control involves such a bewildering assortment of exponential and trigonometric functions that the average engineer cannot afford the time necessary to plow through them to a solution of his current problem.

It is the purpose of this paper to examine the action of the three principal control effects found in present-day instruments, assign practical values to each effect, see what adjustment of each does to the final control, and give a method for arriving quickly at the optimum settings of each control effect. The paper will thus first endeavor to answer the question: "How can the proper controller adjustments be quickly determined on any control application?" After that a new method will be presented which makes possible a reasonably accurate answer to the question: "How can the setting of a controller be determined before it is installed on an existing application?"

Except for a single illustrative example, no attempt will be made to present laboratory and field data, to develop mathematical relations, or to make acknowledgment of material from published literature. A paper covering the mathematical derivations would be quite lengthy as would also a paper covering laboratory and field-test results. Work on these phases of the subject is still under way, and it is expected that the results will be published at a later time when convenient. It is believed advisable to publish the present paper without delay in order to make the information available for use by the many persons interested in the application of automatic-control instruments. To these persons the present subject matter is of much greater interest than the other phases of the study which are being omitted.

To simplify terminology we will take the most common type of control circuit in which a controller interprets the movement of its recording pen into a need for corrective action, and, by varying its output air pressure, repositions a diaphragm-operated valve. The controller may be measuring temperature, pressure, level, or any other variable, but we will completely divorce the measurement portion of the control circuit and speak only of the pen movement in inches; 1 in. of pen movement might represent 1 or 1000 deg F, or a flow of 1 or 1000 gpm. The actual graduation will be of no moment in a study of control.

Our controller will translate pen behavior into behavior of a valve; the relation between the two behavior patterns is determined by the setting of each control effect. The term valve covers any similar device, i.e., a damper or rheostat which must be operated by the controller in order to maintain correct process conditions.

Proportional Response

In spite of the multitude of air, liquid, and electrically operated controllers on the market, all are similar in that they incorporate one, two, or at most three quite simple control effects. These three can be called "proportional," "automatic reset," and "pre-act."

Proportional Response. By far the most common effect is "proportional response," found in practically all controllers. It gives a valve movement proportional to the pen movement, that is, a 2-degree pen movement gives twice as much valve movement as a 1-degree pen movement. Simple spring-loaded pressure-reducing valves are really proportional-response controllers in that, over a short range of pressure, the valve is moved proportionally from one extreme to the other.

Sensitivity. The measure of proportional response is called "sensitivity" or "throttling range;" the former being valve movement per pen movement, the latter its reciprocal or the pen movement necessary to give full valve movement. Either sensitivity or throttling range describes the magnitude of proportional response, though in this paper each response will be measured in units which increase as the relative valve action per pen action increases. In the case of proportional response, the unit will accordingly be called "sensitivity."

Proportional-response sensitivity in some controllers is not adjustable; in most, however, it may be adjusted either continuously or in steps over a considerable range. If we define sensitivity as the output pressure change per inch of pen travel, it is apparent that the limits would be from zero (manual control) to infinitely high (on-off control). Perhaps the widest range of adjustment is found in one controller with sensitivity continuously variable from 1000 to 1 psi per in. A sensitivity of 1000 gives 1 psi output change for each 0.001 in. of pen travel.

Sensitivity adjustment is necessary if optimum control stability is to be attained. It is common knowledge that control with infinitely high proportional response is always unstable, oscillating continuously. True, on certain applications the oscillation may be of such small magnitude that it is not objectionable and, if the surges in supply are not serious in their effect on other portions of the process, the control obtained may be entirely acceptable.

Industry generally demands control of the "throttling" type rather than "on-off" since a proportional-response controller, set in any sensitivity below some maximum, will produce a damped oscillation and eventually straight-line control.

Amplitude Ratio. Sensitivity adjustment affects primarily the stability of control. On any application there is a definite and

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Note: Statements and opinions advanced in papers are to be understood as individual expressions of their authors and not those of the Society.

Sensitivity $= K_c$

Reset rate $= 1/\tau_I$

Preact time $= \tau_D$

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easily determined point called the "ultimate sensitivity" (S_u), above which any oscillation will increase to some maximum amplitude, and below which an oscillation of any size will diminish to straight-line control. Stability may be measured in terms of "amplitude ratio," the relative amplitude of any wave to that of the wave which preceded it. A controller set at the ultimate sensitivity gives an oscillation with an amplitude ratio

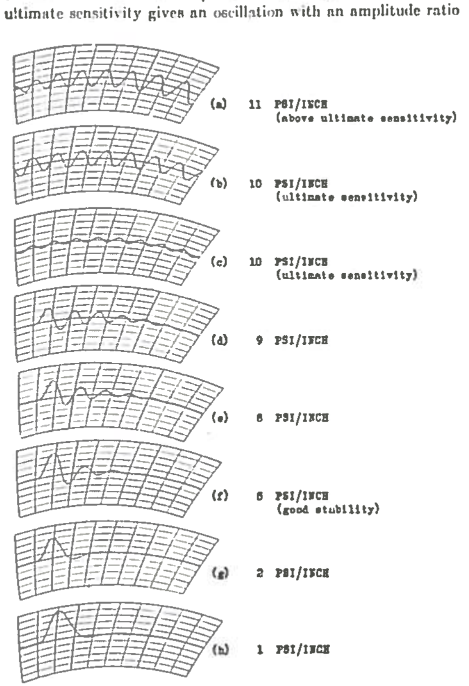

FIG. 1 AMPLITUDE RATIO VERSUS SENSITIVITY
(Effect of disturbance.)

of 1; above the ultimate sensitivity, an amplitude ratio greater than 1; and below the ultimate, an amplitude ratio less than 1.

Amplitude Ratio Versus Sensitivity. Fig. 1 shows the effect of sensitivity adjustment on a typical application. The oscillation was started by a momentary change in valve position. Curves (*b*) and (*c*) were produced at the ultimate sensitivity, which in this case was 10 psi per in. Curve (*a*) was produced at a sensitivity of 11 psi per in. (110 per cent of S_u). Curves (*d*) to (*h*) show the successively smaller amplitude ratios produced as the sensitivity was lowered to 90, 80, 50, 20, and 10 per cent of the ultimate (9, 8, 5, 2, and 1 psi per in.).

In Fig. 1 and succeeding charts, each division is 0.1 in. and each time interval represents 0.625 min.

Regardless of the ultimate sensitivity of any control application, the relationship between amplitude ratio and sensitivity, given as per cent of ultimate sensitivity, remains about as shown in Fig. 2. The ultimate sensitivity thus appears to be a good common point for consideration of sensitivity adjustment on most control applications.

Offset and Load Change. In considering the curves of Fig. 1, the most desirable setting from a stability standpoint would be (*h*), produced at quite a low sensitivity (10 per cent of ultimate). It should be noted in passing, however, that as sensitivity is reduced the period of oscillation increases slightly, which in itself is undesirable. The real drawback of using sensitivity settings a great deal lower than the ultimate value stems from the limitation of proportional response, e.g., that only one valve position can be maintained when the pen is at the desired set point. A "load change," any disturbance in the process requiring a sustained alteration of valve position, will cause the pen to shift away from the set point far enough to give the required valve movement. The magnitude of this shift or "offset" varies inversely with the sensitivity setting used and directly with the required change in

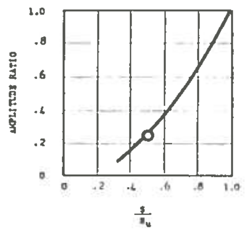

FIG. 2 AMPLITUDE RATIO VERSUS SENSITIVITY

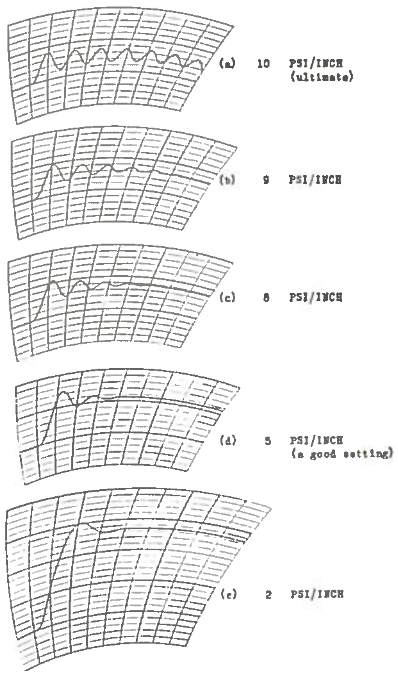

FIG. 3 OFFSET VERSUS SENSITIVITY
(Effect of load change.)

valve position. Fig. 3, cu[illegible] (*a*) to (*e*), illustrates this point. Curve (*a*) shows the offset caused by a load change requiring a 2.8 psi change in output pressure with sensitivity at 10 psi per in. Since this is the ultimate setting, an amplitude ratio of 1 results and a lower setting is indicated. As the sensitivity is decreased to 9, 8, 5, and then 2 psi per in., the offset from this load change increases and the amplitude ratio decreases.

Amplitude Ratio Versus Offset. The rational adjustment of proportional-response sensitivity is then simply a matter of balancing the two evils of offset and amplitude ratio. For most applications a good compromise is the sensitivity which gives an amplitude ratio of 25 per cent. This sensitivity will be very nearly one half that of the ultimate sensitivity, as shown in Fig. 2. An excellent and rapid method of sensitivity adjustment is to find the ultimate sensitivity and then simply cut it in half. Fig. 1, curve (*f*), shows that an amplitude ratio of 25 per cent is achieved by this setting on the application under test. Fig. 3, curve (*d*), shows the result of a load change requiring a 2.8 psi change in controller output pressure. The sensitivity setting of 5 psi per in. allows an offset of 2.8/5 or 0.56 in. with a 25 per cent amplitude ratio.

On most air-operated controllers, the sensitivity adjustment is calibrated either in terms of sensitivity or throttling range. On such instruments the trick of halving the sensitivity to obtain a good setting is quite simple; on those calibrated in throttling range the setting should be doubled, since this unit is the reciprocal of sensitivity. The sensitivity of older instruments with arbitrary adjustment scales may be easily found by moving the pen a definite distance and noting the resulting output-pressure change. This test run at a few points will enable the user to plot a sensitivity-conversion scale.

The statement that a sensitivity setting of one half the ultimate with attendant 25 per cent amplitude ratio gives optimum control must be modified in some cases. At times a lower sensitivity is preferable. For example, the actual level maintained by a liquid-level controller might not be nearly as important as the effect of sudden valve movements on further portions of the process. In this case the sensitivity should be lowered to reduce the amplitude ratio even though the offset is increased by so doing. On the other hand, a pressure-control application giving oscillations with very short period could be set to give an 80 or 90 per cent amplitude ratio. Due to the short period, a disturbance would die out in a reasonable time, even though there were quite a few oscillations. The offset would be reduced somewhat though it should be kept in mind that it can never be reduced to less than one half of the amount given at our previously defined optimum sensitivity of one half the ultimate.

On processes involving wide changes in load, one condition is often encountered which must be considered here. A controller perfectly adjusted for one load condition may start oscillating under another load. If the ultimate sensitivity is checked at the new more difficult load, it will be found lower than at the original easy load condition. Consequently, the sensitivity must always be adjusted so that the correct stability is achieved under the most difficult load condition. Obviously the amplitude ratio will then be lower at the easy load.

AUTOMATIC RESET RESPONSE

The second most common response found in modern controllers is "automatic reset." Its only purpose is to eliminate offset. In action it detects any disparity between pen and set point and gives a slow continuous valve movement in the proper direction to correct the offset. Furthermore, the rate of valve movement is proportional to the distance between pen and set point. Automatic reset then may be defined as a response giving valve velocity proportional to pen displacement from set point.

Some controllers give a constant valve velocity with the direction depending upon whether the pen is above or below the set point. This is a special case and will not be considered further. Neither will those controllers having automatic reset alone (floating response) be considered in this paper. It appears that the floating response controller is most useful on partially "self-controlling" processes.

Reset Rate. As sensitivity was the measure of proportional response, "reset rate" becomes the corresponding measure of automatic-reset response. The units of reset rate are minutes^{-1} or the number of times per minute that automatic reset duplicates the proportional-response correction caused by the disparity between pen and set point.

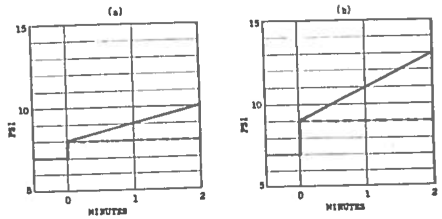

FIG. 4 RESET RATE
(Reset rate = 1 per min.)

Fig. 4(*a*) and (*b*) shows the course of output pressure with time for a reset rate of 1 per min. The dotted lines show the corresponding proportional response pressure. In Fig. 4(*a*), the pen was moved and held far enough from the set point to give a 1 psi change in proportional response. The reset proceeds at the rate of 1 psi per min per 1 psi original change. Fig. 4, curve (*b*), shows a reset rate of 2 psi per min per 2 psi original change. In both cases the reset rate is 1 per min.

In most controllers using automatic reset, some adjustment of the reset rate is provided, though continuous adjustment appears in only a few. In one, the reset rate is adjustable from zero to 20 per min. In order to determine reset rates on an instrument without a calibrated dial, it is only necessary to move the pen away from the set pointer far enough to cause a 1 psi output change and note the additional output-pressure change per minute. The same value can be put on the reset adjustment in controllers other than those of the air-operated type, by making a sustained pen change from the set point, noting the altered valve position which results from proportional response and the additional travel at the end of 1 min from automatic reset. The reset rate is the travel from reset divided by the travel from proportional.

Optimum Reset Rate. Fig. 5(*a*) to (*e*) shows the effect of reset-rate adjustment on control. Fig. 5, curve (*a*), resulted from a load change equivalent to 2.8 psi output pressure with a reset rate of zero, in other words, only proportional response. This curve is the same as Fig. 2(*d*) except that the sensitivity is reduced from 50 per cent of ultimate to 45 per cent of ultimate. A reset rate of 0.5 per min gives the slow return toward the set point shown in Fig. 5(*b*). As the reset rate is increased to 1, to 1.5, and to 2, in Fig. 5(*c*), (*d*), and (*e*), the return becomes more and more rapid. At the same time, instability and period of oscillation increase. In general, curve (*d*) of Fig. 5 would be considered the optimum in that it gives reasonably rapid return without excessive loss of stability or excessive increase in period.

Optimum Reset-Rate Adjustment. The actual reset rate which gives a recovery curve similar to Fig. 5(*d*) varies widely on different control applications. As will be pointed out later, the reset rate appears to vary inversely as the time lag of the application. At present, however, we are more interested in finding a simple method for determining the correct setting.

It has been found that the period of oscillation (P_u) produced at the ultimate sensitivity (S_u) is a good index of required reset-rate adjustment. This period should be measured when the

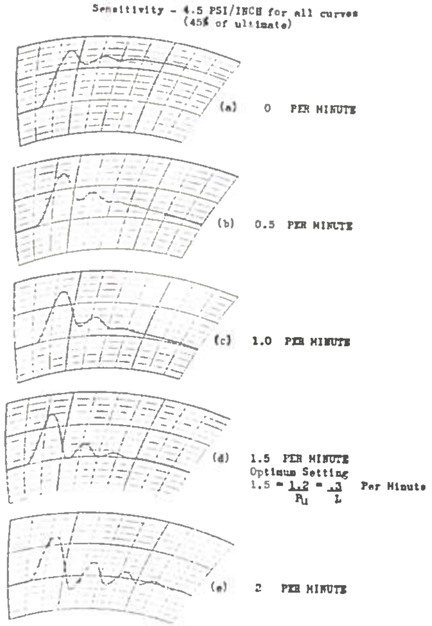

Fig. 5 Reset Rate Versus Recovery
(Load change.)

amplitude of oscillation is quite small, such as on curve (c) of Fig. 1, where the period is about 0.8 min. The optimum setting of reset rate, that which produces a recovery curve similar to Fig. 5(d), is usually about $1.2/P_u$. On the process being tested, the reset rate of 1.2/0.8 or 1.5 was used for curve Fig. 5(d).

In adjusting a controller with proportional and automatic-reset responses, the sensitivity which just gives a small sustained oscillation should be determined (S_u), and the period of oscillation (P_u) in minutes noted. Optimum controller settings will then be approximately

$$\text{Sensitivity} = 0.45S_u$$
$$\text{Reset rate} = 1.2/P_u$$

Note that the recommended sensitivity has been reduced from $0.5S_u$ to $0.45S_u$. Were this not done, the addition of automatic reset would have increased markedly the amplitude ratio. This tendency of automatic reset to decrease stability is one of its bad features; the other is its tendency to increase the period of oscillation.

While a reset rate of $1.2/P_u$ is generally recommended, recovery curves with the same amplitude ratio may be obtained at a higher reset rate and lower sensitivity. In general, however, this procedure results in recovery curves with longer period and greater initial deviation, both of which are detrimental.

Pre-Act Response

The latest control effect made its appearance under the trade name "Pre-Act." On some control applications the addition of pre-act response made such a remarkable improvement that it appeared to be an embodiment of mythical "anticipatory" controllers. On other applications it appeared to be worse than useless. Only the difficulty of predicting the usefulness and adjustment of this response has kept it from being more widely used.

This pre-act effect is as distinct a response as proportional and automatic reset. Pre-act simply gives an additional valve movement proportional to the rate of pen movement. It is used only in conjunction with proportional response.

Pre-Act Time. Since pre-act response is an additional output pressure change per rate of pen movement, its unit is the "pre-act time" in minutes

$$\text{(psi) per (psi per min)} = \text{min}$$

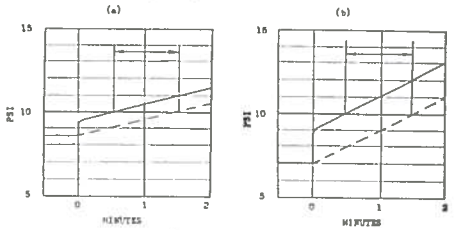

Fig. 6 Pre-Act Time
(Pre-act time = 1 min.)

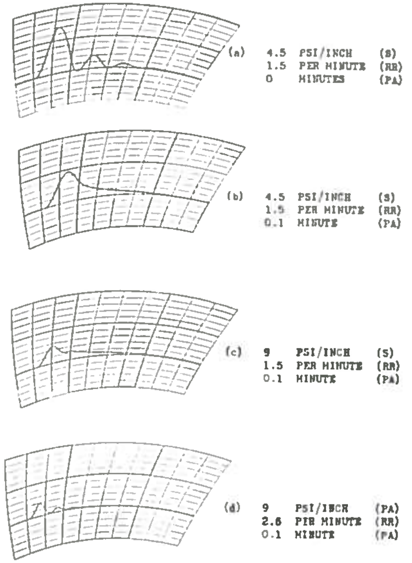

Fig. 7 Control With Pre-Act
(Load change.)

To visualize this unit, assume a controller pen moving away from the set point at such a rate that a proportional-response output change of 1 psi per min results (dotted line of Fig. 6(a)). Addition of 1 min pre-act time will cause the controller output to follow the solid line 1 psi higher, i.e., the pre-act response is 1 psi additional for 1 psi per min proportional-response change. Without altering the pre-act setting, a pen velocity twice as great would give 2 psi additional pressure, as shown in Fig. 6(b). The time by which the solid line of Fig. 6(a) and (b) leads the dotted line is the pre-act time, in this case 1 min.

Recently, several industrial instrument companies have made this control effect available in a more or less adjustable form. In one, the dial is calibrated in terms of pre-act time over a range of 0.2 to 10 min.

Use of Pre-Act Response. Pre-act response has been successfully used on applications which give a period of oscillation greater than about 0.4 min. It is not generally useful on pressure- or flow-control applications and rarely on control of liquid level, though this is not a hard and fast rule. To date, it has been used most widely on temperature-control applications.

The effect of pre-act on control is shown in Fig. 7. Fig. 7 curve (a) repeats curve (d) of Fig. 5, which represented about the optimum control obtainable with proportional and reset responses only. Without altering these settings, the addition of 0.1 min pre-act time changes the recovery curve for the same 2.8 psi load change to that shown at (b). The increased stability is an indication that a higher sensitivity may be used, so it is accordingly increased to 9 psi per in. The resulting curve (c) shows a much smaller initial deviation without excessive amplitude ratio, but an excessively slow return toward the set point, indicating that a faster reset rate is needed. (Compare with Fig. 5(b).) Increasing the reset rate to 2.6 per min produced the curve Fig. 7(d), representing approximately optimum control using the three responses.

A comparison of curves, Fig. 7(a) and (d), discloses that the pre-act response has improved control in several respects. Maximum deviation from the set point has been cut 71 per cent, period of oscillation has been reduced 43 per cent, and the time required for the oscillation to die out has been halved.

Pre-act response does not replace automatic-reset response since it ceases to act when the pen becomes stationary. However, while reset increases period of oscillation and decreases stability, the effect of pre-act is just the opposite. On the debit side for pre-act lies only the increased difficulty of adjusting three responses instead of two, but the use of the basic unit, pre-act time, allows the setting to be determined from the period of oscillation.

Optimum Pre-Act Time Adjustment. It has been found that, for a wide range of control applications, the optimum pre-act time depends directly upon the period of oscillation used to determine the adjustment of the reset rate. In fact the pre-act time should be about $1/8$ of the period of a small-amplitude oscillation at the ultimate sensitivity.

To adjust a controller with proportional, automatic reset, and pre-act responses, determine the ultimate sensitivity (S_u) and note the period (P_u) of a small-amplitude oscillation at this sensitivity. The optimum settings will then be approximately

$$\text{Sensitivity} = 0.6S_u$$
$$\text{Reset rate} = 2/P_u \text{ per min}$$
$$\text{Pre-act time} = P_u/8 \text{ min}$$

On some applications, the sensitivity with pre-act can be greater than $0.6S_u$. This is illustrated by the test application which allowed a sensitivity of $0.9S_u$ (Fig. 7(d)). We have found that the setting is generally between $0.6S_u$ and $1S_u$; in many applications, a sensitivity of $0.6S_u$ will be sufficiently near the optimum setting.

If, at these settings, the amplitude ratio is too high, each adjustment should be reduced slightly. When using the system of units proposed in this paper, a decrease in the setting of any response increases stability. (Actually pre-act increases stability up to its optimum setting and, above that, again gives less stability.) In general, oscillations with a period approximately the same as those occurring at the ultimate sensitivity are due to too high a sensitivity; automatic reset gives longer periods and pre-act shorter periods.

Process-Reaction Curves

A control circuit consists of a controller and a process, the valve being considered a portion of the latter. Pen movement

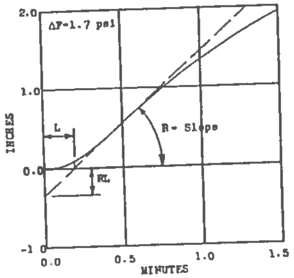

Fig. 8 Reaction Curve

$L = \theta$

$R' = k'$

gives an output-pressure change, which affects the process, which in turn affects the pen. So far, we have considered control effects, the portion of the control circuit tying pen movement to output-pressure-behavior pattern. We have also considered the effect of altering this pattern on the entire control circuit, taking as evidence the pen recovery from disturbances and load changes.

We will now eliminate the controller from the circuit, make certain output-pressure changes, and show how the resulting pen behavior can be used to evaluate controllability of the process and predict optimum controller settings.

Process-Reaction Curve. In any control circuit, there are several time lags. The lag of inflating the valve is present in all. Some time lag occurs in the measuring portion between a change at the thermometer bulb or pressure connection and the indication of that change at the pen. Added to these two may be series of lags in the apparatus under control.

The difficulty of dealing mathematically with processes involving a series of lags or even of applying values to the various lags and adding them is very great indeed. However, having a process, a pen, and a means of controlling the process (a valve), it becomes possible to get the summation of all the lags by simply altering the valve position and analyzing the resulting curve traced by the pen.

To be more explicit, suppose that we have an application with a controller installed and cut the air line connecting the controller to the diaphragm valve. Then, if we connect an air-reducing valve to the diaphragm-operated control valve, it will be possible to apply the air pressure necessary to hold the control valve in any position. We will thus be able to make a change in the pressure applied to the control valve in the same manner as the controller would do it (this can still be called an output pressure because its effect will be the same as though it came from the controller) and note the resulting pen behavior.

With a control circuit so arranged, we may, by applying the orrect pressure to the control valve, first bring the recording en to the desired point on the chart. If then a sudden sustained hange in pressure on the control valve of ΔF psi is made, the en will trace an S-shaped curve which we will call a "reaction urve." Fig. 8 shows a reaction curve for the process which we ave been considering.

While Fig. 8 represents a typical reaction curve, an infinite umber of variations are possible. On some applications, notably iquid-level control, the curve may come to a maximum slope nd continue indefinitely (or until the tank runs over). This ype of process is not "self-controlling." On others a definite ead period or velocity-distance lag exists and the reaction curve hows no pen movement for a finite time after the change in alve position; it then either starts at the maximum rate or uilds up to the maximum.

In discussing optimum controller settings, when using pre-act esponse, we noted that a sensitivity between $0.6S_u$ and $1S_u$ ould be used. The best value appears to depend upon the shape f the reaction curve prior to the maximum slope; a lag preominantly of the dead-period type calls for sensitivities toward .6 S_u.

Optimum Settings From Reaction Curve

Two characteristics of the reaction curve are used to fix the roportional-response sensitivity. The "reaction rate" (R), e., the maximum rate at which the pen moves occurs at the point f inflection in the reaction curve. A line drawn tangent to this oint intersects the initial pen position a certain length of time fter the change in valve position. This time we will call the lag" (L) of our control circuit. The optimum setting of sensitivity for a controller is inversely related to the product of R nd L, determined from the reaction curve. If the tangent line projected until it intersects the vertical axis, the product RL graphically determined, as shown in Fig. 8. Good control is enerally obtained when proportional-response sensitivity is so djusted that a pen movement of RL in. gives a pressure change ΔF psi.

On the reaction curve of Fig. 8, a 1.7 psi valve change was ade so the optimum sensitivity setting is approximately

$$\text{Sensitivity} = \frac{\Delta F}{RL} \text{ psi per in.}$$

here

$$R = 1.7 \text{ in. per min}$$
$$L = 0.2 \text{ min}$$
$$RL = 0.34 \text{ in.}$$
$$\Delta F = 1.7 \text{ psi}$$

he predicted sensitivity of 1.7/0.34 or 5 psi per in. gave curves g. 1(f) and Fig. 3(d). These curves were previously selected as ving good stability, that is, an amplitude ratio of approximately 0.25.

Unit Reaction Rate. No justification has been given for calling e distance L on the reaction curve the lag of the process, but ere appears to be a good reason. On most processes, reaction rves, caused by different valve-pressure changes ΔF, are simi- in shape, differing only in the value of R, that is, the reaction te caused by a 1 psi change is about twice as great as that from 0.5 psi change, but the intersected distance L remains constant gardless of ΔF.

When taking a reaction curve, it is sometimes necessary to ake ΔF quite small, in order to prevent undue disturbance to e process being tested. The resulting reaction rate is then nverted to a "unit reaction rate" (R_1), that which would be used by 1 psi pressure change on the control valve. This is ne by dividing the reaction rate found by ΔF

$$R_1 = \frac{R}{\Delta F} \frac{\text{in. per min}}{\text{psi}}$$

The formula for a good sensitivity setting may then be written

$$\text{Sensitivity} = \frac{1}{R_1L} \text{ psi per in.}$$

The ultimate sensitivity will be about twice as great

$$S_u = \frac{2}{R_1L} \text{ psi per in.}$$

At the ultimate sensitivity, the period of oscillation is about $4L$ min, increasing to about $4.6L$ as the sensitivity is lowered to one half the ultimate.

An approximate description of the characteristics of a process is given by values of the two quantities, unit reaction rate and lag. True, these two are only a rough measure of the entire reaction curve, telling nothing about its shape before and after the point of inflection, but they give enough of the story to allow a prediction not only of optimum sensitivity and period of oscillation but of optimum reset rate and pre-act time settings as well.

It should be kept clearly in mind that the controller settings are determined from the reaction curve caused by an output-pressure change (control-valve-position change) and not by the reaction curve which is caused by a load change.

Reset-Rate Determination From Reaction Curve. Since the period of oscillation at the ultimate sensitivity proves to be 4 times the lag, a substitution of $4L$ for P_u in previous equations for optimum reset rate gives an equation expressing this reset rate in terms of lag. For a controller with proportional and automatic-reset responses, the optimum settings become

$$\text{Sensitivity} = \frac{0.9}{R_1L} \text{ psi per in.}$$

$$\text{Reset rate} = \frac{0.3}{L} \text{ per min}$$

At these settings the period will be about $5.7L$, having been increased by both the lowering of sensitivity and the addition of automatic reset.

Pre-Act Time Determination From Reaction Curve. Using again the relationship between L and P_u, we find that the optimum pre-act time depends directly upon the lag and is normally equal to $L/2$. This tells us that pre-act will not normally be used on applications in which the reaction curve shows a lag smaller than 0.2 min, since the minimum pre-act time available on industrial controllers is about 0.1 min. It will be useful on all applications with lags greater than 0.2 min.

The optimum settings determined previously for all three control effects, when expressed in terms of unit reaction rate and lag, appear as follows

$$\text{Sensitivity} = \frac{1.2}{R_1L} \text{ to } \frac{2}{R_1L} \text{ psi per in.}$$

$$\text{Reset rate} = \frac{0.5}{L} \text{ per min}$$

$$\text{Pre-act time} = 0.5L \text{ min}$$

Control-Valve Characteristics

In general, any change of a control circuit which allows a higher controller sensitivity and faster reset rate to be used will improve the control results obtained. We have seen that the addition of pre-act response gives both of these improvements.

At times certain changes in the pr[illegible] can be made which allow a higher sensitivity and reset rate.

Any decrease in the lag of a process permits an increase in reset rate and attendant reduction in period of oscillation, since the reset rate is inversely related to lag and the period directly related. Any decrease in the lag of a process if it is not attended by an increase in reaction rate permits an increase in sensitivity since the sensitivity is inversely related to the lag. Any decrease in the unit reaction rate of a process, if not attended by an increase in lag, allows higher sensitivities, since sensitivity is inversely related to reaction rate.

Stated more concisely, any decrease in the value of R_1L increases the optimum sensitivity, and any decrease in L increases the optimum reset rate. Also any decrease in L decreases the period of oscillation.

Some applications, as we have already noted, call for widely different sensitivity settings at different load conditions. In these cases, we have said the sensitivity must be set low enough to give stability at the most difficult load even though the control is penalized at easy load conditions. This phenomenon is due to the fact that the unit reaction rate generally changes with load. The lag normally remains about constant. Control valves with special flow-lift characteristics have been used in an attempt to correct for this change in unit reaction rate with load. The optimum characteristics vary with the application under control and are not always "logarithmic" or "equal percentage" as is commonly thought.

Process Classification

Since either the ultimate sensitivity and attendant period or the unit reaction rate and the lag may be used to determine optimum controller settings, it follows that the latter values may be determined from the former. This suggests that, without running a reaction curve on a process, values of R_1 and L may be determined during adjustment of the controller.

Knowing the ultimate sensitivity (S_u) and the period at this sensitivity (P_u), a rearrangement of preceding equations shows how these values may be converted into L and R_1

$$L = P_u/4 \text{ min}$$

$\theta = \frac{P_u}{4}$ (integrating process)

$$R_1 = \frac{8}{P_uS_u} \frac{\text{in. per min}}{\text{psi}}$$

$K' = \frac{8}{P_u} K_u$

Classification of processes in terms of their unit reaction rates and lags would appear to be a decided improvement over present arbitrary methods.

Conclusions

We have proposed a system of units for measuring the control effects which are now in common use. When using these units, the values of the sensitivity, reset rate, and pre-act time all increase as the relative valve action per pen action increases.

The lag and unit reaction rate have been introduced as a quantitative measure of the controllability of processes, and we believe they form a good basis for a classification of processes.

Formulas have been presented which enable the controller settings to be obtained from an analysis of the process-reaction curves (that is, unit reaction rate and lag).

We have presented a simple method for adjusting the controller when it is installed on an application, making use of the ultimate sensitivity and period. Having shown that the controller settings can be obtained from the reaction curve, it will be possible for the equipment designer to calculate an approximate reaction curve for certain applications and thus determine the controller settings even before the equipment is built.

The usefulness of each particular control effect has been shown by examining its effect on the quality of control.

It has been pointed out that valve characteristics should be matched to each process so that a constant unit reaction rate prevails at all loads. This incidentally gives a rational explanation for the use of valves with special flow-lift characteristics.

Examination of pre-act response has shown that it improves control by increasing stability, reducing period, and allowing larger settings for the other responses. The relation between the pre-act setting and lag (or ultimate period) has simplified its adjustment. A summary of control effects is given in Table 1.

TABLE 1 SUMMARY OF CONTROL EFFECTS

Response	Action	Measure	Unit
Proportional	$\frac{\text{Valve movement}}{\text{Pen movement}}$	Sensitivity	Psi per in.
Automatic reset	$\frac{\text{Valve velocity}}{\text{Pen movement}}$	Reset rate	Per min
Pre-act	$\frac{\text{Valve movement}}{\text{Pen velocity}}$	Pre-act time	Min

Note that proportional response action may also be expressed as a valve velocity per pen velocity.

Summary of Controller Adjustments

Determine the ultimate sensitivity (S_u) and period (P_u), or the unit reaction rate R_1 and lag L. For the three types of controllers the optimum settings are as follows:

Proportional

$$\text{Sensitivity} = 0.5S_u = \frac{1}{R_1L}$$

Proportional plus reset

$$\text{Sensitivity} = 0.45S_u = \frac{0.9}{R_1L}$$

$$\text{Reset rate} = \frac{1.2}{P_u} = \frac{0.3}{L}$$

Proportional plus reset plus pre-act

$$\text{Sensitivity} = 0.6S_u = \frac{1.2}{R_1L}$$

$$\text{Reset rate} = \frac{2.0}{P_u} = \frac{0.5}{L}$$

$$\text{Pre-act time} = \frac{P_u}{8} = 0.5L$$

Discussion

E. S. Bristol.[3] The authors have presented a procedure for analyzing control and process characteristics which is logical, comparatively simple, and avoids the use of involved mathematics. The paper thus constitutes a worth-while contribution to the literature sponsored by the Committee on Industrial Instruments and Regulators in its endeavors to formulate standardized methods of approaching automatic-control problems.

Some of the terms and relations employed by the authors can be modified to advantage, in order to make the treatment more general in scope. From this point of view, it is believed preferable to express control action in terms of valve travel rather than in terms of actuating pressure on a diaphragm-operated valve. The latter procedure affords a basis for direct comparison of re-

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sults only for fluid-operated control valves having the same working pressure range. On the other hand, measurement of control action in percentage of full valve travel would apply to electrically operated, as well as fluid-operated power elements, regardless of the range or mode of application of the actuating media, and should not result in complicating the terminology. As a corollary of such a change the authors' "unit reaction rate," or rate of change resulting from 1 psi at the valve diaphragm, would be expressed as rate of change corresponding to full valve travel or to a stated fraction of full valve travel.

While it may be desirable in studying a controller mechanism to consider merely the action resulting from a pen movement measured in inches, this simplification presents difficulties when applied to any specific installation. Thus, on a temperature-control application, the significant characteristic is actual temperature variation and not the resultant pen motion of the particular recorder employed, which motion would vary with the individual scale range without reference to the inherent characteristics of the process. Possibly it is the authors' intention that the actual scale interval equivalent to 1 in. of pen travel be substituted in their relations, when dealing with any specific application.

It is noted that the authors' relation "reaction rate" multiplied by "lag" or R_1L is actually equivalent to the pen deviation that would occur in time L, with the pen moving at rate R. In other words, control sensitivity is found to be inherently related to the reciprocal of a hypothetical pen deviation. Attention is called to the fact that a simple manual simulation of two-position control can be imposed upon a process to investigate its reaction rate and lag characteristics. This can be done by watching a recorder measuring the variable to be controlled and manually opening or closing the valve whenever the pen crosses an arbitrarily selected control point. The slope of the resultant oscillating record where it crosses the control point constitutes a significant reaction rate. Also, the period of the resultant oscillation is related to the time required for valve change to affect the controlled variable. The product of the rate of change and the period as thus obtained constitutes another hypothetical pen deviation which can be used for a basis of correlation with the optimum throttling range or control sensitivity. The width of the pen band, obtained on a two-position test of this nature, is related to the rate of pen motion at the control point, and the period of oscillation, so that the pen band in itself is also a significant term for correlation with the optimum throttling range. The two-position test method for field checks is believed to be a particularly simple means for obtaining an indication of the response characteristics of a process.

G. A. Philbrick.[4] The authors exhibit the response given by the proportional-control action, on the one hand, when augmented by a differentiation, "pre-act," and on the other, when augmented by an integration, "reset." While such responses serve graphically to define these characteristics, it is striking that different generating functions are used in the two cases. Cannot these various classical control actions be better compared on the basis of some common impressed condition? To dispel the illusion of subterfuge, it is suggested that the authors exhibit in their closure the response of both sets of control actions when both varieties of change are imposed; or more simply, perhaps, the composite response of the three-term characteristic itself, for typical values of the three adjustables, when a sudden deviation occurs.

P. W. Keppler.[5] The authors have made a much-needed and highly useful contribution to the problem of setting regulators. However, in connection with the type of optimum transient curve recommended, it should be kept in mind that requirements vary over a wide range regarding uniformity of controlled flow, maximum deviation, average deviation, and stability. For example, any oscillation though damped may be hazardous if resonance can be set up by some other regulator connected to the same process.

The authors have also made a valuable comparison of control functions. To complete this comparison we should consider control based on measurement of the independent energy flow that causes the disturbance. This control function is widely used, generally by proportioning the controlled flow in some exact manner with the independent disturbing flow, and has therefore been called "exact correction."

While of course countless modifications are possible for this control function, in this exact form it requires no adjustment whatever and cannot possibly support any oscillation. It makes the admittedly undesirable "automatic reset" function unnecessary.

To illustrate "exact correction," a specific example is necessary, although it is universally applicable. For this purpose the writer has chosen a single-capacity process with dead time (velocity-distance) lag. In Fig. 9 of this discussion regulator E controls temperature T_4 of tank G, which is kept uniform by mixer H. Regulator E varies temperature T_3 entering pump F by moving gates C and D. Tanks A and B are assumed kept full with fluid at temperatures T_1 and T_2. Pump F maintains constant mass flow through the long pipe line M that introduces dead time lag. The manually operated gate K produces the independent energy flow that causes the disturbance. Float-controlled gate I keeps tank G full, but the constant temperature T_5 is below T_4. Temperature T_4 is transmitted to regulator E by bulb N for proportional and other control functions, and "exact correction" is obtained by suitably connecting E with the orifice L. This control function keeps the heat flow at E equal to the difference between that at K (neglecting variations in outlet temperature) and that at I. It is also assumed that T_4 must be measured so closely that the effect of self-regulation of T_4 is negligible.

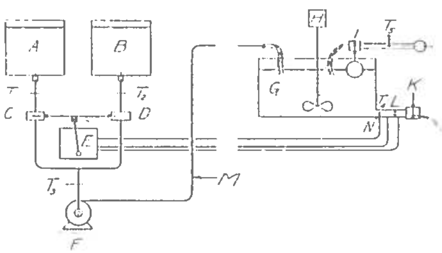

Fig. 9 Single-Capacity Process With Velocity-Distance Lag to Demonstrate "Exact Correction"

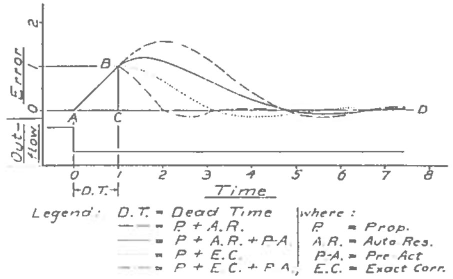

Fig. 10 Nondimensional Curves, Showing Advantages of "Exact Correction"

The nondimensional curves, shown in Fig. 10 of this discussion, obtained by graphical step-by-step integration, demonstrate the advantage of "exact correction" for any single-capacity process having dead time lag and lacking self regulation. The unit of time is the dead time of the process, and the unit of error is the error at time 1. The disturbance is the indicated drop in outflow, and the ideal accuracy obtainable is the broken line A-B-C-D.

The curves shown give very nearly the minimum average error obtainable from the control function of each curve. The regulators containing exact correction are not only shown to give much better accuracy, but they are also much easier to adjust since they contain fewer adjustments.

The added cost of orifice, etc., is of course a disadvantage, but there are ways of overcoming this, so that in many cases a marked reduction in first cost results from exact correction.

J. J. Grebe.[6] This paper gives the simple rules for adjusting the control constants of commercial instruments to have the proper characteristics for any one plant. These rules have been checked in actual plant operations on many types of instruments made by different manufacturers and one homemade unit that the writer described in 1933. The much disputed assertion that a good automatic control system using deviation, rate of change, and second-derivative responses, which are also called reset, proportional, and third response or pre-act, should be able to bring about a new balance in the system, within less than twice the elapsed time of the velocity distance lag or the dead time, has been proved by the work of the present authors.

The third response, which in general is a damped second-derivative function so as to fade out at the time when the second derivative works against good control, serves to counteract the effect of the dead time or the velocity-distance lag called L in Fig. 8 of the paper. Contrary to the opinions of some individuals, such lags are quite common, especially in the chemical industry where long dead times up to several minutes are encountered in processes where considerable time is required to make a change felt through chemically resistant but poor thermal conductors, or where it takes considerable time for solid reagents such as lime slurry to come to equilibrium with the solution.

For this reason, the importance of the third response cannot be overemphasized. In fact, if one were to build a universal instrument suitable for any application, it would be better to have a wide range of adjustment on the third response and reduce the flexibility of adjustment of the second, the proportional-position response. In other words, with a good third-response element, the throttling range can be quite narrow for almost any condition.

Let us hope that the authors may continue to develop the art and improve the maintenance and operation of control installations by following up this good work.

Authors' Closure

Mr. Bristol's suggestion that valve travel replace pressure on a diaphragm-operated valve is sound and should be further considered. In the opinion of the authors, it stems from a universal desire to express sensitivity in terms of a dimensionless unit or at least in terms of a unit applicable to all types of controllers. For this paper "psi per inch" was chosen rather than "per cent valve travel per per cent pen travel" principally because the latter did not appear to be a very euphonious combination. In addition, the "per cent per per cent" unit gives the false impression of being dimensionless. One disadvantage of using percentage of full valve travel is that limiting the stroke of a control valve would alter the sensitivity given in that unit but would not change the sensitivity given as psi per inch.

Inches of pen movement was used rather than per cent of scale range or degrees Fahrenheit since the former was thought to be a more general unit. Degrees Fahrenheit would be a good basis for comparing temperature-control applications, but there would be no analogy between that and the feet of water change in liquid level on another application.

The search for a dimensionless sensitivity ratio is not new. Ivanoff[7] had one in his "Over-All Sensitivity," the ratio of uncontrolled or potential deviation to controlled deviation. In the language of this paper, that would be the final deviation in inches of a reaction curve for a one psi pressure change divided by the reciprocal of controller sensitivity or the inches of pen movement necessary to give a one psi change in output. Ivanoff, however, was dealing with "self-controlling" processes which had a definite potential deviation for each valve opening. On some processes, valve movement determines only the reaction rate and the reaction curve never levels out. The potential deviation on these processes is infinite and Ivanoff's over-all sensitivity is infinite regardless of the controller sensitivity setting and hence meaningless. Even on this type of process, however, the authors' value of R_1L is finite and their ultimate sensitivity a definite value. It appears that controller sensitivity settings can be more universally referred to either ultimate sensitivity or R_1L than to potential deviations. In fact, a controller setting given as "per cent of ultimate sensitivity or as sensitivity $\times$ R_1L is dimensionless and is possibly the answer to the problem.

Another clue in the search for a sensitivity yardstick comes from a scrutiny of control quality. The area under curves such as Fig. 5(d) might be taken as a measure of poorness of control on either a temperature or liquid-level control application. This area in inch-minutes, easily convertible to either "feet-of-water minutes" or "degrees-Fahrenheit minutes" will be directly related to the product of R_1L, L, and ΔF, where ΔF is the difference in output pressure before and after the largest sudden load change to which the process will be subjected. On any process, a load change will give an area under the recovery curve of $(K)(\Delta F)(R_1)(L^2)$, where K is a constant determined by the point in the process at which the load change occurs and by the dimensionless quantities of controller settings, namely, sensitivity $\times$ R_1L; reset rate $\times$ L; and pre-act time/L. It can be seen that any valve-motion unit may be selected for use in R_1, ΔF, and sensitivity as long as it is used consistently in all three.

A method of interpreting the oscillating record obtained by impressed two-position control would certainly be a worthwhile contribution to the study of automatic control. It is hoped that Mr. Bristol will soon publish a detailed method of quantitatively determining application data by such a test. It would be extremely useful if small valve movements giving a record like Fig. 1, curve (c) could be accurately interpreted. Generally industrial processes cannot be disturbed by making large valve movements.

The old concept of pre-act response as a "kicker" may have

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[6] Director, Physical Research Laboratory, The Dow Chemical Company, Midland, Mich. Mem. A.S.M.E.

[7] "Theoretical Foundations of the Automatic Regulation of Temperature," by A. Ivanoff, *Journal of the Institute of Fuel*, Vol. 7, no. 33, Feb., 1934.

prompted Mr. Philbrick's request for showing its response to a sudden pen movement. In the interest of clarity the authors used a sustained pen deviation to show reset rate and a constant rate of pen movement to illustrate pre-act time. The course of output pressure from a controller with proportional plus automatic reset responses for a constant rate of pen movement would be as shown in Fig. 11. The proportional response is 2 psi per minute as in Fig. 6(*b*) and the reset rate one per minute. At any instant the output pressure from automatic reset is rising at a rate equal to the proportional-response output change times the reset rate. The addition of pre-act response will give an additional output pressure equal to the rate of output pressure change due to the proportional response times the pre-act time.

Analysis of pre-act response from an impressed sudden pen

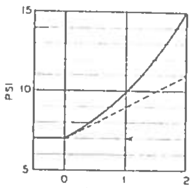

Fig. 11

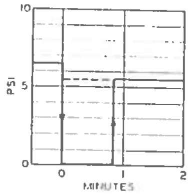

Fig. 12

movement is purely hypothetical because instantaneous pen movements are not met with in practice. A true derivative mechanism would give, for such a pen action, an infinite output change. Actually air-operated controllers do not give an output pressure lower than atmospheric nor higher than their supply pressure. A controller with proportional and pre-act responses would give an output pressure change as shown in Fig. 12 if a sudden pen motion were impressed equivalent to 1 psi proportional-response change. The pre-act time in Fig. 12 is 5 min.

As Mr. Keppler points out, control requirements on certain applications may be so strict that the improvement given by pre-act response may still not hold a pen within the tolerance required. In these cases it is necessary to cast about for another variable upon which a separate or related response may be based. While the study of these multiple controller systems is beyond the scope of this paper, it may be said that they are commonly used and are often very necessary to achieve desired control results. Grebe[8] has called this "metered control."

The type of multiple controller system shown by Mr. Keppler makes use of a separate flow measurement as an indication of demand, to reset the temperature controller. This removes the need for an automatic-reset response working on a basis of temperature pen deviation. The elimination of automatic reset in the temperature controller, however, would allow an offset if any other load change came into the system, for example, a change in temperature of one of the three incoming flows. Also, it would be rather difficult mechanically to convert the reading of flow into an exact mixed liquid temperature unless gates *C* and *D* reproduced flows exactly.

The more common multiple controller system is one in which one controller calls, not for a valve opening, but for a set point change on another controller capable of correcting for the major load change from a measurement at a point of favorable lag. Explaining this from Fig. 9, if the major load change in the system were not the position of gate *K* but temperature T_1, the control system would consist of two temperature controllers. One temperature controller would measure T_2 and operate gates *C* and *D* to maintain T_3. The second controller measuring T_4 would call for the required T_3 necessary to maintain T_4. The first controller would quickly correct for changes in T_1 and T_2 or partial clogging of gates *C* and *D*. The second controller would raise or lower T_3 to correct for the minor load changes such as temperature T_4 or flow through *I*.

[8] "Elements of Automatic Control," by John J. Grebe, *Industrial and Engineering Chemistry*, vol. 29, Nov., 1937, p. 1225.

IV
Giant Strides