6. REFRIGERATION PROCESS CONTROL: A FORMAL DESIGN APPROACH

In the previous Chapter, the performance of the two-stage refrigeration system was examined using different control approaches. The cases examined were chosen by applying heuristic methods. Here, several techniques introduced in Chapter 2 to analyse the dynamic behaviour of control systems are applied to the refrigeration system. These techniques calculate measures to assess the controllability, resiliency, interaction and stability of the system utilising the system open loop transfer function matrix, ***G****, only, i.e. without considering the controller design. The results of the tests are compared to the performance of the system presented in Chapter 5 in order to assess the applicability of the techniques (e.g. can agreement be obtained between the predictions of analysis techniques and the observations in Chapter 5?).*

6.1. INTRODUCTION

In this Chapter, the analysis is conducted in two steps. First, the Base Case of each of the three control approaches is analysed. Then, detailed analysis of the cases examined under the cascade temperature approach is conducted. The cascade temperature approach was chosen for further analysis, as its performance was considerably better than the other two approaches. The cases studied in the subsequent sections of this chapter are summarised in Table 6.1 below.

In applying the control analysis methods, the linearised model derived in Chapter 4 is used as a basis. In addition to that several modified versions of the model are used to examine the characteristics of the system under different considered alternatives (e.g. when using cascade flow controllers). The analysis is carried out at both the steady state, using the steady state gain matrix **G**(0), and over an appropriate frequency range, using the full transfer function matrix **G**(s). The frequency range (0.001 – 10) rad/s is determined to include the characteristic process resonant frequencies (7.536E-3 – 0.608) rad/s determined from the eigenvalues of **A**.

Table 6.1: Cases studied in the control analysis

No	Case	Measured Variables	Manipulated Variables	Pairing used in Performance Analysis*
1	Base-Con	L1, L2, P1, P2, P3	FCP3, N, XV1, XV2, XV3	L1-XV2, L2-XV3, P1-N, P2-XV1, P3-FCP3
2	Base-Dir	L1, L2, TP1o, P2, P3	FCP3, N, XV1, XV2, XV3	L1-XV2, L2-XV3, TP1o-N, P2-XV1, P3-FCP3
3	Base-Cas	L1, L2, TP1o, P2, P3	FCP3, P1, XV1, XV2, XV3	L1-XV2, L2-XV3, TP1o-P1 (P1-N), P2-XV1, P3-FCP3
4	L1/L3-Cas	L1, L2, TP1o, P2, P3	FCP3, P1, XV1, XV2, XV3	L1-XV2, L3-XV3, TP1o-P1 (P1-N), P2-XV1, P3-FCP3
5	L2/L3-Cas	L1, L2, TP1o, P2, P3	FCP3, P1, XV1, XV2, XV3	L2-XV2, L3-XV3, TP1o-P1 (P1-N), P2-XV1, P3-FCP3
6	C/Liq-Cas	L1, L2, TP1o, P2, P3	FCP3, P1, XV1, FL2, FL3	L1-FL2 (FL2-XV2), L2-FL3 (FL3-XV3), TP1o-P1 (P1-N), P2-XV1, P3-FCP3
7	C/Vap-Cas	L1, L2, TP1o, P2, P3	FCP3, P1, FG2, XV2, XV3	L1-XV2, L2-XV3, TP1o-P1 (P1-N), P2-FG2 (FG2-XV1), P3-FCP3

* pairing in brackets refer to the inner cascade loop

In the subsequent sections, the cases are referred to by their numbers. It is important to note that when using **controllability** and **resiliency** measures, each case refers only to a set of measured and manipulated variables and not to a specific pairing. On the other hand, when **interaction** is analysed, the case number refers to the specific pairing listed in Table 6.1 when the method used applies only to specific pairings.

A program was developed using MATLAB to perform the chosen tests on the linear system. Two alternatives were developed, the first alternative performs the tests on the steady state gain matrix **G**(0), whereas the second alternative uses the matrices **A**, **B**, **C**, and **D** to compute the system transfer function matrix **G** at specified points within a frequency range, and uses it to perform some of the dynamic tests. A full listing of this program is found in Appendix B.

6.2. LINEAR MODEL SCALING

Several of the measures used in the interaction analysis are scaling dependent, i.e. depend on the variable magnitudes and hence on the choice of units in the model. This dependence arises largely as a result of the scaling dependence of SVD. Scaling was discussed in Section 2.5.8, where methods to address the problem were reviewed.

In this work, the scaling method used places all variables to be in an interval of [-1,1]; input variables are scaled with respect to their constraints; output variables with respect to their expected variations, and disturbances with respect to their expected range. The same constraints and ranges were used also in specifying maximum allowable limits for the controller gains, and are found in Tables 5.1 and 5.2.

The scaled process gain matrix $\mathbf{G}_s$ is calculated as

$$\mathbf{G}_s = \mathbf{T}_1^{-1}\ \mathbf{G}\ \mathbf{T}_2 \qquad (6.1)$$

where **G** is the non-scaled process gain matrix obtained from Equation 2.3, $\mathbf{T}_1$ is a diagonal matrix of the output scaling factors and $\mathbf{T}_2$ is a diagonal matrix of the input scaling factors.

It should be noted that in this study the pressure P1 is treated as both a measured and manipulated variable depending on the approach examined. Also the liquid and vapour flowrates FL2, FL3 and FG2 are treated as manipulated variables in cascade flow cases. This can be explained as follows. Actually, the flowrates FL2, FL3, and FG2, and the pressure P1 are measured in the process when control schemes using cascade loops are implemented. However, these variables are measured variables for the inner control loops, and hence manipulated variable for the master loop.

6.3. INPUT / OUTPUT VARIABLE SETS

In the refrigeration system, the number of candidate measured variables is 9 (L1, L2, L3, P1, P2, P3, TP1o, TP2o, TP3o), and 9 candidate manipulated variables (FCP3, N, XV1, XV2, XV3, FL2, FL3, FG2, P1). However, not all can be included in the linearised model at the same time. The linearised model derived in Chapter 4 includes only 5 candidate input variables and 7 candidate output variables, as follows:

a. Measured variables: L1, L2, L3, P1, P2, P3, TP1o
b. Manipulated variables: FCP3, N, XV1, XV2, XV3

Applying Equation 2.23, results in 3937 possible control scheme combinations. However, as shown earlier, this number can be reduced significantly by applying heuristic methods. See also, Laush *et al.* (1998). The methods discussed in Section 2.5.3 are also applied here.

6.3.1. Input and Output Effectiveness

With 5 degrees of freedom available, the choice of inputs is fixed, so there is no need to use any of the possible automatic selection methods available in the literature. To assess the choice of outputs, the output effectiveness method suggested by Cao *et al.* (1997b) is applied. The results of the analysis are expressed in terms of the output effectiveness η_O, which is calculated using Equation 2.25. The values are found in Table 6.2. It should be noted that the variables that have η_O greater than 0.5 are selected, the variables with η_O less than 0.5 are eliminated.

Table 6.2: Results of output effectiveness (OE) method

Output Variable	η_O *7 candidate variables*	η_O **(recalculated)** *6 candidate variables*
L1	0.6758	0.6755
L2	0.9785	0.9785
L3	0.3474	0.3481
P1	0.8909	0.9983
P2	1.0000	1.0000
P3	0.9996	0.9996
TP1o	0.1077	-
Recommendation:	Eliminate TP1o and recalculate	Eliminate L3

The results of this method recommend eliminating TP1o and L3 as measured variables, and hence use L1, L2, P1, P2, and P3 as the 5 measurements in controlling the process. This recommendation is translated to the conventional control scheme "Base-Con" (Case 1) described in Chapter 5. This arrangement was previously used as the control scheme for this process (see Wilson and Jones, 1994; Dacey, 1994; Asmar *et al.* 1997; 1998a). However, it recommends eliminating the temperature TP1o as a measured, and hence, as a controlled variable. The performance results demonstrated clearly, that this elimination results in a permanent offset in TP1o, which is not acceptable, and requires operator intervention to eliminate it. The performance results in Chapter 5 demonstrated also that applying approaches in which the temperature is measured perform considerably better.

6.3.2. Controllability

The concepts of controllability were discussed in Chapter 2 (Sections 2.3.2). The main objective of conducting this analysis is to reduce the number of candidate measured and manipulated variables by screening all possible combinations of input / output variables. The measures used here to assess controllability are the condition number and the minimised condition number of **G**. These measures have been widely accepted and used within the control community. For the condition number and the minimised condition number, the better set of variables is the one that has smaller value. Two points are emphasised here, first, these measures are applied to a specific set of input and output variables with no pre-determined or fixed pairing between the variables. Second, the condition number is scaling dependent, whereas the minimised condition number is scaling independent. In this study, the minimised condition number is determined by its upper bound proposed by Grosdidier *et al.* (1985). See Equation 2.21.

In the two-stage refrigeration system, only two levels can be stabilised as this will automatically stabilise the third level. This implies that in the choices examined only 2 levels need to be included in each choice. In the conventional control of this process, the three pressures in the vessels were always considered as measurements. However, since the outlet process stream temperature is of main interest in the control structure, it was decided to include it as a measurement at the expense of the pressure P1 as both of them characterise the conditions in the first evaporator. Consequently the pressures P2 and P3 are treated as measurements in all the choices considered. As a result, the set of measured and manipulated variables left to be examined is reduced dramatically. This argument was used in Chapter 5 to determine the different cases examined. In this section, the cases listed in Table 6.1 are examined.

The results of the analysis are shown in both Table 6.3 and Figures 6.1 and 6.2, where Table 6.3 includes the results obtained from analysing the steady state gain matrix **G**(0), and Figures 6.1 and 6.2 show the results in the chosen frequency range 0.001 to 10 rad/s.

Table 6.3: Controllability and resiliency analysis at steady state

Case	Condition number γ	Minimised condition number γ_{min}	Minimum singular value σ_{min}
1 Basic-Con	5.25E+03	3.674	1.18E-01
2 Basic-Dir	1.47E+04	4.721	4.20E-02
3 Basic-Cas	2.05E+04	4.721	5.27E-02
4 L1/L3-Cas	2.55E+04	20.689	4.41E-02
5 L2/L3-Cas	1.75E+04	4.003	4.40E-02
6 C/Liq-Cas	2.47E+04	4.721	4.61E-02
7 C/Vap-Cas	2.05E+04	4.721	5.27E-02

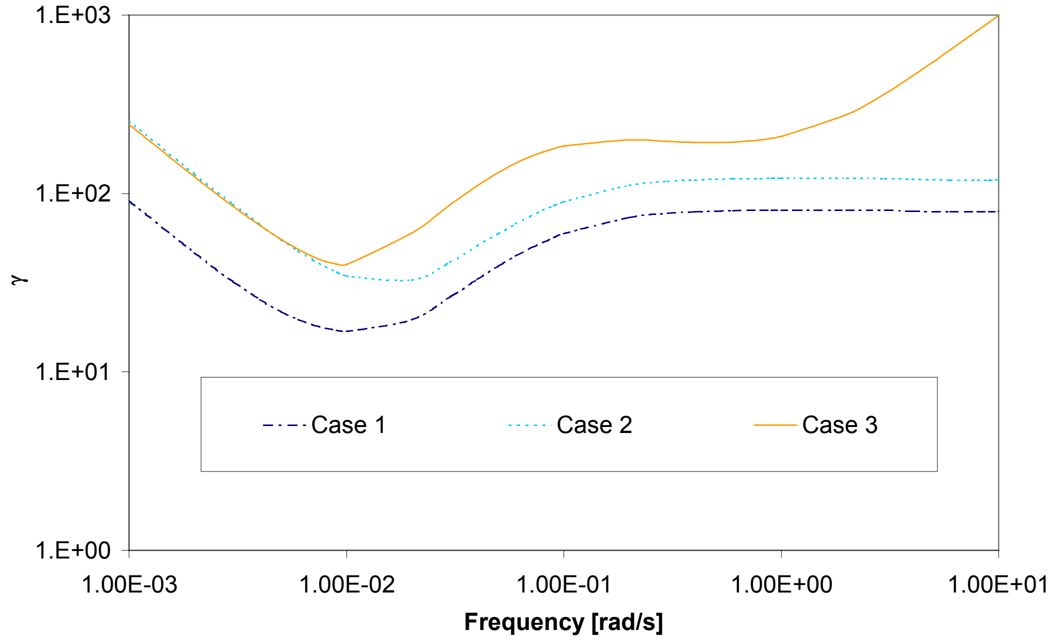
The steady state condition number results are large in all analysed cases. The values are considerably higher than the 10-100 limit of good controllability suggested by several researches (see Joseph and Brosilow, 1978 ; McAvoy, 1983b; Lau *et al.*, 1985), and therefore indicate that the system can be termed as “ill-conditioned” regardless of the case used. In the frequency range investigated, all cases show a minimum condition number in **G**(s) around 0.01 rad/s, where γ drops lower than the 100 limit. This suggests that the system will be more controllable if the designs were made around this frequency.

On a relative basis, considering Cases 1 to 3 which refer to the three different approaches investigated in Chapter 5, the results rank Case 1 as the best controllable case, followed by Case 2, then Case 3. This ranking appears to contradict the performance observed of the three cases in Chapter 5 where the ranking was the opposite. However, this ranking was done based on the ISE of TP1o which is the prime objective, and without considering the other system loops. In fact, the performance of the control loops of L1, L2, P2 and P3 agree with the controllability ranking.

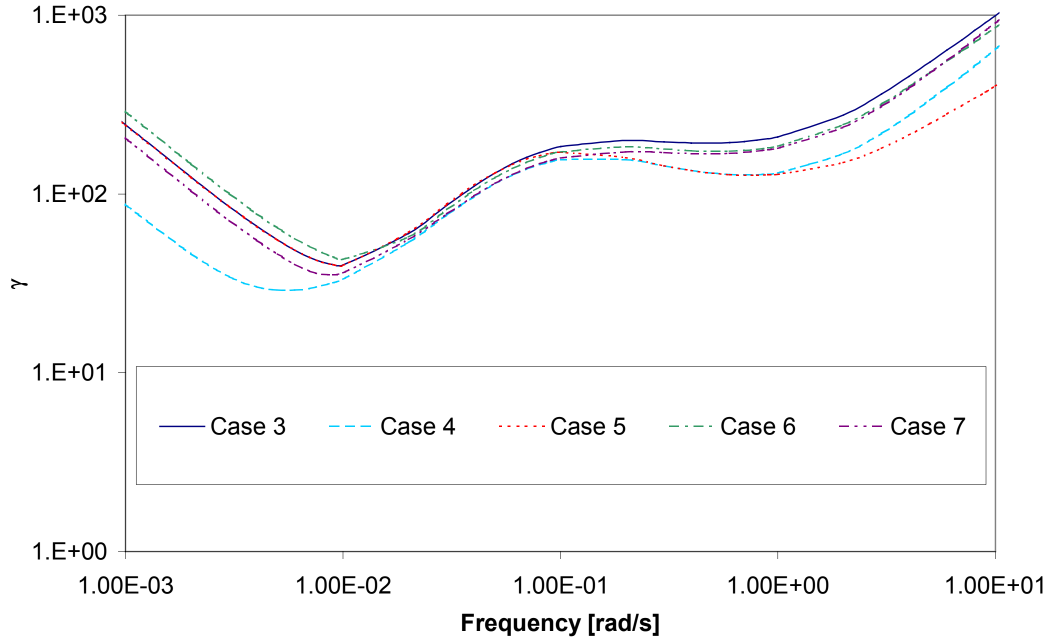
Considering Cases 3 to 7, referring to the five cases investigated under the cascade temperature approach in the previous chapter, shows that Case 5 has the lowest condition number and thus is the most controllable. This agrees with the performance of the corresponding case based on its ISE in TP1o value, and also on the performance of other loops in the system. Cases 6 and 7, referring to the cascade flow cases, show little change from the Base Case 3 in the frequency range investigated, although Case 6 has higher condition number at steady state compared to the other two cases. The performance results agree with the observations and show only negligible change in the performance of the three cases. Case 4 shows the highest steady state condition number, suggesting it is the worst case, however, this is reversed in the frequency range investigated, where its shows lower condition number. The latter observation agrees with the performance of the corresponding case in Chapter 5, which performs better than the Base Case but worse than L2/L3 Case.

The minimised condition number results show very similar results to the condition number analysis. In Cases 1 to 3, the ranking of the approaches is the same at steady state conditions with virtually no difference between Case 2 and 3. The same occurs in the frequency range investigation, where also better controllable performance is predicted at lower frequencies up to 0.01 rad/s.

The minimised condition number values coincide for Cases 3, 6 and 7 indicating that there is no difference in controllability between the three cases. This as explained earlier agrees with the observed performance. Here again, Case 4 shows the highest steady state minimised condition number (with a big margin), suggesting it is the worst case, however, this is reversed in the frequency range investigated, where it shows the lowest minimised condition number. The latter observation agrees with the observed performance of the corresponding case, which performs better than the Base Case but worse than L2/L3 Case. Once again, Case 5 with the lowest steady state minimised condition number is the most controllable, and as shown earlier this agrees with its observed performance. However, it is noticed that in the dynamic range, its minimised condition number is slightly higher than the other cases at lower frequencies, though nothing in the performance results reflect this.

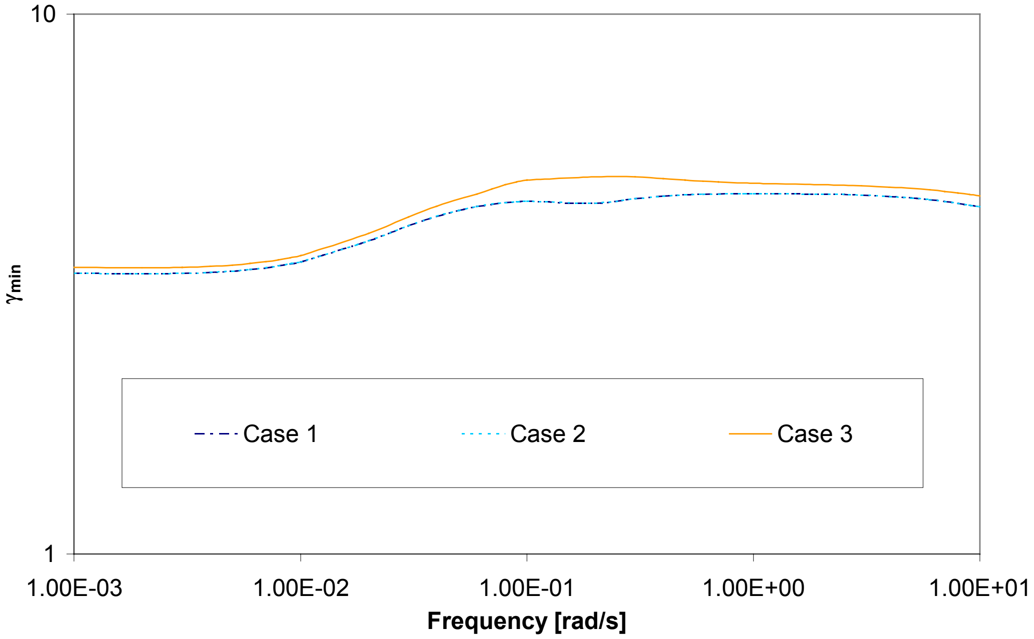


(a) Cases 1-3

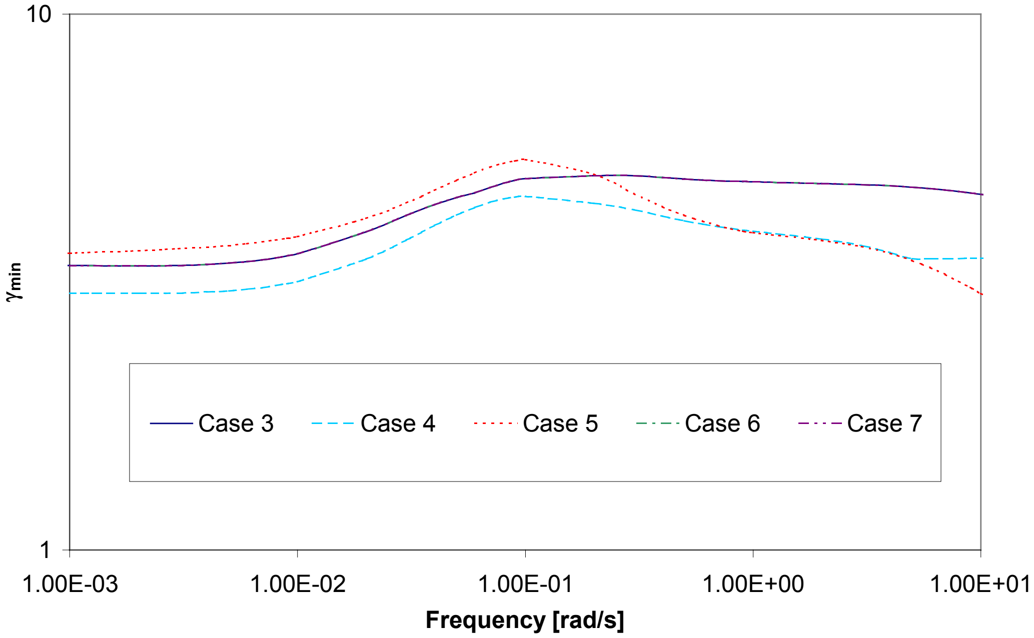


(b) Cases 3-7

Figure 6.1: Results of controllability analysis using the condition number (for Cases 1-7 defined in Table 6.1)



(a) Cases 1-3



(b) Cases 3-7

Figure 6.2: Results of controllability analysis using the minimised condition number (for Cases 1-7 defined in Table 6.1)

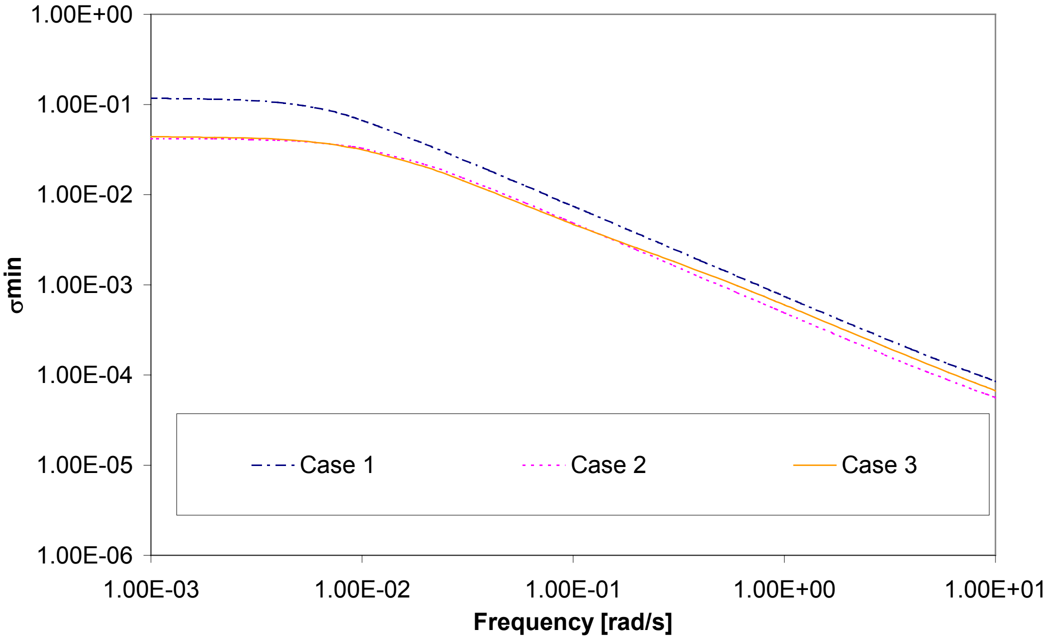
6.3.3. Resiliency

The concepts of resiliency were discussed in Chapter 2 (Section 2.5.2). The minimum singular value results are used as resiliency indicators, where higher values mean more resilient systems. This is understandable as resiliency simply gives an indication of the inherent controllability (Luyben, 1990). Mathematically, the larger the singular value of a matrix, the farther it is from being singular and the easier it is to find its inverse. From a control point of view, the best controller is the inverse of the plant, so the easier it is to find the inverse of the plant matrix **G** measured by the singular value, the better is the controllability. It should be noted though that the minimum singular values are scaling dependent

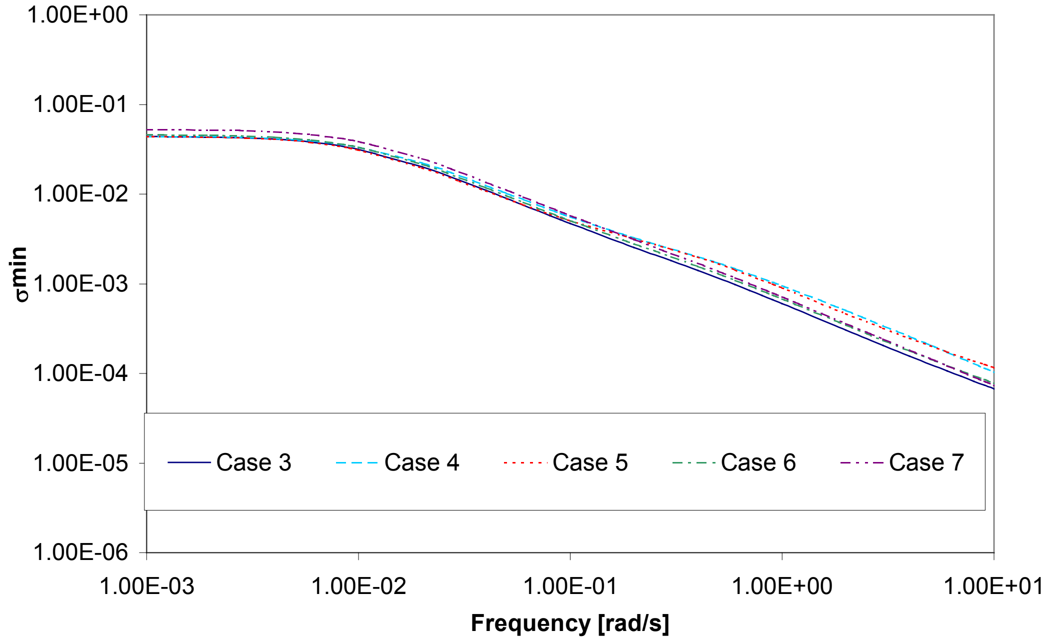
The results of the analysis are shown in both Table 6.3 and Figure 6.3, where Table 6.3 includes the results obtained from analysing the steady state gain matrix **G**(0), and Figure 6.3 shows the results in the chosen frequency range 0.001 to 10 rad/s.

The minimum singular values for all the cases analysed are relatively small, which suggest the system will be difficult to control. The comparison of the three control approaches represented by Cases 1 to 3 results shows different ranking from that obtained using both the condition number and the minimised condition number. Here, Case 1 remains the best, but Case 3, referring to the cascade temperature approach, shows better resiliency than Case 2. The results apply to both steady state and the investigated frequency range.

In examining Cases 3 to 7, the minimum singular values give different ranking suggesting that both Cases 4 and 5 are the worst. Case 6, referring to the cascade liquid flow, is slightly better, while Cases 3 and 7 are the best. This ranking does not agree with the performance obtained in the previous chapter. In the frequency range, the results of all five cases are very close to each other, and show a deterioration in the process resiliency at frequencies higher than 0.01 rad/s.



(a) Cases 1-3



(b) Cases 3-7

Figure 6.3: Results of resiliency analysis using the minimum singular values (for Cases 1-7 defined in Table 6.1)

6.4. GUIDANCE ON INPUT / OUTPUT PAIRINGS

The selection of variable pairing is an important step in designing multi SISO controllers. In Chapter 5, the selection of the pairings was decided based on heuristics and engineering practice. The pairing choices are being investigated here using several pairing methods introduced in Chapter 2. These methods arrange pairing to minimise control loop interaction so that the multivariable system resembles independent SISO loops. As a result, a full interaction analysis is needed, and its results constitute an integral part of the pairing selection.

The measures applied here are the RGA, and the RIA. As shown earlier, these two measures are related closely to each other. The interaction and pairing rules can be found in Chapter 2. The results of the analysis are found in Tables 6.4 and 6.5. In these tables, the variables were arranged in such a way that the cases examined in the performance analysis are all paired on the diagonal of the RGA or RIA matrices.

In determining the best pairing for each case, the results of the RGA and the RIA came to the same conclusions (See Tables 6.4 and 6.5). For Case 1, referring to the conventional approach, the analysis confirmed the validity of the conventional "Base" five-loop scheme as the best pairing that minimises interaction. For Cases 2 and 3, referring to the other two approaches, the pairing used in the performance analysis was valid in each case, however, it was not the recommended pairing for the 5 loops. Three pairings (L2-XV3, P2-XV1, P3-FCP3) were recommended, but both measures predict that considerable interaction exists when the loops L1-XV2 and Tp1o-N or TP1o-P1 are closed. Instead, the two methods both suggest reversing the above two pairings, and pair L1-N or L1-P1 and TP1o-XV2.

The previous performance analysis, however, does not agree with the RGA and the RIA recommended pairing. As shown previously, the "Base Case" pairings for both the direct temperature approach (Case 2) and the cascade temperature approach (Case 3) perform well, despite having two loops paired on very low RGA elements. On the other hand, examining the two cases applying the recommended pairing showed clearly, that both the RGA and the RIA failed again in predicting the best pairing.

In Case 2, with RGA recommended pairing the system performed poorly. Using the tuning method described in Chapter 5, the system was unstable, and the gain factor on the TP1o-XV2 has to be halved to –0.05 to guarantee stability. However, the transient responses were too oscillatory, and the ISE values considerably increased, where ISE in TP1o was 177 compared to 34.8 in the diagonal pairing case.

In Case 3, the system's performance was considerably better compared to Case 2. The ISE values were lower compared to even the diagonal pairing of Case 2. However ISE in TP1o was much higher (31.8) compared to the value obtained from the diagonal pairing (1.49) of the same Case.

A pairing in which the two loops L1-N and P1-XV2 were paired the opposite way round to Case 1 was also examined. This is analogous to the recommendations of RGA for Case 2 and 3, which performance results were reported in Section 5.5. The two loops pairing is valid as it is done on positive but very small RGA elements thus indicating high interaction. The simulation results surprisingly show relatively good performance (TP1o is not controlled). However, the tuning settings found by applying the method of Chapter 5 were unstable, and the gain on the P1-XV2 loop has to be cut to a fraction of its recommended value. Furthermore, its sign had to be reversed. This finding contradicts RGA predictions which state that a sign change may be necessary only if the pairing is done on negative RGA elements.

In examining Cases 6 and 7, referring to cascade flow cases, both RGA and RIA matrices were identical to the matrices obtained from Case 3. Hence, the recommendations of both RGA and RIA were identical to the recommendations of Case 3, i.e. they recommended the same pairing to Case 3, but replaced each valve position, as manipulated variable, with the corresponding flowrate.

In Cases 4 and 5, the pairings used in the performance examinations could not be validated with the RGA and the RIA. In both Cases, only P2 and P3 loops were validated, and the diagonal pairing used results in one pairing (L3) on a negative RGA element (RIA element less than –1). However, in the performance investigation, there was no need to reverse the sign of the controller, when the five loops were in operation.

The two pairings predicted by RGA and RIA for the above cases were:

- Case 4: L1-XV3, L3-P1 (P1-N), TP1o-XV2, P2-XV1, P3-FCP3
- Case 5: L2-XV3, L3-P1 (P1-N), TP1o-XV2, P2-XV1, P3-FCP3

In practice, the two pairings resulted in unstable responses regardless of the controllers tuning, and this is an example of RGA and RIA failure in defining appropriate pairings.

A very important note here is that the above two diagonal pairings are done on negative Niederlinski Index values (as shown in Table 6.6). This contradicts the definition of NI which states that in a case of a negative NI, the system will be unstable for any controller settings (Niederlinski, 1971; Luyben, 1990; Zhu and Jatan, 1993; Zhu, 1996; Luyben and Luyben, 1997). On the contrary, the performance of these two cases is ranked first and second amongst the investigated cases in Chapter 5.

6.5. PERFORMANCE MEASURES FOR FIXED INPUT / OUTPUT PAIRINGS

Several additional interaction measures were applied to analyse the results further. These measures are calculated based on a fixed pairing. The first two applied measures are the extensions of the RIA and the Niederlinski Index suggested by Zhu and Jatan (1993) and Zhu (1996). Other measures applied are the Jacobi Eigenvalue Criterion JEC, the dynamic interaction measure DIM, the performance interaction measure PIM and the μ-interaction measure μIM.

In Table 6.6, the values for the total RIA and NI are listed for the 7 cases using both the diagonal pairing and the recommended RIA pairing. The values of relevance in this study are the diagonal ones since they reflect the actual pairings investigated in the performance analysis. Total RIA suggests than interaction is considerably higher in Cases 2 and 3 compared to Case 1. In Cases 3 to 7, it suggests that Case 4 is the best, Case 5 is the worst, while it indicates that no difference in interaction exists between Cases 3, 6 and 7.

*Table 6.4: Results of the RGA analysis **

Case		RGA					Comments
1 Base-Con		XV2	XV3	N	XV1	FCP3	
	L1	**0.896**	-0.192	0.060	0.137	0.100	Diagonal pairing valid and recommended
	L2	0.013	**1.405**	-0.063	-0.162	-0.194	
	P1	0.056	-0.199	**0.988**	-0.003	0.157	
	P2	0.035	0.011	-0.079	**1.008**	0.026	
	P3	0.000	-0.025	0.093	0.021	**0.912**	
2 Base-Dir		XV2	XV3	N	XV1	FCP3	
	L1	0.280	-0.655	**0.967**	0.113	0.295	Diagonal pairing valid but not recommended
	L2	0.013	**1.405**	-0.063	-0.162	-0.194	
	TP1o	**0.671**	0.264	0.082	0.021	-0.038	
	P2	0.035	0.011	-0.079	**1.008**	0.026	
	P3	0.000	-0.025	0.093	0.021	**0.912**	
3 Base-Cas		XV2	XV3	P1	XV1	FCP3	
	L1	0.193	-0.655	**0.968**	0.240	0.254	Diagonal pairing valid but not recommended
	L2	0.015	**1.405**	-0.064	-0.165	-0.191	
	TP1o	**0.761**	0.264	0.088	-0.083	-0.031	
	P2	0.034	0.011	-0.053	**0.977**	0.031	
	P3	-0.002	-0.025	0.061	0.030	**0.937**	
4 L1/L3-Cas		XV2	XV3	P1	XV1	FCP3	
	L1	0.713	**4.470**	0.490	-3.521	-1.151	Diagonal pairing invalid
	L3	-0.505	-3.719	**0.415**	3.596	1.214	
	TP1o	**0.761**	0.264	0.088	-0.082	-0.030	
	P2	0.034	0.011	-0.053	**0.977**	0.031	
	P3	-0.002	-0.025	0.061	0.030	**0.937**	
5 L2/L3-Cas		XV2	XV3	P1	XV1	FCP3	
	L2	0.020	**1.226**	0.065	-0.154	-0.157	Diagonal pairing invalid
	L3	0.187	-0.475	**0.839**	0.229	0.219	
	TP1o	**0.761**	0.264	0.088	-0.083	-0.031	
	P2	0.034	0.011	-0.053	**0.977**	0.031	
	P3	-0.002	-0.025	0.061	0.030	**0.937**	
6 C/Liq-Cas		XV2	XV3	P1	FG2	FCP3	
	L1	0.193	-0.655	**0.968**	0.240	0.254	Diagonal pairing valid but not recommended
	L2	0.015	**1.405**	-0.064	-0.165	-0.191	
	TP1o	**0.761**	0.264	0.088	-0.083	-0.031	
	P2	0.034	0.011	-0.053	**0.977**	0.031	
	P3	-0.002	-0.025	0.061	0.030	**0.937**	
7 C/Vap-Cas		XV2	XV3	P1	FG2	FCP3	
	L1	0.193	-0.655	**0.968**	0.240	0.254	Diagonal pairing valid but not recommended
	L2	0.015	**1.405**	-0.064	-0.165	-0.191	
	TP1o	**0.761**	0.264	0.088	-0.083	-0.031	
	P2	0.034	0.011	-0.053	**0.977**	0.031	
	P3	-0.002	-0.025	0.061	0.030	**0.937**	

** Bold values indicate recommended RGA pairing*

*Table 6.5: Results of the RIA analysis **

Case		RIA					Comments
1 Base-Con		XV2	XV3	N	XV1	FCP3	
	L1	**0.117**	-6.202	15.616	6.308	9.034	Diagonal pairing valid and recommended
	L2	73.315	**-0.288**	-16.989	-7.174	-6.152	
	P1	16.879	-6.037	**0.012**	-315.630	5.355	
	P2	27.531	89.866	-13.648	**-0.007**	38.182	
	P3	3.479E+06	-40.377	9.750	47.029	**0.097**	
2 Base-Dir		XV2	XV3	N	XV1	FCP3	
	L1	2.569	-2.527	**0.034**	7.872	2.389	Diagonal pairing valid but not recommended
	L2	73.315	**-0.288**	-16.989	-7.174	-6.152	
	TP1o	**0.490**	2.788	11.220	46.761	-27.305	
	P2	27.531	89.866	-13.648	**-0.007**	38.182	
	P3	3.479E+06	-40.377	9.750	47.029	**0.097**	
3 Base-Cas		XV2	XV3	P1	XV1	FCP3	
	L1	4.185	-2.527	**0.033**	3.169	2.938	Diagonal pairing valid but not recommended
	L2	67.135	**-0.288**	-16.676	-7.071	-6.227	
	TP1o	**0.315**	2.788	10.308	-13.110	-33.775	
	P2	28.259	89.895	-19.729	**0.023**	31.286	
	P3	-425.700	-40.389	15.506	32.111	**0.067**	
4 L1/L3-Cas		XV2	XV3	P1	XV1	FCP3	
	L1	0.403	**-0.776**	1.043	-1.284	-1.869	Diagonal pairing invalid
	L3	-2.979	-1.269	**1.410**	-0.722	-0.176	
	TP1o	**0.315**	2.791	10.309	-13.152	-33.801	
	P2	28.260	88.708	-19.727	**0.023**	31.313	
	P3	-424.830	-40.709	15.503	32.003	**0.068**	
5 L2/L3-Cas		XV2	XV3	P1	XV1	FCP3	
	L2	48.709	**-0.184**	14.330	-7.484	-7.380	Diagonal pairing invalid
	L3	4.336	-3.104	**0.192**	3.360	3.558	
	TP1o	**0.315**	2.788	10.309	-13.113	-33.779	
	P2	28.259	89.620	-19.726	**0.023**	31.294	
	P3	-426.060	-40.428	15.500	32.105	**0.067**	
6 C/Liq-Cas		XV2	XV3	P1	FG2	FCP3	
	L1	4.185	-2.527	**0.033**	3.169	2.938	Diagonal pairing valid but not recommended
	L2	67.142	**-0.288**	-16.678	-7.071	-6.227	
	TP1o	**0.315**	2.788	10.308	-13.110	-33.775	
	P2	28.260	89.756	-19.728	**0.023**	31.290	
	P3	-425.730	-40.388	15.506	32.112	**0.067**	
7 C/Vap-Cas		XV2	XV3	P1	FG2	FCP3	
	L1	4.185	-2.527	**0.033**	3.169	2.938	Diagonal pairing valid but not recommended
	L2	67.135	**-0.288**	-16.676	-7.071	-6.227	
	TP1o	**0.315**	2.788	10.308	-13.110	-33.775	
	P2	28.259	89.895	-19.729	**0.023**	31.286	
	P3	-425.700	-40.389	15.506	32.111	**0.067**	

** Bold values indicate recommended RIA pairing*

The NI interaction measure as suggested by Zhu and Jatan (1993) contradicts the RIA rankings, and fails to give any conclusive results in Cases 3 to 7, as it has negative values in Cases 4 and 5, and has the same value in Cases 3, 6 and 7. Here again, it suggests that Cases 2 and 3 exhibit much higher interaction compared to Case 1.

Table 6.6: Total RIA and NI results

Pairing	Diagonal		Recommended		
Case	**NI**	**RIA**	**NI**	**RIA**	**Comments**
1 Base-Con	0.856	-0.070	0.856	-0.070	
2 Base-Dir	10.343	13.589	0.856	0.325	
3 Base-Cas	9.572	14.296	0.855	0.150	
4 L1/L3-Cas	-3.616	9.534	0.461	0.672	NI predicts unstable diagonal closed loop system
5 L2/L3-Cas	-128.150	56.004	3.459	0.413	NI predicts unstable diagonal closed loop system
6 C/Liq-Cas	9.572	14.296	0.855	0.150	
7 C/Vap-Cas	9.572	14.296	0.855	0.150	

Both the diagonal pairing and the RGA/RIA recommended pairings were also examined using the Jacobi Eigenvalue Criterion (JEC). The results of the examination are found in Table 6.7. They show that JEC ranks the cases in an identical order using the diagonal or the RGA/RIA recommended pairing, though it predicts that using the recommended pairings will result in less interaction in the systems. As shown earlier, the system did not perform better when these recommended pairings were used.

In comparing the approaches, JEC predicts that the least interaction will happen in Case 1 (referring to the conventional approach), the difference between the other two cases is very small, but in Case 3 (referring to the cascade temperature approach) it shows less interaction. This agrees with RIA ranking.

In comparing Cases 3-7, referring to different schemes in the cascade temperature approach, the JEC failed to differentiate between Cases 3, 6 and 7. This failure repeats the same behaviour of the total RIA and the NI interaction measure. It agrees also with the total RIA measure by predicting that Cases 4 and 5 will show the highest interaction.

Table 6.7: JEC results

Case	Diagonal pairing	Recommended pairing
1 Base-Con	0.4664	0.4664
2 Base-Dir	4.2615	1.2521
3 Base-Cas	3.9953	1.2511
4 L1/L3-Cas	4.4363	2.3127
5 L2/L3-Cas	7.5397	2.3137
6 C/Liq-Cas	3.9953	1.2511
7 C/Vap-Cas	3.9953	1.2511

In applying the remaining three measures, it should be noted that the DIM determines an optimum pairing at each frequency based on SVD pairing method, and uses it to calculate the interaction measure, whereas the PIM and the μIM operate with the assumption that variables are arranged to give pairings on the diagonal of **G**.

To apply the DIM on the refrigeration system, the optimum pairing it uses was examined at the steady state using the SVD pairing method. Table 6.8 shows the recommended optimum pairings. It can be seen that the method failed to suggest a suitable pairing in 5 cases (Cases 1, 2, 3, 4, and 7), as it recommends pairing two measurements with the same manipulated variable. In Case 3, for example, it recommends pairing both the process outlet temperature TP1o and the condenser pressure P3 with the condenser cooling FCP3. In Cases 5 and 6 it produced the following pairings:

- Case 5: L2-XV2, L3-P1, TP1o-FCP3, P2-XV3, P3-XV1
- Case 6: L2-FL2, L1-P1, TP1o-FCP3, P2-FL3, P3-XV1

Physically, it is almost unimaginable to control the process outlet temperature with the condenser cooling. Attempts at simulations using the above pairings failed.

Results of applying the SVD pairing method cast doubt over the DIM results as it shows that the DIM evaluates interaction with invalid sets of pairings. Therefore, an alteration was made to its calculation such that its optimised pairing is substituted by a fixed pairing. The results of the calculation are found in Table 6.9, and Figure 6.4. According to the measure definition, interaction is given by a value between 0 and 90, and values above 15 indicate high interaction. As can be seen, the modified DIM values are very high (close to 90), suggesting the system is highly interactive.

Table 6.8: Recommended pairing using SVD method

Case		XV2	XV3	N	XV1	FCP3	Comments
1 Base-Con	L1		X				Method failed
	L2	X					
	P1					X	
	P2				X		
	P3					X	

Case		XV2	XV3	N	XV1	FCP3	Comments
2 Base-Dir	L1		X				Method failed
	L2	X					
	TP1o					X	
	P2				X		
	P3					X	

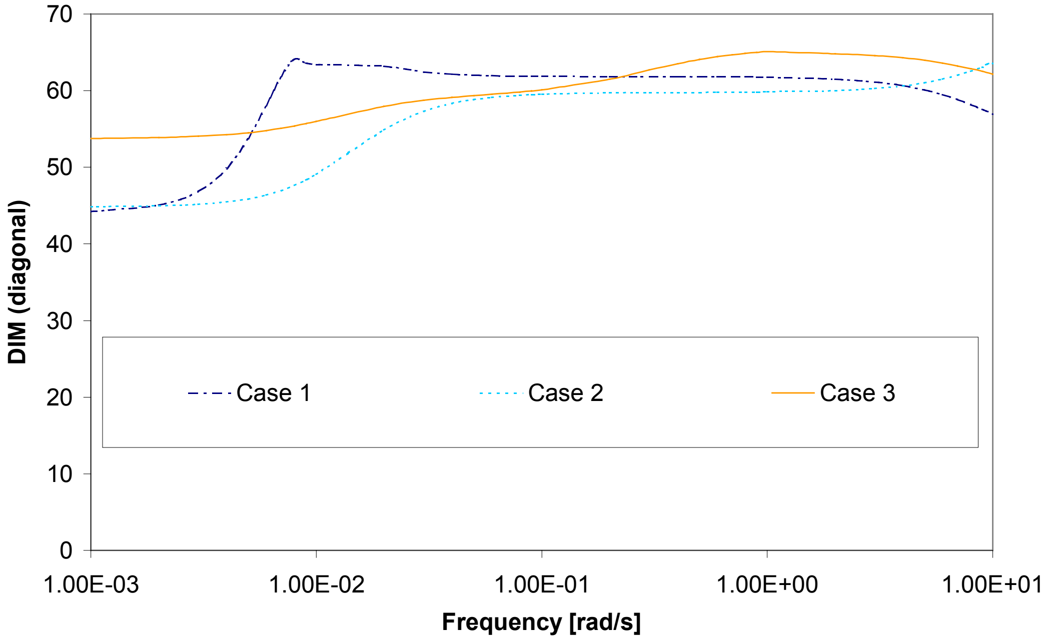
Case		XV2	XV3	P1	XV1	FCP3	Comments
3 Base-Cas	L1			X			Method failed
	L2	X					
	TP1o					X	
	P2				X		
	P3					X	

Case		XV2	XV3	P1	XV1	FCP3	Comments
4 L1/L3-Cas	L1			X			Method failed
	L3		X				
	TP1o					X	
	P2		X				
	P3				X		

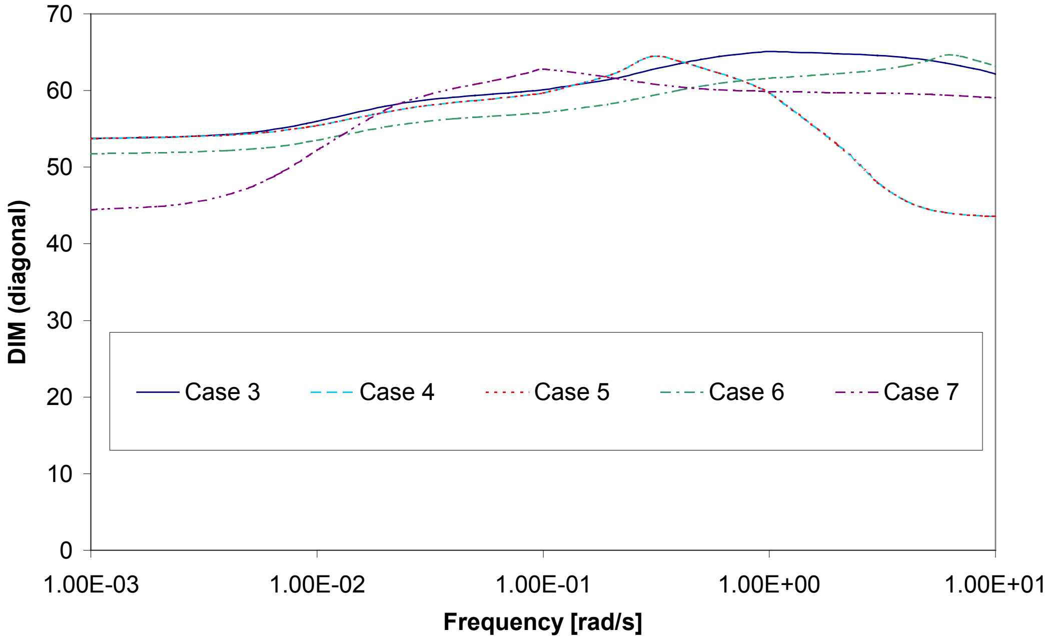
Case		XV2	XV3	P1	XV1	FCP3	Comments
5 L2/L3-Cas	L2	X					`
	L3			X			
	TP1o					X	
	P2		X				
	P3				X		

Case		FL2	FL3	P1	XV1	FCP3	Comments
6 C/Liq-Cas	L1			X			
	L2	X					
	TP1o					X	
	P2		X				
	P3				X		

Case		XV2	XV3	P1	FG2	FCP3	Comments
7 C/Vap-Cas	L1			X			Method failed
	L2	X					
	TP1o					X	
	P2				X		
	P3					X	



(a) Cases 1-3



(b) Cases 3-7

Figure 6.4: DIM analysis results

(for Cases 1-7 defined in Table 6.1)

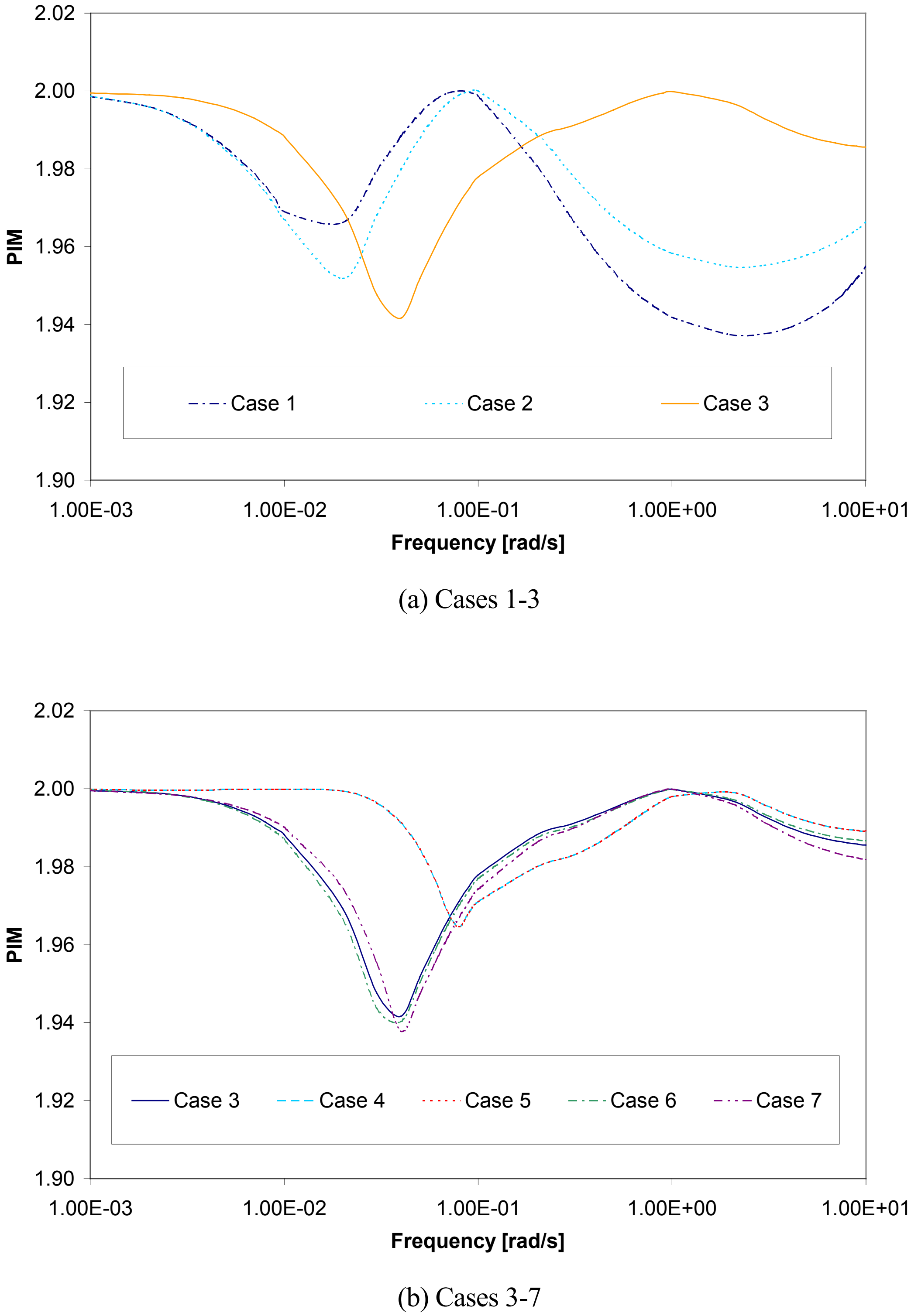
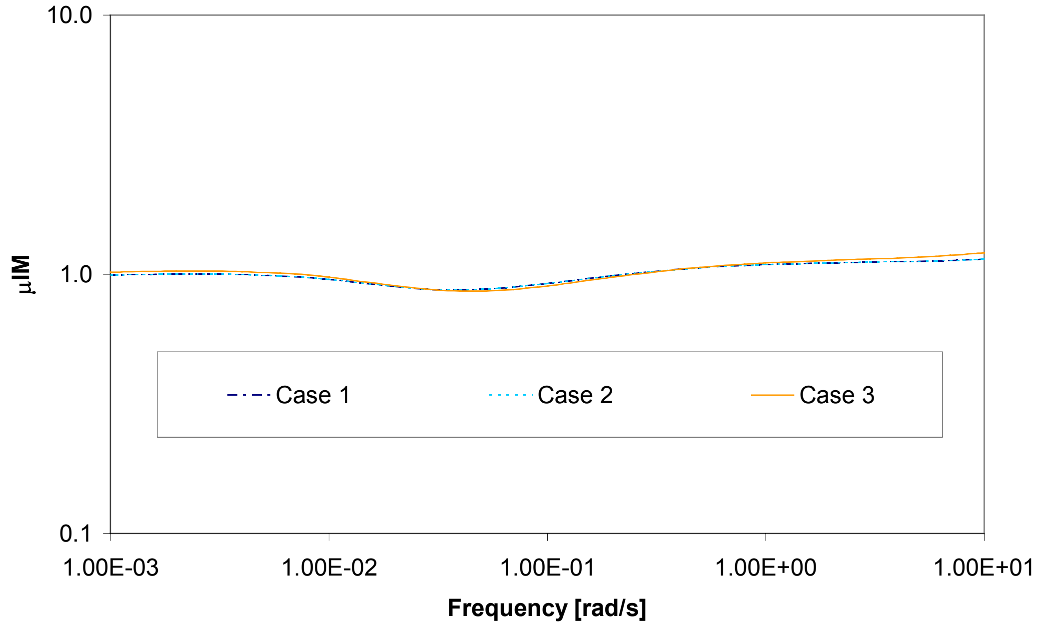
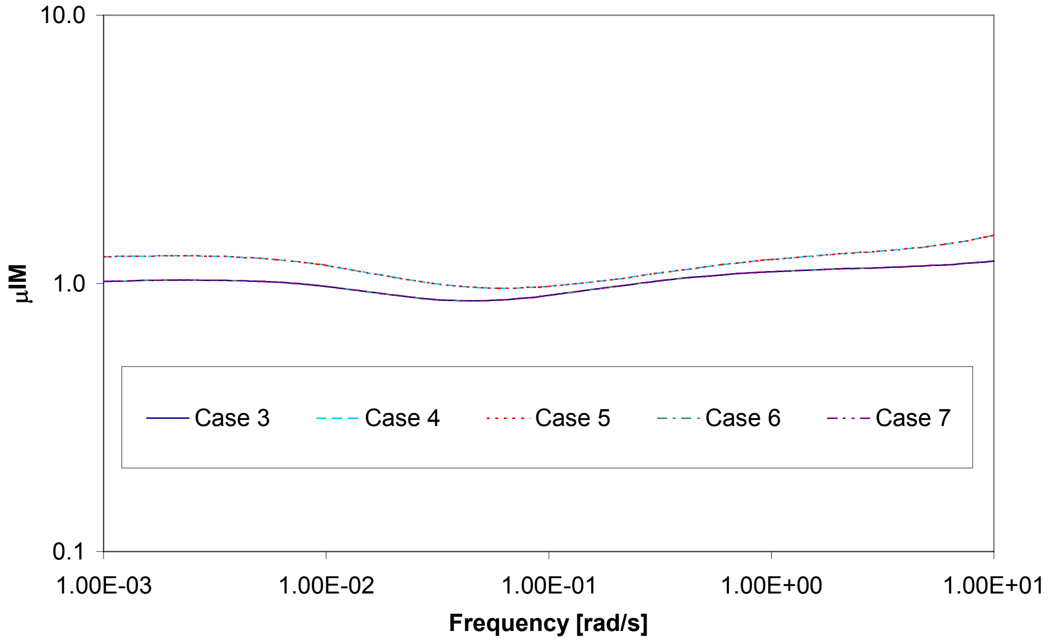


Figure 6.5: PIM analysis results

(for Cases 1-7 defined in Table 6.1)



(a) Cases 1-3



(b) Cases 3-7

Figure 6.6: μIM analysis results

(for Cases 1-7 defined in Table 6.1)

In comparing Cases 1-3, DIM shows that Case 1, referring to the conventional approach with diagonal pairing, shows the least interaction, followed by Case 2, referring to the direct temperature approach, and then by Case 3, referring to the cascade temperature approach. DIM values in Case 3 are very close to 90 which translates to a fully interactive system according to the measure definition. See Table 6.9. In comparing Cases 4-7, the difference in DIM values is very small, and no conclusion can be drawn from the analysis.

PIM analysis was also performed on all cases, but the results at steady state were inconclusive (See Table 6.9). According to the measure definition the interaction in each system is given by a value between 0 and 2, where 2 indicates total interaction. The results of the analysis for all investigated cases were above 1.995, hence indicating very highly interactive systems. However, the difference in PIM between the cases was minute, and no relative conclusions can be drawn from it. In looking at the results in the frequency range in Figure 6.5, a noticeable minimum can be seen in all cases at a frequency of 0.02 to 0.05 rad/s. This represents the lowest level of interaction attainable by the system, but even this is strongly interactive (PIM of 1.94).

The last measure tested is the μIM. In its definition, μIM measures how far the control structure is from a fully decentralised controller, and states that values of μIM less than 1 indicate that integral action cannot be included in the controller design. μIM was applied on the tested cases with diagonal controllers. It was calculated using the upper bound defined by Morari and Zafiriou (1989) found in Equation 2.23. The results in Table 6.9 show that at steady state all cases fail this test, and "theoretically" cannot have integral action in the system. However this is not the case, as all cases performed well using proportional integral controllers, as can be seen in Chapter 5. In ranking the cases, the method suggests that Case 1 is far better than the other cases, followed by Case 3 then Case 2. Differences between Cases 3-7 are small, with Cases 3, 6 and 7 better than Case 4 and then 5.

The results of μIM in the frequency range are shown in Figure 6.6. In examining the results, all cases perform satisfactorily with values slightly above 1, apart from a slight drop below 1 at a frequency around 0.05 rad/s. These values indicate high interaction. A detailed analysis of the cases show that μIM values are almost identical in Cases 1 to 3, and show only small differences in Cases 4 to 7, with Cases 4 and 5 performing slightly better.

Table 6.9: Interaction measures results at steady state

Case	DIM	PIM	μIM
1 Base-Con	80.273	1.9995	0.809
2 Base-Dir	81.591	1.9997	0.206
3 Base-Cas	89.978	1.9989	0.220
4 L1/L3-Cas	89.955	2.0000	0.179
5 L2/L3-Cas	89.967	1.9999	0.101
6 C/Liq-Cas	89.959	1.9965	0.220
7 C/Vap-Cas	89.978	1.9989	0.220

6.6. DISCUSSION

A comparison of the recommendations of all methods considered is summarised in Table 6.10. The performance of the cases is ranked for each method from best to worst, where number 1 means the best. The ranking is divided into two groups. Cases 1-3, referring to the three approaches, constitute one group, and Cases 3-7, referring to the cases under the cascade temperature approach, form the other group.

As can be seen in Table 6.10, the recommendations of the methods do not agree with the performance presented in Chapter 5. Furthermore, the methods contradict each other. This does not mean that all the usage of these methods should be discouraged. On the contrary, the analysis can give insight into the behaviour of the system, and some useful data can be obtained.

Controllability and resiliency analysis results indicate that the system is ill-conditioned and difficult to control regardless of the approach used. The two controllability measures investigated (the condition number and the minimised condition number) were relatively successful in identifying the more controllable systems. However, this selection is done on mathematical grounds and does not account for the physical conditions or control objectives of the process. In the refrigeration system, both measures showed that omitting the process outlet temperature from the control structure will lead to a more controllable systems. The performance results showed that this is true for the pressure and level control loops, but the prime object of the process control, i.e. the process outlet temperature, performs badly and an undesired offset persists in its final response. Hence a control system that includes temperature control will be needed.

Table 6.10: Comparison of investigated indexes

Measure	**Approach**			**Case**				
	Conventional	**Direct Temperature**	**Cascade Temperature**	**Base**	**L1/L3**	**L2/L3**	**C/Liq**	**C/Vap**
Case number	1	2	3	3	4	5	6	7
Performance	**3**	**2**	**1**	**4**	**2**	**1**	**5**	**3**
γ	1	2	3	2=	5	1	4	2=
γ_{min}	1	2=	2=	2=	5	1	4	2=
σ_{min} (MRI)	1	3	2	1=	5	4	3	1=
NI	2=	2=	1	1=	4	5	1=	1=
NI (diagonal)	1	3	2	1=	-	-	1=	1=
RIA	1	3	2	1=	5	4	1=	1=
RIA (diagonal)	1	2	3	2=	1	5	2=	2=
JEC	1	3	2	1=	4	5	1=	1=
JEC (diagonal)	1	3	2	1=	4	5	1=	1=
DIM (diagonal)	1	2	3	4=	1	3	2	4=
μIM (diagonal)	1	3	2	1=	4	5	1=	1=
PIM	2	3	1	2=	5	4	1	2=

The minimum singular value analysis agreed with the controllability analysis in recommending using a control scheme without the process outlet temperature, hence the same argument above applies.

The results of the three interaction measures DIM, PIM and μIM agreed only on one thing: the system is highly interactive. However all three methods failed to differentiate conclusively between the cases investigated. Furthermore, the DIM results could not be used using the optimal SVD pairing method as it has been shown that the pairings it produces are not reliable and are invalid on many occasions.

PIM and μIM contradicted each other in different parts of the frequency range. At frequency around 0.05 rad/s PIM predicted that the system exhibits its lowest interaction. μIM on the other hand came to the opposite conclusion and suggested that the worst condition occurs around the same frequency.

In this study, the RGA and the RIA gave identical recommendations. However, their recommendations failed to identify the best performing pairings in six out of the seven cases examined. Furthermore, the results obtained contradict the widely accepted characteristics of the RGA in several ways. In Cases 4 and 5, the recommended RGA/RIA pairings resulted in unstable systems regardless of the tuning used, whereas using the diagonal pairing that includes one pairing on a negative RGA element was not only successful but the best performing cases. In addition to that, no sign change was required in the controller gain of the loops paired on the negative RGA element.

Applying a similar configuration on the conventional approach (i.e. pairing L1-N and P1-XV2, with other three loops unchanged) resulted in an interesting case. As can be seen from Table 6.4, this pairing is valid, though not recommended due to high predicted interaction. However, in applying the control scheme, the tuning of the loop P1-XV2 had to be altered significantly to guarantee stability as the gain factor has to be cut in magnitude, and furthermore its sign had to be reversed.

As shown earlier, two diagonal pairings of Cases 4 and 5 were applied to cases with negative Niederlinski Index values as shown in Table 6.6. This contradicts the definition of NI which states that in a case of a negative NI, the system will be unstable for any controller settings. On the contrary, the two cases performance is ranked first and second amongst all investigated cases.