

### **3. REFRIGERATION PROCESS CONTROL: CASE STUDY MODEL**

*This chapter introduces the two-stage refrigeration system case study. It describes in detail the mathematical model developed, highlighting the assumptions used and the correlations derived to describe the behaviour of some elements of the equipment.*

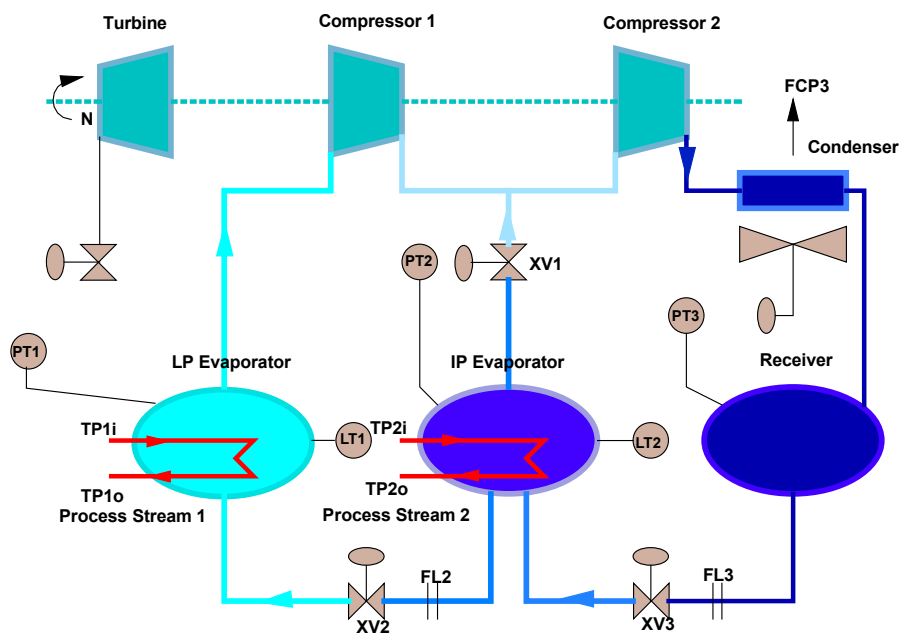
#### **3.1. PROCESS DESCRIPTION**

The cycle considered here is a two-stage side-load refrigeration unit with propylene as a working fluid. The equipment dimensions are based on an existing installation at a large petrochemical plant. Figure 3.1 shows the process flowsheet.

The main process heat load is removed in the low pressure (LP) evaporation stage. An additional independent heat load is removed in the intermediate pressure (IP) evaporation stage, and this helps to reduce compressor power consumption through its economiser flash of saturated liquid letdown from the high pressure (HP) refrigerant receiver. The LP evaporator is a kettle reboiler, whilst the IP evaporator is a flash drum with a submerged coil. The volumes of the two vessels differ by a factor of 5. The LP and IP evaporators are designed to provide adequate liquid inventory to ensure the evaporator tubes are always covered by liquid so that effective heat transfer can take place. Also the evaporators are designed to allow for sufficient vapour space for liquid disengagement to avoid excessive liquid droplet carry over to the high speed compressor, thus demisting pads are needed, and the required allowances in design are made to account for this.

The vapour refrigerant is compressed in a two stage centrifugal compressor driven by a steam turbine. The fraction of the refrigerant that flows around the second compression stage (IP-HP side) is compressed at a smaller ratio than the fraction that enters the first compression stage. Therefore less energy is consumed in the process, as less steam is needed to drive the compressor turbine.

A forced draft air cooler is used to condense the HP refrigerant vapour, followed by a receiver tank which provides storage for the condensed liquid, sufficient to accommodate any float in the inventory of the two evaporators.



*Figure 3.1: Two-stage refrigeration system*

The conventional control scheme on this unit consists of the following five loops: the pressure in the first evaporator is controlled by manipulating the compressor speed. This is essential as this pressure directly affects the refrigerant saturated pressure and consequently the heat transfer. The pressure of the second evaporator is controlled to avoid over-chilling the process stream, this is done by manipulating the flowrate of the refrigerant vapour exiting the IP evaporator via a control valve. Manipulating the air flowrate across the condenser controls the refrigerant pressure in the condenser (discharge pressure of the second compressor). The levels in the evaporator drums are controlled to ensure that the evaporator tubes remain submerged in refrigerant

liquid to maintain good heat transfer, hence two liquid flow control valves are manipulated to achieve this.

## 3.2. MODEL DERIVATION

A mathematical model of the refrigeration system can be built from a set of algebraic and differential equations. The state of the refrigerant at each control volume is established by applying the principles of mass and energy conservation, alongside the state equations that correlate the various refrigerant properties at each control volume. Typical values for heat transfer coefficients, valve constants, etc., which must be determined experimentally, or estimated if this is not possible, are substituted into the equations to develop a combined model of the process.

The model has been developed in generalised form as an  $n$ -stage refrigeration system. It has then been applied to the case of a two-stage system, where several relations are fitted for the physical and chemical characteristics and for the compressor characteristics.

### 3.2.1. Assumptions

The basic simplifying assumptions used in this model are listed below.

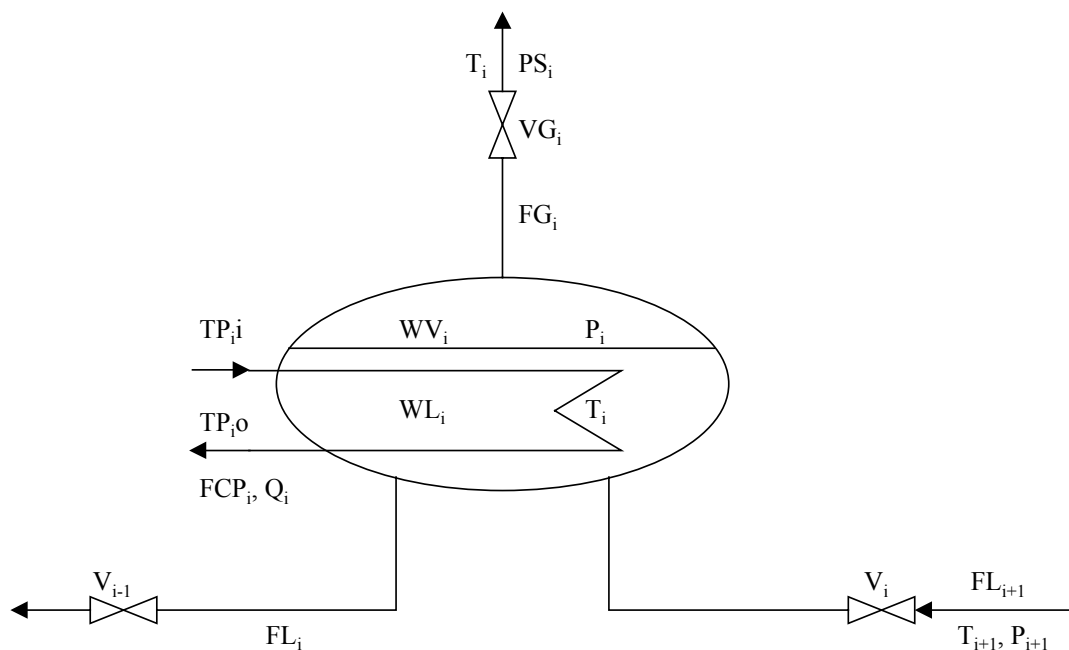
- The process streams are modelled as single phase and undergoing sensible cooling only.
- Instantaneous movement between steady state conditions is assumed in the heat exchangers (i.e. temperatures around the evaporator tube bundles and the condenser), at the compressor inter-stage pressure node, and in the compressor speed (the driver speed is assumed to move instantaneously).
- Heat loads are imposed as ‘warm’ process streams, which flow through the tube sides of the tube bundles, which are considered as always submerged.
- The thermal inertia of the metal equipment in the refrigeration cycle is neglected.

- The vessels are assumed insulated, so no heat transfer occurs between the vessels, the piping and the surrounding environment.
- The heat transfer areas and overall heat transfer coefficients are constant.
- No head losses occur from refrigerant flow in the pipelines.
- Adiabatic expansion of liquid refrigerant occurs as it flows through expansion valves. Although the process is isenthalpic, the refrigerant temperature is reduced as the liquid is cooled down as a result of providing latent heat for the formation of the flash-vapour.

### 3.2.2. Equations:

Based on the above assumptions, a generalised model is developed. For the special case of two-stage model, it involves six first order differential equations, in addition to a set of nearly 30 algebraic equations that correlate the other parameters together.

#### 3.2.2.1. Evaporator:



**Figure 3.2: General evaporator**

For the evaporator  $i$  in Figure 3.2, mass and energy balance are evaluated. They give

the variation in the total refrigerant hold-up  $W_i$  and the corresponding enthalpy content  $H_i$ .

In general, the mass balance equation is written as:

$$\frac{dW_i}{dt} = FL_{i+1} - FL_i - FG_i \quad (3.1)$$

where  $FL_i$  and  $FG_i$  are the liquid and vapour refrigerant mass flowrates from the evaporator,  $FL_{i+1}$  is the liquid refrigerant mass flowrate into the evaporator.

It should be noted that no liquid refrigerant flows from the first evaporator, so  $FL_1$  is always zero.

The general energy balance equation is written as:

$$\frac{dH_i}{dt} = (HFL_{i+1} FL_{i+1}) - (HFL_i FL_i) - (HFG_i FG_i) + Q_i \quad (3.2)$$

where  $HFL_i$  and  $HFG_i$  are the specific enthalpies of the liquid and vapour refrigerant streams respectively, and  $Q_i$  is the heat transferred to the evaporator.

The liquid refrigerant hold-up  $WL_i$  is calculated from the mass balance

$$WL_i = W_i - WV_i \quad (3.3)$$

where  $WV_i$  is the vapour refrigerant hold-up, which is calculated iteratively from the equation of state for the vapour refrigerant in the evaporator (Equation 3.33).

The liquid refrigerant volumetric hold-up  $WLV_i$  is then simply evaluated as follows

$$WLV_i = WL_i \nu f_i \quad (3.4)$$

where  $\nu f_i$  is the specific volume of the liquid refrigerant.

Assuming a fixed vessel cross section, this volumetric liquid hold-up is considered equivalent to the liquid level in the drum and hence  $WLV_i$  is used here as an indication of liquid level. So in the rest of this study, whenever a reference to any level  $L_i$  is used, it actually means the volumetric liquid hold-up  $WLV_i$ . For large variations in liquid level, the shape of the vessel needs to be accounted for.

The liquid refrigerant flowing into evaporator  $i$  is at a lower temperature (the saturation temperature at the pressure  $P_i$ ) than that of the process stream in the same evaporator, this is necessary to provide the driving force for heat transfer. The pressure of this feed liquid is reduced as the liquid passes through the control valve. Here a part of the refrigerant vaporises in order to reduce the temperature of the remaining liquid to the evaporator temperature  $T_i$ .

In this model it is assumed that the mass flowrate through the valve is proportional to the fractional valve opening  $XVL_i$  (i.e. linear characteristic) and the square root of the pressure drop across the valve, thus

$$FL_{i+1} = XVL_i CVL_i \sqrt{P_{i+1} - P_i} \quad (3.5)$$

where  $CVL_i$  is taken as the constant valve flow coefficient, and  $P_i$  is the pressure in the evaporator  $i$  and  $P_{i+1}$  is the valve's inlet pressure.

Generally, Equation 3.5 can be also applied to flashing liquids if the actual pressure drop is less than the maximum allowable pressure drop,  $\Delta Pa$ , that is effective in producing flow (after which an increase in pressure drop does not cause flow increase). If the above condition is not fulfilled then Equation 3.6 should be used (GPSA, 1994) to determine the pressure drop.

$$\Delta Pa = km (P_{i+1} - rc P_v) \quad (3.6)$$

where  $P_v$  is the liquid vapour pressure,  $rc$  is the critical pressure ratio for the liquid and  $km$  is a flow characteristic constant that depends on the valve style and size.

The evaporator saturation temperature  $T_i$  is calculated iteratively starting from the total enthalpy of the refrigerant in the vessel

$$H_i = WL_i HFL_i + WV_i HFG_i \quad (3.7)$$

Assuming a value for  $T_i$ , its saturated pressure  $P_i$  is calculated using the Antoine Equation (Equation 3.8), where A, B, C are constants.

$$P_i = \exp \left( A - \frac{B}{(T_i - C)} \right) \quad (3.8)$$

At any instant, the heat absorbed by the refrigerant in the evaporator,  $Q_i$ , is that given by the process stream

$$Q_i = FCP_i (TP_{i,i} - TP_{i,o}) \quad (3.9)$$

where  $FCP_i$  is the product of mass flowrate and specific heat capacity of the process stream, and  $TP_{i,i}$  and  $TP_{i,o}$  are the process stream inlet and outlet temperatures respectively

The process stream outlet temperature depends on the heat transfer area  $AR_i$ , the overall heat transfer coefficient  $U_i$ , the refrigerant temperature inside the evaporator  $T_i$  and the mass flowrate of the process stream according to the relationship (Kay, 1963)

$$TP_{i,o} = (1-a_i) T_i + a_i TP_{i,i} \quad (3.10)$$

where  $a_i$  is defined as

$$a_i = \exp \left( \frac{-U_i AR_i}{FCP_i} \right) \quad (3.11)$$

The flowrate of vapour refrigerant leaving the evaporator  $FG_i$  is saturated, and its

temperature and pressure are the same as those of the boiling liquid in the evaporator. This vapour is in general combined at an inter-stage mixing node with the vapour discharged from a previous compressor stage and drawn into the compressor (see Figure 3.4). With a valve  $VG_i$  present, then the refrigerant mass flowrate is proportional to the fractional valve opening  $XVG_i$  (i.e. linear characteristic), and the square root of the pressure drop across the valve, where  $CVG_i$  is the valve flow coefficient and  $PS_i$  is the suction pressure of the compressor  $i$  (at the mixing node).

$$FG_i = XVG_i CVG_i \sqrt{P_i - PS_i} \quad (3.12)$$

$FG_i$  is always determined solely by the compressor suction, as there is no valve and no mixing node.

### 3.2.2.2. Condenser and Receiver:

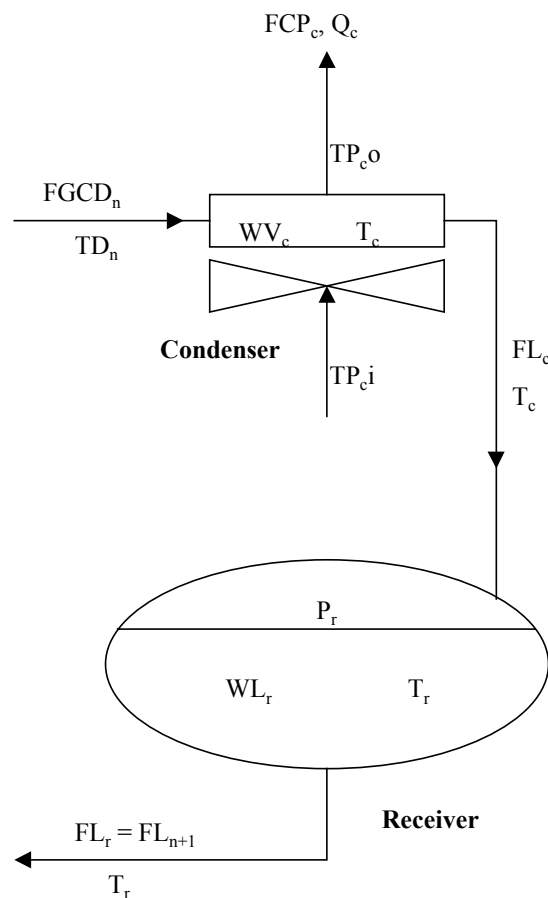


Figure 3.3: Condenser and receiver



Figure 3.3 shows an illustration of the condenser and receiver installation. The vapour refrigerant discharged from the compressor stage  $n$  enters the condenser in a superheated state at pressure  $PD_n$  and temperature  $TD_n$ . As a result of giving heat to the cooling air, it condenses at the saturation temperature  $T_c$  corresponding to the pressure  $PD_n$ . This temperature is calculated iteratively from the equation of state (by trial and error with Antoine's Equation) for the saturated vapour refrigerant. The vapour refrigerant in the condenser and the receiver are lumped together at this pressure and temperature. It is assumed that no liquid refrigerant remains in the condenser. Therefore a mass balance in the condenser gives the variation in the refrigerant's vapour hold-up  $WV_c$ , where  $FGCD_n$  is the vapour refrigerant discharge flowrate from compressor stage  $n$ , and  $FL_c$  is the liquid refrigerant condensate flowrate from the condenser

$$\frac{dWV_c}{dt} = FGCD_n - FL_c \quad (3.13)$$

This vapour occupies a volume  $VV$  equal to the condenser volume  $V_c$  plus the vapour space of the receiver.  $VV$  is expressed in the equation below

$$VV = V_c + V_r - (WL_r v_{f_r}) \quad (3.14)$$

where  $V_r$  is the volume of the receiver,  $WL_r$  is the liquid hold-up in the receiver and  $v_{f_r}$  is the specific volume of the liquid refrigerant in the receiver.

The liquid refrigerant is stored in the receiver under the same pressure  $PD_n$ , but with slightly a different temperature  $T_r$  than the saturation temperature  $T_c$ . The receiver liquid refrigerant hold-up  $WL_r$  is calculated from an overall mass balance on the refrigerant total inventory  $W$ .

$$WL_r = W - \sum_i^n W_i - WV_c \quad (3.15)$$

The mass and energy balances across the receiver liquid space together give the

variation in the receiver's liquid temperature as\*

$$\frac{dT_r}{dt} = \frac{FL_c}{WL_r} (T_c - T_r) \quad (3.16)$$

where  $FL_c$  is the liquid refrigerant mass flowrate from the condenser, which is calculated as follows

$$FL_c = \frac{Q_c}{HFG_c - HFL_c} \quad (3.17)$$

where  $HFG_c$  and  $HFL_c$  are enthalpies of the refrigerant vapour discharging from the compressor and the saturated refrigerant liquid exiting the condenser respectively.

The heat rejected by the refrigerant in the condenser is calculated using the same method as for heat absorbed in the evaporators (i.e. Equation 3.9-3.11 with the subscript "c" replacing "i").

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\* The mass balance through the receiver is

$$\frac{dWL_r}{dt} = FL_c - FL_r \quad (3.16a)$$

The energy balance is

$$\frac{dH_r}{dt} = FL_c HL_c - FL_r HL_r = CP \frac{d(WL_r T_r)}{dt} = CP (FL_c T_c - FL_r T_r) \quad (3.16b)$$

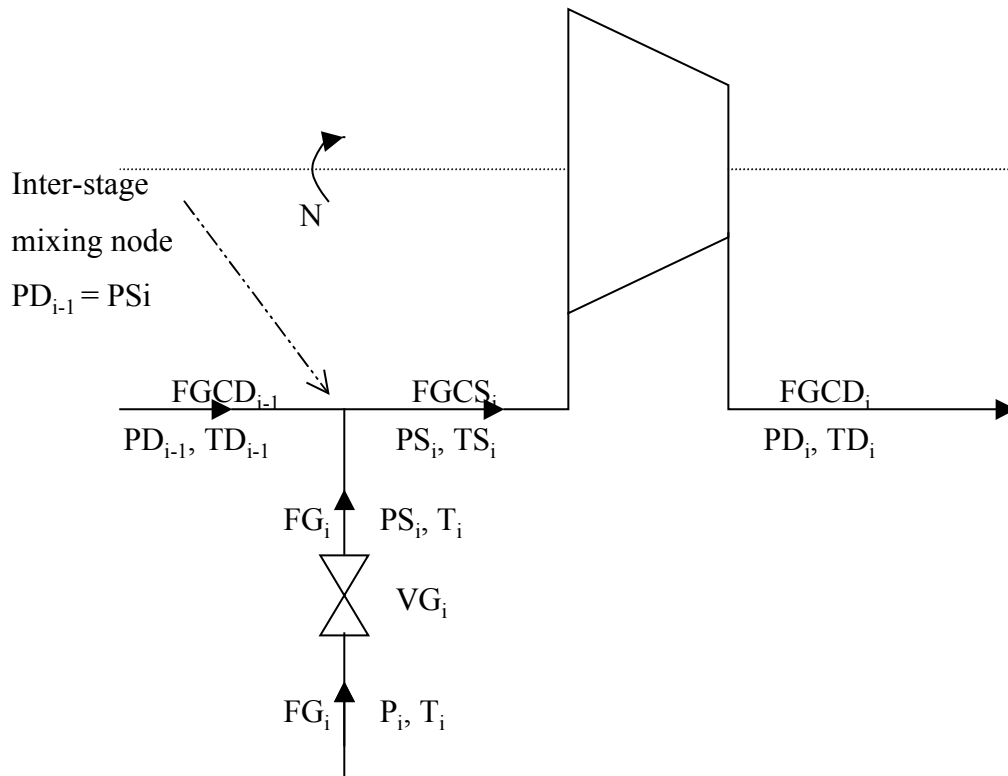
From 3.16b

$$T_r \frac{dWL_r}{dt} + WL_r \frac{dT_r}{dt} = FL_c T_c - FL_r T_r \quad (3.16c)$$

Substituting Equation 3.16a in Equation 3.16c, collecting terms and rearranging

$$\frac{dT_r}{dt} = \frac{FL_c}{WL_r} (T_c - T_r) \quad (3.16)$$

## 3.2.2.3. Compressor



**Figure 3.4: General compressor and inter-mixing node**

The mass flowrate at the  $i^{\text{th}}$  compressor (see Figure 3.4) suction  $FGCS_i$  is calculated from the equation of state.

$$FGCS_i = FGV_i \frac{M_w}{TS_i} \frac{PS_i}{R z_i} \quad (3.18)$$

where  $FGV_i$  is the volumetric flowrate of the refrigerant vapour entering the compressor stage,  $M_w$  is the refrigerant RMM,  $R$  is the gas constant and  $z_i$  is the compressibility factor.  $TS_i$  and  $PS_i$  are the temperature and pressure at the refrigerant vapour inlet.

This flow  $FGSC_i$  consists of the vapour refrigerant from the evaporator  $i$ , combined with the vapour refrigerant discharged from the compressor stage  $i-1$  at the same node pressure  $PD_{i-1}$ . Therefore the vapour refrigerant entering the suction line to the compressor  $i$  must satisfy the refrigerant mass balance performed at the inter-stage

mixing node as follows

$$FGCS_i = FGCD_{i-1} + FG_i \quad (3.19)$$

Equation 3.19 is crucial for the convergence of the inter-stage pressure loop.

The inter-stage pressure is calculated iteratively after substituting Equation 3.18 in Equation 3.19.

A simple energy balance calculates the temperature of  $FGCS_i$  stream  $TS_i$  on the inter-stage mixing node assuming that the temperature of vapour refrigerant  $FG_i$  does not change across the valve  $VG_i$ . This is true for ideal gases. Here we assume the degree of non-ideality is not large, and since the pressure drop through the valve is not large, the variation in this temperature is negligible and this assumption is justified.

$$TS_i = \frac{TD_{i-1} FGCD_{i-1} + T_i FG_i}{FGCD_{i-1} + FG_i} \quad (3.20)$$

It should be noted that  $PS_1$  equals  $P_1$  and  $TS_1$  equals  $T_1$  as there is no mixing node for the vapour refrigerant before the first compressor.

In general, the performance of a centrifugal compressor can be represented through a modification of the "Affinity Laws" (GPSA, 1994) which take the form

$$\frac{FGV_i}{N^q} = f_i(hs_i) \quad (3.21)$$

where  $hs$  is the scaled head defined as

$$hs_i = \frac{h_i}{N^p} \quad (3.22)$$

$N$  is the fractional compressor speed based on normal operation, and  $q$  and  $p$  are

constants. The performance curve  $f_i()$  is independent of speed, but for a compressor,  $p$ ,  $q$  and  $f_i()$  are specific to a particular machine.

The above relations are valid only inside the stable region of the compressor performance curve bounded by the surge condition, and the stonewall condition, also known as the sonic barrier. Stonewall is not usually a problem when compressing air and lighter gases, but it becomes more prevalent as the molecular weight of the gas increases (Evans, 1979).

Similarly, the compressor polytropic efficiency  $\eta_i$  (which is used later) is taken as:

$$\eta_i = f_{2_i}(h_{s_i}) \quad (3.23)$$

The relationship is obtained by fitting the real machine data supplied by the manufacturers.

The compressor head difference is proportional to the temperature rise across it, and is expressed as

$$h_i = k_i \frac{R}{g M_w} (TD_i - TS_i) \quad (3.24)$$

where  $g$  is the gravitational acceleration.

The vapour discharged from the compressor is always in a superheated state. The discharge temperature of a compressor  $TD_i$  is calculated using the relation

$$TD_i = \frac{TS_i}{\left(\frac{PD_i}{PS_i}\right)^{\frac{1}{k_i}}} \quad (3.25)$$

where  $TS_i$  is the suction temperature of the compressor,  $PS_i$  and  $PD_i$  are the suction and discharge pressures respectively, and  $k_i$  is a constant defined as

$$k_i = \frac{n}{n - 1} = \frac{\eta_i \gamma_{av,i}}{\gamma_{av,i} - 1} \quad (3.26)$$

where  $n$  is called the polytropic exponent and  $\gamma_{av}$  is averaged between the suction and discharge conditions as

$$\gamma_{av,i} = 0.5 \left( \frac{CPI_i}{CPI_i - R} + \frac{CPO_i}{CPO_i - R} \right) \quad (3.27)$$

$CPI_i$  and  $CPO_i$  are the stage  $i$  suction and discharge specific heat capacities respectively.

The specific heat capacity  $CP$  is considered to be a function of temperature only

$$CP = C_1 + C_2 T + C_3 T^2 + C_4 T^3 \quad (3.27)$$

where  $C_1$ - $C_4$  are constants.

#### 3.2.2.4. Equilibrium relations

The compressibility factor, liquid specific enthalpy, vapour specific enthalpy and liquid specific volume of the saturated refrigerant are obtained by linearising the corresponding saturation curves at specific pressure values  $LP$ ,  $IP$  and  $HP$ . Usually, the uniform pressure in each section and the smoothness of these curves permit a quite accurate linear approximation inside a defined range. Hence, linear equations are written to calculate these quantities as follows.

The refrigerant compressibility factor

$$z = \phi_1 + \phi_2 T \quad (3.29)$$

The vapour refrigerant specific enthalpy

$$\text{HFG} = \zeta_1 + \zeta_2 T \quad (3.30)$$

The liquid refrigerant specific enthalpy

$$\text{HFL} = \delta_1 + \delta_2 T \quad (3.31)$$

The liquid refrigerant specific volume

$$v_f = \lambda_1 + \lambda_2 T \quad (3.32)$$

The vapour refrigerant specific volume is calculated from the corresponding equation of state

$$v_g = \frac{z_i R T_i}{P_i M_w} \quad (3.33)$$

### 3.3. THE MATHEMATICAL MODEL APPLIED TO THE TWO-STAGE REFRIGERATION SYSTEM

The application of the derived model to the two-stage refrigeration system is straightforward. The following nomenclature changes are made to agree with the nomenclature used in the simulation model (see Chapter 4).

- The two liquid level control valves are denoted V2 and V3 respectively. Their corresponding fractional valve openings are denoted by XV2 and XV3, and the valve flow coefficients are denoted as CV2 and CV3 respectively.
- The vapour control valve is denoted V1. Its corresponding fractional valve opening and the valve flow coefficient are denoted by XV1 and CV1 respectively.
- The liquid refrigerant flowing into the second evaporator FL<sub>3</sub> is coming from the receiver, which is denoted originally as FL<sub>r</sub>.
- The vapour refrigerant flowing into the condenser is denoted as FG<sub>3</sub>
- The pressure in the receiver is denoted as P<sub>3</sub>.

- $WV_c$  is referred to as  $W_4$ ,  $T_r$  as  $T_3$ , and  $T_c$  as  $T_D$ . For other variables in the condenser and the receiver, the subscript 3 replaces the subscript  $c$  or  $r$  generally.

Noting the above changes, the six differential equations in the model become.

In the first evaporator:

$$\frac{dW_1}{dt} = FL_2 - FL_1 - FG_1 \quad (3.34)$$

$$\frac{dH_1}{dt} = (HFL_2 FL_1) - (HFL_1 FL_1) - (HFG_1 FG_1) + Q_1 \quad (3.35)$$

In the second evaporator:

$$\frac{dW_2}{dt} = FL_3 - FL_2 - FG_2 \quad (3.36)$$

$$\frac{dH_2}{dt} = (HFL_3 FL_3) - (HFL_2 FL_2) - (HFG_2 FG_2) + Q_2 \quad (3.37)$$

In the condenser and the receiver:

$$\frac{dW_4}{dt} = FG_3 - FL_4 \quad (3.38)$$

$$\frac{dT_3}{dt} = \frac{FL_4}{W_3} (T_D - T_3) \quad (3.39)$$

### 3.3.1. Compressors Equations

Equations 3.21 and 3.23 for each compressor stage are fitted using the actual data supplied by the manufacturers. For the two compressors used in this model,  $q$  is equal to 1.56 and 1.79 for the first and second stage compressor respectively, and  $p$  is equal to 2.19 and 2.11 for the first and second stage compressor respectively.



In the two-stage model, applying Equation 3.21, the functions  $f_1$  and  $f_2$  were obtained by fitting empirical curves for the specific compressors involved from the manufacturers data (see Figures 3.5 and 3.6)).

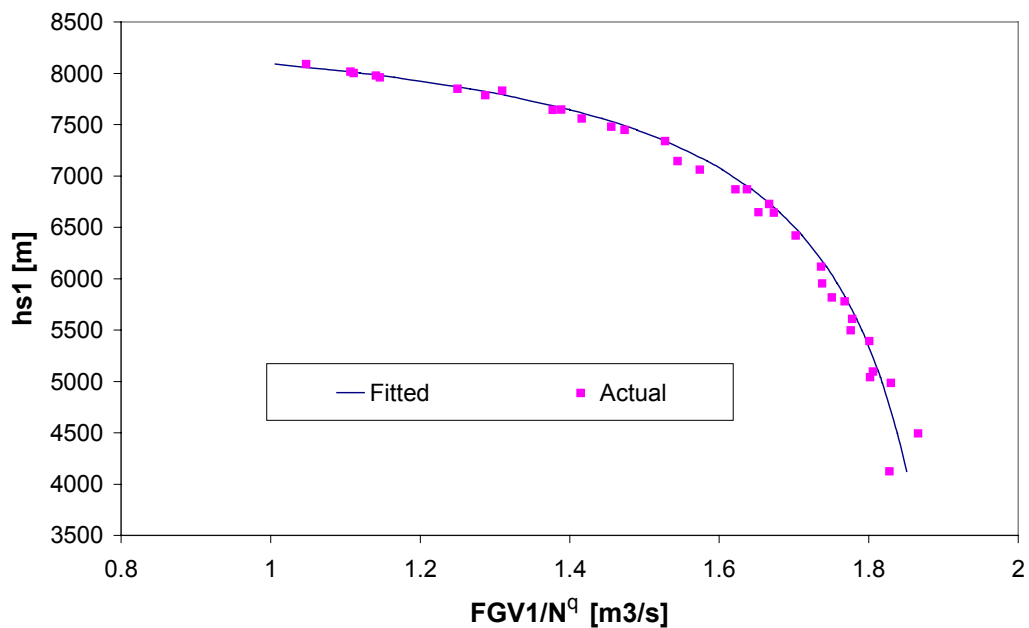
For the first compressor:

$$f_1(h_{s1}) = \frac{FGV_1}{N^q} = \frac{C_{11} h_{s1} - C_{12}}{h_{s1} - C_{13}} \quad (3.40)$$

where  $C_{11}$ ,  $C_{12}$ ,  $C_{13}$  are constants, whose values are found in the Table 3.1.

**Table 3.1: Coefficients for Equation 3.40**

Constant	$C_{11}$ [m <sup>3</sup> /s]	$C_{12}$ [m <sup>4</sup> /s]	$C_{13}$ [m]
Value	1.9928	16791	8756.4



**Figure 3.5: Comparison between actual data and fitted curve for Equation 3.21 for the first compressor**

And for the second compressor

$$f_2(h_{s2}) = \frac{FGV_2}{N^q} = C_{24} + C_{25} \log_{10} \left( \left( \sqrt{TX^2 + 1} \right) - TX \right) \quad (3.41a)$$

where TX is

$$TX = \frac{C_{21}}{\tan \theta} \quad (3.41b)$$

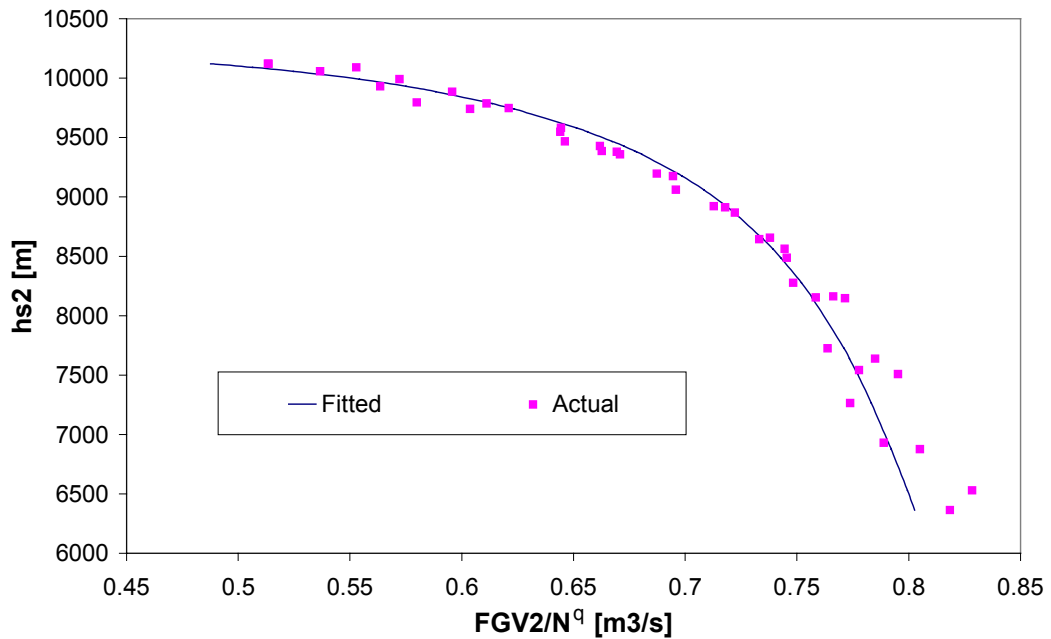
where  $\theta$  (radians) is calculated as

$$\theta = \left( \frac{C_{22} - h_{s2}}{C_{23}} \right) \quad (3.41c)$$

and  $C_{21}, C_{22}, C_{23}, C_{24}, C_{25}$  are constants, whose values are found in the Table 3.2

**Table 3.2: Coefficients for Equation 3.41**

Constant	$C_{21}[-]$	$C_{22} [m]$	$C_{23} [m]$	$C_{24} [m^3/s]$	$C_{25} [m^3/s]$
Value	0.45615	10296	2956.2	0.816051	0.27585



**Figure 3.6: Comparison between actual data and fitted curve for Equation 3.21 for the second compressor**

The stonewall conditions for these compressors are:

- $h_{s1 \max} = 8425$
- $h_{s2 \max} = C_{22}$

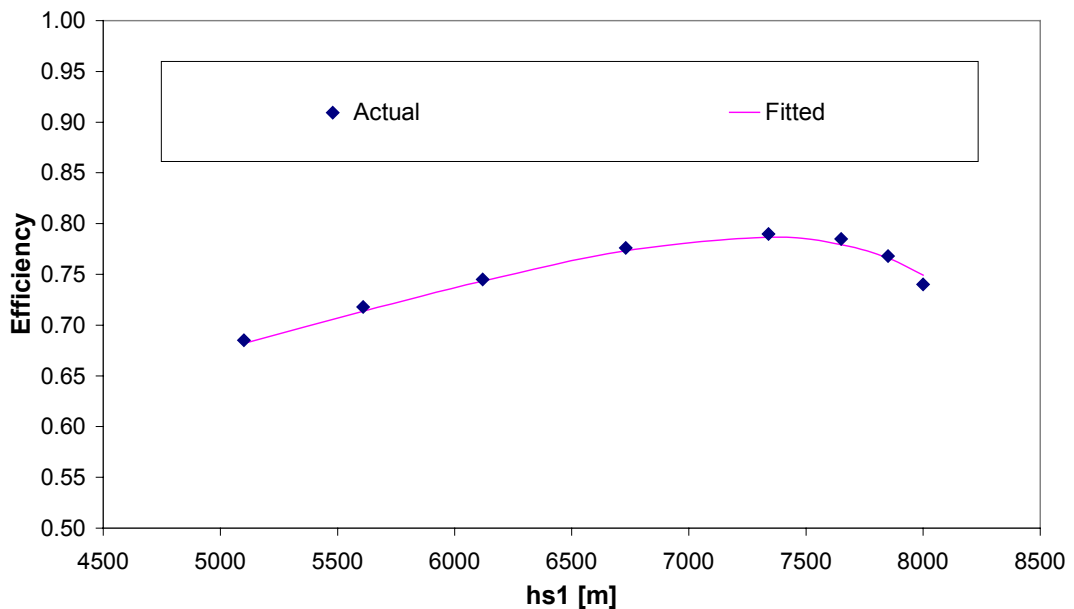
Applying Equation 3.28, efficiency functions following the form of Equation 3.42 were obtained by fitting empirical curves for the specific compressors involved from the manufacturers data (see Figures 3.7 and 3.8)).

$$\eta_i = e_1 h_{si} + e_2 - 10^{(e_3 h_{si} - e_4)} \quad (3.42)$$

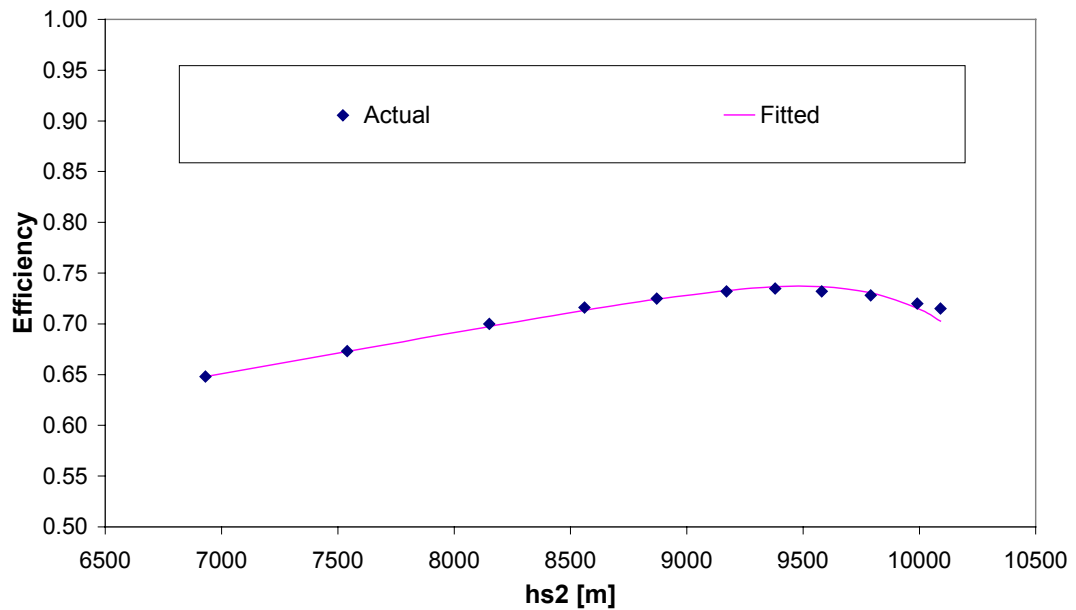
where  $h_{si}$  is the scaled head, and  $e_1, e_2, e_3, e_4$  are constants, and are different for each compressor. The constants are found in the Table 3.3 below.

**Table 3.3: Coefficients for Equation 3.42**

Compressor no.	$e_1$ [ $m^{-1}$ ]	$E_2$ [-]	$e_3$ [ $m^{-1}$ ]	$e_4$ [-]
1	$6.47 \times 10^{-5}$	0.353	$7.101 \times 10^{-4}$	6.5963
2	$4.098 \times 10^{-5}$	0.364	$1.1218 \times 10^{-3}$	12.445



**Figure 3.7: Comparison between actual data and fitted curve for Equation 3.23 for the first compressor**



**Figure 3.8:** Comparison between actual data and fitted curve for Equation 3.23 for the second compressor

### 3.3.2. Propylene Characteristics:

In the two-stage model, the refrigerant used is propylene. The following data are therefore used.

For propylene A, B and C constants for Antoine Equation (Equation 3.8) are given in Table 3.4.

**Table 3.4:** Antoine Equation coefficients for propylene. Here the temperature is in K and the pressure in bar

A	B	C
9.0825	1807.53	26.15

The specific heat capacity is expressed as

$$C_P = C_1 + C_2 T [ C_3 + C_4 T ( C_5 + C_6 T ) ] \quad (3.43)$$

where  $C_1$ - $C_6$  are constants. Their values are given in Table 3.5.

**Table 3.5: Constants for the specific heat capacity relationship (in J/(kg.K))**

$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
3.707	0.01	23.439	0.001	-11.594	$2.2033 \times 10^{-3}$

Linearised equations of the form of Equations 3.29-3.32 are used to calculate the compressibility factor, liquid specific enthalpy, vapour specific enthalpy and liquid specific volume of the saturated refrigerant. The values of the constants in the above equations are given in Table 3.6.

**Table 3.6: Coefficients for the equilibrium relations**

Equation	Constant	LP	IP	HP
29	$\phi_1$ [-]	0.995	0.566	$2.97 \times 10^{-3}$
	$\phi_2$ [ $K^{-1}$ ]	0	$1.26 \times 10^{-3}$	-0.194
30	$\zeta_1$ [J/kg]	747.441	663.375	474.0188
	$\zeta_2$ [J/(kg.K)]	1.36908	1.63599	2.09759
31	$\delta_1$ [J/kg]	7.16036	7.16036	95.5601
	$\delta_2$ [J/(kg.K)]	2.63768	2.63768	2.32367
32	$\lambda_1$ [ $m^3/kg$ ]	$8.05418 \times 10^{-4}$	$5.91479 \times 10^{-4}$	$7.493 \times 10^{-6}$
	$\lambda_2$ [ $m^3/(kg.K)$ ]	$3.934 \times 10^{-6}$	$4.777 \times 10^{-6}$	$-2.00589 \times 10^{-4}$

### 3.3.3. Controllers

The dynamics of the refrigeration process are considered to be slow relative to the dynamics of the measuring sensors/transmitters and the final control elements, which are much faster and hence are neglected. The same applies to the transport lag in the process pipe-work.

The initial control strategy considered involves two levels and three pressure control loops as explained earlier. It is assumed that each loop acts as a single input single output SISO proportional-integral PI controller.

The PI action for the loop  $j$  is described by the equation below

$$u_j = K_j \left( \epsilon_j + \frac{\int \epsilon_j dt}{TI_j} \right) \quad (3.44)$$

where  $u_j$  is the controller action,  $K_j$  is the controller gain,  $TI_j$  is the integral time action, and  $\epsilon_j$  is the error of the measured (controlled) variable and is defined as

$$\epsilon_j = y_{spj} - y_j \quad (3.45)$$

where  $y_{spj}$  is the set point of controlled variable and  $y_j$  is its measured value.