Analytical design of feedback compensators based on Robustness/Performance and Servo/Regulator trade-offs

Utility in PID control applications

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Outline

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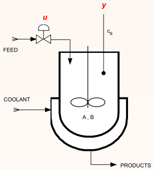
Outline

The control problem



Given a process (plant) P:

The objective of a control system is to make the output y behave in a desired way by manipulating the plant input u.

In addition, the desired behaviour should be achieved (at least approximately) in the face of uncertainty and external disturbances acting on the process.

Assumptions:

- P is Single-Input Single-Output (SISO)

- A model of P is available in the form of an ODE with constant coefficients, so that:

$$P(s) = \frac{Y(s)}{U(s)} = \frac{n_p(s)}{d_p(s)} e^{-sh} \tag{1}$$

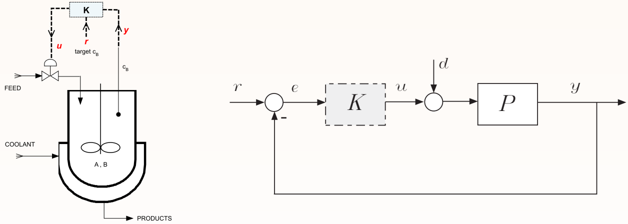
Feedback compensators

As recently explained:

Feedback: still the simplest and best solution

S. Skogestad, *Modeling, Identification and Control*, 30(3), pp. 149–155, 2009

Feedback is a simple (pretty old) concept indeed:



The feedback controller K determines u from the error e. Here, we stick to the linear control setting:

$$K(s) = \frac{U(s)}{E(s)} = \frac{n_k(s)}{d_k(s)} \qquad (2)$$

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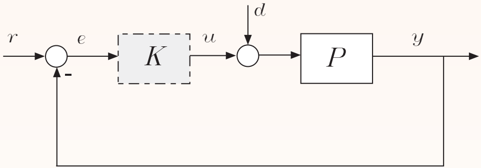
Design trade-offs

What makes the design task hard?

The choice of the feedback controller is part of a complex web of design trade-offs. Understanding and balancing these trade-offs is the essence of control system design.

G. Goodwin, S. Graebe, M. Salgado, *Control System Design*, Prentice Hall, 2000.

Robustness vs Performance



$$y = \underbrace{(1+PK)^{-1}PK}_{T}\, r + \underbrace{(1+PK)^{-1}}_{S} Pd \quad (3)$$

$$e = \underbrace{(1+PK)^{-1}}_{S}\, r - \underbrace{(1+PK)^{-1}}_{S} Pd \quad (4)$$

- For Performance we want $S \approx 0$ (equivalently $T \approx 1$) at low frequencies Perfect control: $y \approx 1r + 0d$, $e \approx 0r - 0d$.
- However, for Robustness we want $T \approx 0$ at high frequencies.

Robustness vs Performance

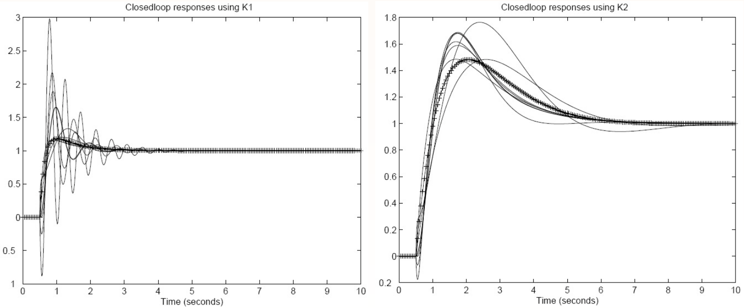
Consider:

- A plant P and an uncertain set $\mathcal{F} \ni P$:

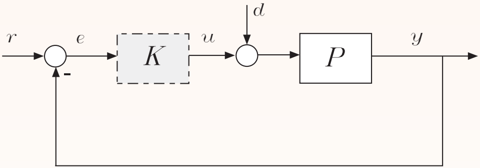
$$P = \frac{1}{s-1} \qquad \mathcal{F} = \{P(1+\Delta_m), |\Delta_m(j\omega)| \leq \Delta_m^*(j\omega)\}$$

- $K_1 = -10\frac{0.9s+1}{s}$: *high-performance* controller
- $K_2 = -1\frac{2.8s+1}{s}$: *robust* controller

Time domain responses (y) for a set-point change in r:



Servo vs Regulation (case i)



On the other hand, the effect of the reference and disturbance signals on the output are given by:

$$e = Sr - SPd \tag{5}$$

Therefore, when S and SP have significantly different dynamics, there is an inherent trade-off between servo (S) and regulatory (SP) performance.

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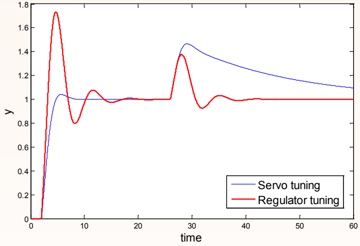

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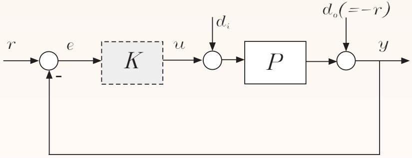
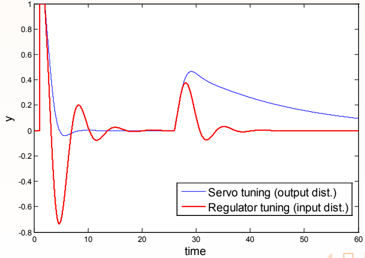
Servo vs Regulation (case ii)



As $e = -Sd_o - SPd_i$, output disturbances d_o can be interpreted as changes in the reference r. Then, the same kind of *servo/regulator* trade-off exists between *input/output* disturbances:



Outline

Problem statement

Objective

The goal of this thesis is to provide *analytical* model-based design procedures in terms of both the *Robustness/Performance* and *Servo/Regulator* trade-offs, and give insight into how the tuning depends on the process parameters.

Note that, quite commonly, only one of the aforementioned compromises is considered:

- Tuning rules aimed at *set-point tracking* and *disturbance rejection* have been derived based on minimizing common performance indices (Zhuang and Atherton, 1993; Visioli, 2001).
- Some methodologies only consider the Robustness/Performance trade-off (e.g., IMC).

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Outline

IMC (*rough* overview)

Step 1: Take $P = P_a P_m$, where $P_a \in \mathcal{RH}_\infty$ is all-pass and P_m is MP.
Step 2: Specify the closed-loop relation:

$$T_{yr} = T = P_a f \implies K = P_m^{-1} \frac{1}{f^{-1} - P_a} \quad (5)$$

where f is the so-called IMC filter:

$$f = \frac{1}{(\lambda s + 1)^n} \quad (6)$$

The main purpose of f is twofold:

- First, to ensure the *properness* of K (take n large enough, $n \geq \delta(P)$).
- Second, λ is used to balance *Robustness* and *Performance*.

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New filter

$$f = \frac{\sum_{i=1}^{k} a_i s^i + 1}{(\lambda s + 1)^{n+k}} \quad (6)$$

The main purpose of f is twofold:

- First, to ensure the *properness* of K and **internal stability** (select $a_1, \ldots, a_k$ so that $T = 1$ at the k unstable poles).
- Second, λ is used to balance *Robustness* and *Performance*.

IMC: the role of λ

- As $\lambda \to 0$, the closed-loop tends to be *optimal* w.r.t. the ISE index: $\int_0^\infty e^2(t)dt$.
- However, uncertainty imposes a limit to the closed-loop bandwidth, making *detuning* of the controller necessary.

Multiplicative Uncertainty Case:

If the real plant belongs to $\mathcal{F} = \{P(1+\Delta_m), |\Delta_m(j\omega)| \leq \Delta_m^*(j\omega)\}$, then the closed-loop is *robustly stable* provided that $\|T\Delta_m^*\|_\infty < 1$ ($|T(j\omega)| < |1/\Delta_m^*(j\omega)| \; \forall\omega$):

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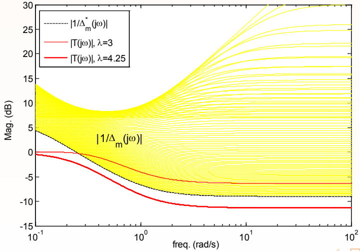


IMC: summary

The main advantages of IMC are its *simplicity* and *analytical character*, while some of its drawbacks are listed below:

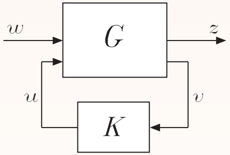
- For stable plants, the poles of P are cancelled by the zeros of the controller K $\Longrightarrow$ good results in terms of set-point tracking but sluggish disturbance attenuation when P has slow/integrating poles.
- For unstable plants, the pole-zero pattern of f can lead to large peaks on the sensitivity functions $\Longrightarrow$ poor (mid-freq) robustness and large overshoots in the transient response.
- In general, poor *servo/regulator* performance compromise is obtained.

To put it simple: only the *Robustness/Performance* trade-off is considered (through T).

Outline

$\mathcal{H}_\infty$ control

Modern $\mathcal{H}_\infty$ control is based on a generalized feedback setup:



(Signal-based) design philosophy : to press-down the peaks of several closed-loop relations captured by G. Mathematically, the synthesis problem can be expressed as

$$\min_{K\in\mathcal{C}} \|T_{zw}\|_\infty = \min_{K\in\mathcal{C}} \|\mathcal{F}_l(G,K)\|_\infty \tag{7}$$

where

$$T_{zw} = \mathcal{F}_l(G,K) \doteq G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \tag{8}$$

$\mathcal{H}_\infty$ control: blending $\mathcal{H}_\infty$ and IMC

Most of the IMC drawbacks can be overcome by considering the rest of closed-loop transfer functions, and not only T as IMC does. Following this idea, Dehghani et al. (2006) proposed an $\mathcal{H}_\infty$ design to generalize IMC:

$$\begin{aligned}\rho &= \min_{K\in\mathcal{C}} \|T_{zw}\|_\infty \\ &= \min_{K\in\mathcal{C}} \left\| \mathcal{F}_l\left(\left[\begin{array}{cc:c} -P_a f & \epsilon_2 P & P \\ 0 & \epsilon_1\epsilon_2 & \epsilon_1 \\ \hdashline 1 & -\epsilon_2 P & -P \end{array}\right], K \right) \right\|_\infty \\ &= \min_{K\in\mathcal{C}} \left\| \begin{array}{cc} T - P_a f & \epsilon_2 SP \\ \epsilon_1 KS & \epsilon_1\epsilon_2 S \end{array} \right\|_\infty \end{aligned} \tag{9}$$

The optimization aims at minimizing the closeness between T and $P_a f$ along the lines (but with more flexibility) of the standard IMC. In addition, care is taken now of $SP = T_{yd}$ and $KS = T_{ur}$.

$\mathcal{H}_\infty$ control: summary

- The revised design method has great versatility, blending IMC and $\mathcal{H}_\infty$ ideas elegantly.
- In exchange, the resulting procedure inevitably loses part of the IMC simplicity (even if $f, \epsilon_1, \epsilon_2$ can be chosen in a systematic way) and its analytical character. This may translate into design pitfalls as noted in (Lee and Shi, 2008).
- Another disadvantage of the $\mathcal{H}_\infty$ machinery is that it usually gives high-order controllers (the more frequency weights used, the higher the resulting controller's order).

Outlook

The proposed design methodology will share the *analytical* character of IMC and much of its simplicity, but extra design parameters will be introduced to deal with *servo/regulation* issues.

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Weighted Sensitivity

The proposed methodology stems from the WSP:

$$\min_{K \in \mathcal{C}} \|WS\|_\infty \tag{10}$$

where

- $S = (1 + PK)^{-1}$ is the *sensitivity* transfer function.
- W is a frequency weight used to shape S conveniently.

Interesting facts

- The sensitivity function S is a very good (closed-loop) performance indicator: its main advantage is that because we ideally want $S \approx 0$, it suffices to consider $|S|$, i.e., we don't need to worry about its phase (Skogestad, 2005).
- The WSP is a well-studied problem, so there is a lot of insight on how to select the weight W. Furthermore, the analytical solution is easily obtained.

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Outline

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Selection of W

Let us denote by $\tau_1, \ldots, \tau_k$ the time constants of the *unstable* or slow poles of P. Then, the following choice of W is made:

$$W = \frac{(\lambda s + 1)(\gamma_1 s + 1)\cdots(\gamma_k s + 1)}{s(\tau_1 s + 1)\cdots(\tau_k s + 1)} \tag{11}$$

with

$$\lambda > 0 \qquad \gamma_i \in [\lambda, |\tau_i|] \tag{12}$$

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Rationale: term $1/s$

The integrator is included to impose $S(0) = 0$, that is, to satisfy the requirement of **integral action** (zero steady-state error for step inputs).

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Rationale: dipoles $\frac{\gamma_i s + 1}{\tau_i s + 1}$

Take $\lambda \approx 0$, then:

- If $\gamma_i = |\tau_i|$, then $|W| \approx |1/s|$ and $\min_{K \in \mathcal{C}} \|WS\|_\infty \approx \min_{K \in \mathcal{C}} \left\|\frac{1}{s} S\right\|_\infty$ (**Servo**)
- If $\gamma_i = \lambda$, then $|W| \approx |P/s|$ and $\min_{K \in \mathcal{C}} \|WS\|_\infty \approx \min_{K \in \mathcal{C}} \left\|\frac{1}{s} SP\right\|_\infty$ (**Regulator**)

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Note: In practice, one can consider a single tuning parameter γ such that

$$(\gamma_1, \ldots, \gamma_k)^T = (1-\gamma)(\lambda, \ldots, \lambda)^T + \gamma(|\tau_1|, \ldots, |\tau_k|)^T$$

UAB

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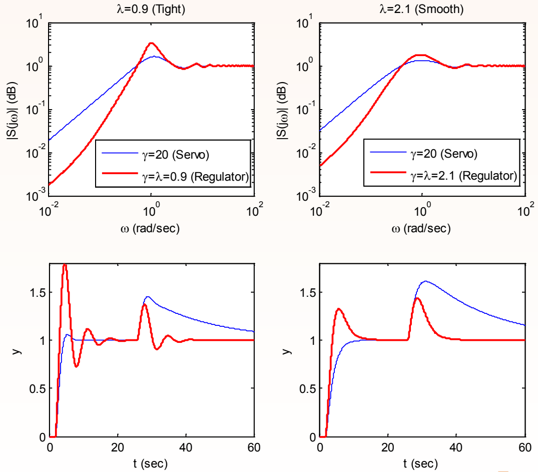
Rationale: term $(\lambda s + 1)$

The λ parameter is used to adjust the **Robustness/Performance** trade-off. As we increase the value of λ, W asks for lower values of $|S|$ at middle-high frequencies, which ultimately translates into slower response and better robustness.

Selection of W: illustrative example

Consider $P = \frac{e^{-s}}{20s+1}$, for which $W = \frac{(\lambda s+1)(\gamma s+1)}{s(20s+1)}$ and $\lambda > 0, \gamma \in [\lambda, 20]$



Analytical Solution (a preliminary result)

Lemma

Assume that P is purely rational (i.e., there is no time delay in P), and that W is a MP weight ***including the unstable poles of*** *P. Then, the optimal weighted sensitivity subject to internal stability ($K \in \mathcal{C}$) is given by*

$$\mathcal{N}^o = WS = \rho \frac{q(-s)}{q(s)} \tag{13}$$

where ρ and $q = 1 + q_1 s + \cdots + q_{\nu-1} s^{\nu-1}$ (Hurwitz) are uniquely determined by the interpolation constraints:

$$W(z_i) = \mathcal{N}^o(z_i) \qquad i = 1 \ldots \nu, \tag{14}$$

being $z_1 \ldots z_\nu$ ($\nu \geq 1$) the RHP zeros of P.

Analytical Solution

Consider the following factorizations:

$$P = \frac{n_p}{d_p} = \frac{n_p^+ n_p^-}{d_p^+ d_p^-} \qquad W = \frac{n_w}{d_w} = \frac{n_w}{d'_w d_p^+} \tag{15}$$

Then,

$$S = \mathcal{N}^o W^{-1} = \rho \frac{q(-s) d_w}{q(s) n_w} \qquad T = 1 - \mathcal{N}^o W^{-1} = \frac{n_p^+ \chi}{q(s) n_w} \tag{16}$$

where χ is a polynomial satisfying

$$q(s) n_w - \rho q(-s) d_w = n_p^+ \chi \tag{17}$$

The feedback controller

$$K = \arg\min_{K \in \mathcal{C}} \|WS\|_\infty = \frac{T}{S} P^{-1} = \frac{d_p^- \chi}{\rho n_p^- q(-s) d'_w} \tag{18}$$

Application to PID

PID controllers are named after the *Proportional, Integral* and *Derivative* control modes they combine. According to (Kano and Ogawa, 2010), the ratio of applications of PID, advanced, and MP Control is **100:10:1**. For simple models, it is easy to derive tuning rules based on the proposed methodology. For instance, assuming the *series* form:

$$K_{pid} = K_c \left(1 + \frac{1}{T_i s}\right)(T_d s + 1) \tag{19}$$

Tuning rules for simple, **possibly unstable**, 1st and 2nd order models ($e^{-sh} \approx -sh + 1$).

Model	K_c	T_i	T_d	
$K_g \frac{e^{-sh}}{\tau s+1}$	$\frac{1}{K_g} \frac{T_i}{\lambda+\gamma+h-T_i}$	$\frac{\tau(h+\lambda+\gamma)-\lambda\gamma}{\tau+h}$	-	$\gamma \in [\lambda, \lvert\tau\rvert]$
$K_g \frac{e^{-sh}}{s}$	$\frac{1}{K_g} \frac{T_i}{\lambda\gamma+hT_i}$	$h+\lambda+\gamma$	-	$\gamma \in [\lambda, \infty)$
$K_g \frac{e^{-sh}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{1}{K_g} \frac{T_i}{\lambda+\gamma+h-T_i}$	$\frac{\tau_1(h+\lambda+\gamma)-\lambda\gamma}{\tau_1+h}$	$T_d = \tau_2$	$\gamma \in [\lambda, \lvert\tau_1\rvert]$

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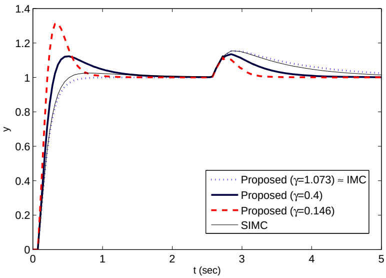
Example (stable plant)

Let us consider $P = \frac{e^{-0.073s}}{1.073s+1}$, $\lambda = 0.146$. Three values for γ:

- $\gamma = \tau = 1.073$ (servo tuning).
- $\gamma = \lambda = 0.146$ (regulator tuning).
- $\gamma = 0.4$ (**balanced** tuning)



The SIMC tuning rule (Skogestad, 2003):

$$K_c = \frac{1}{K_g}\frac{\tau}{\lambda + h} \qquad T_i = \min\{\tau, 4(\lambda + h)\} \tag{20}$$

Example (unstable plant)

For unstable plants, the IMC filter may cause large overshoot and somewhat poor robustness due to the large peak in the filter frequency response (Campi et al., 1994; Dehghani et al., 2006).

Here, we consider

$$P = \frac{e^{-s}}{-20s+1}$$

The IMC controller is given by $K_{imc} = (-20s+1)\frac{f}{1-e^{-s}f}$, where

$$f = \frac{a_1 s + 1}{(\lambda s + 1)^2} \tag{21}$$

with $a_1 = 20\left(e^{1/20}(\lambda/20+1)^2 - 1\right)$. For $\lambda = 2$, and taking $e^{-s} \approx -sh+1$, we finally obtain

$$K_{imc} = \frac{-11.53s^2 - 1.542s + 0.1059}{s^2 - 0.04669s} \tag{22}$$

Example (unstable plant)

As for the proposed method (PI controller):

- $\lambda = 2, \gamma = \lambda$ (regulator tuning) ($\approx$ IMC)
- $\lambda = 0.9, \gamma = 9 \in [0.9, 20]$

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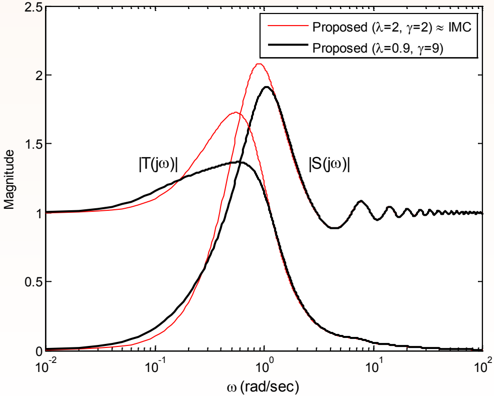
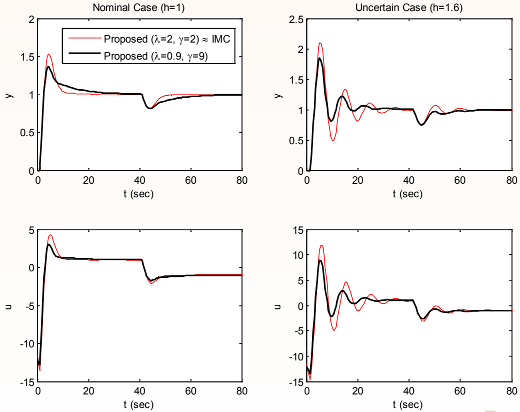
- $\lambda = 0.9, \gamma = 9 \in [0.9, 20]$



Example (unstable plant)

As for the proposed method (PI controller):

- $\lambda = 2, \gamma = \lambda$ (regulator tuning) ($\approx$ IMC)
- $\lambda = 0.9, \gamma = 9 \in [0.9, 20]$



Outline

Selection of W ($\mathcal{H}_2$ case)

So far we considered the $\mathcal{H}_\infty$ norm. Another widely extended norm is the $\mathcal{H}_2$ one, so one can similarly consider:

$$\min_{K \in \mathcal{C}} \|WS\|_2 \tag{21}$$

Following analogous arguments:

$$W = \frac{(\lambda s + 1)^n}{s} \prod_{i=1}^{k} \frac{\gamma_i s + 1}{\tau_i s + 1} \qquad \lambda > 0, \gamma_i \in [\lambda, |\tau_i|] \tag{22}$$

- "n" is selected large enough to ensure the properness of the final controller ($n \geq \delta(P)$)

NOTE: Here we are considering step signals, for other types of inputs one can replace $1/s$ with $1/d_d$, where d_d denotes the generating polynomial of the input (s for steps, s^2 for ramps, etc.)

Analytical solution

Set $P = P_a P_m$, where $P_a \in \mathcal{RH}_\infty$ is all-pass and P_m is MP. The solution to the $\mathcal{H}_2$ WSP is given by (Morari and Zafiriou, 1989):

$$K = \frac{Q}{1 - PQ}, \; Q = (P_m W)^{-1} \left\{ P_a^{-1} W \right\}_* \tag{23}$$

where the operator $\{\}_*$ denotes that after a partial fraction expansion (PFE) of the operand, all the terms involving the poles of P_a^{-1} are omitted.

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Remark

If $P_a = 1$ (P is MP), then $Q = P_m^{-1}$ and $K = "\infty" \times P^{-1}$ (Perfect control: $T = 1, S = 0$). We want to avoid this because:

- The resulting K will be generally improper.
- The weight W has no influence in this case.

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Proposed modification

If P is delay-free, we modify (23) as follows:

$$Q = (P_m W)^{-1} \left\{ P_a^{-1} W \right\}_\star, \; \left\{ P_a^{-1} W \right\}_\star = \left. \left\{ \Big(\underbrace{e^{-sh}}_{P'_a} \Big)^{-1} W \right\}_* \right|_{h=0} \tag{24}$$

Analytical solution: new insight into IMC filter design

The final controller is such that

$$T = P_a f \tag{25}$$

with $f = W^{-1}\left\{P_a^{-1}W\right\}_\star$. We can alternatively express f as

Standard filter

$$f = \frac{\sum_{i=0}^{k} a_i s^i}{(\lambda s+1)^n \prod_{i=1}^{k}(\gamma_i s+1)} \tag{26}$$

where $a_0, \ldots, a_k$ are determined from the following system of linear equations

$$T|_{s=\pi_i} = P_a f|_{s=\pi_i} = 1 \qquad i = 0, \ldots, k \tag{27}$$

being $\pi_i = -1/\tau_i, i = 0, \ldots, k$ the poles of W.

NOTE: As long as the a_i coefficients satisfy (27), λ and γ_i can be selected freely without any concern for *nominal stability*.

Extension to plants with complex poles

So far we have only considered real poles in P. However, the proposed methodology can be extended to deal with complex poles too. Consider:

$$P = K_g \frac{e^{-sh}}{(s/\omega_n)^2 + 2\xi/\omega_n s + 1} \tag{28}$$

Then,

$$W = \frac{(\lambda s + 1)^2(\gamma_{1,2}s^2 + \gamma_{1,1}s + 1)}{s\left((s/\omega_n)^2 + 2\xi/\omega_n s + 1\right)} \tag{29}$$

which yields

$$f = \frac{a_2 s^2 + a_1 s + a_0}{(\lambda s + 1)^2\left(\gamma_{1,2}s^2 + \gamma_{1,1}s + 1\right)} \tag{30}$$

and the a_i coefficients satisfy that $P_a f = e^{-sh} f = 1$ at the poles of W.

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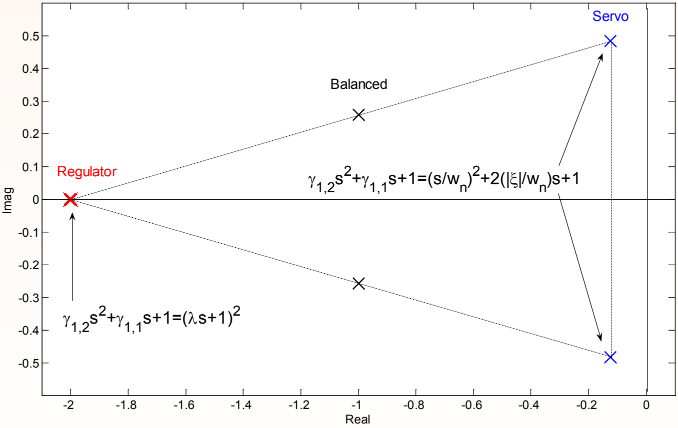
Extension to plants with complex poles

Tuning "intervals" for $\gamma_{1,2}, \gamma_{1,1}$

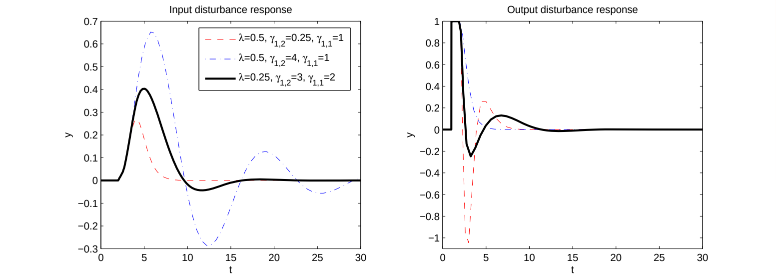


Example (application to input/output disturbances)

Model: $P = 4\frac{e^{-s}}{4s^2+s+1}$, $K_g = 4, h = 1, \omega_n = 0.5, \chi = 0.25.$
Tuning type:

- servo (output dist.): $\lambda = 0.5, \gamma_{1,2} = 4, \gamma_{1,1} = 1$, $f = \frac{1}{(\lambda s+1)^2}$ (standard IMC filter)
- regulator (input dist.): $\lambda = 0.5, \gamma_{1,2} = \lambda^2 = 0.25, \gamma_{1,1} = 2\lambda = 1$, $f \approx \frac{2.9s^2+2.3s+1}{(\lambda s+1)^4}$
- ***balanced***: $\lambda = 0.25, \gamma_{1,2} = 3, \gamma_{1,1} = 1$, $f \approx \frac{4.7s^2+2s+1}{(\lambda s+1)^2(3s^2+2s+1)}$

Nominal Case



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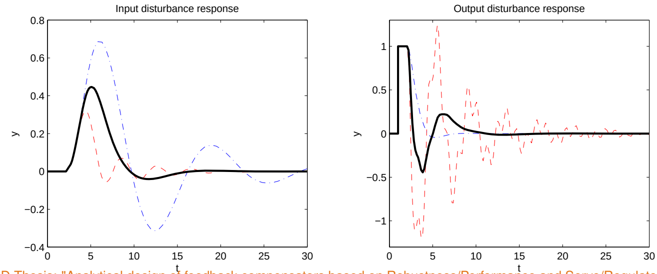
Uncertain case ($h = 1.15$)



Outline

Summary

- In this thesis, the design of SISO LTI feedback controllers has been addressed focusing on the robustness/performance and servo/regulator trade-offs.
- An analytical, fairly general, approach has been adopted based on the weighted sensitivity problem and a systematic selection of W.
- The tuning of the resulting feedback compensators involves two types of parameters (λ, γ_i), one for each considered trade-off. Qualitative tuning guidelines are:
  - Augmenting λ tends to make the system slower (more robust) to the detriment of *performance* (better high-frequency *robustness*).
  - Decreasing γ_i improves the *regulatory* performance at the expense of larger M_S, M_T values, and *vice versa*, increasing γ_i improves the *servo* response by pressing down the peaks of $|S|$ and $|T|$ (better mid-frequency *robustness*).
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Summary: Thesis publications (Journal papers)

1. S. Alcántara, C. Pedret, R. Vilanova, and W. Zhang. *Simple Analytical min-max Model Matching Approach to Robust PID Tuning with Smooth Set-Point Response.* **Industrial & Engineering Chemistry Research**, 49(2):690–700, 2010. (Chapter 2)
2. S. Alcántara, C. Pedret, R. Vilanova. *On the model matching approach to PID design: Analytical perspective for robust Servo/Regulator tradeoff tuning.* **Journal of Process Control**, 20(5):596–608, 2010. (Chapters 3,4)
3. S. Alcántara, W. Zhang, C. Pedret, R. Vilanova, and S. Skogestad. *IMC-like analytical design with S/SP mixed sensitivity consideration: Utility in PID tuning guidance.* **Journal of Process Control**, 21(6):976–985, 2011. (Chapter 6)
4. S. Alcántara, C. Pedret, R. Vilanova, and S. Skogestad. *Generalized Internal Model Control for balancing input/output disturbance response.* **Industrial & Engineering Chemistry Research**, 50(19):11170–11180, 2011. (Chapter 7)

Summary: Thesis publications (Conference papers)

1. S. Alcántara, C. Pedret, R. Vilanova, and W. Zhang.
Unified Servo/Regulator design for robust PID tuning.
IEEE **International Conference on Control Applications (CCA'10)**,
pp. 2432 – 2437, 8–10 Sept. 2010, Yokohama (Japan).
(Chapter 5)

2. S. Alcántara, S. Skogestad, C. Grimholt, C. Pedret and R. Vilanova.
Tuning PI controllers based on $\mathcal{H}_\infty$ *weighted sensitivity*.
19th Mediterranean Conference on Control & Automation (MED'11),
pp. 1301–1306, 20–23 June 2011, Corfu (Greece).
(Chapter 6)

Outline

Perspectives

- For high-order plants, the presented methodology can be used as the first step of the design procedure. Then, the controller's order can be reduced if necessary.
- In the **PID control** context:
  - Revisit the SIMC rule (Skogestad, 2003).
  - Derive PID tuning rules for other common models.
  - Extension to **fractional PID** setting.
- Application to **RESET-based** control (Appendix 6A illustrates this point by comparing the *PI+CI* and *2DOF PI* strategies).
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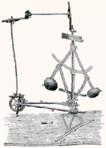

Thank you for your attention!

¡Gracias por su atención!

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