Chapter 7

ON-LINE OPTIMIZATION AND SELECTION OF MEASUREMENTS

This is the last of three chapters that discuss optimal operation of a general heat exchanger network. A method that combines the use of steady state optimization and decentralized feedback control is proposed. A general steady state model is developed, which is easily adapted to any heat exchanger network. Using this model periodically for optimization, the operating conditions that minimize utility cost are found. Setpoints are constant from one optimization to the next, and special attention is paid to the selection of measurements such that the utility cost is minimized in the presence of unknown disturbances and model errors. As opposed to the two preceding chapters, the method in this chapter results in a *fixed* control structure. In addition to heat exchanger networks, the proposed method may also be applied to other types of processes where the optimum lies at the intersection of constraints.

The method is proposed by Glemmestad *et al.* (1997), and much of the content in this chapter is taken from that paper.

7.1 Introduction

The problem being addressed is the same as in the two previous chapters, i.e. optimal operation of a given HEN. The approach in this chapter, however, is different. A fixed control structure for control of the target temperatures is selected, and the optimization is carried out by utilizing the extra manipulations and extra measurements internally in the HEN. The method uses steady state optimization which is carried out on-line with regular time intervals. The result of this optimization is then implemented by specifying the optimal value (setpoint) of some variables ("optimization variables"). It will be shown that the

selection of which optimization variables that are controlled affect the performance of the HEN when unknown disturbances are present. A procedure for optimal selection of these variables is presented.

In the following, it is assumed that the stream data (heat capacity flowrates and supply/target temperatures), network structure and heat exchanger areas are given and that the HEN is sufficiently flexible. To manipulate the network it is assumed that utility duties can be adjusted and that a variable bypass is placed across each process-to-process heat exchanger. In case of stream splits, we may also assume that split fractions can be varied.

The remaining part of the chapter is organized as follows: First, the complete method is outlined in section 7.2 and it is demonstrated that the selection of which variables that are kept at setpoints influences the performance. In section 7.3, the procedure for selection of optimization variables will be described in detail, and the term robust optimum will be introduced in section 7.4. The steady state optimization model is presented in section 7.5, then the complete method is applied to an example in section 7.6 and finally some conclusions are drawn.

7.2 Outline of method

In order to perform a meaningful on-line optimization, it is required that there is at least one extra degree of freedom during operation. As shown in chapter 4, most HENs have this feature.

Figure 7.1a shows a schematic block diagram of the method that will be described. The optimizer contains a scalar objective function (criterion) J which indicates how well the HEN is operated, and a steady state model of the HEN. As the objective function we will use total utility cost of the HEN. The model is optimized regularly and reference values for the optimization variables are passed to the controller K_2. The reference values (setpoints) are constant in the period between each optimization.

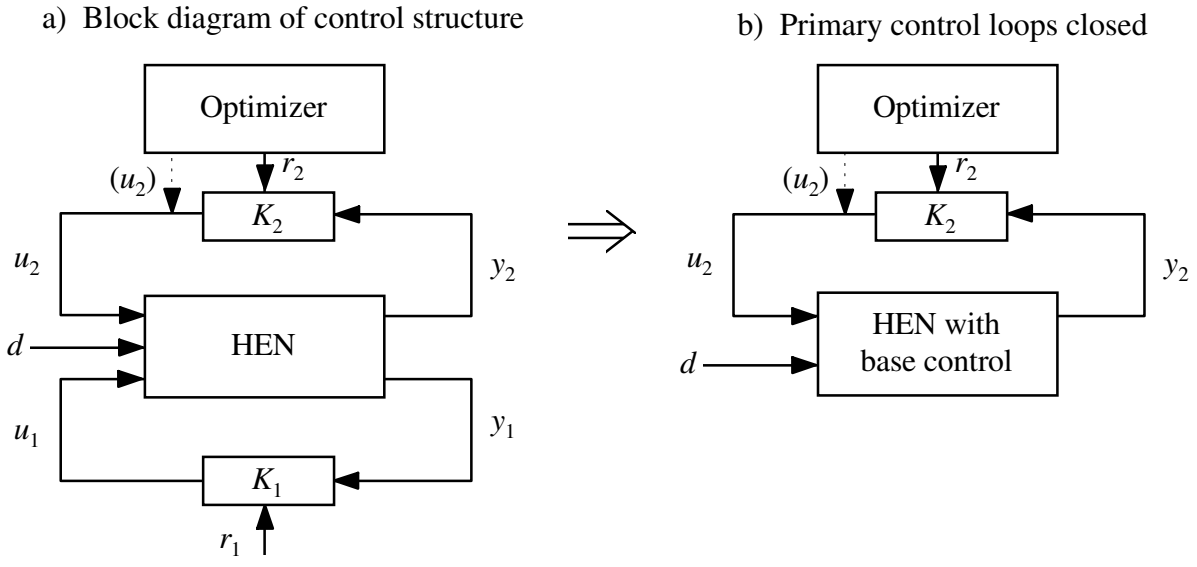

Figure 7.1 General optimizing control structure.

All inputs (manipulations) u and outputs (measurements) y are separated into $u = [u_1 \quad u_2]^T$ and $y = [y_1 \quad y_2]^T$, respectively. y_1 are those outputs which have given target (reference) values and u_1 are those manipulations dedicated to keep y_1 at their target values. Satisfying the targets for y_1 is simply the primary goal of optimal operation (definition 2.1). Now, we close the control loops for the primary outputs with controller K_1 ("base control") and assume integral action in the loops (no steady state control error). This leads to figure 7.1b where focus is on the remaining part of the system, i.e. the secondary variables (u_2, y_2 and setpoints r_2) and the optimizer. It is simply assumed that the base control is implemented and that it works.

We want to focus on the secondary goal of optimal operation; utility cost minimization (variables associated with this goal have index 2). u_2 is the "excess" manipulation(s) which represent the degree(s) of freedom that we will use to minimize utility cost. Of course, one could compute optimal values for u_2 and apply these directly (open-loop implementation) as indicated by the dashed line in figure 7.1a and 7.1b. Alternatively, the optimizer could pass reference values for some "extra" measurements y_2 (closed-loop implementation). If the disturbance d was perfectly known (and constant), it would not matter (at steady state) which variables were chosen. However, from the explanation below it will be clear that the selection of *which* variables that are passed from the optimizer down to the control level affects how close to optimum the HEN can be operated.

The variables (setpoints) r_2 that are passed from the optimizer to the control level will be denoted *optimization variables*.

Let the disturbance d be partitioned into the following two contributions:

$$d = d_0 + d_u \tag{7.1}$$

where d_0 is the information that the optimizer has about the disturbances when it performs an optimization, and d_u (unknown disturbances) are all deviations from d_0 and the real disturbance until a new optimization is carried out. That is, d_u consists of for example unknown disturbances and model errors in addition to changes of the disturbances in the period between two optimizations (optimization interval). Measurement/estimation errors will not be handled explicitly in this chapter, but these errors may be included in d_u and treated as any other deviation.

Since the optimizer has no specific information about d_u, the optimization is based on $d = d_0$. In practice, however, d_u may vary within some known (or selected) bounds. The effect of $d_u \neq 0$ should be taken care of in the optimizer in order to avoid that the HEN becomes infeasible (primary goal can not be satisfied) for some disturbances. Figure 7.2 shows a typical situation for a general plant with one degree of freedom (one extra manipulation) and an objective function J that should be minimized. The plant has one disturbance input and two candidate measurements A and B ($y_2 = [y_{2,A} \quad y_{2,B}]^T$) that can be controlled to a desired value using the extra manipulation u_2. (Since subscripts 1 and 2 are used to distinguish between the primary and secondary sets of inputs and outputs, we uses letters A, B etc. to denote individual elements of u and y). Also, remember that base control to keep primary outputs at fixed setpoints is already implemented.

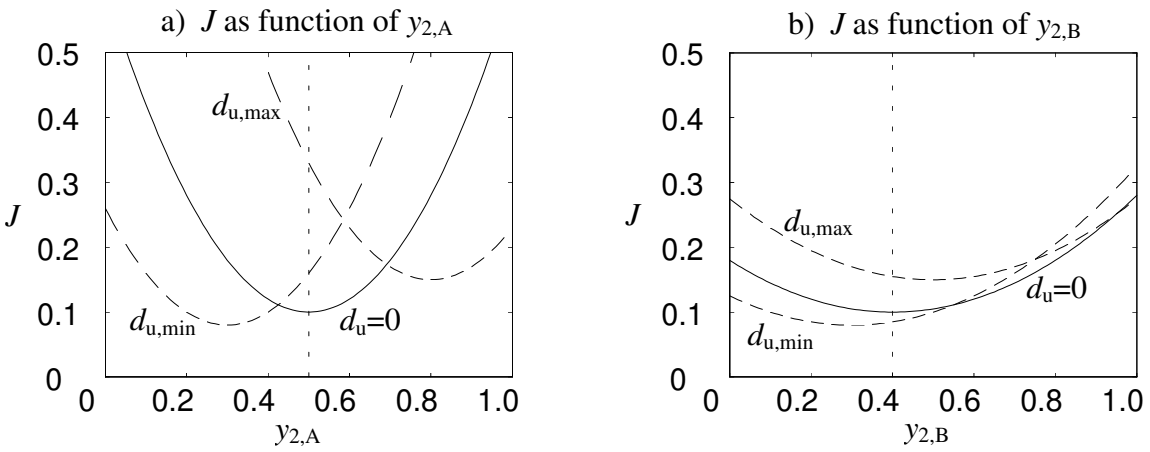

Figure 7.2 Unconstrained process.

Figure 7.2a shows J as a function of $y_{2,A}$ with the disturbance as a parameter. The solid line is for $d_u = 0$, and the two dashed lines represent the extremes for d_u. Figure 7.2b shows similar curves as a function of $y_{2,B}$. Since we have to base our optimal values on $d_u = 0$, we can choose to keep either $y_{2,A} \approx 0.5$ or $y_{2,B} \approx 0.4$ using feedback control. From the figure, however, we see that when keeping $y_{2,B}$ constant, J is *less sensitive to both variations in* y_2 (control error) *and to unknown disturbances,* than when keeping $y_{2,A}$ constant. Therefore we prefer to keep $y_{2,B}$ constant between the optimizations. This simple example illustrates how the choice of optimization variables affects the objective function for an unconstrained process. Figure 7.3 shows similar curves as in figure 7.2 for a process where the optimum is *constrained*, which is typical for most HENs. (Minimum utility consumption corresponds to maximum utilization of process-to-process exchangers which again means that some bypasses are closed).

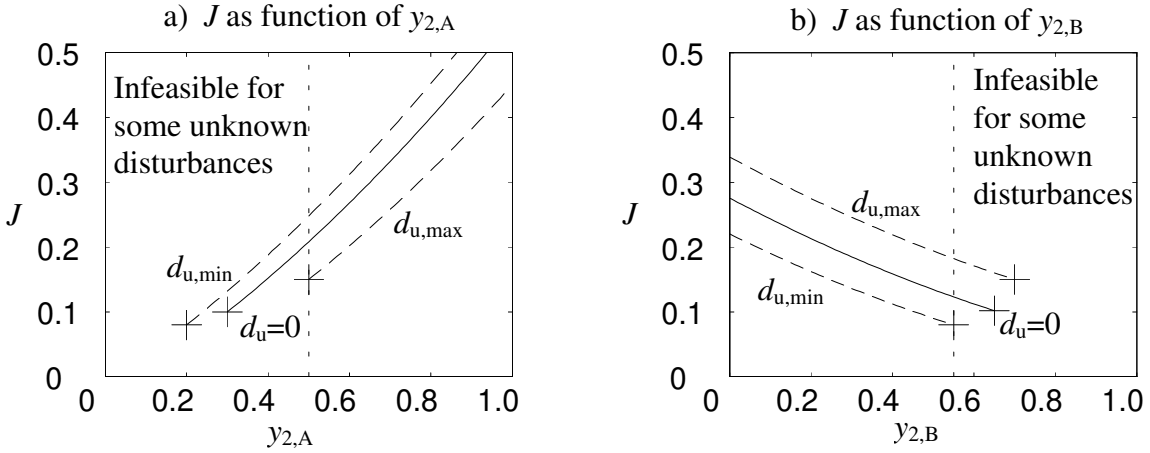

Figure 7.3 Constrained process (typical for HENs).

In figure 7.3a the process is infeasible when $y_{2,A}$ becomes too small (marked with "+"). In a HEN this typically happens when a bypass saturates such that a target temperature no longer can be met. When $y_{2,B}$ is kept at a given value (figure 7.3b) the process is infeasible when the value becomes too *large*. More interesting in the constrained case, however, is that the *nominal* optimum ($d_u = 0$) is *infeasible* for some unknown disturbances. That is, we have to "back off" from the nominal optimum and find the optimal values that are feasible for all

unknown disturbances. These values are indicated with the vertical dashed lines in figure 7.3a and 7.3b. Again we see that it is preferred to keep $y_{2,B}$ (rather than $y_{2,A}$) constant. However, a new obstacle has occurred; we need a remedy in the optimizer to find the values that are optimal also in the presence of the unknown disturbances. We do not want to compute optimum for all possible disturbances during operation since this may be too time consuming. This problem can be solved by computing proper constraints ("safety margins") on u_1 that are implemented in the optimizer. The optimal value of these safety margins is strongly correlated to the choice of optimization variables.

The following steps summarize the main parts of the complete procedure for on-line optimization of HENs:

1. Determine which manipulations (u_1) that should be used to control the primary outputs y_1 and design a control configuration and controllers for the primary goal (base control).
2. For each excess manipulation u_2 choose a measurement y_2 (among all candidates) such that the operation is insensitive to disturbances (see more details in next section). The additional constraints (safety margins) on u_1 are also found. Design decentralized controllers to control y_2.
3. Implement the steady state model including the constraints found in step 2 in the optimizer.

During operation, the optimizer computes setpoints for the optimization variables and apply these to the controller K_2 at regular intervals.

7.3 Selection of measurements

This section describes a procedure for selection of optimization variables (step 2 in the complete method given above). The selection of outputs for optimizing control is discussed in Morud (1995, chapter 8) and in Skogestad and Postlethwaite (1996, chapter 10). In the latter, a method that is based on choosing outputs that maximize $\underline{\sigma}(G_{22})$ (smallest singular value) for a properly scaled system is proposed. In this chapter a more direct method is applied (which is also mentioned in Skogestad and Postlethwaite, 1996). Before the procedure is presented, the following notation is introduced:

$y_{2,\text{cand}}$ is a vector containing all candidates to y_2.

y_{opt} is the optimal value of $y_{2,\text{cand}}$ for a given d_u.

y_{opt}^s is a fixed value of $y_{2,\text{cand}}$ such that the objective function is minimized while the network is feasible for all d_u.

J^s is $J(y_{\text{opt}}^s)$ for a given value of d_u.

Δu_1 is the constraint imposed on u_1 such that an optimization problem based on $d_u = 0$ gives feasibility for all d_u within prespecified bounds.

The steps in the procedure are listed below and some of the points will be further explained. For simplicity, we will assume there is only one degree of freedom (one optimization variable).

a) Select (i) minimum and maximum values for d_u, (ii) the objective function J, (iii) the entries of $y_{2,cand}$, (iv) the values for d_u that should be included in the computations and (v) define the type of J_{mean} that will be used for choosing optimization variable.

b) Compute y_{opt} and J_{opt} for "all" cases of d_u (i.e. the values from step (iv) in the previous point), see table 7.1. This table may also include row(s) for $u_{2,opt}$ (open-loop implementation). Note that $d_{u,j}$ is *case j* of d_u while $y_{opt,i}$ denotes *element i* in y_{opt}.

	$d_{u,1}$	$d_{u,2}$	$d_{u,j}$
$y_{opt,\,A}$	$y_{opt,\,A}(d_{u,1})$	$y_{opt,\,A}(d_{u,2})$	$y_{opt,\,A}(d_{u,j})$
$y_{opt,\,B}$	$y_{opt,\,B}(d_{u,1})$	$y_{opt,\,B}(d_{u,2})$	$y_{opt,\,B}(d_{u,j})$
$y_{opt,\,i}$	$y_{opt,\,i}(d_{u,1})$	$y_{opt,\,i}(d_{u,2})$	$y_{opt,\,i}(d_{u,j})$
J_{opt}	$J_{opt}(d_{u,1})$	$J_{opt}(d_{u,2})$	$J_{opt}(d_{u,j})$

Table 7.1 y_{opt} and J_{opt} for all cases of d_u .

c) Keep $y_{2,cand,i} = y^s_{opt,i}$ for each output candidate, and evaluate $J_i^s(d_{u,j})$ and the resulting J_{mean}. In general, the setpoint $y^s_{opt,i}$ should be optimized in order to minimize J_{mean}, but for constrained processes it will be some extreme value from table 7.1 (to ensure feasibility for all d_u).

	$d_{u,1}$	$d_{u,2}$	$d_{u,j}$	J_{mean}
J^s_A	$J^s_A\,(d_{u,1})$	$J^s_A\,(d_{u,2})$	$J^s_A\,(d_{u,j})$	$J_{mean,\,A}$
J^s_B	$J^s_B\,(d_{u,1})$	$J^s_B\,(d_{u,2})$	$J^s_B\,(d_{u,j})$	$J_{mean,\,B}$
J^s_i	$J^s_i\ (d_{u,1})$	$J^s_i\ (d_{u,2})$	$J^s_i\ (d_{u,j})$	$J_{mean,\,i}$

Table 7.2 J^s for all cases of d_u .

d) Choose the variable that gives the smallest J_{mean} from the last column in table 7.2 as optimization variable, i.e. this measurement should be controlled to a setpoint which is updated periodically by the optimizer.

We have now found the best optimization variables. To simplify the on-line optimization we may want to use only the nominal disturbance set, $d_u = 0$. To ensure that we find the correct value of y^s_{opt} (which ensures feasibility for all disturbances), we may impose some constraint ("safety margin") for the optimizer, e.g. $u_1 \geq \Delta u_1$. This will be explained in more detail for a simple example in section 7.6. (See also remark 7.1 at the end of this section). The "safety margin" on u_1 should of course not be implemented in the regulatory control level.

Until now we have only considered one degree of freedom. If there were 2 degrees of freedom, 2 elements of $y_{2,cand}$ would have to be fixed at a time. Table 7.2 would need as many rows as there are possibilities to pick two variables out of the total number of candidate

measurements. For example, if there are 6 candidate measurements and 2 degrees of freedom, the number of possibilities is $\frac{6!}{2!4!} = 15$.

REMARK 7.1. It is clear that the value of Δu_1 may depend on d_0. We assume that this change is small and that the value can be used for all d_0. In practice, one should carry out the procedure for selection of optimization variables for different d_0, to verify that Δu_1 does not change too much. The worst case value should be chosen if it is not acceptable to violate the primary goal, while a mean value can be used if a small violation to the targets is tolerable.

7.4 Robust optimum

This section introduces the term *robust optimum*. It will be clear that, when we have made a selection of variable(s) for y_2, we seek the value of this variable that corresponds to the robust optimum. The robust optimal value is the optimal value when unknown disturbances and model errors are encountered. The robust optimum will often be different from the nominal optimum, where no disturbances (only nominal disturbance values) or model errors are considered.

For the unconstrained and smooth objective function in figure 7.2, the value of y_2 resulting in robust optimum is approximately equal to the value resulting in nominal optimum. For the constrained objective function in figure 7.3, the situation is different. We require that the process is feasible for the unknown disturbances. When $y_{2,A}$ is selected, the *nominal* optimal value is $y_{2,A} = 0.30$ whereas the value resulting in *robust* optimum is $y_{2,A} = 0.50$. When $y_{2,B}$ is selected, the *nominal* optimal value is $y_{2,B} = 0.65$ whereas *robust* optimum is achieved for $y_{2,A} = 0.55$. It is clear that the nominal optimal value cannot be applied since this will result in infeasiblity for some unknown disturbances.

Mathematically, if we have an objective function $J(x, d)$ where the value of the variable (argument) x can be selected/manipulated and d represent the disturbances, we can write

$$\text{Nominal optimum:} \quad J_{\text{nopt}} = \text{opt}(J(x, d = d_0))$$

$$\text{Robust optimum:} \quad J_{\text{ropt}} = \text{opt}(J(x, d \in D))$$

In operation, we are not interested in the objective value directly, however, the argument (e.g. manipulations or setpoints) that minimize the objective is more relevant:

$$\text{Nominally optimal argument:} \quad x_{\text{nopt}} = \arg \operatorname*{opt}_x (f(x, d = d_0))$$

$$\text{Robust optimal argument:} \quad x_{\text{ropt}} = \arg \operatorname*{opt}_x (f(x, d \in D))$$

The set D represent the possible values of the unknown disturbances. D may also represent a *probability distribution* of the unknown disturbances. The disturbances may for instance be assumed to be normally distributed with a given mean (nominal) value and a given standard deviation. If the probability distribution is not bounded, we cannot require the process to be feasible for *all* possible disturbances. Instead, we could require feasibility with a specified probability, such as requiring feasibility in 99% of the operating time. We shall, however, use a different approach where the disturbances are specified by probability distributions:

When a HEN is announced infeasible during operation, this normally does not imply that the HEN cannot be operated. Typically, the reason for infeasibility is that it is impossible to reach the targets for the primary measurements (outlet temperatures). Instead of requiring that the targets are satisfied, we choose to penalize the deviation (control error in primary control loop). In this way, the constrained problem is transformed into an unconstrained problem, however, the new objective function will often be *asymmetric*. This implies that the robust optimum is different from the nominal optimum.

In the search for the robust optimum, we simply use Monte Carlo simulations. That is, random disturbances are generated from the given probability distribution. For each set of random disturbances the objective J is computed as a function of the selected secondary variable. This is repeated for a large set of randomly generated disturbances, and the robust optimum is the optimum of the mean value of J for the set of disturbances.

An example is shown in figure 7.4. below. There are 2 different disturbances and they are both normally distributed. The thin solid line shows the objective J as a function of the selected secondary measurement y_2 for the nominal disturbances. Starting from a high value of y_2, the objective decreases when y_2 decreases until the nominal optimum where $y_2 = 150$. Decreasing y_2 further, the primary outputs can no longer be kept at target. This is penalized in the objective function resulting in a steep increase of the objective.

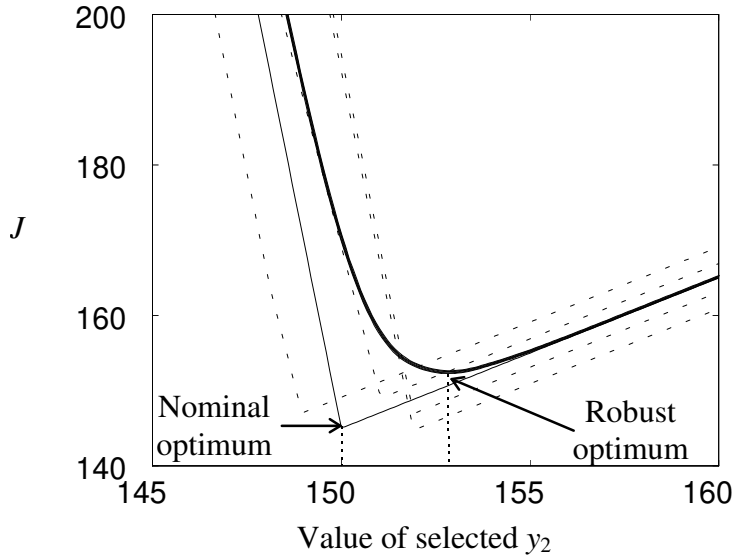

Figure 7.4 Robust optimum

The 4 dashed lines show the objective for 4 different sets of the disturbances. The location where the primary targets are violated and objective increases steeply, depends on the value of the disturbances. The most interesting curve in figure 7.4, however, is the thick solid curve. This curve is the average of 1000 randomly generated sets of disturbances. While each fixed disturbance result in a sharp break of the corresponding curve, the average curve is smooth. To reach the robust optimum, a setpoint of $y_2 \approx 153$ should be applied. The robust optimum is slightly above 150, which is higher than the nominal optimum of $J = 145$. However, applying the nominally optimal values of $y_2 = 150$, the *expected* value of the objective is roughly $J \approx 170$. Since we do not know the exact value of the disturbances, only

the probability distribution, the optimal setpoint for y_2 is the value corresponding to the robust optimum ($y_2 \approx 153$).

The curves in figure 7.4 are actually based on the results for the HEN in the example in section 7.6, when the disturbances are normally distributed. The supply temperature of stream H1 has a mean value of 190°C and a standard deviation of 3°C, while CP_{C2} has a mean value of 0.50kW/°C and a standard deviation of 0.01kW/°C. The variable selected for y_2 is the setpoint for temperature T_1 (in figure 7.6) and the objective function is:

$$J = \text{utility consumption [kw]} + 25\left|T_{C2}^{o} - T_{C2}^{t}\right| + 25\left|y_2 - r_2\right|$$

That is, a deviation of 1°C in the bypass controlled temperature or in T_1 is assumed to "cost" the same as 25kW.

If the Monte Carlo method is not used and one requires the process to be feasible (primary targets met) for all possible unknown disturbances, a number of difficulties may be encountered in practice:

1. The worst case may not be at the corner points for the disturbances, thus finding the worst case disturbances may be a difficult task itself.
2. Even a corner point check may be time consuming and even prohibitive when there are many independent disturbances.
3. If there are more than just a few disturbances, the probability that all have the worst case values at the same time is very small.

Taking these 3 points into account, it is assumed that Monte Carlo simululations will give reasonable results for most practical cases. Using the Monte Carlo approach, it is also straightforward to include dependencies between different disturbances when such information is available. Despite this preference for the Monte Carlo approach, in the example in section 7.6 it is required that the HEN is feasible at the corner points for the 2 disturbances.

A disadvantage with the Monte Carlo method is that a large number of disturbance sets may have to be generated in order to get good results, and it is difficult to say in advance how many disturbance sets that are necessary. However, as it is explained in the previous section, the robust optimum is only determined once (or for a few cases) and this is done off-line. The difference in the nominal and robust optimal values are used to find a constraint on the primary manipulations u_1. With this constraint, the (approximate) robust optimum is found periodically from the measured and inaccurate values of the disturbances.

7.5 Steady state optimization model

This section presents a steady state model that can be adapted to any HEN. It is developed primarily for implementation in the optimizer, however, it may also be used in the procedure for selection of optimization variables (to generate tables 7.1 and 7.2).

Before we present the general model, consider the two alternatives (equations (7.2) and (7.3), respectively) to model a single heat exchanger with bypass, see figure 7.5.

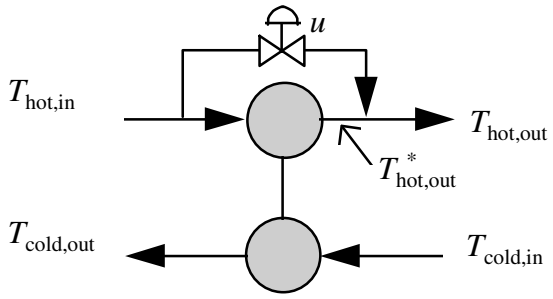

Figure 7.5 Single heat exchanger with bypass.

At steady state it is of no consequence whether the bypass is placed across the hot side or cold side, and the choice in the figure is arbitrary. The temperature driving force $\Delta T_m(\cdot)$ may be the logarithmic mean or some approximation to it. Note particularly the difference between equations (7.2a) and (7.3) regarding the arguments of $\Delta T_m(\cdot)$.

$$Q = UA\Delta T_m(T_{hot,\,in}, T_{cold,\,in}, T^*_{hot,\,out}, T_{cold,\,out}) \tag{7.2a}$$

$$T_{hot,\,out} = uT_{hot,\,in} + (1-u)T^*_{hot,\,out} \tag{7.2b}$$

$$Q \le UA\Delta T_m(T_{hot,\,in}, T_{cold,\,in}, T_{hot,\,out}, T_{cold,\,out}) \tag{7.3}$$

Equation (7.2) includes the hot exit temperature before it is mixed with the bypass stream and this results in bilinearities in (7.2b). The inequality in (7.3) expresses a constraint on Q when the boundary is placed *outside* the bypass splitter and mixer. The bypass fraction u does not even occur in (7.3), but the equality part of (7.3) corresponds to $u = 0$. In the optimization model, we choose the second alternative for each heat exchanger since this eliminates the bilinearities in the bypass mixer. If u is needed, it can be found after the optimization of the network by solving one nonlinear equation for each bypass fraction. This equation can be found from solving (7.2b) for $T^*_{hot,\,out}$ and inserting this expression into (7.2a), which is solved for u through iteration. (Solving one unknown in one nonlinear equation n times is much simpler than solving n unknowns in n nonlinear equations simultaneously). As it will be shown, the value of u is often *not* required explicitly as it normally is the manipulated input in a feedback control loop.

The steady state model for a general HEN uses the following sets of heat exchangers:

PHX : Set of all Process-to-process Heat eXchangers.

HBT : Subset of PHX with Hot side outlet directly entering a Bypass controlled Target.

CBT : Subset of PHX with Cold side outlet directly entering a Bypass controlled Target.

HUT : Subset of PHX with Hot side outlet entering a Utility controlled Target (through a cooler).

CUT : Subset of PHX with Cold side outlet entering a Utility controlled Target (through a heater).

HS : Subset of PHX with Hot side inlet directly entering from a (hot) Supply.

CS : Subset of PHX with Cold side inlet directly entering from a (cold) Supply.

The general HEN model shown below (equations 7.4 to 7.12b) is an NLP problem. The variable c in equation (7.4) denotes the cost (pr. energy unit) for the utilities.

$$\min\Big(\sum_{i\in\mathrm{HUT}} c_i^{\mathrm{coolers}} Q_i^{\mathrm{coolers}} + \sum_{j\in\mathrm{CUT}} c_j^{\mathrm{heaters}} Q_j^{\mathrm{heaters}}\Big) \tag{7.4}$$

subject to

Equalities, (7.5) to (7.9)

$$Q_i = CP_i^{\mathrm{hot}}\,(T_i^{\mathrm{hot,in}} - T_i^{\mathrm{hot,out}}) \qquad i \in \mathrm{PHX} \tag{7.5a}$$

$$Q_i = CP_i^{\mathrm{hot}}\,(T_i^{\mathrm{hot,in}} - T_i^{\mathrm{hot,out}}) \qquad i \in \mathrm{PHX} \tag{7.5b}$$

$$Q_i^{\mathrm{coolers}} = CP_i^{\mathrm{hot}}\,(T_i^{\mathrm{hot,out}} - T_i^{t}) \qquad i \in \mathrm{HUT} \tag{7.6a}$$

$$Q_i^{\mathrm{heaters}} = CP_i^{\mathrm{cold}}\,(T_i^{\mathrm{t}} - T_i^{\mathrm{cold,out}}) \qquad i \in \mathrm{CUT} \tag{7.6b}$$

$$T_i^{\mathrm{hot,out}} = T_i^{\mathrm{t}} \qquad i \in \mathrm{HBT} \tag{7.7a}$$

$$T_i^{\mathrm{cold,out}} = T_i^{\mathrm{t}} \qquad i \in \mathrm{CBT} \tag{7.7b}$$

$$T_i^{\mathrm{hot,in}} = T_i^{\mathrm{s}} \qquad i \in \mathrm{HS} \tag{7.8a}$$

$$T_i^{\mathrm{cold,in}} = T_i^{\mathrm{s}} \qquad i \in \mathrm{CS} \tag{7.8b}$$

Interconnection equations (problem specific) (7.9)

Inequalities, (7.10) to (7.12b)

$$Q_i \le \alpha_i U_i A_i \Delta T_{\mathrm{m}\,i} \qquad i \in \mathrm{PHX} \tag{7.10}$$

$$Q_i \ge 0 \qquad i \in \mathrm{PHX} \tag{7.11}$$

$$Q_i^{\mathrm{coolers}} \ge 0 \qquad i \in \mathrm{HUT} \tag{7.12a}$$

$$Q_i^{\mathrm{heaters}} \ge 0 \qquad i \in \mathrm{CUT} \tag{7.12b}$$

Note that the index denotes heat exchangers and *not* streams (which is common in many other models), and that ΔT_{m} denotes the temperature driving force *outside* the bypass stream as in equation (7.3). As an example, the network in figure 7.6 will lead to the following sets: PHX = {A,B}, HUT = {B}, CUT = {A}, HBT = ∅, CBT = {B}, HS = {A} and CS = {A,B}, and the only interconnection equation (7.9) is $T_{\mathrm{A}}^{\mathrm{hot,out}} = T_{\mathrm{B}}^{\mathrm{hot,in}}$.

During each optimization, T^{t}, T^{s}, *CP* and *UA* for each heat exchanger are treated as constants. The model is valid without modifications for networks with fixed stream split fractions since *CP* denotes heat flow capacity in each heat exchanger. For networks with variable stream splits, *CP* in the split streams can be regarded as variables, and equations that

preserve the mass balance in the splitter(s) and energy balance in the mixer(s) must be included. During operation, variable stream splits can be used as manipulated inputs.

The constant α in equation (7.10) is a factor that may limit the duty of a heat exchanger somewhat below its theoretical maximum. This is simply the way that the constraint on u_1 is implemented. Instead of implementing $u_1 \geq \Delta u_1$ directly (which is impossible since the model does not include u_1), the corresponding value for α has to be computed. This is done separately for each heat exchanger that controls a primary output. The example below explaines how this can be done. For heat exchangers associated with u_2, we have $\alpha = 1$.

The model does not include any upper constraints on the duty of the utility exchangers, and this implies the assumption that these are designed to handle the required duty. If this is not the case, additional constraints have to be added to the model, e.g. an upper limit on the duty.

The only possible source of nonlinearities in the model (for networks without variable splits) is the term ΔT_m in equation (7.10). In other words, if arithmetic mean (as opposed to logarithmic mean) is used as the temperature driving force, the model can be solved as an LP problem. The following procedure for solving the model has proven to be reliable: First, use arithmetic mean in (7.10) for all exchangers and solve the corresponding LP problem. Second, replace arithmetic mean with logarithmic mean (or e.g. Paterson or Chen approximations) and solve the NLP problem using the LP solution as the initial value.

7.6 Example

The HEN used in the example is shown in figure 7.6. The primary outputs are the outlet temperatures of each stream which should be controlled to their target values of 30, 160 and 130°C for streams H1, C1 and C2, respectively. That is, we have

$$y_1 = \begin{bmatrix} T_{H1}^o & T_{C1}^o & T_{C2}^o \end{bmatrix}^T$$

where superscript ^{o} denotes outlet temperature. There is a total of four manipulations (two bypasses and two variable utility duties) which gives

$$u = \begin{bmatrix} u_A & u_B & q_c & q_h \end{bmatrix}^T$$

There are two disturbances; ±10°C in the supply temperature of stream H1 and ±0.05 kW/°C in the *CP* of stream C2. These values represent the maximum variations d that may be present. The smaller variations/errors (d_u) that may occur within the optimization interval is defined in step 2a) of the procedure. *UA* for heat exchangers A and B are 0.523 and 1.322 kW/°C, respectively. For simplicity, it is assumed that the utility exchangers are able to deliver sufficient duty for all possible cases. With this assumption and the given *UA*-values, all target temperatures can be reached for all combinations of disturbances mentioned above.

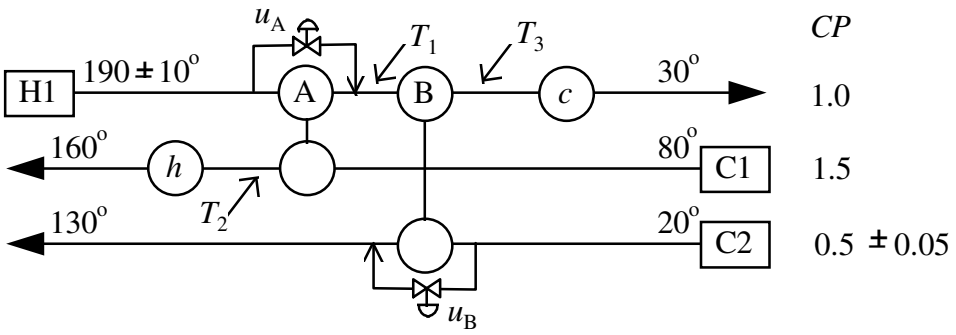

Figure 7.6 Heat exchanger network used in example.

Applying the procedure step by step yields:

Step 1. Assign primary manipulations.
We use the main rule for selection of manipulations in HENs which is to choose the manipulation closest to the measurement (e.g. Mathisen, 1994, chapter 4). This implies that the primary manipulations u_1 become q_c, q_h and u_B and these control the outlet temperatures of streams H1, C1 and C2, respectively.

Step 2. Selection of optimization variable.
There is one excess manipulation, $u_2 = u_A$, and the steps a) to d) below illustrate the selection of optimization variable.

2a) We assume:

(i) $d_u = [\pm 3°C, \pm 0.01kW/°C]^T$ (maximum variations/errors of the disturbances within the optimization interval).

(ii) The objective function is $J = q_c + q_h$ (utility consumption)

(iii) Possible candidates to y_2 are $y_{2,cand} = [T_1 \ T_2 \ T_3 \ u_A]^T$ (see figure 7.6). Note that the open-loop implementation (u_A) is an alternative.

(iv) The computations are done for the four "corner points" of d_u in addition to $d_u = 0$. J_{mean} is the arithmetic mean of these five cases. (We require that target temperatures have to be reached for the five cases).

2b) y_{opt} and J_{opt} for different d_u are shown in table 7.3. The table is generated for $d_0 = [0 \ 0]^T$, i.e. for nominal values of the disturbances (190°C and 0.5kW/°C). Also a row for $u_{B,opt}$ is included for extra information.

2c) Table 7.4 shows J for optimal fixed values of $y_{2,cand}$. Note that in this example, the values for $y_{2,cand}$ can be found without optimization, but simply from table 7.3 and physical insight (see remark 7.2 at the end of this section). If there is a possibility that the optimum is not constrained one would have to resort to conventional optimization.

2d) From the last column of table 7.4 it is clear that keeping T_1 constant is preferred.

Step 3. Implementation of optimizer.
The model (including the sets and connection equations) was described in the previous section. The constraint ("safety margin") that should be included in the optimizer is

$u_B \geq 0.025$. We will explain how this value is obtained, but first we explain the details in the implementation of this constraint. To implement the constraint, we first find $q_B = 55$kW for $d_u = 0$ (55kW is the deficit heat of stream C2). Then we find $\alpha_B = 0.946$ from $q_B = \alpha_B \, UA_B \Delta T_{m,B}$, where the last term is the logarithmic mean for heat exchanger B for $d_u = 0$ and $T_1 = 151.9°C$. Implementing $\alpha_B = 0.946$ (and $\alpha_A = 1.0$) in equations (7.10) will ensure the required safety margin on u_B when unknown disturbances d_u are present.

	Case 1 $d_{u,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	Case 2 $d_{u,2} = \begin{bmatrix} -3 \\ -0.01 \end{bmatrix}$	Case 3 $d_{u,3} = \begin{bmatrix} -3 \\ +0.01 \end{bmatrix}$	Case 4 $d_{u,4} = \begin{bmatrix} +3 \\ -0.01 \end{bmatrix}$	Case 5 $d_{u,5} = \begin{bmatrix} +3 \\ +0.01 \end{bmatrix}$
$T_{1,\,opt}$	150.0	149.0	151.0	151.9	151.9
$T_{2,\,opt}$	106.7	105.4	104.0	107.4	107.4
$T_{3,\,opt}$	95.0	95.1	94.9	98.0	95.8
$u_{A,\,opt}$	0.000	0.105	0.292	0.000	0.000
$u_{B,\,opt}$	0.000	0.000	0.000	0.038	0.011
J_{opt}	145.0	147.0	149.0	146.9	144.7

Table 7.3 Values for y_{opt} and J_{opt} for all cases of d_u in the example. Case 1 is the nominal disturbance.

	Case 1	Case 2	Case 3	Case 4	Case 5	J_{mean}
$J(T_1^s = 151.9)$	148.9	153.0	150.8	147.0	144.8	148.9
$J(T_2^s = 104.0)$	153.0	151.2	149.0	159.2	155.0	153.0
$J(T_3^s = 98.0)$	151.0	152.9	155.1	146.9	149.1	151.0
$J(u_A^s = 0.292)$	151.1	151.2	149.0	153.2	151.0	151.1

Table 7.4 J^s for the possible choices of measurement and for all cases of d_u in the example.

Note that all values for u_B (and the constraint Δu_B) differ from the values given in Glemmestad *et al.* (1997) for the same example. The values in that paper actually refer to the bypass fractions on the hot side of heat exchanger B (which is inconsistent with figure 7.6), while the correct values for the bypass fractions on the cold side are given here.

The actual value for the safety margin ($\Delta u_B = 0.025$) is obtained as follows: The values of u_A and u_B for the five cases in table 7.4 corresponding to $T_1 = 151.9°C$ are given in table 7.5. For cases 4 and 5, u_A saturates at zero which implies that it is no longer possible for u_A to maintain $T_1 = 151.9°C$. The optimizer uses d_0 (case 1) where u_B takes the value of 0.025. Thus, in order to handle cases 4 and 5, a safety margin of $\Delta u_B = 0.025$ has to be used by the optimizer. Note that if we accepted that T_1 deviated from its setpoint (due to saturation in u_A) it would be possible to further reduce utility consumption somewhat. Then the setpoint for T_1 could be reduced slightly below 151.9°C until u_B saturated for some disturbance. In this example we require that setpoints for secondary measurements have to be satisfied.

The reason for implementing the "safety margin" on u_B as an *inequality* constraint is that other values of d_0 may give $u_{B,opt} > 0.025$. Requiring $u_B = 0.025$ in such cases will result in infeasibility.

	Case 1	Case 2	Case 3	Case 4	Case 5
u_A	0.207	0.354	0.354	0	0
u_B	0.025	0.038	0.012	0.038	0.012

Table 7.5 Values of u_A and u_B when T_1 = 151.9°C.

The value for J_{mean} of 148.9 kW in table 7.4 should be compared to the mean value of J_{opt} from table 7.3 which is 146.5 kW. That is, it costs 1.6% of the utility consumption to guarantee feasibility for the worst case unknown disturbance.

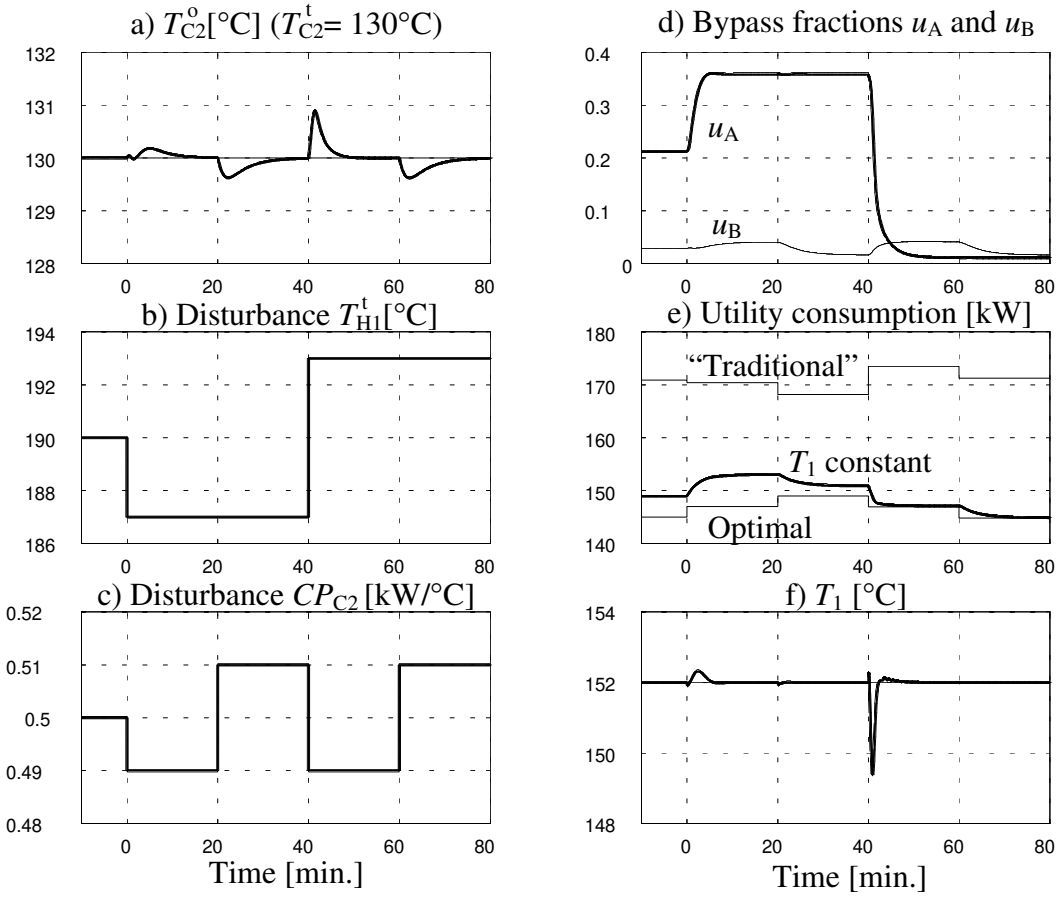

Figure 7.7 Results for example 7.1. (a) Controlled output T_{C2}^o with setpoint 130°C. (b) Disturbance T_{H1}^s. (c) Disturbance CP_{C2}. (d) Bypass u_A (thick line) and bypass u_B (thin line). (e) Utility consumption. (f) Temperature T_1 (secondary measurement), thick line shows actual value while thin line shows setpoint.

Figure 7.7 shows the results for the example when T_1 is selected as secondary measurement (optimization variable). Figure 7.7a shows that T_{C2}^o can be controlled to its setpoint for all unknown disturbances around the nominal operating point $d_0 = [0\ \ 0]^T$. Note from figure 7.7b and 7.7c, that the time up to zero corresponds to case 1 of d_u, from 0 to 20 minutes corresponds to case 2, from 20 to 40 minutes is case 3 and so on. Bypass fractions are shown in figure 7.7d, and u_A drops to close to zero after 40 minutes (case 4 and 5). The perhaps most important curves are shown in figure 7.7e. The steady state values for the utility consumption corresponds to the values in table 7.4 (upper row). The optimal values (when d_u is perfectly known) from the lower row of table 7.3 is also plotted for comparison. The utility

consumption should be compared with the "traditional" scheme *without* optimization also shown in figure 7.7e. The latter is implemented by fixing u_A at a value such that the network is just feasible for *all* possible disturbances, i.e. $d = [\pm 10, \pm 0.05]^T$, using u_B only. (This requirement gives $u_A = 0.680$). From the results given in figure 7.7 and table 7.4, it is clear that the main reduction in utility consumption compared to the traditional case is due to the periodic optimization (about 13% nominally), whereas the selection of optimization variable squeezes another 2.75% (between best and worst case).

Figure 7.7 has shown results around the nominal operating point corresponding to the cases 1 to 5 in tables 7.3 and 7.4. Only one optimization is done prior to the simulations, and the optimal setpoint for T_1 found here is maintained during the simulation (see figure 7.7f). Figure 7.8 shows similar results, but for a larger part of the operating region and with optimizations updating the setpoint for T_1 at 0, 20 and 40 minutes.

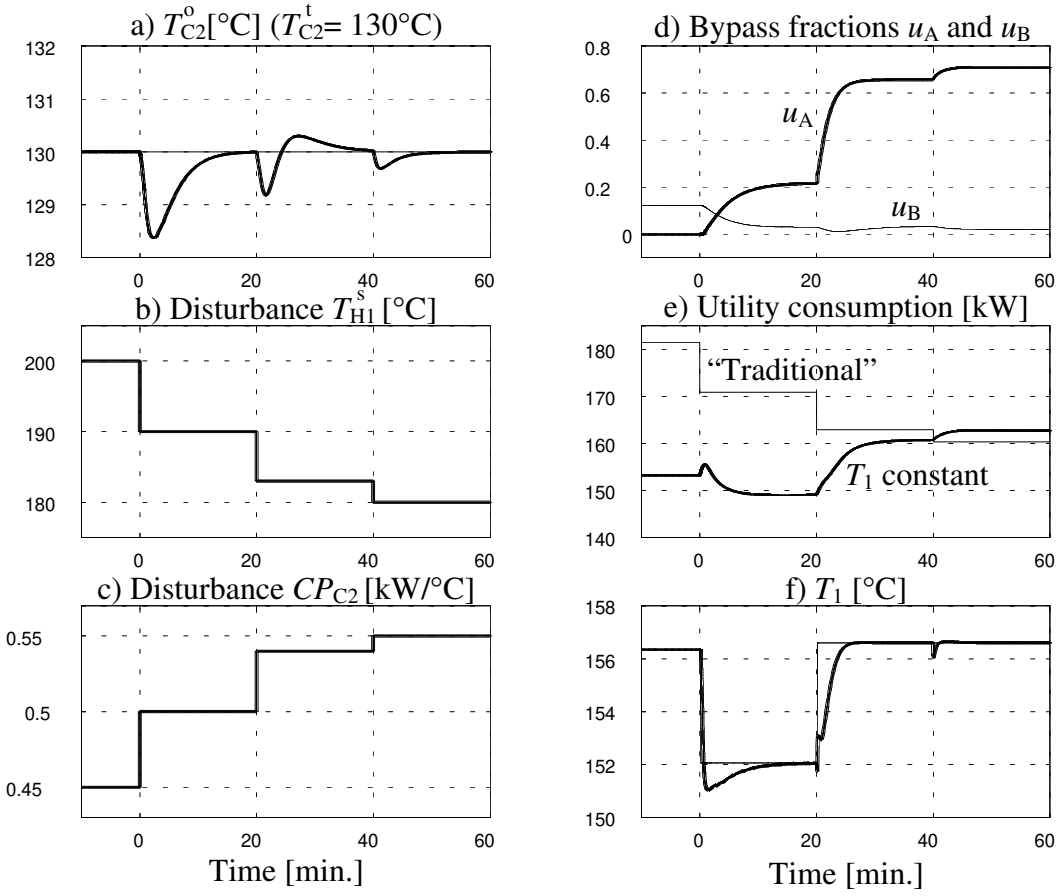

Figure 7.8 Results for example 7.1. The different curves show the same as figure 7.7 but for a larger part of the operating region, and with steady state optimization done at 0, 20 and 40 minutes.

At $t < 0$, we have $T^s_{H1} = 200$°C and $CP_{C2} = 0.45$kW/°C (i.e. $d = [10 \ -0.05]^T$). For this operating state, u_A is saturated at zero. For the steady state optimization carried out before the simulations started, the constraint on u_B was not active. From figure 7.8d we see that u_B cannot saturate if unknown disturbances (within the prespecified bound of $d_u \le [\pm 3 \ \pm 0.01]^T$) occur. At time equal to zero, nominal operating conditions are encountered and an

optimization is performed immediately after. Figure 7.8f shows how T_1 is controlled to its new setpoint (which now has the same value as in figure 7.7). At $t = 20$, the (known) disturbance $d = [-7 \;\; 0.04]^T$ is applied and a new optimization is done. At $t = 40$, an *unknown* disturbance of $d_u = [-3 \;\; 0.01]^T$ is applied which brings the operating state to the opposite corner of where we started. Since the new disturbance is unknown, the optimizer now returns the same setpoint for T_1 as it computed at $t = 20$. The steady state value for u_B at this corner point is close to zero.

Note that the utility consumption at the last part of the simulation is similar for the "traditional" approach (u_A is fixed at 0.680) and the proposed method. This is because this corner point is limiting u_A for the traditional approach, thus this approach is optimal for this corner of the operating region. After 40 minutes (the extreme corner point) the traditional approach actually has lower utility consumption than the proposed method. This can be explained from the requirement we have made in this example that also the setpoint for T_1 should be satisfied when unknown disturbances are present. As mentioned above, relaxing this requirement could reduce the utility consumption somewhat further for this example.

REMARK 7.2. From figure 7.6, it is clear that decreasing T_1, T_3 (by decreasing u_A) or u_A will reduce utility consumption (J). I.e. optimal values for these variables in table 7.3 are minimum values (smaller values will violate the primary goal). Therefore, the case with the *largest* value has to be chosen as this is the smallest value feasible for all d_u. For T_2, a similar (but opposite) argument leads to choosing the *smallest* value in table 7.3.

7.7 Conclusions

A method for optimal operation of heat exchanger networks based on periodic steady state optimization is proposed. Compared to the methods in the two previous chapters, the method here focuses more on the control task. This is accomplished by the first step of the method where a fixed control structure for the outlet temperatures (primary measurements) is selected. Thus, all outlet temperatures are usually controlled by the heat exchanger located immediately upstream, and a fast response is obtained. The periodic steady state optimization concerns the setpoints for measurements *internally* in the HEN, which are controlled by the remaining manipulations. An important issue is the selection of which measurements that are kept at constant setpoints (using feedback control) between each optimization. The objective functions used during operation and for selection of optimization variables are identical. Optimal operating conditions for heat exchanger networks are normally located at the intersection of constraints, and additional constraints ("safety margins") have to be implemented in the optimizer in order to maintain the target temperatures when unknown disturbances are present.

Notation

c	Cost data
CP	Heat capacity flowrate
d	Disturbance
e	Control error
g	Element in transfer function/matrix

G	Process transfer function
K	Transfer function for controller
Q	Heat load (duty) of heat exchanger.
r	Reference (setpoint)
T	Temperature
t	Time
u	Manipulated input (bypass fraction)
y	Output (measurement)

Superscripts:

BP	Bypass
U	Utility
o	Actual output or outlet (temperature)
s	Supply (temperature)
t	Target or reference (temperature)

Subscripts:

0	Nominal
1	Primary
2	Secondary
C*i*	Cold stream *i*
H*j*	Hot stream *j*
m	Mean value
nopt	Nominal optimum
ropt	Robust optimum
u	Unknown

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