Chapter 4

DEGREES OF FREEDOM IN OPERATION OF HENs

In order to perform optimization during operation, there has to be at least one extra degree of freedom (DOF) after regulatory control is implemented. This chapter gives a thorough discussion of DOFs in the operation of HENs, and it will be shown that extra manipulations often cannot be utilized for optimization of utility cost. A rule to quickly verify whether a HEN problem may be optimized during operation is presented, and an equation for the number of DOFs that can be utilized for optimization ($N_{DOF,\,U}$) is derived.

4.1 Introduction

In section 2.4 it was briefly demonstrated that the number of DOFs (N_{DOF}) during operation is different from the synthesis stage. During operation we assume:

- Fixed network structure.
- Fixed installed heat exchanger areas where the effective area can be varied between zero and the installed area by manipulating a bypass. Process exchangers without bypass will also be considered in section 4.4.
- Specified supply and target temperatures and heat capacity flowrates of process streams.

An important question regarding operation of HENs is whether optimization is possible or not. A prerequisite for doing any optimization is that there are degrees of freedom left when manipulations are used for regulatory control. The most crucial question is not how *many* DOFs the HEN has, but simply *whether* we have zero DOF (optimization is not possible), or

one or more DOFs (optimization is possible). To answer this question quickly for a given network, the simple rule being derived in section 4.3 is helpful.

The chapter is organized as follows: The remaining part of this introduction discusses the number of manipulations required for regulatory control of target temperatures, to see if there may be extra manipulations. Section 4.2 shows that extra manipulations (more than needed for regulatory control) often may not be utilized for optimization and the number of DOFs that can be utilized for utility optimization ($N_{\mathrm{DOF,\,U}}$) is defined. Section 4.3 presents the rule to find whether optimization should be considered or not. A quantitative expression for $N_{\mathrm{DOF,\,U}}$ is derived in section 4.4, and some examples are given in section 4.5.

The primary goal of optimal operation requires that the targets are met at steady state and this implies that it has to be possible to control the target temperatures independently. To achieve this, the HEN must be *functionally* controllable (Rosenbrock, 1970, p.170) which for a linear system requires that the rank of the transfer matrix (at steady state) is equal to or greater than the number of outputs. If we assume that manipulations are selected such that independent control of the target temperatures is possible, i.e. the system is non-singular, it is clear that *the number of manipulations for regulatory control is equal to the number of targets*. In practice, it is often straightforward to choose manipulations that yield a regulatory control system without singularities in HENs simply using the main rule for input/output pairing (rule 2.2 in section 2.3). Control of utility controlled targets is trivially non-singular due to the diagonal structure. Potential singularities in control of the bypass controlled targets *may* occur and it is usually a structural property of the HEN. In Mathisen (1994, chapter 3) a number of singular network structures are presented.

A necessity for optimal operation is of course that regulatory control is possible (primary goal in definition 2.1). As shown above this requires at least as many manipulations as targets. Euler's rule from graph theory states that the number of edges in a graph is given by

$$N_{\mathrm{edges}} = N_{\mathrm{nodes}} + N_{\mathrm{loops}} - N_{\mathrm{subnetworks}} \tag{4.1}$$

Applied to HENs, edges represent units (process or utility exchangers) and nodes represent streams (process or utility streams). The discussion starts by considering HENs with minimum number of units (no loops). For a connected network ($N_{\mathrm{subnetworks}} = 1$) with no loops ($N_{\mathrm{loops}} = 0$), the (minimum) number of units is given by

$$N_{\mathrm{units,\,min}} = N_{\mathrm{P}} + N_{\mathrm{U}} - 1 \tag{4.2}$$

where N_{P} is the number of process streams and N_{U} is the number of different utility types. A pinch problem has at least two utility streams (one hot and one cold) and hence the number of units is at least one more than the number of process streams. Each unit (process or utility exchanger) can be manipulated and represents a DOF that can be used for regulatory control. The maximum number of targets is equal to the number of process streams (all outlet temperatures are controlled). Thus, it can be concluded that all pinch problems have at least one extra manipulation after regulatory control is implemented.

Threshold problems may have only one utility stream and therefore the minimum number of units may be equal to the number of process streams. In such cases there are no extra manipulations for optimization. If, however, one or more outlet temperatures do not have any

target values, the number of manipulations required for regulatory control is less than the minimum number of units and there will be extra manipulations. Also, for threshold problems with loops (more than minimum number of units) and problems with multiple hot and/or cold utilities (e.g. different steam levels) there will be extra manipulations. To conclude this section, it has been found that pinch problems *always* have extra manipulations whereas threshold problems *may* have extra manipulations. This is, however, only an intermediate result, based on the assumption that all heat exchanger duties can be manipulated. More useful results will be deduced in the following sections.

4.2 DOF with respect to utility consumption

In this section, it will be shown that introducing loops (more than minimum number of units/more manipulations) may *not* always increase the number of DOFs that can be used for optimization. Let N_t be the number of targets and assume that each target has one manipulation assigned for regulatory control. Marselle *et al.* (1982) and Mathisen (1994, chapter 1) states that N_{DOF} is given by

$$N_{DOF} = N_{units} - N_t \tag{4.3}$$

N_{units} is the total number of manipulations since each unit is regarded as a manipulation (and variable split fractions are disregarded in this discussion). N_{DOF} from equation (4.3) may be thought of as the extra degrees of freedom with respect to bypass fractions. However, it is not always possible to exploit these DOF to anything useful, and the network in figure 4.1 illustrates this.

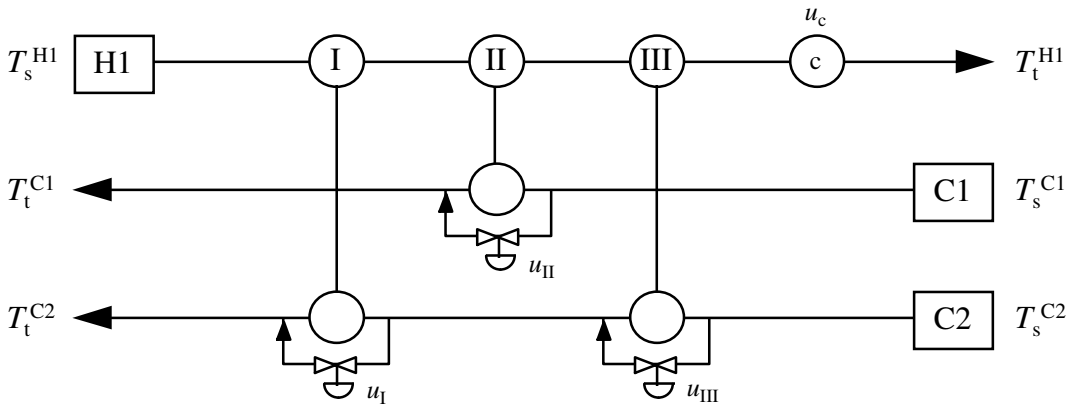

Figure 4.1 Network where DOF cannot affect utility consumption.

In this HEN the three outlet temperatures should be controlled, and with four manipulations we would expect N_{DOF} to be one. Examining the network we see that the duty of heat exchanger II, Q_{II}, (and hence u_{II}) is dictated by the deficit heat of stream C1 and the sum $Q_I + Q_{III}$ is dictated by the deficit heat of stream C2. Now, with the duties fixed for all three process exchangers, it is clear that these duties together with the excess heat in stream H1 dictate the utility consumption Q_c and hence u_c. The DOF in this example can only be used to shift duties between the heat exchangers I and III. It does *not* affect the utility consumption and it can *not* be utilized for any meaningful optimization during operation.

The result for this example is rather obvious since it is a threshold problem with only one utility stream. Thus, the utility consumption can not be manipulated when the target temperatures are fixed. While this case may not be used as a counter example to equation (4.3), it shows that using this equation may give misleading results regarding optimization during operation. While (4.3) gives $N_{DOF} = 1$ (hence one would expect optimization to be possible), the correct statement is that the utility consumption is fixed and optimization should not be considered.

The discussion above motivates the need for a definition of degrees of freedom that is suited for optimal operation of HENs.

Definition 4.1 *Degrees of freedom with respect to utility consumption* $N_{DOF,\,U}$.
The number of DOFs with respect to utility consumption $N_{DOF,\,U}$ is the number of DOFs that can be utilized for optimization of utility consumption when targets are satisfied. ■

Note that with degrees of freedom in the subsequent, we usually mean the *extra* degrees of freedom left when regulatory control is implemented

4.3 A simple rule to check scope for optimization

At the end of section 4.1, we concluded that pinch problems always have extra manipulations while threshold problems may have extra manipulations. In this section, a few simple rules to determine whether extra manipulations can be used for utility optimization will be derived.

From the arguments given previously, the most important aspect is to determine whether $N_{DOF,\,U} = 0$ or whether $N_{DOF,\,U} \geq 1$. This can be done with the total energy balance of a HEN as basis, see figure 4.2.

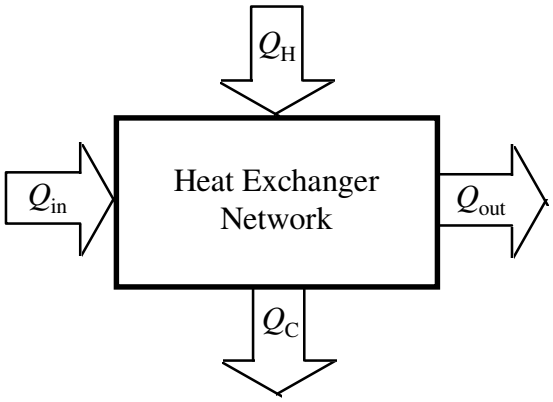

Figure 4.2 Total energy balance in a HEN.

In the figure, Q_{in} and Q_{out} are the energy flow in and out of the HEN via the process streams while Q_H and Q_C are hot and cold utilities. The steady state energy balance when only thermal energy is considered becomes

$$Q_{in} + Q_H = Q_{out} + Q_C \tag{4.4}$$

This total energy balance together with the result above (equation 4.2 and the two paragraphs after that) regarding number of manipulations for pinch and threshold problems will now be used to derive rules to find whether a HEN can be optimized during operation or not. Pinch problems and threshold problems are treated separately.

Pinch problems.
From equation (4.2) it was found that pinch problems always have at least one extra manipulation. Also, pinch problems have $Q_H > 0$ and $Q_C > 0$ and there will always be a path from hot to cold utility since we assume a connected network (disconnected networks can of course be treated as connected subnetworks, both for pinch and threshold problems). We may always increase or decrease Q_H and Q_C with the same amount of heat and still preserve the energy balance. Therefore, the extra manipulation(s) can be used to minimize utility consumption and for pinch problems we always have $N_{DOF,\,U} \geq 1$. If some outlet streams are free, i.e. have no targets, also Q_{out} may be varied and this introduces additional degrees of freedom. Note that the nominal design often is optimal in the nominal operating point. There is, however, no guarantee that a control system will find this operating condition, since it is possible to operate the HEN with unnecessary large utility consumption. Therefore, optimization should be considered in order to operate the HEN with (close to) minimum utility consumption at any operating point.

Threshold problems.
In threshold problems we have $Q_H = 0$ or $Q_C = 0$. First, consider a threshold problem with only one utility stream (e.g. one steam level) and that all outlet temperatures have targets such that Q_{out} is fixed at any operating point. From the total energy balance we see that the utility consumption is fixed (equal to $|Q_{in} - Q_{out}|$) which means that we have $N_{DOF,\,U} = 0$. Above, from equation (4.2) it was stated that threshold problems with minimum number of units and no free outlet temperatures have no extra manipulations and therefore no degrees of freedom. Now, we have found that this holds also for threshold problems independent of the number of units. That is, introducing loops (additional manipulations) in threshold problems does *not* increase $N_{DOF,\,U}$. The extra manipulations can only be used to shift duties internally in the HEN.

The situation changes if some of the outlet temperatures are free. Then, Q_{out} can be varied at any operating point and there always exists a path from the utility exchanger(s) and to the free outlet temperature(s). This means that the HEN may be optimized during operation and that we have $N_{DOF,\,U} \geq 1$. Also note that if there are more than one utility stream (e.g. cooling water and a cryogenic refrigerant), optimization is meaningful also when all outlet temperatures have targets (even if total utility consumption in terms of kW, MW etc. is fixed). In such cases one may minimize the most expensive utility stream, or more general, minimize total utility *cost*.

These results for pinch and threshold problems are summarized in rule 4.1. This rule is very simple to use but as it will be clear from the explanation below, it only serves as necessary conditions for $N_{DOF,\,U} \geq 1$.

Rule 4.1 *Degrees of freedom with respect to utility consumption of HENs.*
Based on the structural properties of a connected HEN problem, the following three rules (one rule for each problem type) states whether the HEN can be optimized or not:

a) Pinch problems (all cases) $N_{DOF,U} \geq 1$

b) Threshold problems with no free outlet temperatures $N_{DOF,U} = 0$

c) Threshold problems with at least one free outlet temperature $N_{DOF,U} \geq 1$

The rules a) and c) are necessary conditions for optimization ($N_{DOF,U} \geq 1$), whereas rule b) is sufficient to state that optimization is not possible ($N_{DOF,U} \leq 0$). ■

According to rule 4.1, the question whether a HEN can be optimized or not, is *independent of the internal HEN structure*, it only depends on the problem type and which outlet temperatures that should be controlled. This implies that in many cases, adding more units (introducing more loops) will not increase $N_{DOF,U}$. However, the only thermodynamic constraint that has been considered is the total energy balance of the HEN. It has been assumed that each unit has positive temperature driving forces such that heat transfer is possible, and it has been assumed that the duty of each unit may be manipulated. If for example the duty of a utility exchanger drops to zero for given operating conditions, this unit can no longer be used for control purposes, and an upstream exchanger has to be bypassed to reach the target temperature. In this way, the HEN will loose one degree of freedom with respect to bypass fractions and it *may* also reduce $N_{DOF,U}$. A reduction in $N_{DOF,U}$ may lead to only one feasible operating condition for a HEN that can be optimized at other operating points. It may also lead to infeasible operation, meaning that the primary goal of optimal operation (reach targets) cannot be met.

Rule 4.1 assumes that a bypass is placed across each process heat exchanger. If this is not the case, $N_{DOF,U}$ may be reduced. As an example consider a pinch design with minimum number of units (one unit more than the number of targets). If one unit cannot be manipulated (e.g. because bypass is not installed), all manipulations must be used for regulatory control, thus optimization is not possible.

Despite the fact that rule 4.1 (a and c) is only necessary conditions for $N_{DOF,U} \geq 1$, it may give valuable information regarding optimization during operation from a quick glance at the network, or simply from the problem type. If the HEN consists of more subnetworks, then rule 4.1 must be applied to each subnetwork.

4.4 A quantitative expression for the number of DOF

A quantitative equation for the number of degrees of freedom that can be utilized for utility optimization ($N_{DOF,U}$) of HENs is derived. The same equation is valid for HENs with loops, splits and multiple utilities, and it is not required that all process exchangers have bypasses for manipulation. Pinch and threshold problems as well as HENs where there are outlet temperatures without target values (free outlet temperatures) are covered. In the general equation for $N_{DOF,U}$, however, internal utility exchangers are disregarded. That is, for streams where a utility exchanger is present, it is assumed to be the last unit in that stream.

With utility optimization, we mean optimization of utility *cost*, and not only the total consumption. It is assumes that different utilities (e.g. different steam levels) have different cost. This implies that for cases where there are multiple utilities (e.g. high and low pressure steam), also the distribution (e.g. HP/LP-ratio) between the different utilities is considered. The expression for $N_{DOF,\,U}$ will be developed starting with a trivial example.

Example 4.1 *Trivial example for* $N_{DOF,\,U}$
The HEN in figure 4.3 has 4 manipulations and 2 targets, thus equation (4.3) gives $N_{DOF} = 2$. For this example one may argue that the two process exchangers are placed side by side, and they may be treated as one unit. Since the heat removed from the hot stream equals the heat received by the cold stream, the two bypasses only contributes with one free variable and therefore the number of DOFs that can be used for optimization is only 1 (identified as the path from the heater *h* to the cooler *c*).

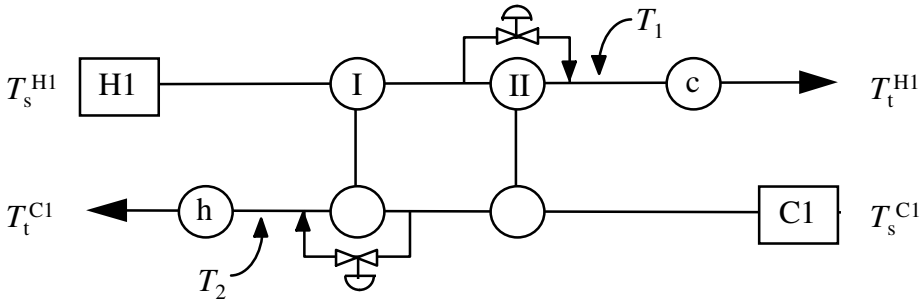

Figure 4.3 Trivial HEN.

An alternative explanation being better suited for generalization to other HENs is the following: Specifying the temperature T_1 freely, T_2 is given by an energy balance around the two process exchangers, and vice versa. Hence, the two variable duties Q_I and Q_{II} only spans a one-dimensional space in T_1 and T_2. Utility consumption takes place in the utility exchangers, and with one free variable (T_1/T_2) together with varying the duties of the utility exchangers, there are totally 3 free variables that affect the utility consumption. With 2 constraints (specifications on the target temperatures) there is one DOF left for utility optimization, thus $N_{DOF,\,U} = 1$. So for a total of 2 degrees of freedom ($N_{DOF} = 2$), only 1 can be used for utility optimization ($N_{DOF,\,U} = 1$). Note, however, that one temperature between the process exchangers may be specified or one bypass may saturated *without reducing* $N_{DOF,\,U}$ (since the process exchangers still spans a one-dimensional space in T_1 and T_2). In other words: Even though the extra unit (loop) in figure 4.3 does not increase $N_{DOF,\,U}$, it serves as a "spare" degree of freedom. ■

To generalize, we divide a HEN into two parts – the "inner" HEN and the "outer" HEN, as shown in figure 4.4. For this general HEN, all outlet temperatures have target values. We will come back to free outlet temperatures at the end of the section.

The inner HEN consists of all process exchangers, splitters and mixers etc. The utility consumption takes place in the outer HEN which consists of the utility exchangers and target values. Streams with target values enter the outer HEN even if they do not have a utility exchanger. It is important to notice that all information from the inner to the outer HEN

passes through the streams entering the outer HEN. Thus, the number of streams put limitations on the number of free variables passed from the inner to the outer HEN. In figure 4.3, the two temperatures T_1 and T_2 constitute the interface between the inner and the outer HEN. The next example demonstrates how the concept of inner and outer HEN can be used to find $N_{\mathrm{DOF,\,U}}$.

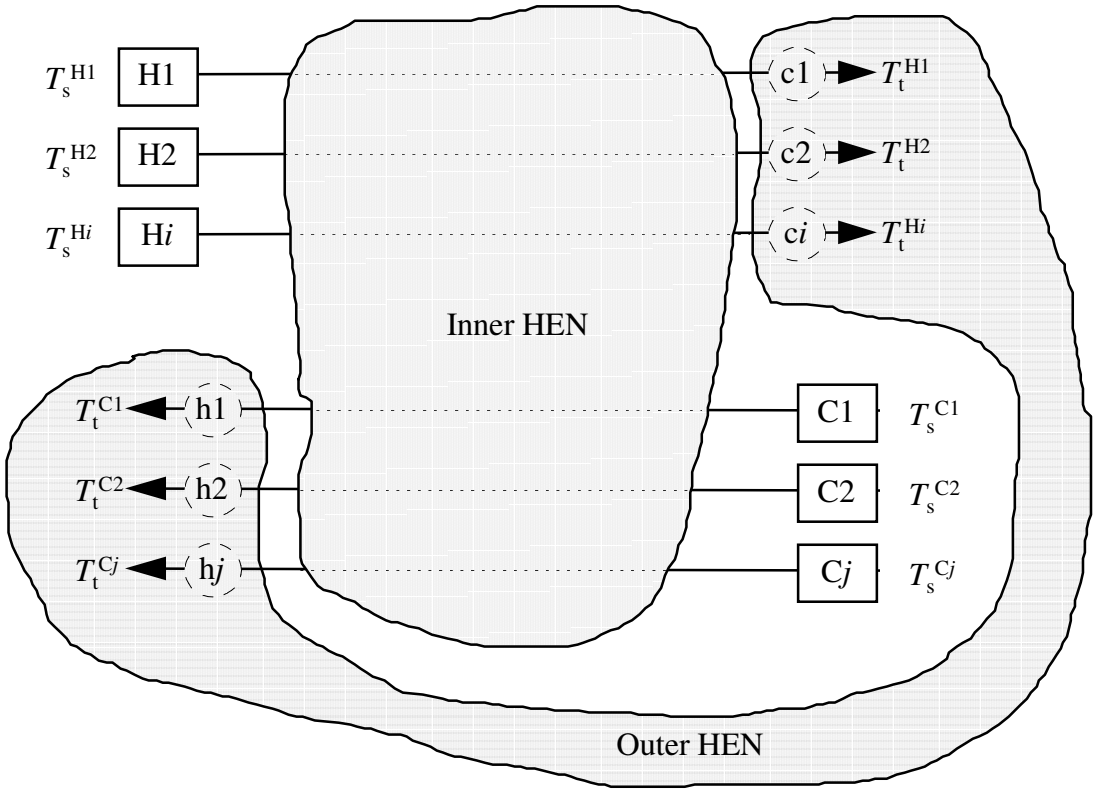

Figure 4.4 Inner and outer HEN.

Example 4.2 *Simple example for $N_{\mathrm{DOF,\,U}}$ using inner and outer HEN.*
The HEN in figure 4.5 has at total of 6 manipulations and 4 targets. The interface between the inner and outer HEN is the four temperatures T_1 to T_4.

Each process exchanger has a bypass and the duties are denoted Q_{I}, Q_{II}, Q_{III} and Q_{IV}. For the 4 streams the following equations for the inner HEN can be written:

$$\left.\begin{aligned} CP_{\mathrm{H1}}\left(T_1 - T_s^{\mathrm{H1}}\right) &= -Q_{\mathrm{I}} - Q_{\mathrm{IV}} \\ CP_{\mathrm{H2}}\left(T_2 - T_s^{\mathrm{H2}}\right) &= -Q_{\mathrm{II}} - Q_{\mathrm{III}} \\ CP_{\mathrm{C1}}\left(T_3 - T_s^{\mathrm{C1}}\right) &= Q_{\mathrm{I}} + Q_{\mathrm{III}} \\ CP_{\mathrm{C2}}\left(T_4 - T_s^{\mathrm{C2}}\right) &= Q_{\mathrm{II}} + Q_{\mathrm{IV}} \end{aligned}\right\} \Rightarrow \underbrace{\begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}}_{\text{Rank} = 3} \begin{bmatrix} Q_{\mathrm{I}} \\ Q_{\mathrm{II}} \\ Q_{\mathrm{III}} \\ Q_{\mathrm{IV}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \tag{4.5}$$

where $b_1 = CP_{\mathrm{H1}}(T_1 - T_s^{\mathrm{H1}})$ etc.

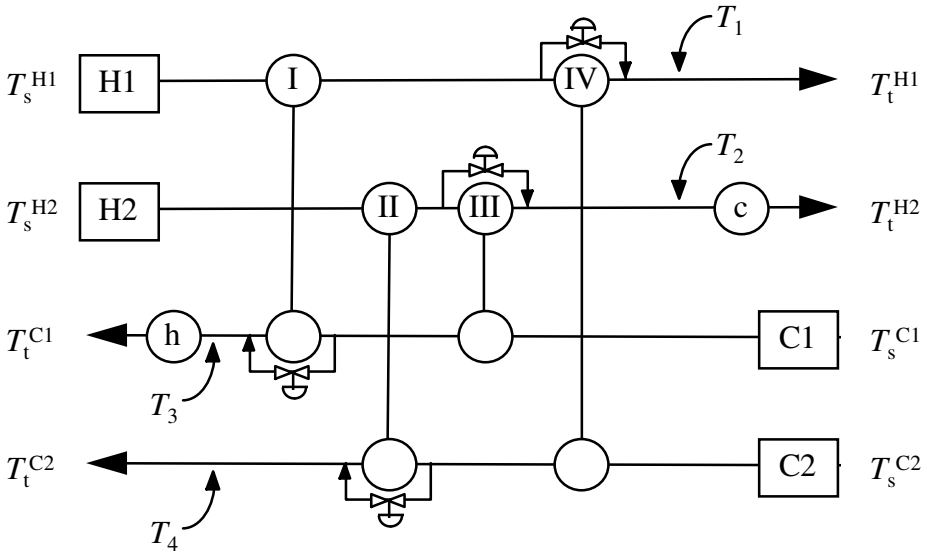

Figure 4.5 Simple HEN for demonstrating $N_{DOF,\,U}$.

The rank of the 4×4 matrix is 3 as indicated in equation (4.5). This implies that the 4 bypasses only spans a 3-dimensional space in the temperatures T_1, T_2, T_3 and T_4. We could actually conclude that the 4 bypasses could not span more than 3 dimensions without putting up equation (4.5), since if three of the temperatures T_1 to T_4 are specified, the fourth is given by an energy balance for the inner HEN. By looking at the set of equations and the corresponding rank, however, we can guarantee that the bypasses span a space with dimension equal to 3. The 3-dimensional space spanned by the bypasses implies that the inner HEN contributes with 3 free variables that may be utilized for utility optimization. The 2 utility exchangers give 2 more free variables, and with the 4 constraints (targets) we have:

$$N_{DOF,\,U} = \underbrace{3+2}_{\text{Free variables}} - \underbrace{4}_{\text{Constraints}} = 1 \tag{4.6}$$

Also in this example, there is one "additional" DOF that cannot be used for utility optimization; one temperature in the inner HEN may be specified or one bypass on the process exchangers may become inactive (e.g. saturated) without reducing $N_{DOF,\,U}$, since the inner HEN still spans a 3-dimensional space. ■

So far, we have assumed that each utility exchanger contributes with one free variable in the optimization of utility cost. This may seem to be an obvious assumption due to the structure of HENs given by equation 2.1. The diagonal structure from the utility exchangers to the utility controlled target temperatures suggests that no singularities can occur here. However, before we can put up the equation for $N_{DOF,\,U}$, it is required to consider the case where utility exchangers are parts of a loop, see figure 4.6.

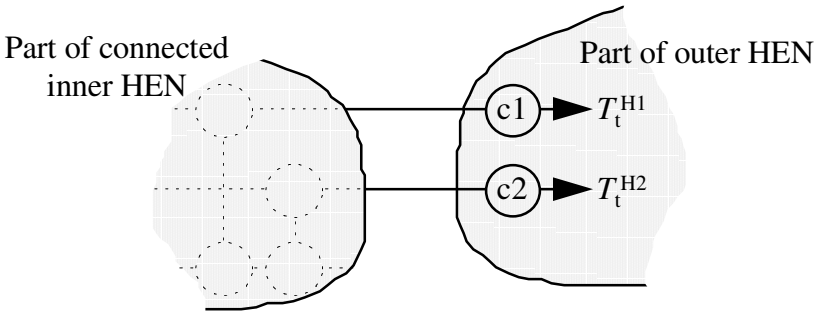

Figure 4.6 Utility exchangers forming a loop.

Assuming that the two coolers use the same type of utility, e.g. cooling water, they form a loop together with the connection in the inner HEN. Manipulating the loop, every second unit may increase or decrease its duty by the same amount while maintaining the energy balance (satisfy targets). When the two coolers use the same type of utility, the distribution of heat between these two units is indifferent for the total utility cost. The two units may be considered to represent two independent variables, while the loop may be considered as an additional constraint. Hence the net contribution of two utility units forming a loop is one free variable. More in general, $N_{\text{utility units}}$ utility exchangers using the same utility stream form $N_{\text{utility units}} - 1$ independent loops, thus contributing with only 1 free variable for utility optimization.

If the two coolers in figure 4.6 use different utility streams (e.g. c1 uses cooling water and c2 uses a cryogenic refrigerant), then they would not form a loop but instead they would form a path from the cooling water to the refrigerant. Manipulating this path will change the distribution of these two utility streams and with different cost, this may be used for optimization. Thus the two coolers contribute with two free variables for utility optimization.

The net contribution of free variables from the outer HEN is equal to $N_{\text{utility units}} - N_{\text{utility loops}}$ (number of utility units minus number of independent loops involving utilities). Above, it was argued that several utility exchangers on the same utility stream only give 1 free variable for optimization due to the formations of loops. This applies for each utility stream and we have

$$N_{\text{utility units}} - N_{\text{utility loops}} = N_{\text{U}} \tag{4.7}$$

As before, N_{U} is the number of different utility types. That is, 2 steam levels and cooling water gives $N_{\text{U}} = 3$, no matter how many utility units that are installed. From equation (4.7), the net contribution of free variables from the outer HEN simply is N_{U}. That is, the number of utility units is not directly involved in $N_{\text{DOF, U}}$, and we do not have to find the number of independent loops involving utility exchangers. Now, the equation for $N_{\text{DOF, U}}$ is given in theorem 4.1.

Theorem 4.1 *Degrees of freedom that can be used for utility optimization*, $N_{DOF,\,U}$
For a HEN with no utility exchangers and no target specifications in the inner HEN, the number of degrees of freedom that can be used for utility optimization in a HEN is given by:

$$N_{DOF,\,U} = R + N_U - N_t \tag{4.8}$$

where R is the dimension of the space spanned by the temperatures of the streams entering the outer HEN, due to all possible manipulations in the inner HEN. N_U is the number of utility streams and N_t is the number of target temperatures.

Proof: R is the number of free variables from the inner HEN. In the outer HEN, utility exchangers represent variables, but more than one unit on the same utility stream (same utility type) form loops. Manipulating such loops where the utility cost is the same do not affect the utility cost, thus each utility stream represent one free variable for utility optimization (equation 4.7). No singularities can occur in the outer HEN due to the diagonal structure, hence the total number of free variables is $R + N_U$. With a total number of N_t constraints, this proves the theorem. ■

In the rest of this section, the role of loops in the inner HEN, split fraction and process exchangers without bypasses will be discussed. Then, HENs that have free outlet temperatures are discussed, and it is shown that theorem 4.1 holds also for such cases. Finally, equation (4.8) will be simplified such that R do not have to be computed as shown in example 4.2.

Loops in inner HEN
Adding more than minimum number of units to a HEN design introduces loops. In most cases, this will not increase $N_{DOF,\,U}$. The reason to this is that the number of process streams between the inner and outer HEN is an upper bound on R. Usually (process exchangers without bypasses may result in exceptions), R is one less than the number of process streams since if all temperatures except one is specified, the last temperature is given by an energy balance. A typical case is the HEN in figure 4.5 which has one loop (one excess unit). However, the extra unit does not increase $N_{DOF,\,U}$ since R is limited by the four process streams and the energy balance of the inner HEN. In fact, one can add as many units and manipulations in the inner HEN as one like without increasing $N_{DOF,\,U}$, since R cannot exceed 3 for this case.

Split fractions
The same argument as for loops in the inner HEN applies for variable split fractions: Usually, $N_{DOF,\,U}$ is not increased since R is limited by the number of process streams and the energy balance. Figure 4.7 shows a configuration that may be part of a larger HEN.

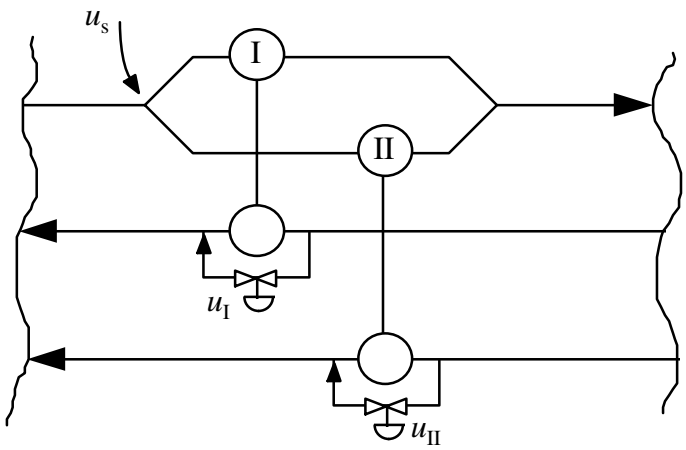

Figure 4.7 Part of HEN with variable stream split (u_s).

Even if there are 3 manipulations (u_I, u_{II} and u_s), only 2 free variables can be transferred to the surroundings. Again, the energy balance for the shown part of the HEN fixes the third of the stream exit temperatures. Addition of the split fraction will not contribute to increasing $N_{DOF,\,U.}$ However, the presence of the split fraction allows one of the bypasses to be saturated without reducing the number of free variables to the surroundings. Also, if one of the process exchangers do not have any bypass installed, then the addition of the split fraction will increase the number of free variables from 1 to 2. This leads to the next topic:

Process exchangers without bypasses

It has been argued above, that R is normally bounded by the number of process streams (N_P) minus one. However, R can of course not exceed the number of manipulations in the inner HEN. In cases where there are process exchangers without bypasses and the total number of manipulations is less than $N_P - 1$, then the number of manipulations will limit R and $N_{DOF,\,U}$ is limited by the process exchangers without bypasses. If there are excess units (loops) in the inner HEN, then removing bypasses will not reduce $N_{DOF,\,U}$ unless R is reduced.

Free outlet temperatures

When considering free outlet temperatures, it is important to recall that only those streams with targets enter the outer HEN. That is, the number of streams entering the outer HEN is always (per definition of outer HEN) equal to N_t. For HENs with free outlet temperatures, we have $N_t = N_P - N_F$, where N_F is the number of free outlet temperatures. For all HENs, an upper bound on R is N_t. This follows trivially since N_t is the number of "information channels" transferred to the outer HEN. Here, we assume that there are enough manipulations to span the whole space with dimension N_t. For a HEN with no free outlet temperatures, however, R is restricted to $N_t - 1$ ($= N_P - 1$) since the energy balance around the inner HEN represents one constraint. Now, adding a process stream with a free outlet temperature (and connecting it to the HEN with one or more units), the free outlet serve as a sink/source for the inner HEN. That is, the streams entering the outer HEN span the full space with dimension N_t, thus R and also $N_{DOF,\,U}$ are increased by one. Going one step further, we may add another process stream with a free outlet temperature and connect it to the HEN. R already spans the full space, and it is now constrained by N_t. That is, R and $N_{DOF,\,U}$ are *not increased any further*. To conclude so far we have found that for HENs where

R is not limited by the manipulations in the inner HEN, *adding one or more process streams with free outlet temperatures increases $N_{DOF,\,U}$ by 1.*

Another approach is to consider the case where a target is relaxed (a target temperature is turned into a free outlet temperature) without introducing any new streams. The result is the same, but the arguments are slightly different: The starting point is a HEN with no free outlet temperatures and we have $R = N_t - 1$. Relaxing one target specification, N_t is reduced by one and one less stream is entering the outer HEN. However, the free outlet temperature serves as a sink/source and R now spans the full space of the new set of streams/temperatures entering the outer HEN. That is, R is unchanged and with N_t reduced by 1, $N_{DOF,\,U}$ is increased by 1. Proceeding by relaxing another target, N_t is reduced again, but now also R is reduced since it is bounded by N_t. With both R and N_t reduced by one, $N_{DOF,\,U}$ is unchanged. Similar to above, we have found that for HENs where R is not limited by the manipulations in the inner HEN, *relaxing one or more target specifications increases $N_{DOF,\,U}$ by 1.* This result is demonstrated by example 4.4 in the next section. Note that the result does not hold when R is limited by the number of manipulations in the inner HEN. In such cases, $N_{DOF,\,U}$ may be increased by more than 1 (until $R = N_t$).

Simplification of equation (4.8)

Equation (4.8) covers all situations discussed above such as loops, splits, units that cannot be manipulated and free outlet temperatures, however, it requires computation of the rank R. Now, this equation will be simplified, yet still it will be valid for most HENs. Focus will be on the term $R - N_t$ which is part of equation (4.8). First, the case where there are no free outlet temperatures is considered.

As pointed out above, an *upper* bound on R for cases with no free outlet temperatures is equal to $N_t - 1$. On the other hand, the smallest number of process exchangers to form a connected inner HEN is $N_P - 1$ ($= N_t - 1$). Assuming that these process exchangers have bypasses and that there are no singularities, a *lower* bound on R is $N_t - 1$. With upper bound equal to lower bound, we can conclude that $R = N_t - 1$, hence the term $R - N_t$ in equation (4.8) equals –1.

For HENs with free outlet temperatures, an upper bound on R is N_t since the free outlet temperature(s) can be considered as a sink/source for the energy balance of the inner HEN. The smallest number of process exchangers to form a connected inner HEN is $N_P - 1$ and with $N_t = N_P - N_F$, we have $N_P - 1 = N_t + N_F - 1$. Assuming that the duties of the process exchangers can be manipulated and that no singularities are present a *lower* bound on R is N_t. Again, the upper bound equals the lower bound and we have $R - N_t = 0$.

Inserting these results in equation (4.8), we have found that $N_{DOF,\,U}$ for HENs where all process exchangers have bypasses and there are no singularities, is

HENs with no free outlet temperatures: $N_{DOF,\,U} = N_U - 1$ (4.9a)

HENs with (one or more) free outlet temperatures: $N_{DOF,\,U} = N_U$ (4.9b)

For HENs with more than minimum number of units, this result also hold if some process exchangers do not have bypass installed or if there are singularities, as long as R is not reduced below its upper bound.

4.5 Examples

In this section, the use and validity of rule 4.1, theorem 4.1 and equation (4.9) is demonstrated by 3 examples.

Example 4.3 *HEN with multiple utilities.*
The HEN in figure 4.8 has a total of 11 manipulations and 6 targets. There are 4 utilities: LP-steam, HP-steam, cooling water (CW) and a refrigerant (R1). The inner HEN has 6 units (1 loop) and all these have bypass installed.

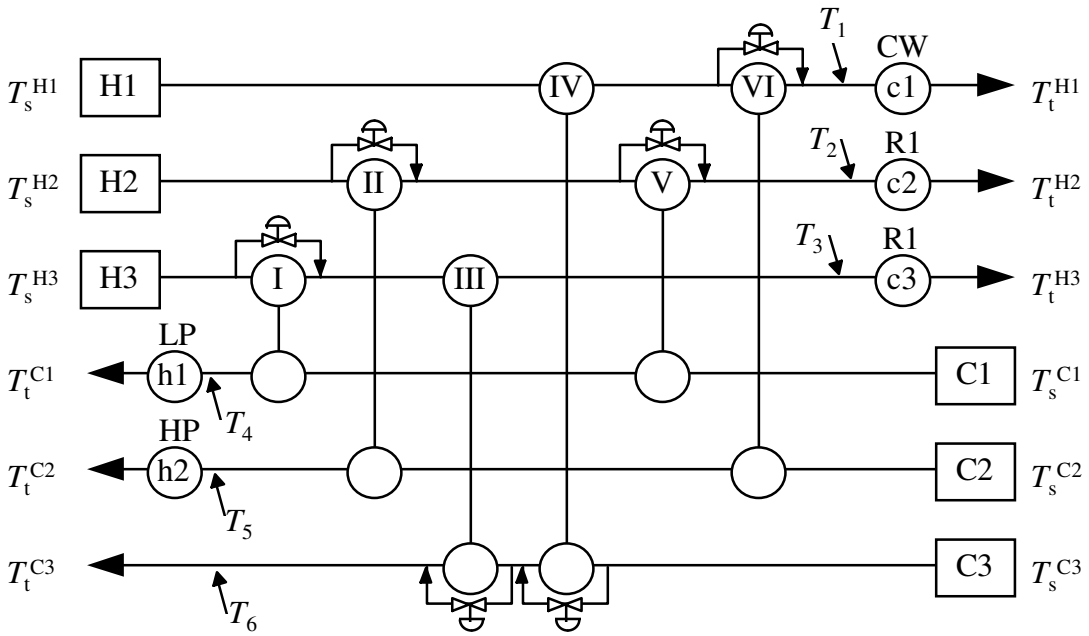

Figure 4.8 HEN with multiple utilities.

Rule 4.1 suggests that optimization is possible. Using theorem 4.1 to find $N_{\mathrm{DOF,\,U}}$, we first compute R by putting up the equations for the inner HEN as in example 4.2. In matrix form, we get

$$\begin{bmatrix} 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Q_{\mathrm{I}} \\ Q_{\mathrm{II}} \\ Q_{\mathrm{III}} \\ Q_{\mathrm{IV}} \\ Q_{\mathrm{V}} \\ Q_{\mathrm{VI}} \end{bmatrix} = \begin{bmatrix} b_1(T_1) \\ b_2(T_2) \\ b_3(T_3) \\ b_4(T_4) \\ b_5(T_5) \\ b_6(T_6) \end{bmatrix} \tag{4.10}$$

The rank of the matrix is 5, thus we have $R = 5$. With $N_{\mathrm{U}} = 4$ and $N_{\mathrm{t}} = 6$, equation (4.8) in theorem 4.1 gives

$$N_{\mathrm{DOF,\,U}} = R + N_{\mathrm{U}} - N_{\mathrm{t}} = 5 + 4 - 6 = 3 \tag{4.11}$$

For this example, equation (4.9a) also gives $N_{\mathrm{DOF,\,U}} = 3$ since the assumptions made in the simplifications of (4.8) are fulfilled. The 3 DOF for utility optimization can be identified as

- Heat recovery level (total utility energy consumption)
- Distribution of LP-steam and HP-steam.
- Distribution of cooling water/refrigerant.

Equation (4.3) suggests a total of 5 DOF (11 manipulations and 6 targets), thus we expect that there are 2 additional DOF that do not affect the utility cost. These are identified as

- Distribution of duties between the coolers c2 and c3 (manipulating loop formed by outer and inner HEN together).
- Distribution of duties in process exchangers (manipulating loop in inner HEN).

■

Example 4.4 *HEN with free outlet temperatures.*
This example demonstrates that more than one free outlet temperature may not increase $N_{\mathrm{DOF,\,U}}$, see figure 4.9. The HEN structure is identical as in example 4.2 where we found $N_{\mathrm{DOF,\,U}} = 1$ when all outlet temperatures had targets.

First, we consider the case where $T_{\mathrm{o}}^{\mathrm{H1}}$ is free. The interface from the inner to the outer HEN is constituted by the temperatures T_2, T_3 and T_4 and N_{t} is reduced to 3. The matrix to compute R is

$$\begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_{\mathrm{I}} \\ Q_{\mathrm{II}} \\ Q_{\mathrm{III}} \\ Q_{\mathrm{IV}} \end{bmatrix} = \begin{bmatrix} b_1(T_2) \\ b_2(T_3) \\ b_3(T_4) \end{bmatrix} \tag{4.12}$$

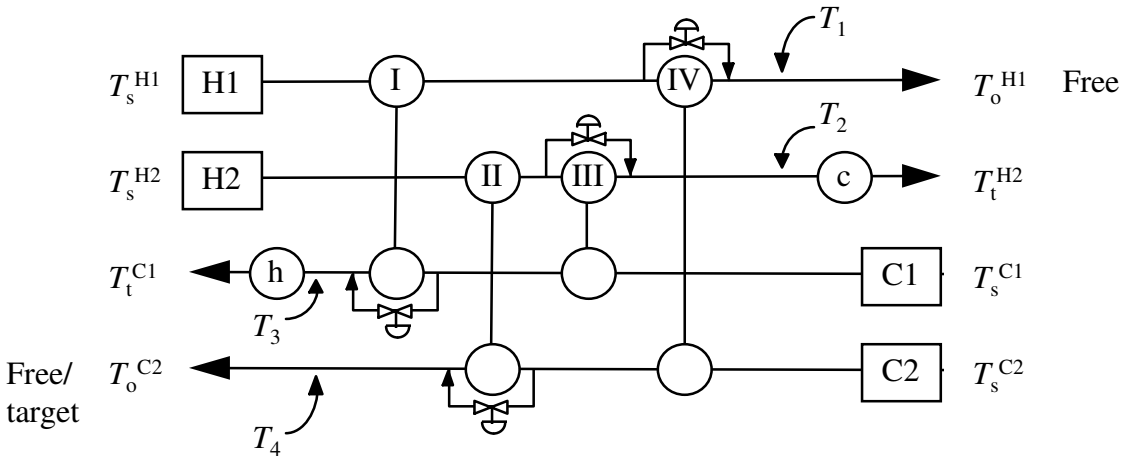

Figure 4.9 HEN with free outlet temperatures.

The rank of the matrix in equation (4.12) is $R = 3$ which is the same as in example 4.2. The matrix is identical to the matrix in equation (4.5) with the first row deleted, and stream H1 serves as a sink/source for the energy balance. The result is

$$N_{\mathrm{DOF,\,U}} = R + N_{\mathrm{U}} - N_{\mathrm{t}} = 3 + 2 - 3 = 2 \tag{4.12}$$

Now, we go one step further and also let $T_{\mathrm{o}}^{\mathrm{C2}}$ be free. The interface to the outer HEN are the 2 temperatures T_2 and T_3, and the corresponding equations for the inner HEN become equal to equation (4.12) with the last row deleted. This yields $R = 2$ and since also $N_{\mathrm{t}} = 2$, the result with 2 free outlet temperatures is

$$N_{\mathrm{DOF,\,U}} = R + N_{\mathrm{U}} - N_{\mathrm{t}} = 2 + 2 - 2 = 2 \tag{4.13}$$

$N_{\mathrm{DOF,\,U}}$ do not increase when the number of free outlets is increased above 1. This only holds when the manipulations in the inner HEN spans the whole space of N_{t} dimensions. In other examples where $R < N_{\mathrm{t}}$, relaxing target specifications may increase $N_{\mathrm{DOF,\,U}}$ until $R = N_{\mathrm{t}}$. ■

Example 4.5 *HEN with split and a process exchanger without bypass.*
The HEN in figure 4.10 has a total of 7 manipulations (including the split fraction u_{s}) and 6 targets. It is a pinch problem with 2 utilities, rule 4.1 suggest that optimization is possible and equation (4.9a) results in $N_{\mathrm{DOF,\,U}} = 1$. Note that process exchanger V does not have a bypass, and we should be careful when using equation (4.9a). Below it is shown that this HEN in fact has $N_{\mathrm{DOF,\,U}} = 0$, thus no optimization of utility cost should be considered for this design. The explanation uses only simple physical insight described in the previous section, no equations are needed.

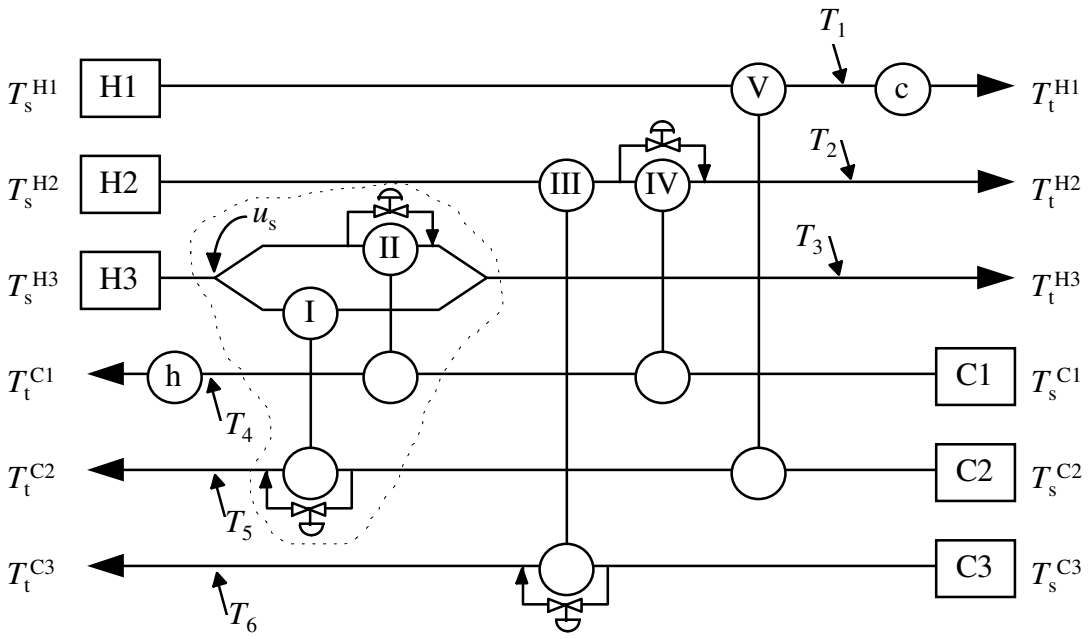

Figure 4.10 HEN with variable stream split (u_{s}) and a process exchanger without bypass.

The reason for $N_{\mathrm{DOF,\,U}} = 0$ is that the 3 manipulations u_{s}, u_{I} and u_{II} only spans a 2-dimensional space in the temperatures T_3, T_4 and T_5. This is easily seen by drawing a system boundary

around the splitted part of stream H3 and the process exchangers I and II on streams C1 and C2 (see dotted line). The energy balance for this subsystem only allows for 2 temperatures to be freely specified. That is, the 3 manipulations u_s, u_I and u_{II} only contributes with 2 free variables to the surroundings. We can already conclude that these 2 free variables have to be used for control of T_3 and T_5, thus we cannot control T_4. The 2 other manipulations in the inner HEN (u_{III} and u_{IV}) gives a total number of free variables from the inner HEN of $R = 4$. From this, we obtain

$$N_{DOF,U} = R + N_U - N_t = 4 + 2 - 6 = 0 \tag{4.14}$$

This example has demonstrated that it may not always be possible to optimize pinch problems even if there are extra manipulations. In addition it has been shown that it is not always necessary to write down equations and compute the rank R mathematically. ■

4.6 Conclusions

It has been shown that degrees of freedom in operation of HENs may not always be utilized for utility optimization and the number of DOFs that can be used for optimization, $N_{DOF,U}$ is defined. Most important for operation is not how many DOFs that can be used for optimization, but whether optimization is possible ($N_{DOF,U} \geq 1$) or not ($N_{DOF,U} = 0$). If $N_{DOF,U} < 0$, the operation of the HEN is infeasible (targets cannot be met) and the HEN design or the target specifications have to be modified.

A rule to quickly find whether optimization is possible or not has been established. We concluded that threshold problems with free outlet temperatures and pinch problems may be optimized, whereas threshold problems with no free outlet temperatures never can be optimized. For threshold problems and pinch problems with free outlet temperatures, this rule only serve as a necessary condition for optimization.

A quantitative expression for $N_{DOF,U}$ is developed and as part of that the HEN was divided into the inner HEN and the outer HEN. In many cases, the number of physical streams entering the outer HEN (streams with target value) represents a bottleneck for $N_{DOF,U}$. The equation requires computation of the rank of the transfer matrix from manipulations in the inner HEN to the temperatures of the streams entering the outer HEN, thus the detailed HEN structure is considered. By making some assumptions about the inner HEN (bypasses on all process exchangers and no singularities), $N_{DOF,U}$ is simply given by the number of utility streams.

The following three chapters concern methods for optimal operation of HENs. In those chapters it is assumed that the HENs to be operated have $N_{DOF,U} \geq 1$. As it has been clear from this chapter, it should always be verified that the HEN has $N_{DOF,U} \geq 1$ before any optimizing control scheme is considered.

Notation

CP_{Cj}	Heat capacity flowrate of cold stream j.
CP_{Hi}	Heat capacity flowrate of hot stream i.
N_{DOF}	Number of degrees of freedom.
$N_{DOF, U}$	N_{DOF} that can be utilized for optimization of utility cost.
N_F	Number of free outlet temperatures.
N_P	Number of process streams.
N_t	Number of target specifications.
N_U	Number of different utility types (same as number of utility streams).
Q_i	Heat load (duty) of heat exchanger i (where i is a Roman numeral)
R	Rank. (Dimension of the space when the temperatures in the streams to the outer HEN are spanned by all manipulations in the inner HEN).
T_o^{Cj}	Outlet temperature of cold stream j.
T_o^{Hj}	Outlet temperature of hot stream i.
T_s^{Cj}	Supply temperature to cold stream j.
T_s^{Hj}	Supply temperature to hot stream i.
T_t^{Cj}	Target temperature of cold stream j.
T_t^{Hj}	Target temperature of hot stream i.
u	Manipulation (often bypass fraction).

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