Systematic design of advanced control structures

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Norwegian University of Science and Technology

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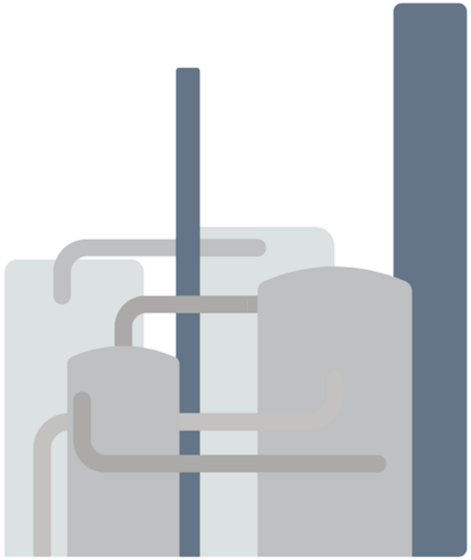
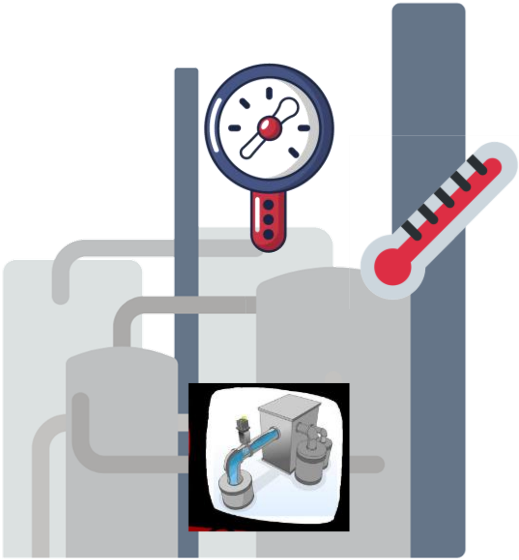
Motivation and scope



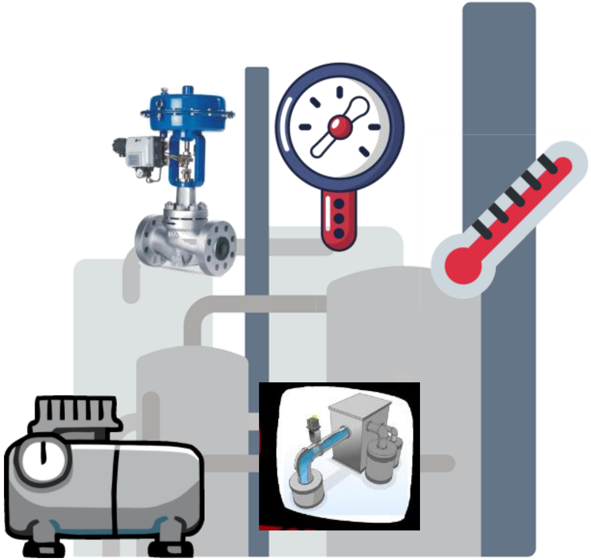
Motivation and scope



CV: controlled variable (output, *y*)

- Temperature
- Pressure
- Concentration

Motivation and scope



CV: controlled variable (output, *y*)

- Temperature
- Pressure
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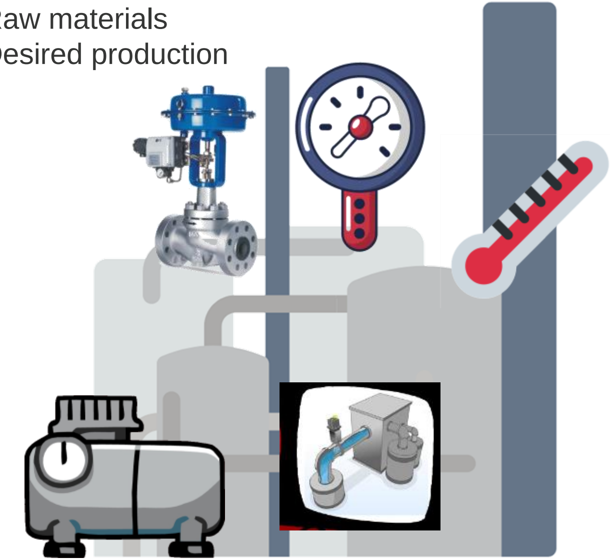
MV: manipulated variable (input, *u*)

- Valve opening
- Compressor rotational speed

Motivation and scope

DV: disturbance variable (*d*)

- Ambient temperature
- Raw materials
- Desired production



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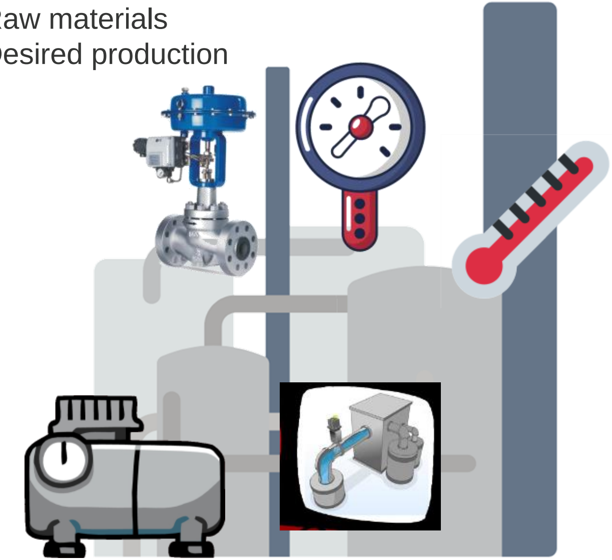

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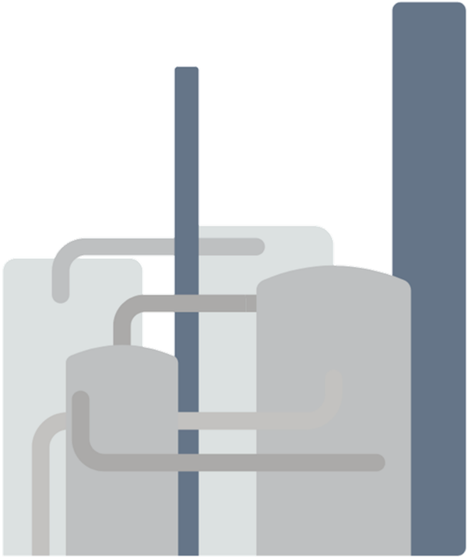
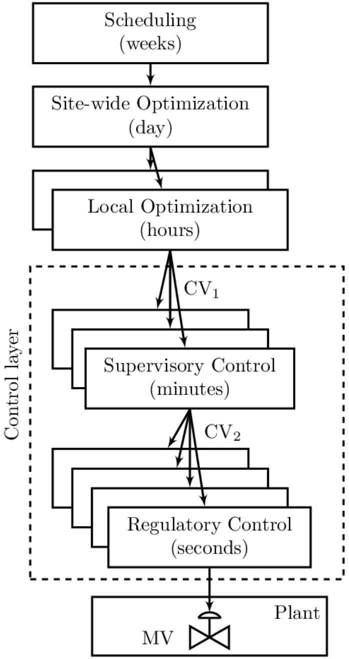
Motivation and scope





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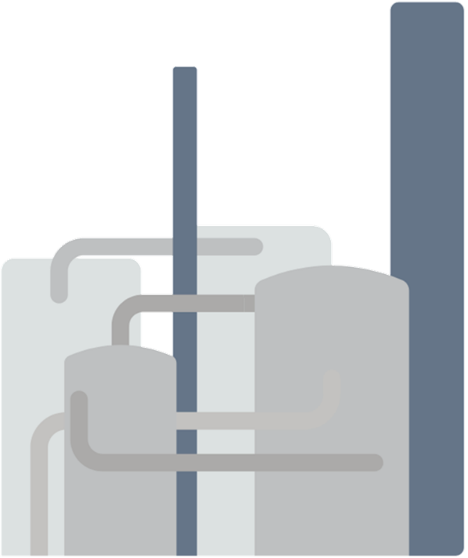
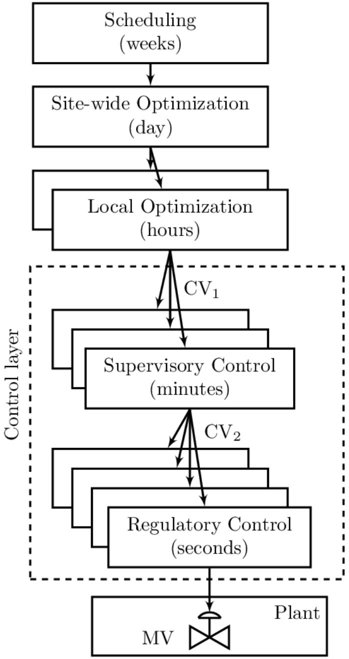




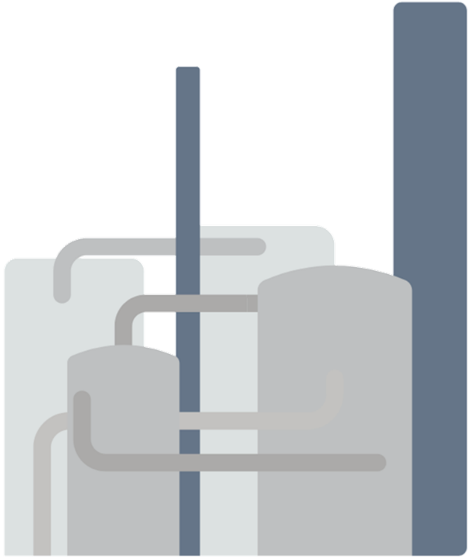
Top-down analysis:

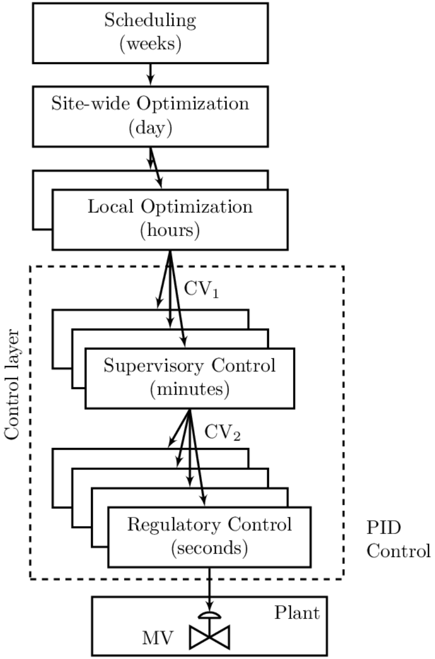
S1-S4: Identify steady-state optimal operation

Bottom-up analysis:

S5-S7: Design control structure

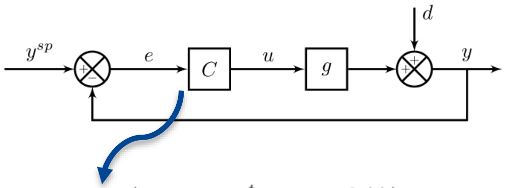
Motivation and scope





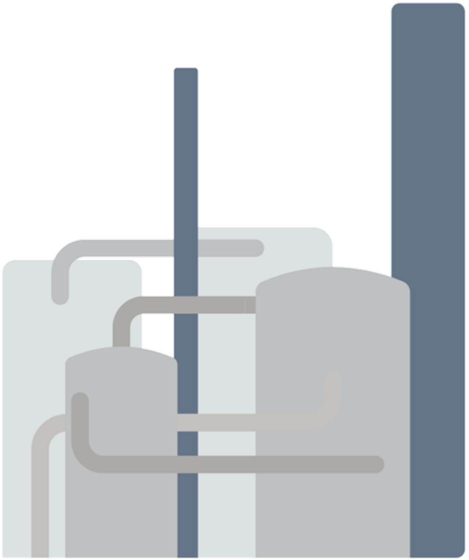
Bottom-up analysis:

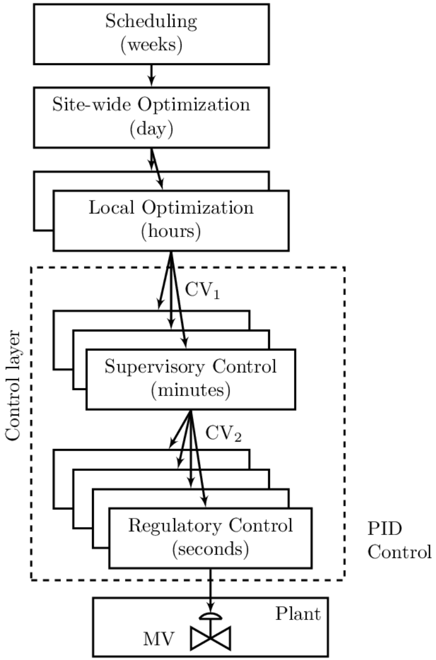
S5: regulatory control layer



$$u(t) = u^0 + K_C \left(e(t) + \frac{1}{\tau_I} \int_0^t e(t) + \tau_d \frac{de(t)}{dt} \right)$$

Motivation and scope



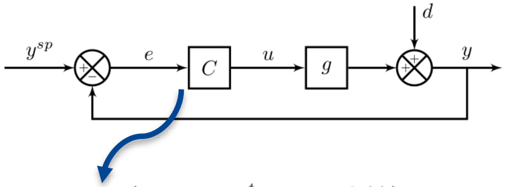


Bottom-up analysis:

S5: regulatory control layer

S6: supervisory control layer

S7: online optimization layer



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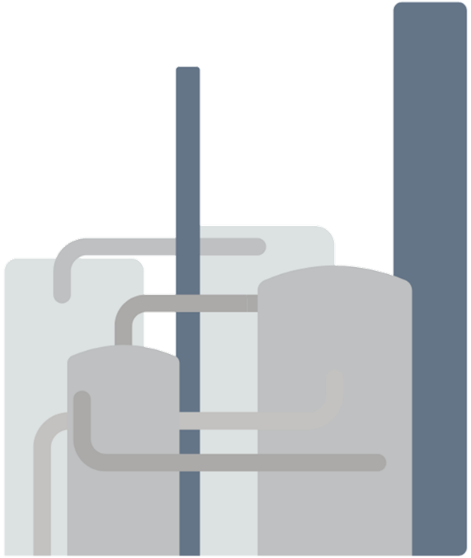

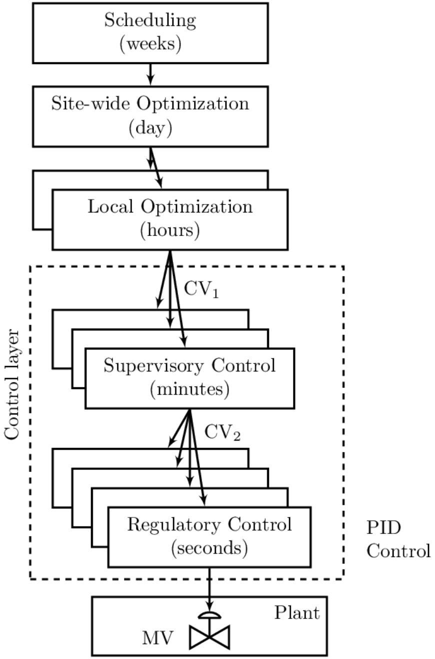

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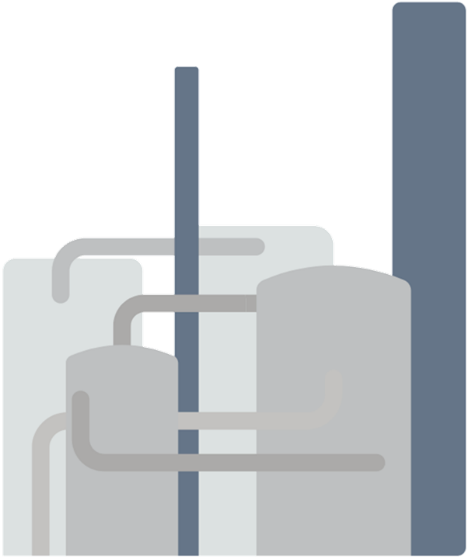
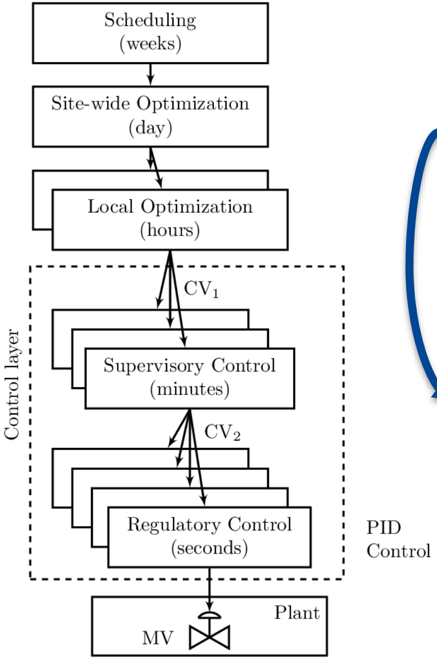
Motivation and scope





Bottom-up analysis:

S5: regulatory control layer

S6: supervisory control layer

S7: online optimization layer

Keeps operation
in the right
active constraint region

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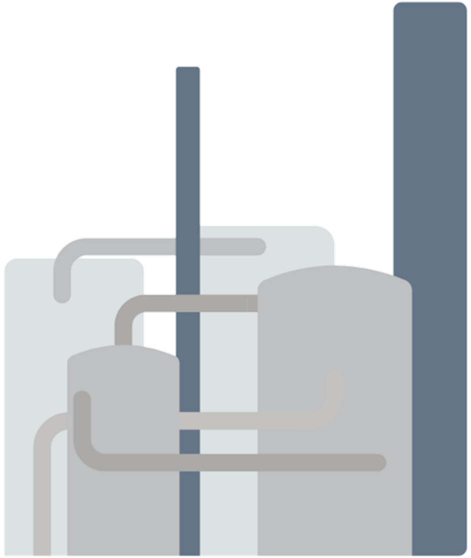
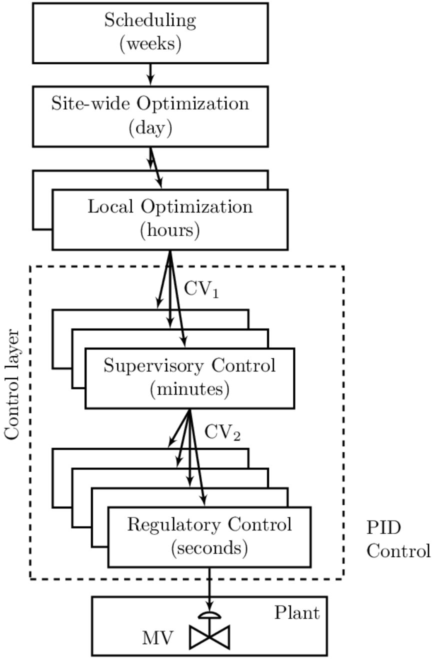
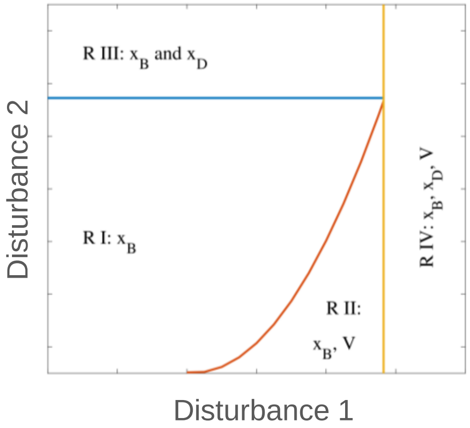
S6: supervisory control layer





Constraint region

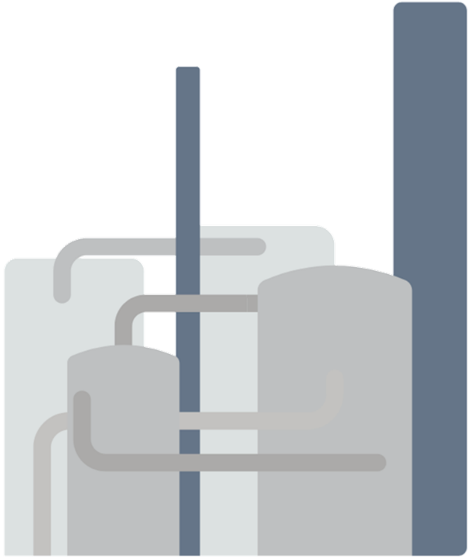
«region in the disturbance space defined by which constraints are active within it»

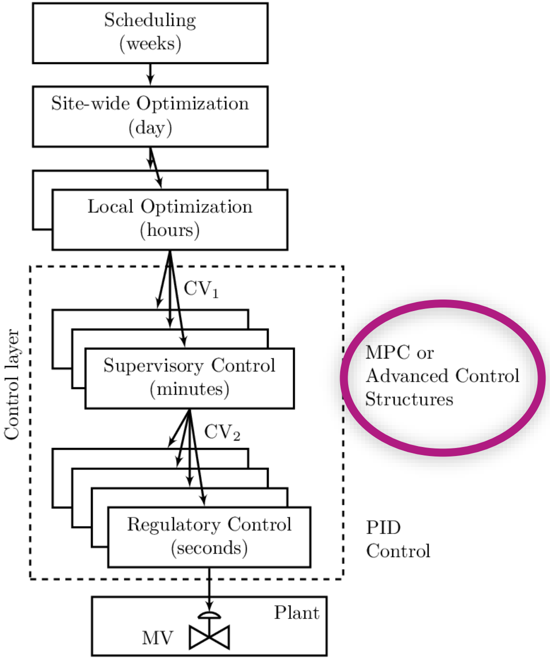


Jacobsen and Skogestad (2011) Active constraint regions for optimal operation of chemical processes. Industrial & Engineering Chemistry Research.

Motivation and scope

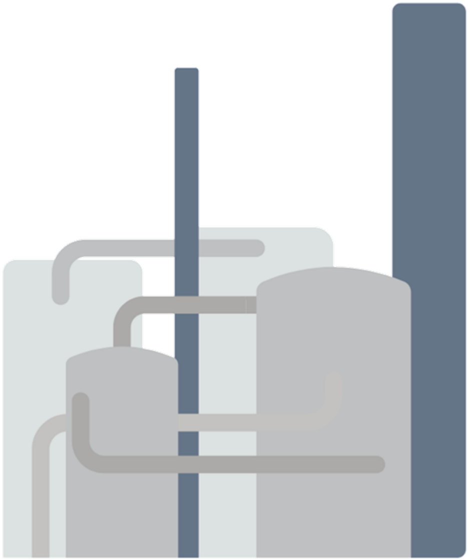
S6: supervisory control layer

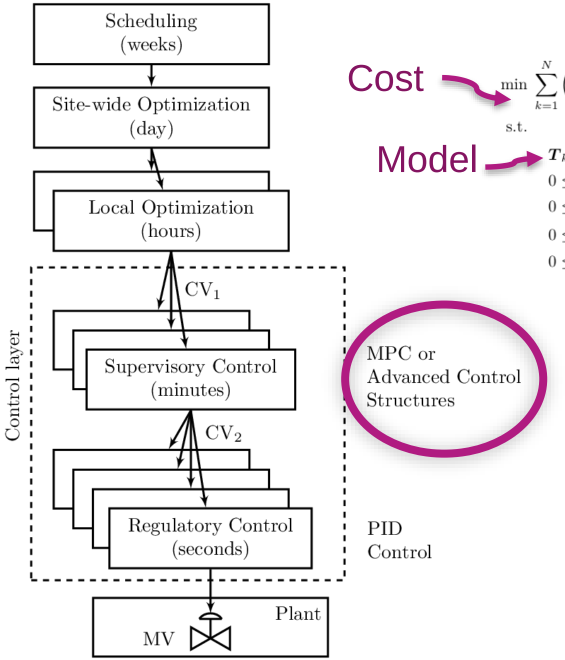




Motivation and scope

S6: supervisory control layer





Model predictive control

Cost

$$\min \sum_{k=1}^{N} \left(\omega_1 \left\|(T_{H_k} - T_H^{sp})\right\|^2 + \omega_2 \left\|(F_{H_k}^{max} - F_{H_k})\right\|^2 \right)$$

s.t.

Model

$$\left.\begin{aligned} &\boldsymbol{T}_{k,i} = f(T_{H_k,i}, T_{H_k,i-1}, T_{C_k,i}, T_{C_k,i+1}, F_{H_k}, F_{C_k}) \\ &0 \leq F_{H_k} \leq F_H^{max} \\ &0 \leq F_{C_k} \leq F_C^{max} \end{aligned}\right\} \quad \forall k \in \{1, \ldots, N\}$$

$$\left.\begin{aligned} &0 \leq \Delta F_{H_k} \leq 0.1 F_H^{max} \\ &0 \leq \Delta F_{C_k} \leq 0.1 F_C^{max} \end{aligned}\right\} \quad \forall k \in \{1, \ldots, N-1\}$$

Constraints

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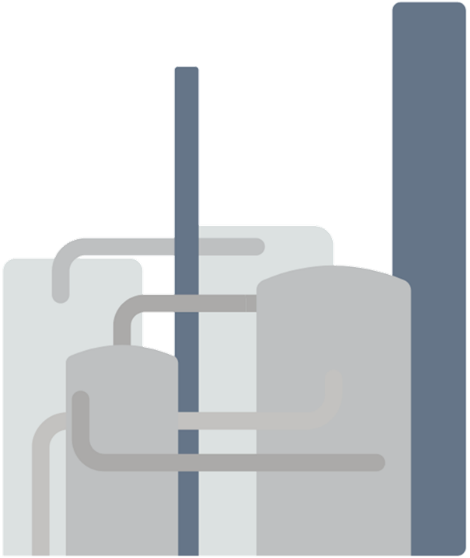
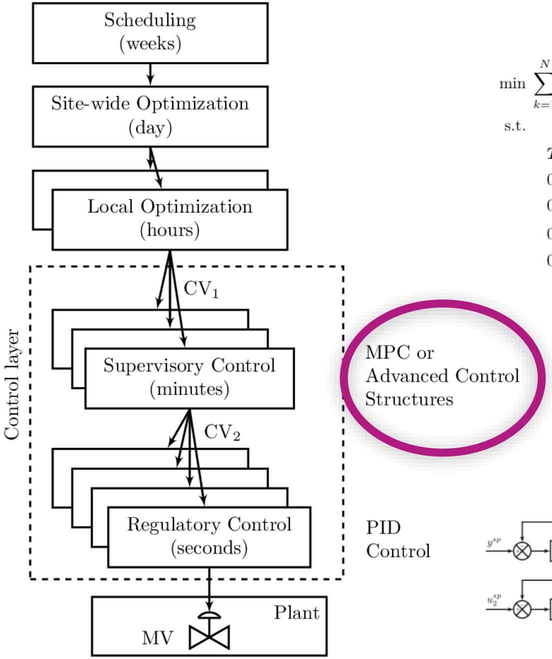
Motivation and scope





S6: supervisory control layer

Model predictive control

$$\min \sum_{k=1}^{N} \left(\omega_1 \left\|(T_{H_k} - T_H^{sp})\right\|^2 + \omega_2 \left\|(F_{H_k}^{max} - F_{H_k})\right\|^2 \right)$$

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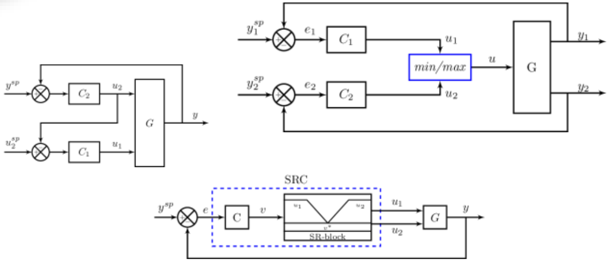
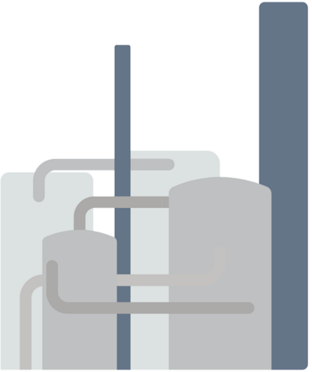
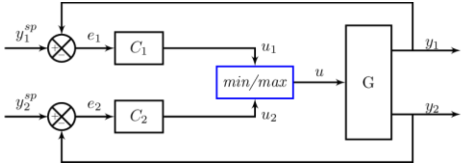
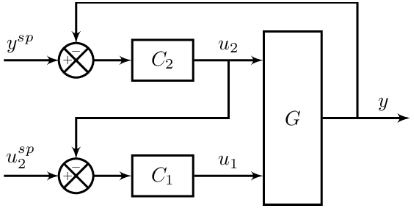
Advanced control structures

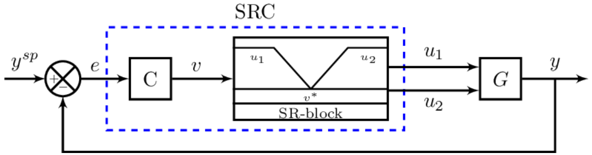


Active constraint switching with classical advanced control structures









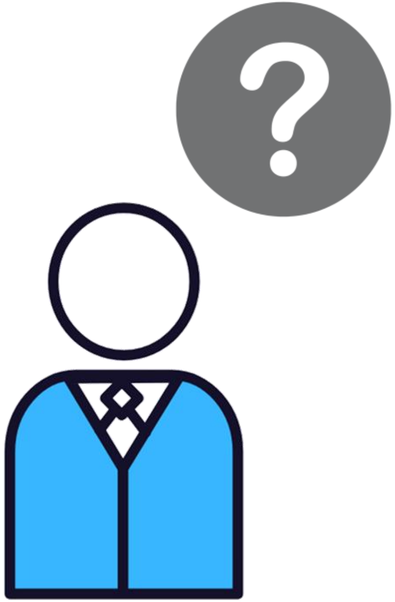


Figure taken from www.transmittershop.com/blog/causes-solutions-annoying-noise-control-valves

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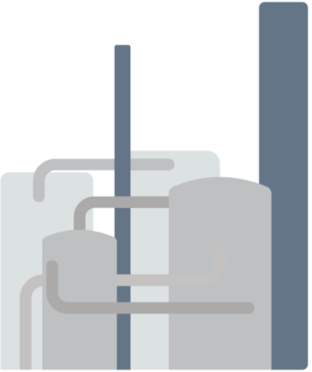
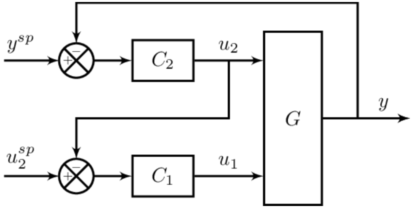
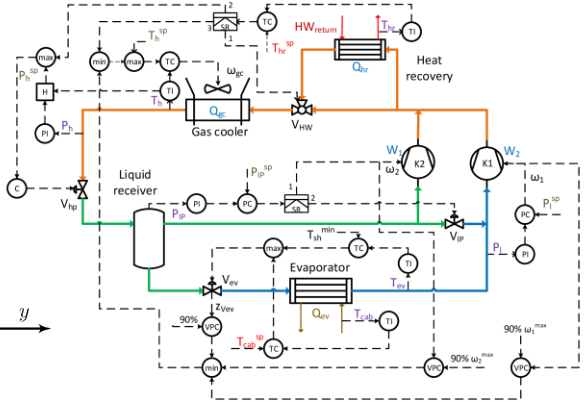
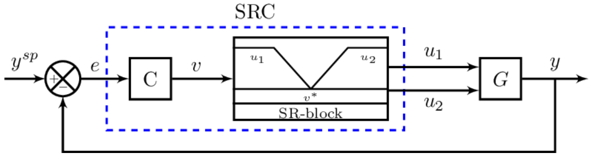
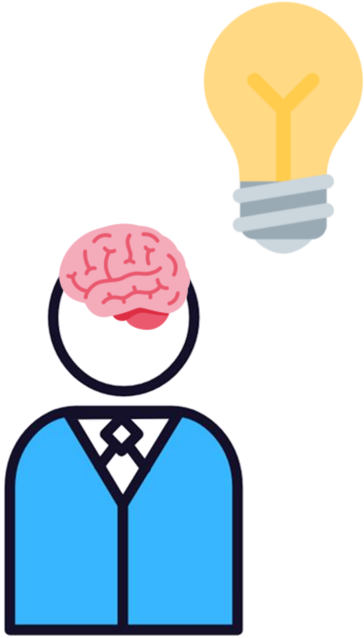
Active constraint switching with classical advanced control structures











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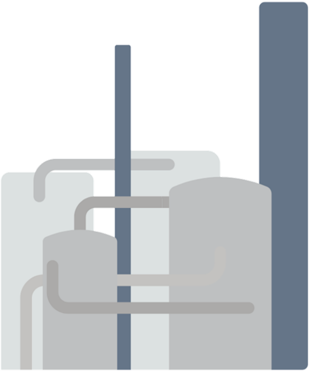

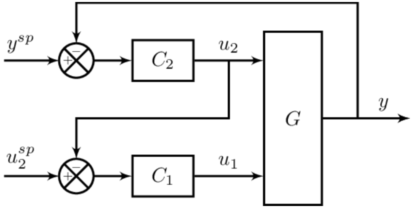

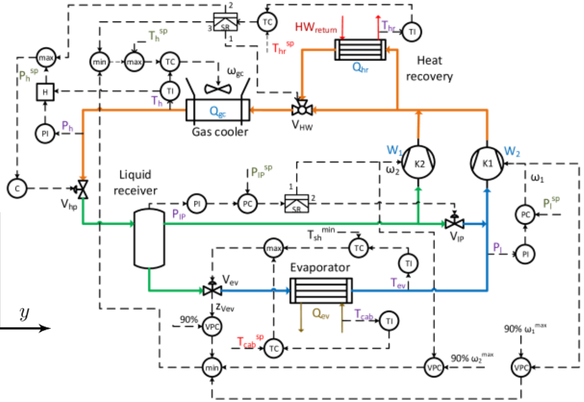

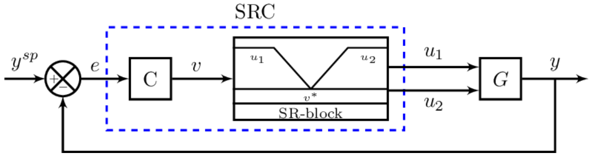

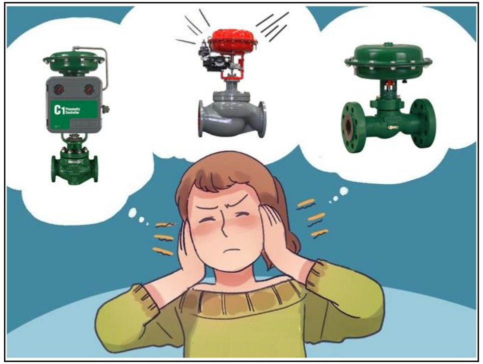

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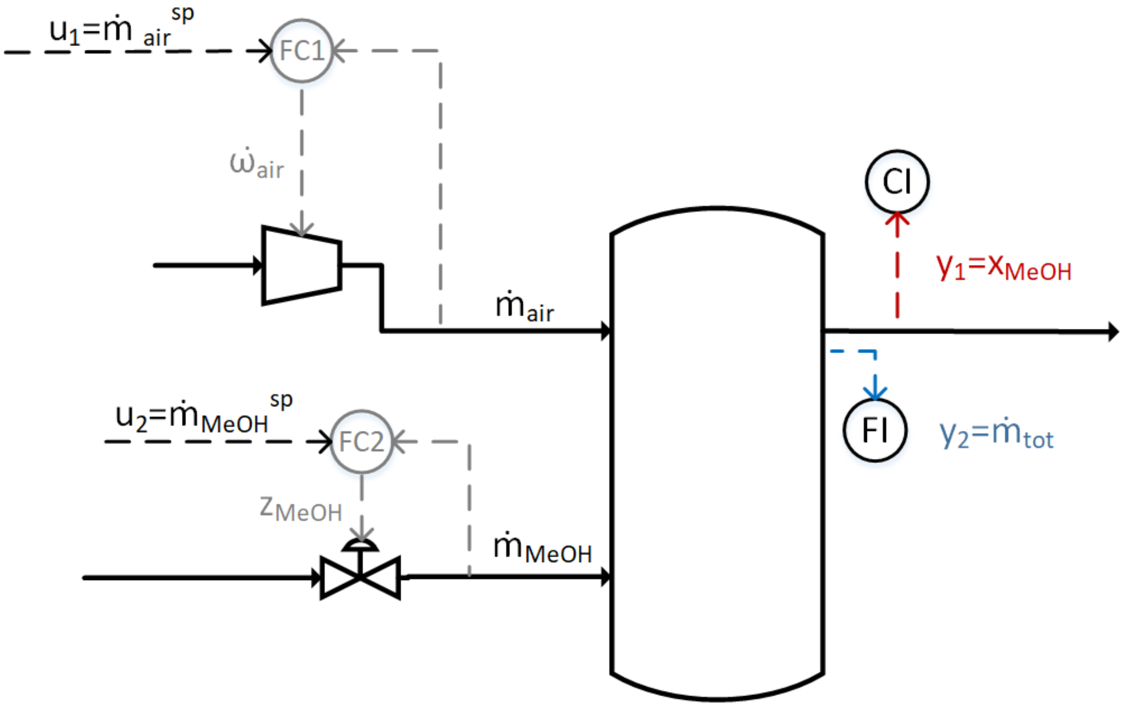
Design procedure for active constraint switching with classical advanced control structures

- A1
  - Define control objectives, CV constraints and MV constraints
- A2
  - Organize constraints in priority list
- A3
  - Identify possible and relevant active constraint switches
- A4
  - Design control structure for optimal operation
- A5
  - Design control structure to handle active constraint switches

Design procedure for active constraint switching

Case study:
Mixing of
air and MeOH

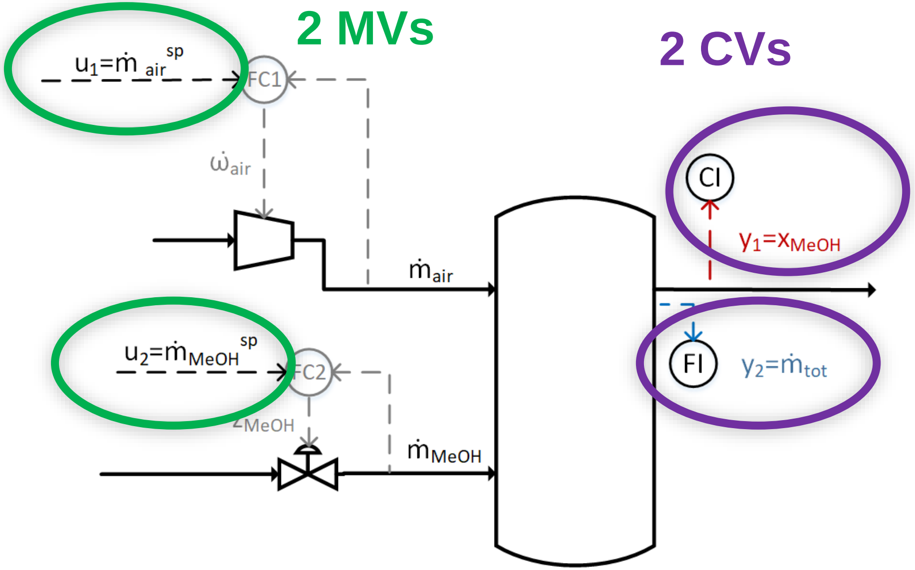


Design procedure for active constraint switching

Step A1: Define control objectives, CV constraints and MV constraints

Design procedure for active constraint switching

Step A1: Define control objectives, CV constraints and MV constraints



Design procedure for active constraint switching

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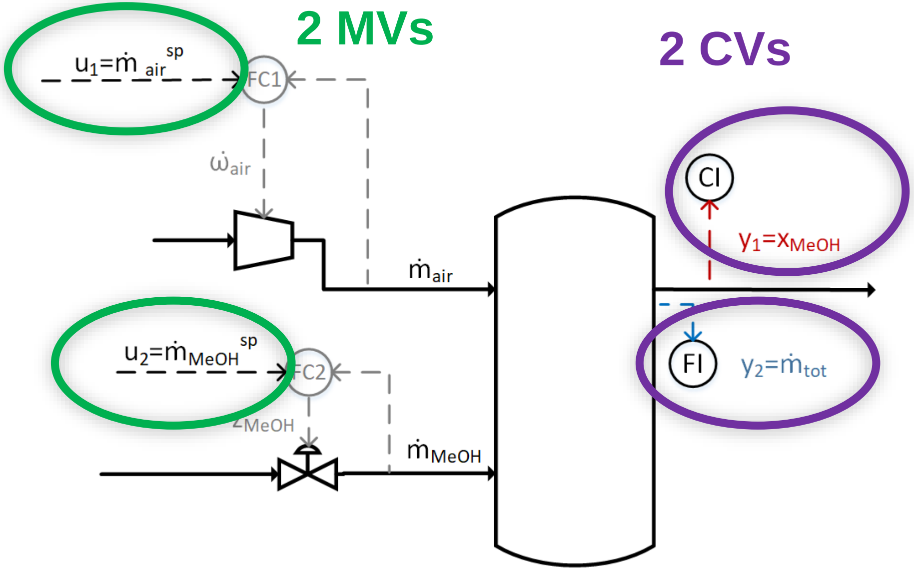
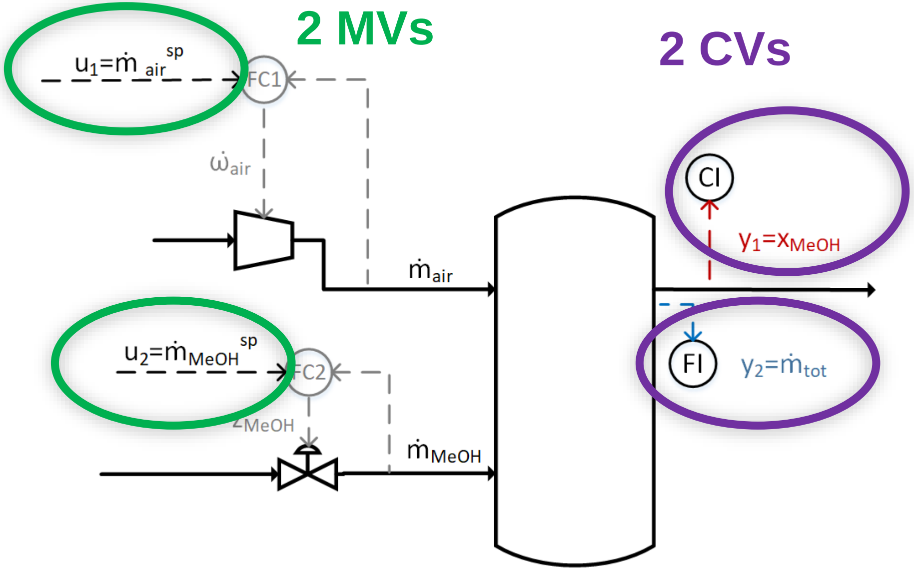


Control objectives:

- Keep $y_1 = x_{MeOH} = 0.10$ ← ideal
- Keep $y_1 = x_{MeOH} > 0.08$
- Control $y_2 = m_{tot}$ ← ideal

Design procedure for active constraint switching

Step A1: Define control objectives, CV constraints and MV constraints



Control objectives:

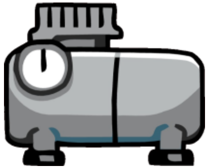
- Keep $y_1 = x_{MeOH} = 0.10$
- Keep $y_1 = x_{MeOH} > 0.08$
- Control $y_2 = m_{tot}$

Variable	Units	Maximum	Nominal
$y_1 = x_{MeOH}$	kmol/kmol	0.10	0.10
$y_2 = \dot{m}_{tot}$	kg/h	-	26860
$u_1 = \dot{m}_{air}$	kg/h	25800	23920
$u_2 = \dot{m}_{MeOH}$	kg/h	-	2940

u_1 is has a maximum value

Design procedure for active constraint switching

Step A2: Organize constraints in priority list



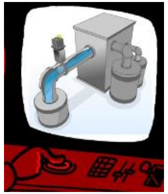


Figure from www.indelac.com/blog/control-valves-vs.-regulators-in-control-applications

Design procedure for active constraint switching

Step A2: Organize constraints in priority list

(P1) Physical MV inequality constraints	• Constraint on air flow (u_1) • Constraint on MeOH flow (u_2)	$\dot{m}_{air}^{min} \leq \dot{m}_{air} \leq \dot{m}_{air}^{max}$ $\dot{m}_{MeOH}^{min} \leq \dot{m}_{MeOH} \leq \dot{m}_{MeOH}^{max}$
(P2) Critical CV inequality constraints	• Constraint (max and min) on x_{MeOH} (y_1)	$x_{MeOH}^{min} \leq x_{MeOH} \leq x_{MeOH}^{max}$
(P3) Less critical CV and MV constraints	• Setpoint on x_{MeOH} (y_1)	$x_{MeOH} = x_{MeOH}^{sp}$
(P4) Desired throughput	• Setpoint on m_{tot} (y_2)	$\dot{m}_{tot} = \dot{m}_{tot}^{sp}$
(P5) Self-optimizing variables	• No unconstrained degrees of freedom	

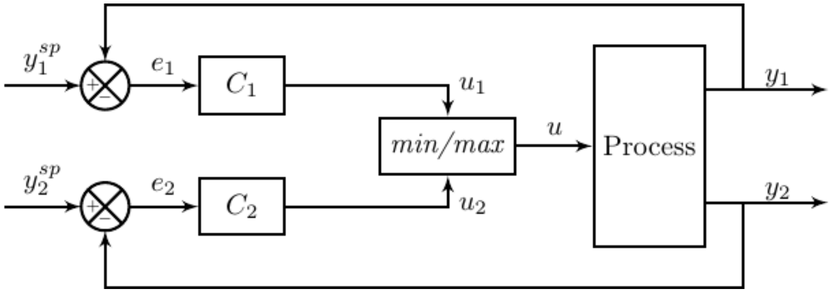
Design procedure for active constraint switching

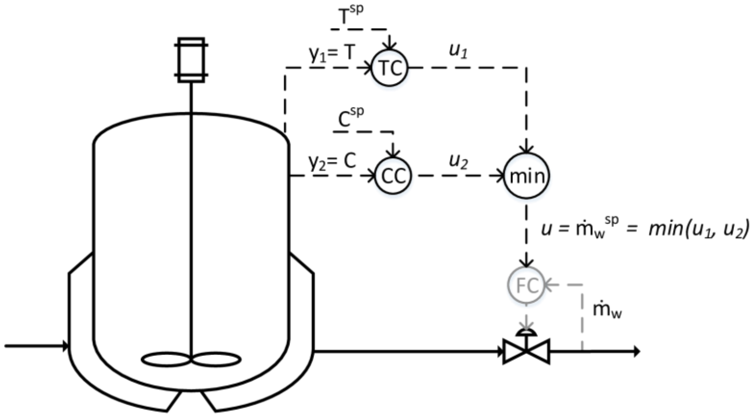
Step A3: Identify possible and relevant active constraint switches

Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

- **Case 1: CV to CV constraint switching**
  One MV switching between two alternative CVs.





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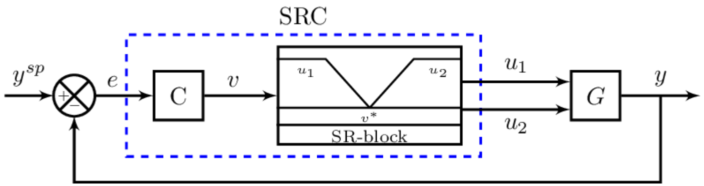
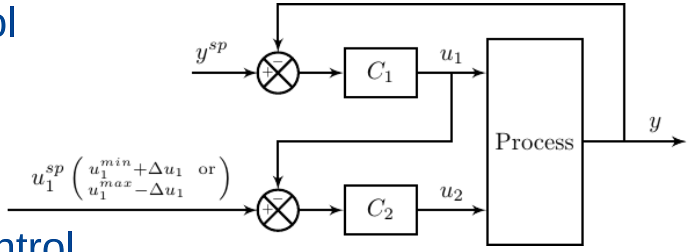
Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

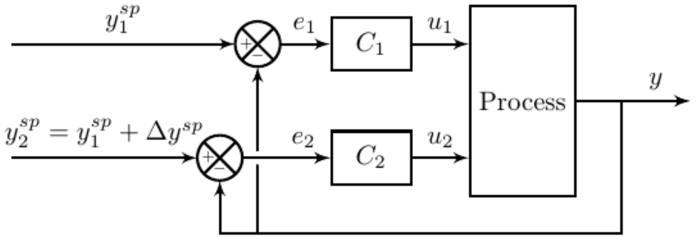
- **Case 2: MV to MV constraint switching**
  More than one MV for one CV.



Split range control



Valve position control

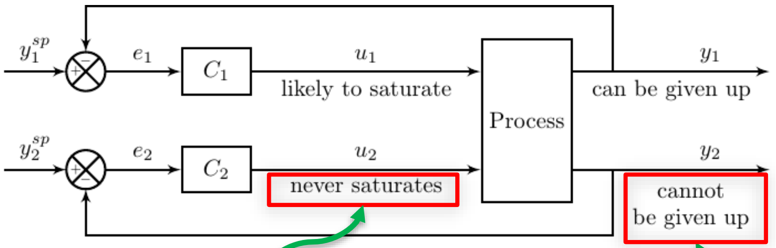


Different controllers with different setpoints

Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

- **Case 3: MV to CV constraint switching**
  MV controlling a CV that may saturate; no extra MVs



MV that does not saturate

High priority CV: always controlled

Input saturation pairing rule

«an MV that is likely to saturate at steady-state should be paired with a CV that can be given up»

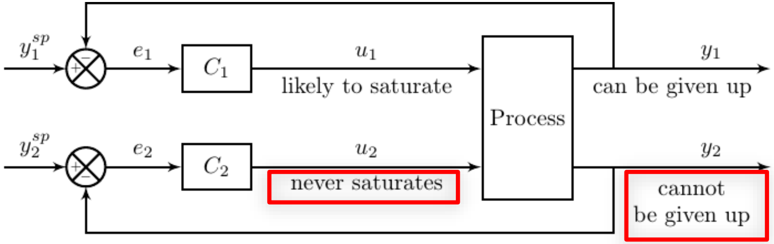
Low priority CV

NTNU

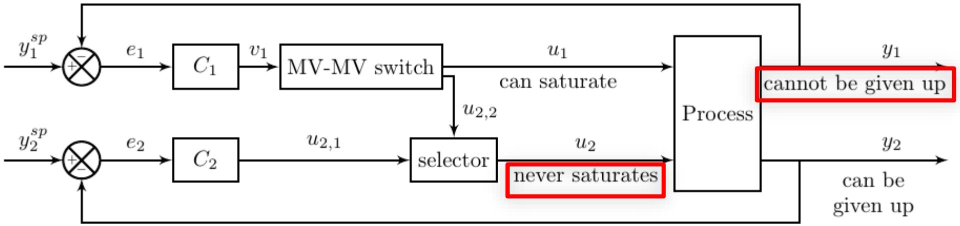
Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

- **Case 3: MV to CV constraint switching**
  MV controlling a CV that may saturate; no extra MVs



Following input saturation pairing rule



NOT following input saturation pairing rule

Design procedure for active constraint switching

Step A3: Identify possible and relevant active constraint switches

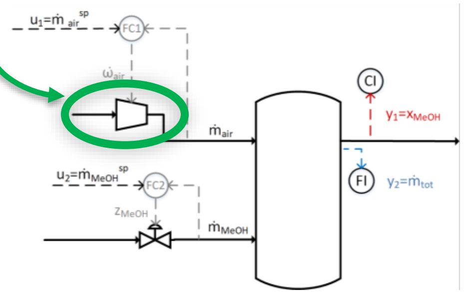
- At nominal operation point all constraints are satisfied
- **Constraint switch:**
  - Reach maximum air flow (u_1)

    ⬇

  - Lose a degree of freedom (**case 3**)
    - Must give up controlling the constraint with the lowest priority (desired throughput)

Variable	Units	Maximum	Nominal
$y_1 = x_{MeOH}$	kmol/kmol	0.10	0.10
$y_2 = \dot{m}_{tot}$	kg/h		26860
$u_1 = \dot{m}_{air}$	kg/h	25800	23920
$u_2 = \dot{m}_{MeOH}$	kg/h		2940

$u_1 = \dot{m}_{air}^{max}$



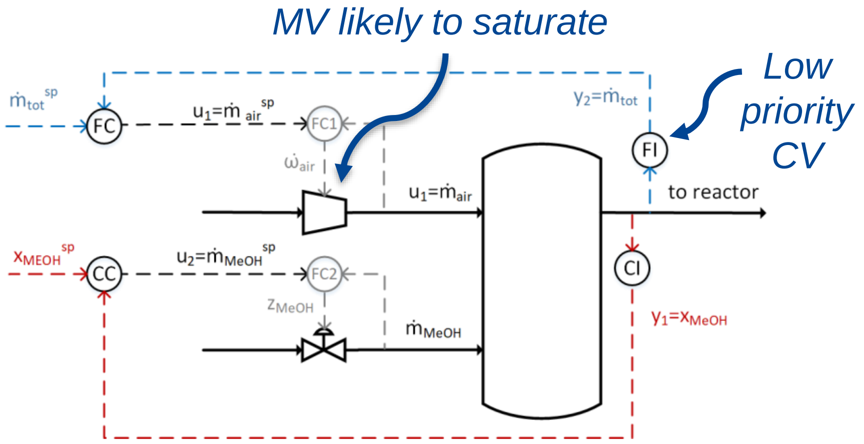
Design procedure for active constraint switching

Step A4: Design control structure for optimal operation

Design procedure for active constraint switching

Step A4: Design control structure for optimal operation

Case A



Following input saturation pairing rule

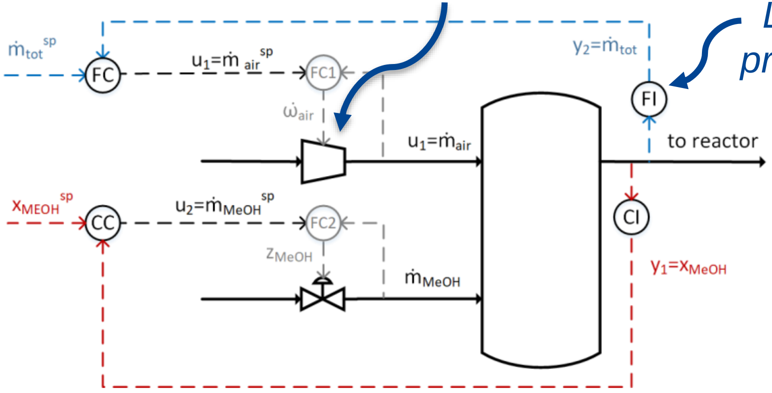
Design procedure for active constraint switching

Step A4: Design control structure for optimal operation

Case A

MV likely to saturate

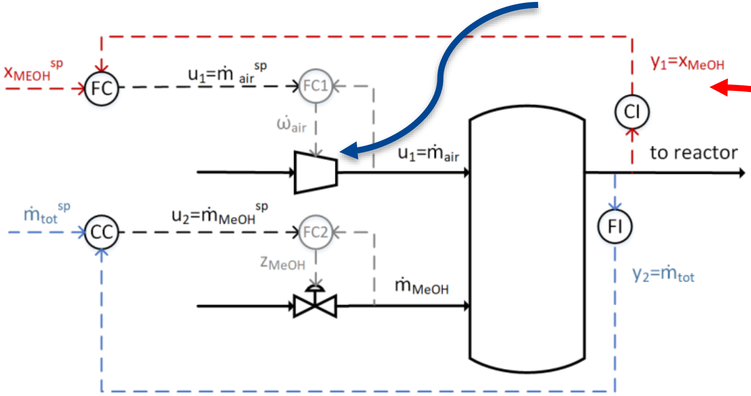
Low priority CV



Following input saturation pairing rule

Case B

MV likely to saturate



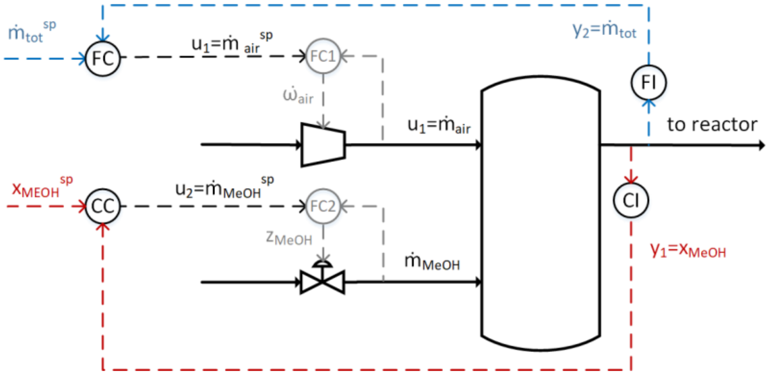
High priority CV not always controlled

NOT following input saturation pairing rule

Design procedure for active constraint switching

Step A4: Design control structure for optimal operation

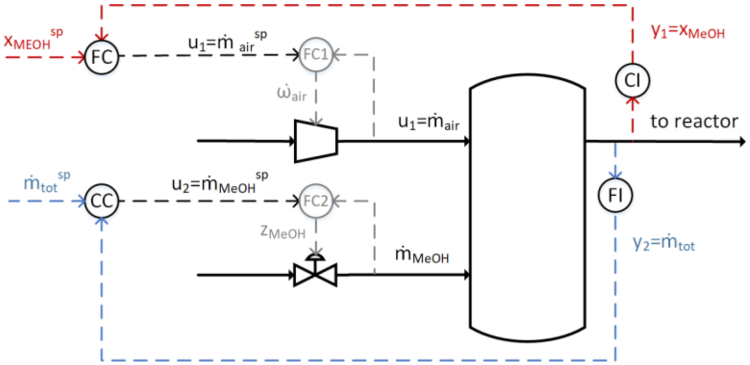
Case A



Following input saturation pairing rule

Case B

Needs MV to CV switching



NOT following input saturation pairing rule

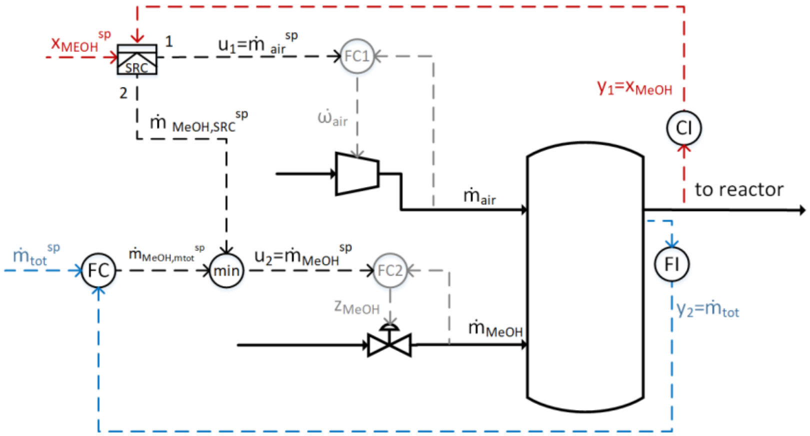
Design procedure for active constraint switching

Step A5: Design control structure to handle active constraint switches

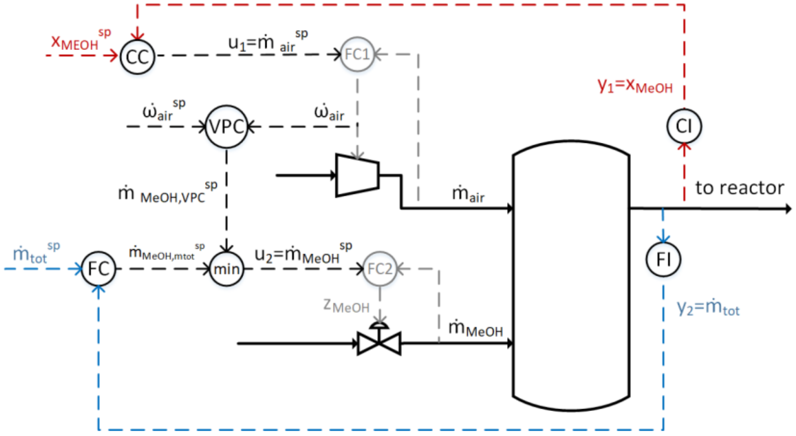
Design procedure for active constraint switching

Step A5: Design control structure to handle active constraint switches

Case B-SRC
Split range control+selector

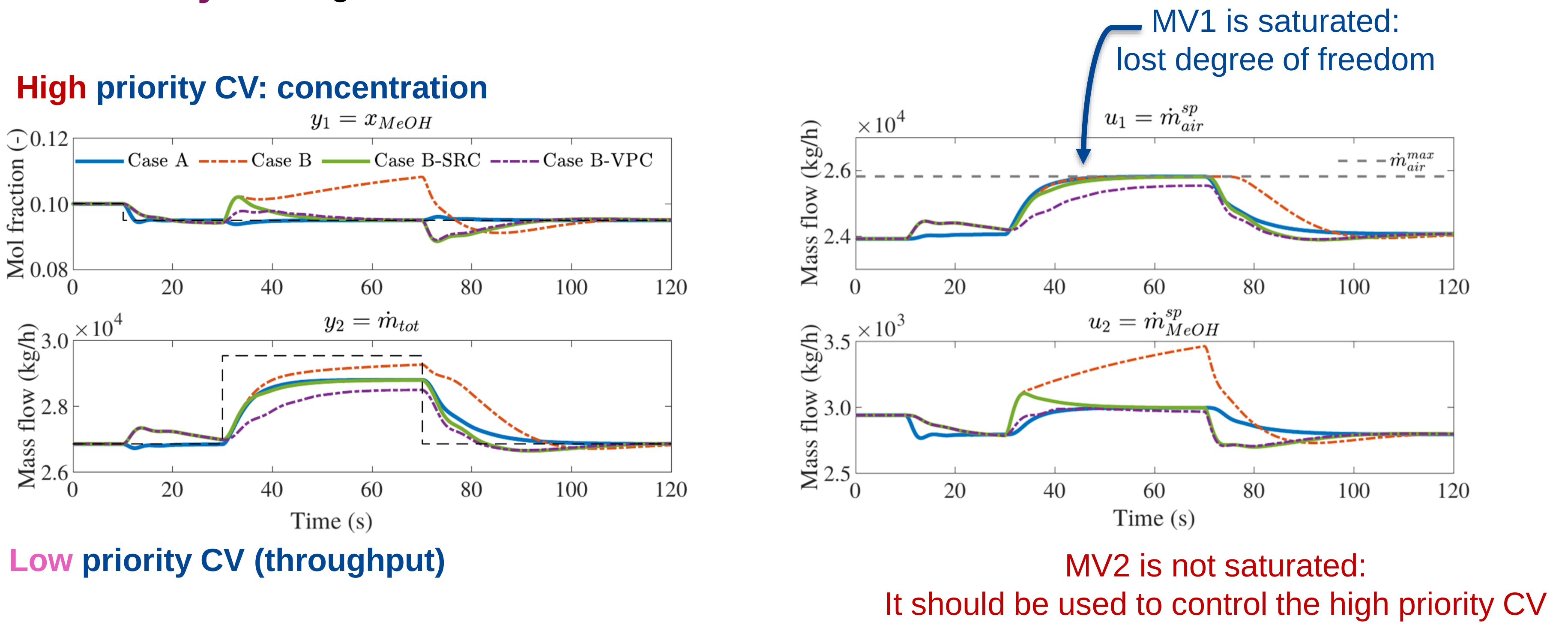


Case B-VPC
Valve position control + selector



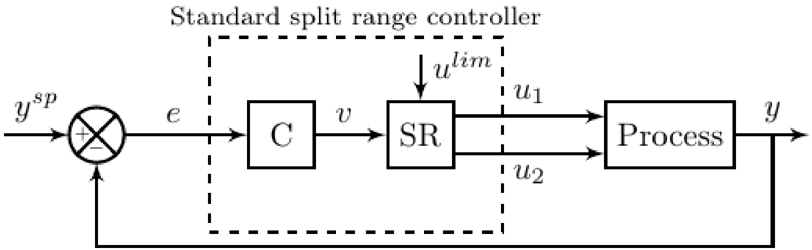
Design procedure for active constraint switching

Case study: Mixing of air and MeOH

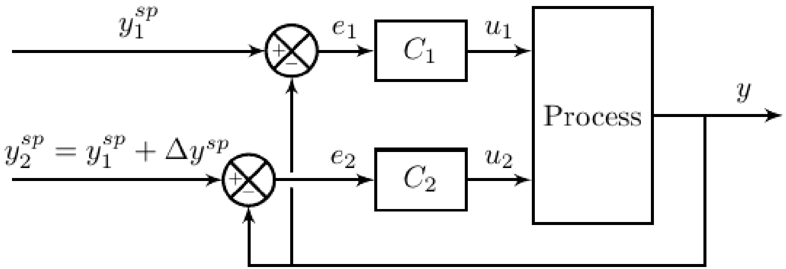


MV to MV constraint switching

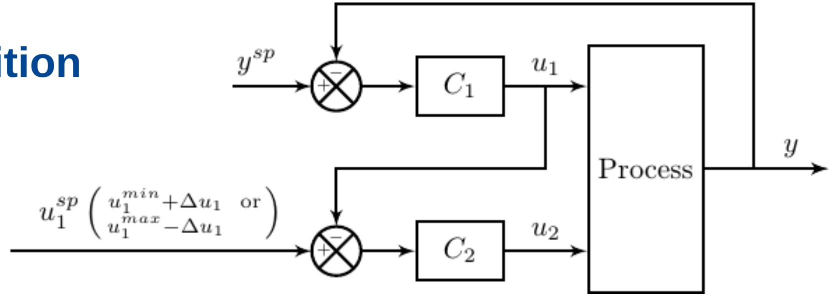
Split range control



Different controllers with different setpoints



Valve position control



NTNU

Classical split range control

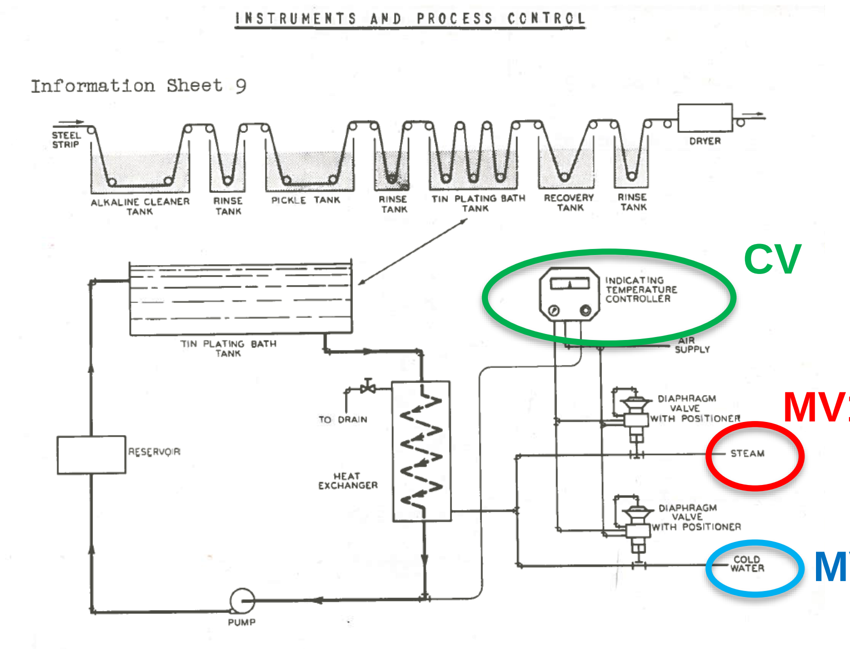
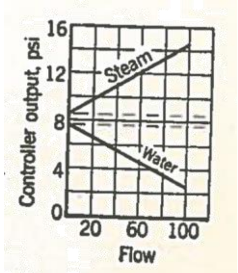
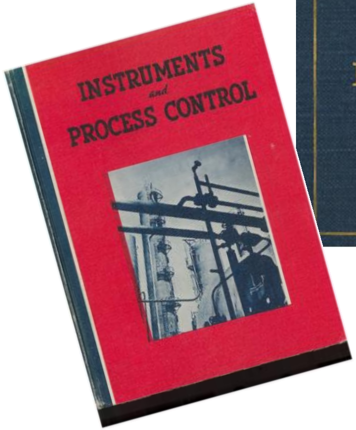
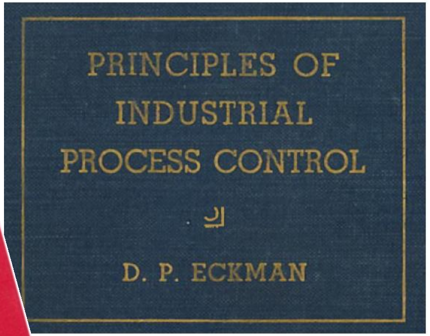


Fig. 125 - Temperature Control for a Tin Plating Bath
Courtesy of Taylor Instrument Companies



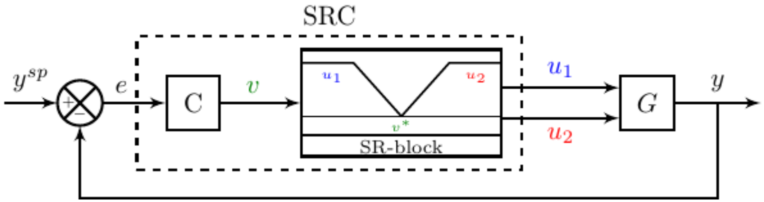


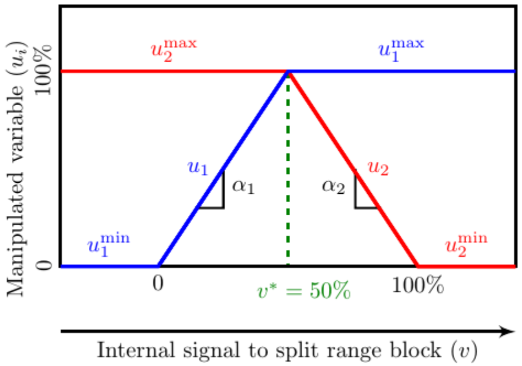


Eckman, D.P. (1945). Principles of industrial control, New York.

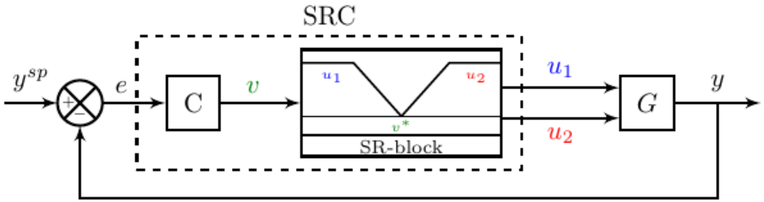
Monogram of Instruments and Process Control prepared at Cornell, NY, in 1945

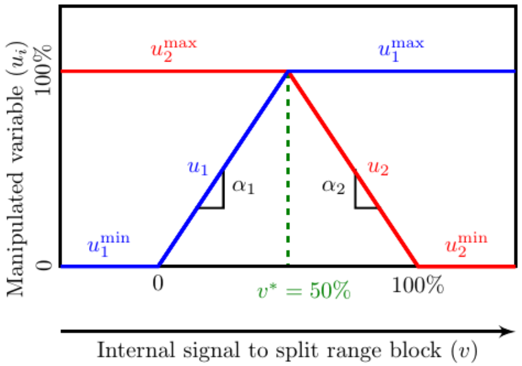
Classical split range control





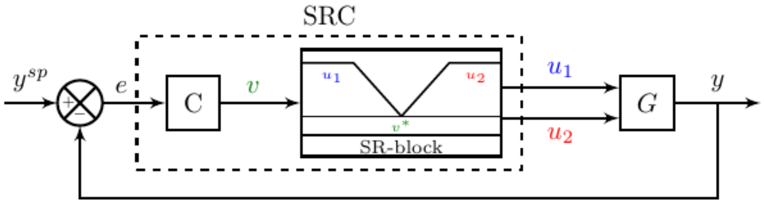
Classical split range control

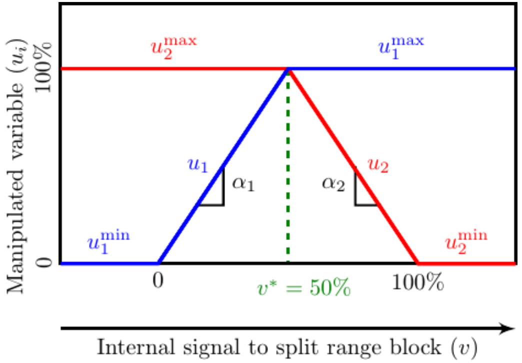




v	internal signal to split range block→ limited physical meaning
v^*	split value
u_i	controller output → physical meaning
α_i	gain from v to ui → slope

Classical split range control



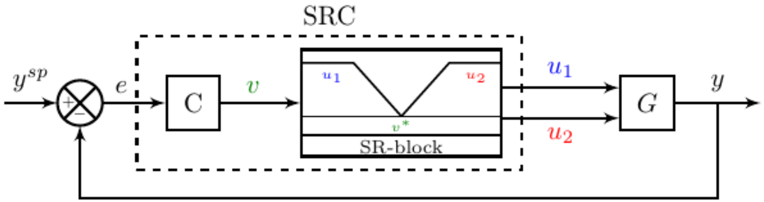


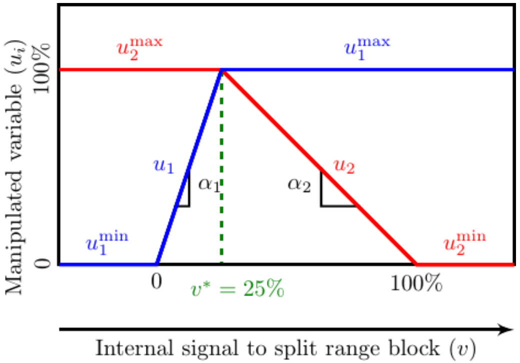
v	internal signal to split range block → limited physical meaning
v*	split value → degree of freedom
u_i	controller output → physical meaning
α_i	gain from v to ui → slope

$$u_i = u_{i,0} + \alpha_i \; v \quad \forall i \in \{1, \ldots, N\}$$

NTNU

Classical split range control





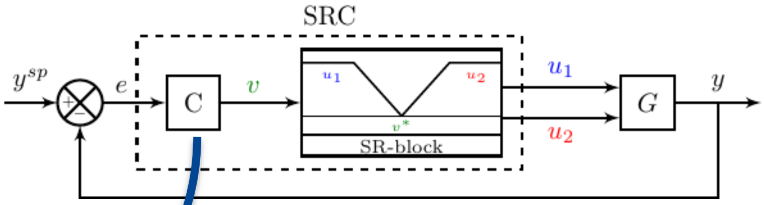
v	internal signal to split range block→ limited physical meaning
v*	split value → degree of freedom
u_i	controller output → physical meaning
α_i	gain from v to ui → slope

$$u_i = u_{i,0} + \alpha_i \; v \quad \forall i \in \{1, \ldots, N\}$$

Design of split range control: select slopes

Goal: get desired loop gain |g C|

at crossover frequency $\omega_c = \frac{1}{\tau_C}$

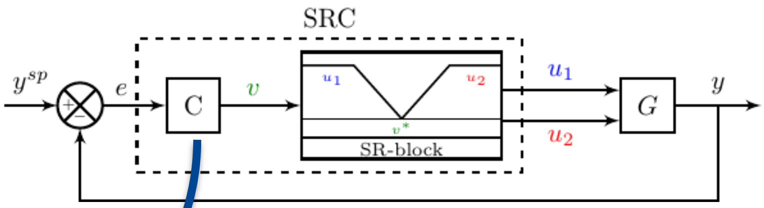


$$C(s) = K_C\left(1 + \frac{1}{\tau_I s}\right)$$

Design of split range control: select slopes

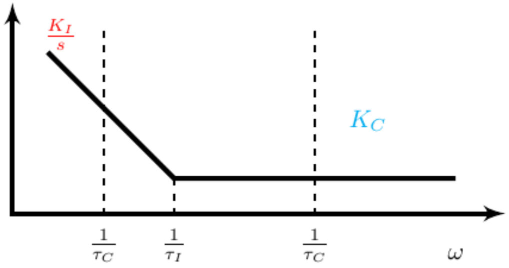
Goal: get desired loop gain $|g\ C|$

at crossover frequency $\omega_c = \frac{1}{\tau_C}$



$$C(s) = K_C\left(1 + \frac{1}{\tau_I s}\right)$$

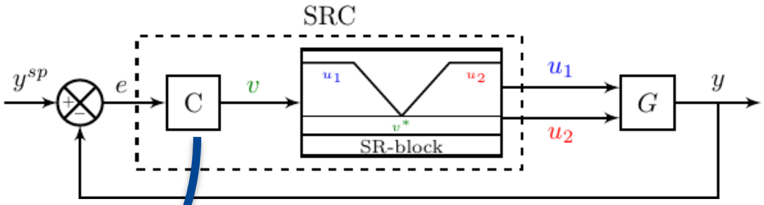
$K_I = \frac{K_C}{\tau_I}$



Design of split range control: select slopes

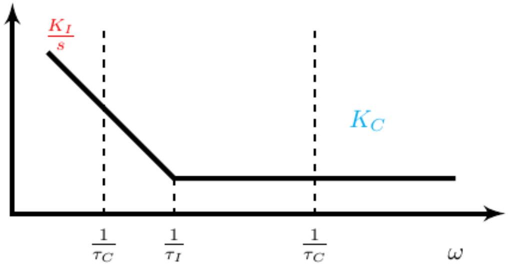
Goal: get desired loop gain $|g\ C|$

at crossover frequency $\omega_c = \frac{1}{\tau_C}$



$$C(s) = K_C\left(1 + \frac{1}{\tau_I s}\right)$$

$K_I = \frac{K_C}{\tau_I}$



Fast process

$$K_{I,i} = \alpha_i K_I$$

Desired gain for u_i — DOF — Common gain in C

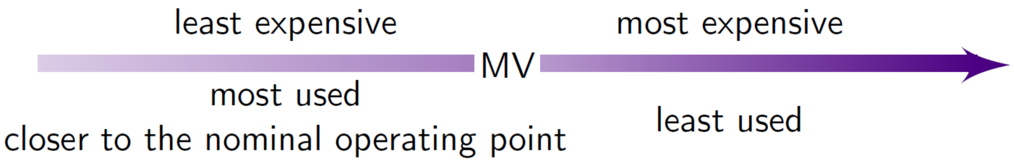
Slow process

$$K_{C,i} = \alpha_i K_C$$

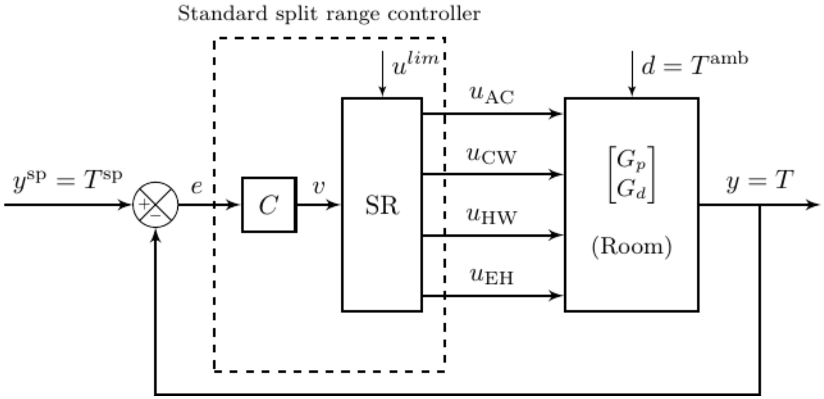
Desired gain for u_i — DOF — Common gain in C

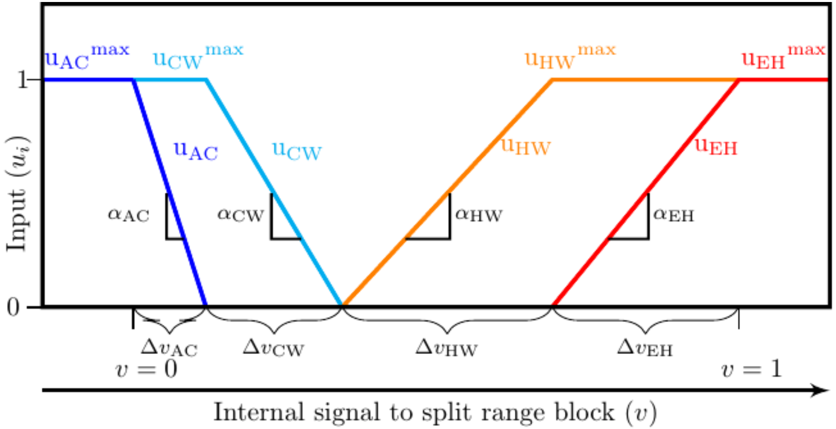
Design of split range control: order of MVs

1. Define the desired operating point for every MV
2. Group the MVs according to the effect on the CV
3. Within each group, define order of use



Design of split range control





$u_1 = u_{AC}$: air conditioning (AC)

$u_2 = u_{CW}$: cooling water (CW)

$u_3 = u_{HW}$: heating water (HW)

$u_4 = u_{EH}$: electrical heating (EH)

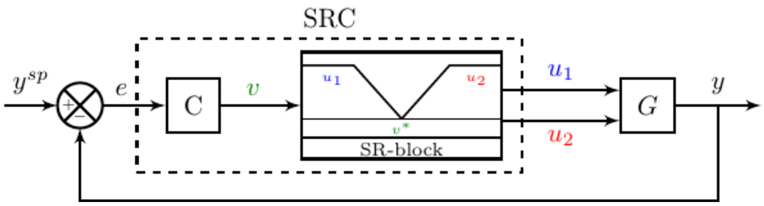
Classical split range control: a compromise

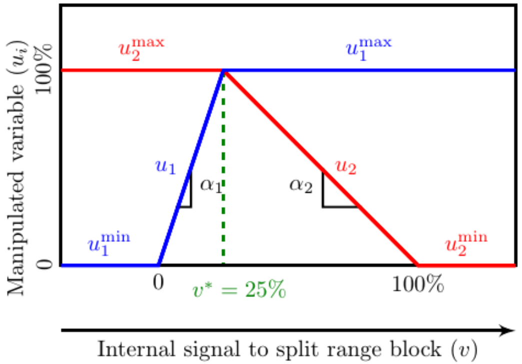
$$C(s) = K_C\left(1 + \frac{1}{\tau_I s}\right)$$

2 tuning parameters

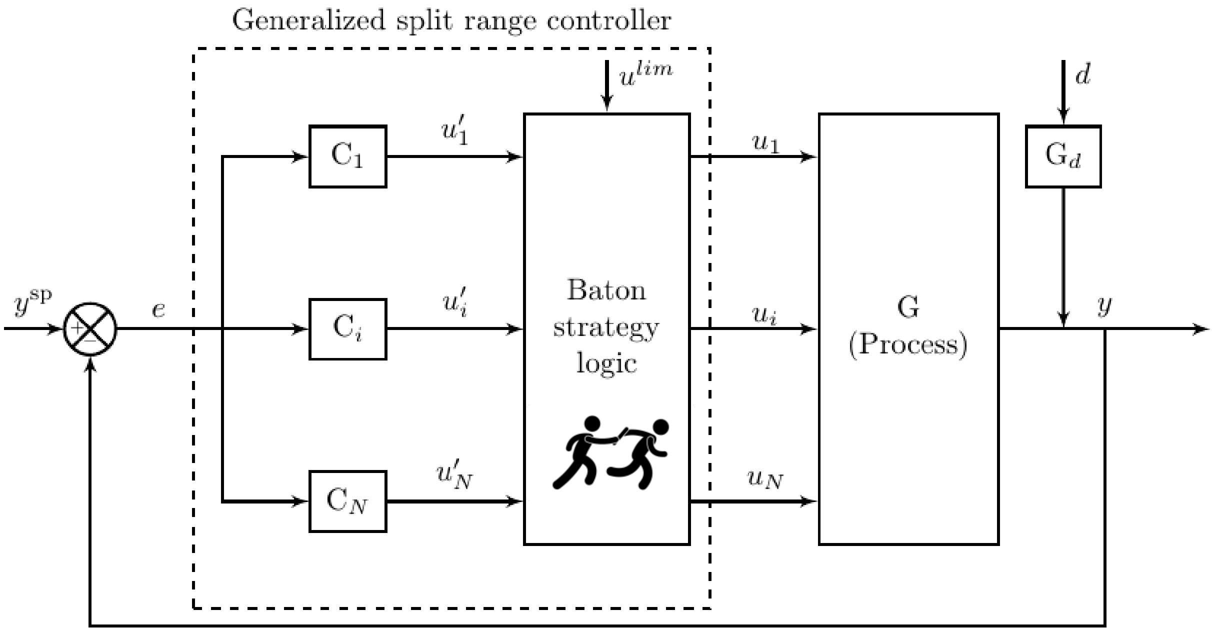
$$K_{C,i} = \alpha_i K_C$$

1 DOF

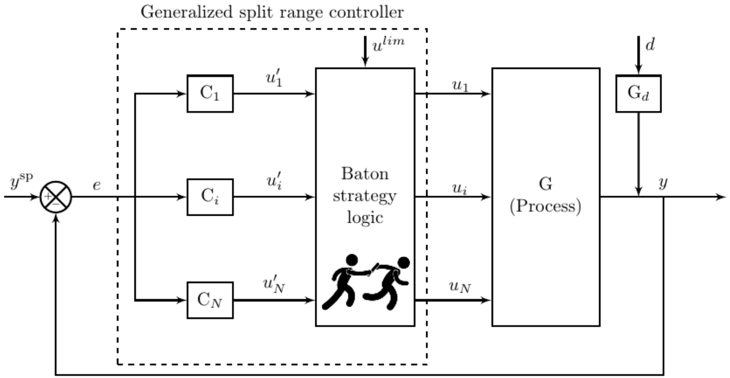




Generalized split range controller



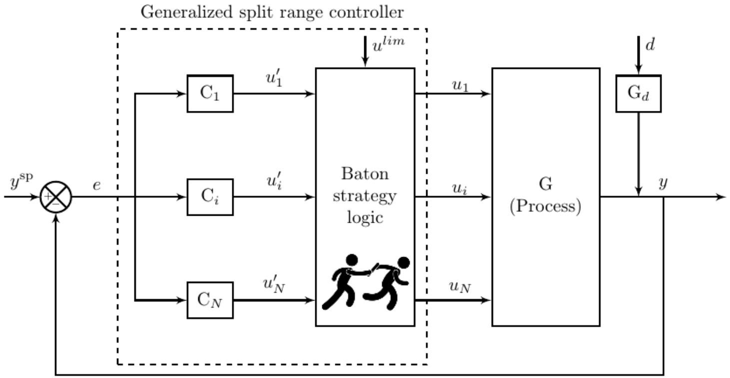
Generalized split range controller



Preliminary step:

- Define order of use of MVs (j=1,…,N)
- Tune controllers

Generalized split range controller



Preliminary step:

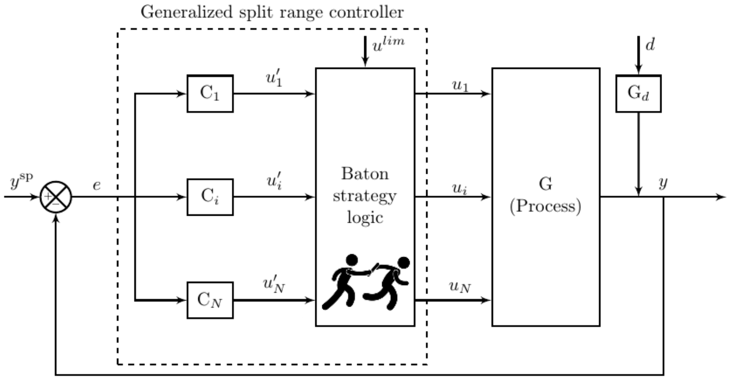
- Define order of use of MVs (*j*=1,…,N)
- Tune controllers

«Baton strategy» logic

k is the active input

- C_k computes u_k' (suggested value for u_k)
- If u_k^{min}< u_k'< u_k^{max}
  - Keep u_k active and u_k← u_k'
  - Keep remaining u_i at limiting value
- else
  - Set u_k= u_k^{min} or u_k< u_k^{max}, depending on the reached limit
  - New active input selected according to predefined sequence (*j*= *k*-1 or *j*=*k*+1)

Generalized split range controller



«Baton strategy» logic

k is the active input

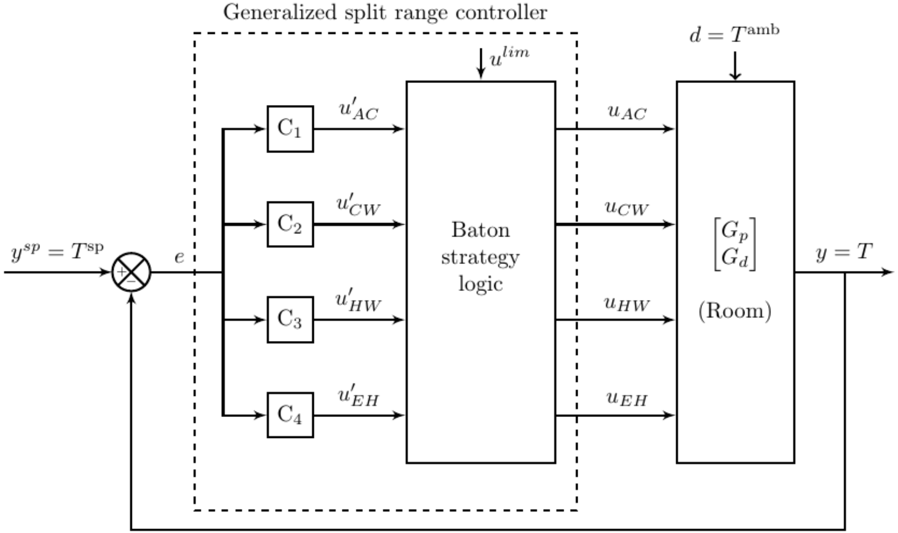
- C_k computes u_k' (suggested value for u_k)
- If $u_k^{min} < u_k' < u_k^{max}$
  - Keep u_k active and $u_k \leftarrow u_k'$
  - Keep remaining u_i at limiting value
- else
  - Set $u_k = u_k^{min}$ or $u_k < u_k^{max}$, depending on the reached limit
  - New active input selected according to predefined sequence ($j = k$-1 or $j = k$+1)

Preliminary step:

- Define order of use of MVs (*j*=1,…,N)
- Tune controllers

> The active input will *decide* when to switch and will remain active as long as it is not saturated.

Generalized split range controller



	Active input (input with *baton*, u_k)			
Value of u'_k	$u_1 = u_{\text{AC}}$	$u_2 = u_{\text{CW}}$	$u_3 = u_{\text{HW}}$	$u_4 = u_{\text{EH}}$
$u_k^{min} < u'_k < u_k^{max}$	keep u_1 active $u_1 \leftarrow u'_1$ $u_2 \leftarrow u_2^{max}$ $u_3 \leftarrow u_3^{min}$ $u_4 \leftarrow u_4^{min}$	keep u_2 active $u_1 \leftarrow u_1^{min}$ $u_2 \leftarrow u'_2$ $u_3 \leftarrow u_3^{min}$ $u_4 \leftarrow u_4^{min}$	keep u_3 active $u_1 \leftarrow u_1^{min}$ $u_2 \leftarrow u_2^{min}$ $u_3 \leftarrow u'_3$ $u_4 \leftarrow u_4^{min}$	keep u_4 active $u_1 \leftarrow u_2^{min}$ $u_2 \leftarrow u_1^{min}$ $u_3 \leftarrow u_3^{max}$ $u_4 \leftarrow u'_4$
$u'_k \geq u_k^{max}$	keep u_1 active (max. cooling)	baton to u_1 $u_1^0 = u_1^{min}$	baton to u_4 $u_4^0 = u_4^{min}$	keep u_4 active (max. heating)
$u'_k \leq u_k^{min}$	baton to u_2 $u_2^0 = u_2^{max}$	baton to u_3 $u_3^0 = u_3^{min}$	baton to u_2 $u_2^0 = u_2^{min}$	baton to u_3 $u_3^0 = u_3^{max}$

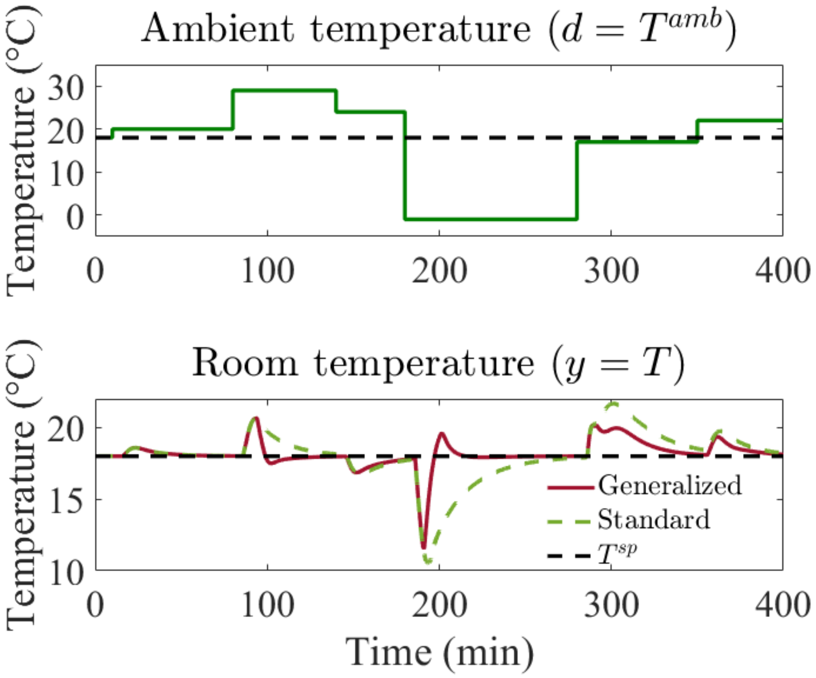
$u_1 = u_{AC}$: air conditioning (AC)

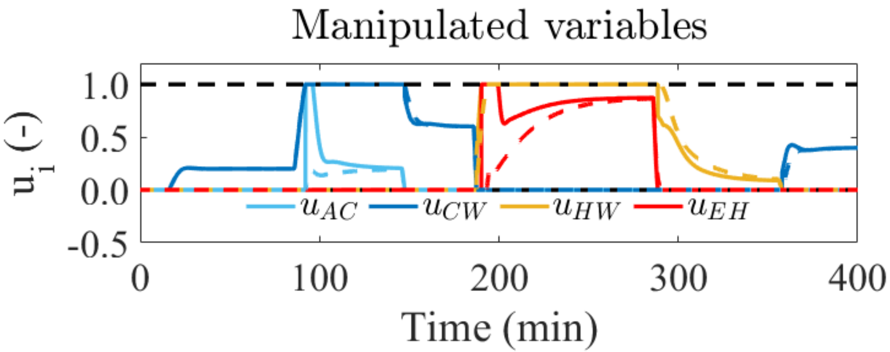
$u_2 = u_{CW}$: cooling water (CW)

$u_3 = u_{HW}$: heating water (HW)

$u_4 = u_{EH}$: electrical heating (EH)

Generalized vs standard split range controller





Generalized split range controller: initialization

k

Generalized split range controller: initialization

k

$$u'_k(t) = u_k^0 + K_{C,k}\left(e(t) + \frac{1}{\tau_{I,k}}\int_{t_b}^{t} e(t)\right)$$

This suggested input was not being applied while input k was not in use

This accumulated error is not due to the previous actions of input k

Generalized split range controller: initialization

k

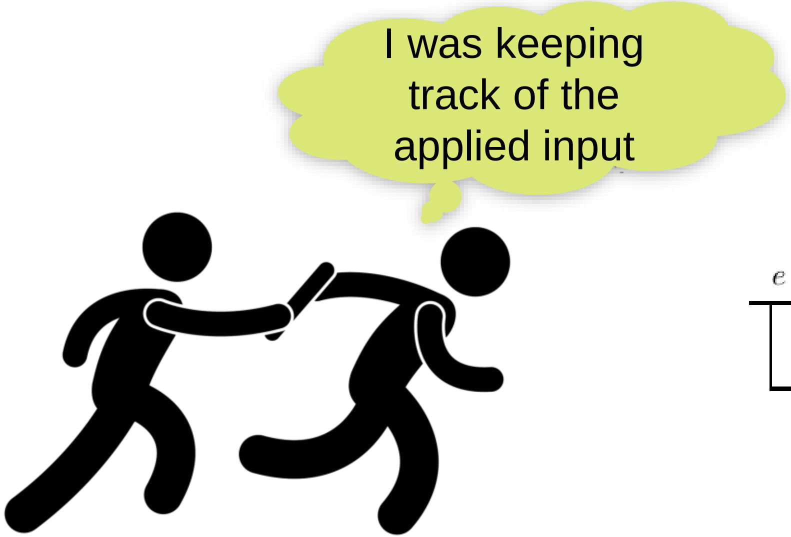
Resetting:

$$u'_k(t) = u_k^0 + K_{C,k}\left(e(t) + \frac{1}{\tau_{I,k}}\int_{t_b}^{t} e(t)\right)$$

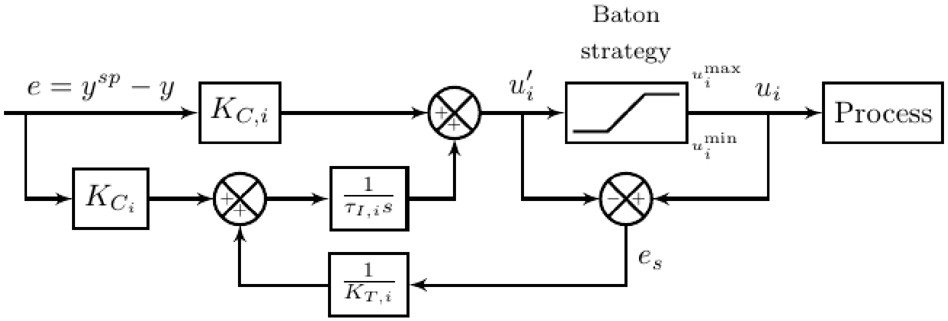
$$u_k(t_b) = u_k^0 + K_{C,k}e(t_b)$$

Initial action proportional to error at t_b

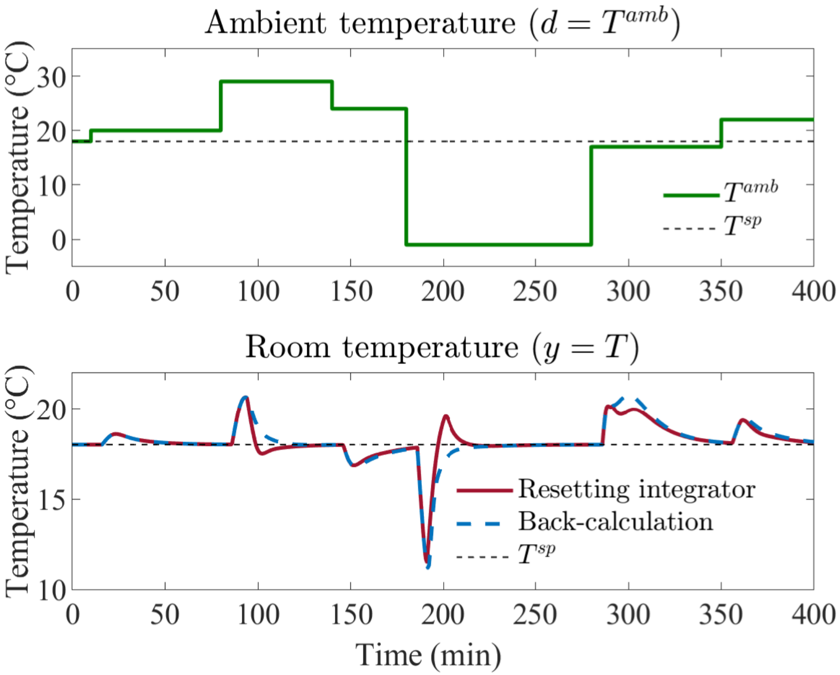
Generalized split range controller: initialization



Back-calculation:

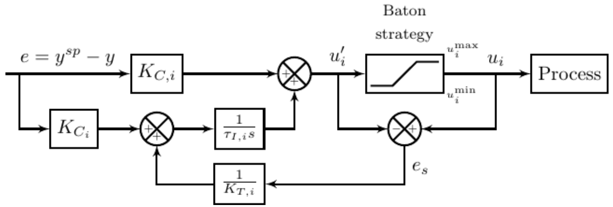


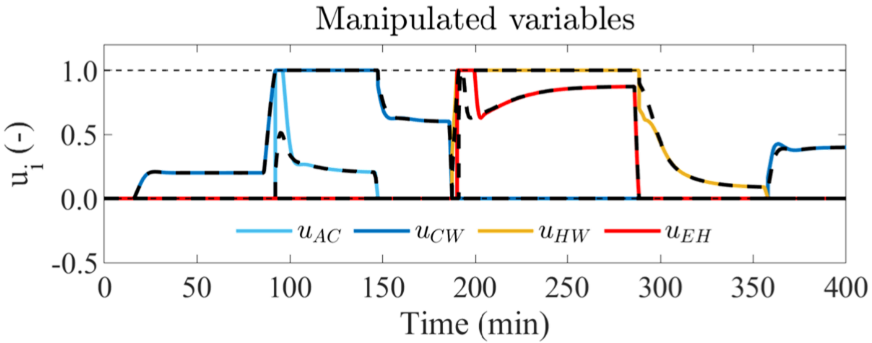
Generalized split range controller: initialization



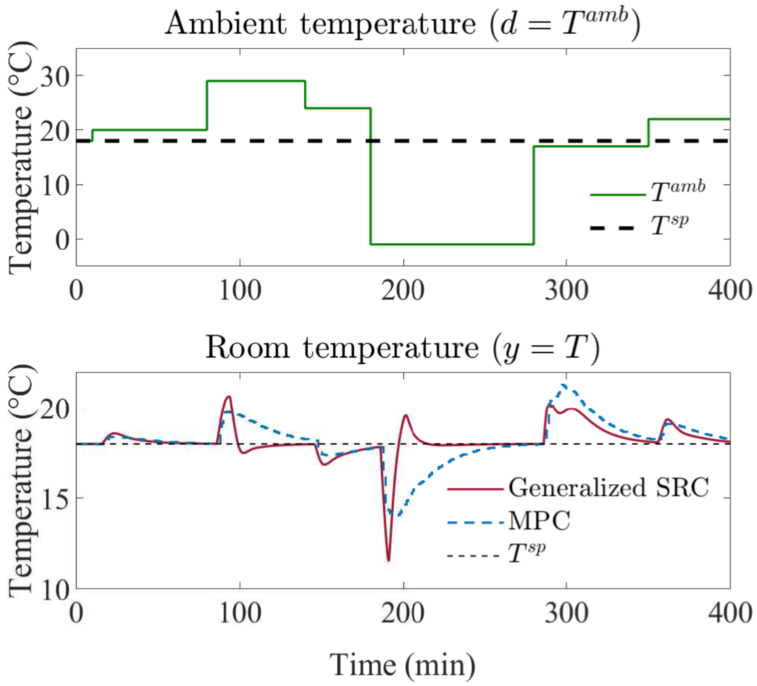
Resetting: $$u_k(t_b) = u_k^0 + K_{C,k}e(t_b)$$

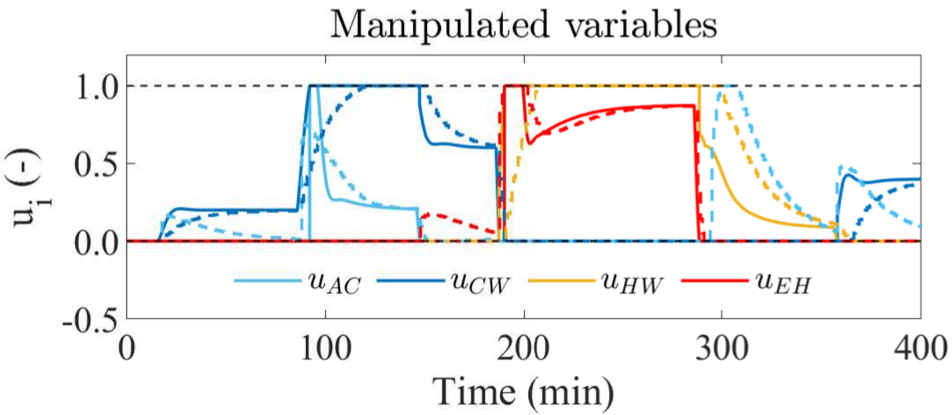
Back-calculation:





Generalized split range controller vs MPC





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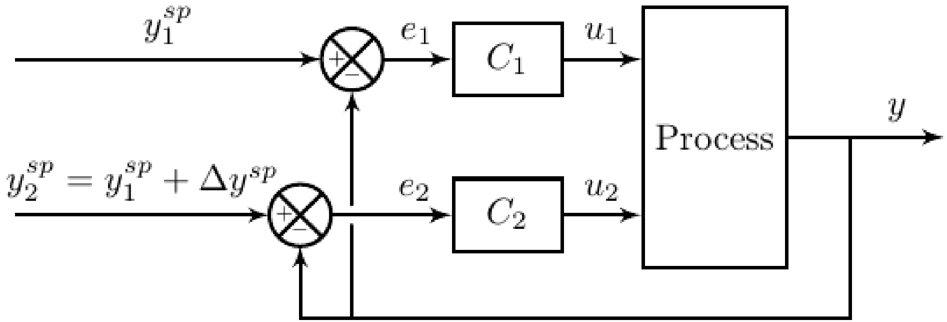
Multiple controllers with different setpoints

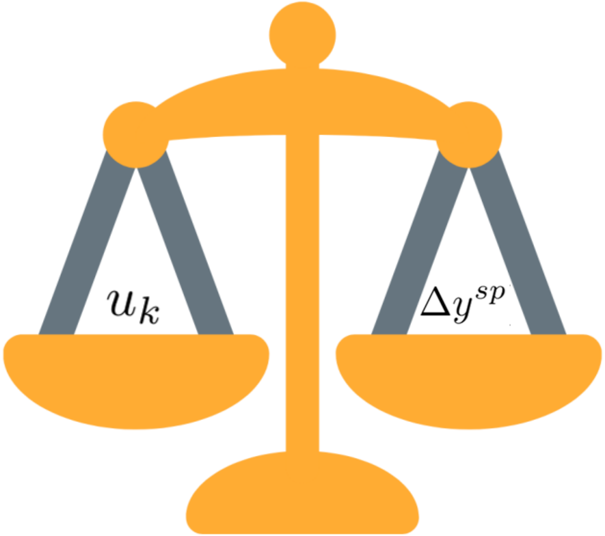
Does this make sense at any point?



Multiple controllers with different setpoints

Input

usage



Setpoint

deviation

$$J(u_k, \Delta y^{sp})$$

Multiple controllers with different setpoints: Optimal setpoint deviation

Linear for *u* and quadratic for Δ*y*

$$J = p_{u_k} u_k + p_y (y - y^{sp})^2 + c$$

Inputs* are a linear function of *output

$$u_i = k_i \; y + u_{i,0}$$

Multiple controllers with different setpoints: Optimal setpoint deviation

Linear for *u* and quadratic for Δ*y*

$$J = p_{u_k} u_k + p_y (y - y^{sp})^2 + c$$

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Cost when using u_k as input

$$J = p_{u_k} k_k \; y + p_y (y - y^{sp})^2 + c_k + p_{u_k} u_{k,0}$$

Multiple controllers with different setpoints: Optimal setpoint deviation

Linear for *u* and quadratic for Δ*y*

$$J = p_{u_k} u_k + p_y (y - y^{sp})^2 + c$$

Inputs* are a linear function of *output

$$u_i = k_i \; y + u_{i,0}$$

Cost when using u_k as input

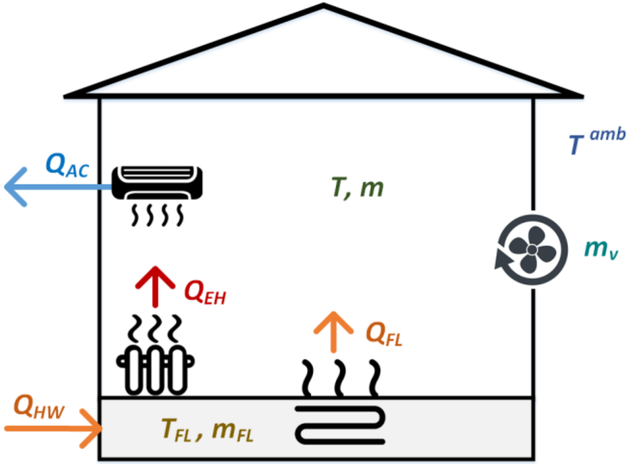
$$J = p_{u_k} k_k \; y + p_y (y - y^{sp})^2 + c_k + p_{u_k} u_{k,0}$$

$$\frac{dJ}{dy} = 0$$

Optimal setpoint deviation minimizing cost

$$\Delta y^{sp^*} = y^* - y^{sp} = -\frac{p_{u_k} k_k}{2 p_y}$$

Multiple controllers with different setpoints: Case study

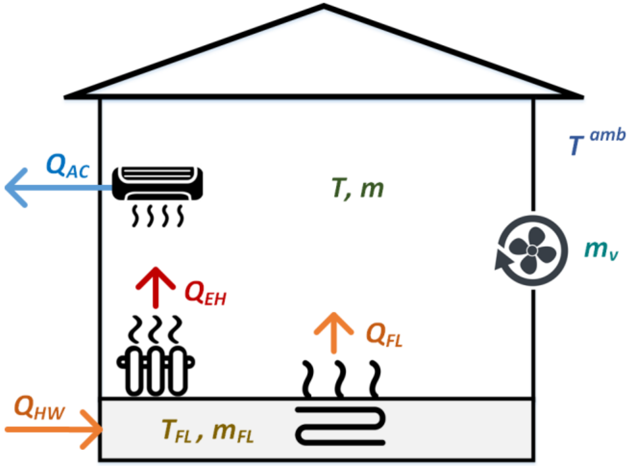


Q_{AC} : air conditioning

Q_{HW} : heating water

Q_{EH} : electrical heating

Multiple controllers with different setpoints: Case study



Q_{AC} : air conditioning

Q_{HW} : heating water

Q_{EH} : electrical heating

Cost: linear for *u* and quadratic for Δ*y*

$$J = \overbrace{\underbrace{p_{AC}Q_{AC}}_{p_1 u_1} + \underbrace{p_{HW}Q_{HW}}_{p_2 u_2} + \underbrace{p_{EH}Q_{EH}}_{p_3 u_3}}^{J_{\text{energy}}} + \underbrace{p_T(T - T^{sp})^2}_{p_y(y-y^{sp})^2} \quad [\$/s]$$

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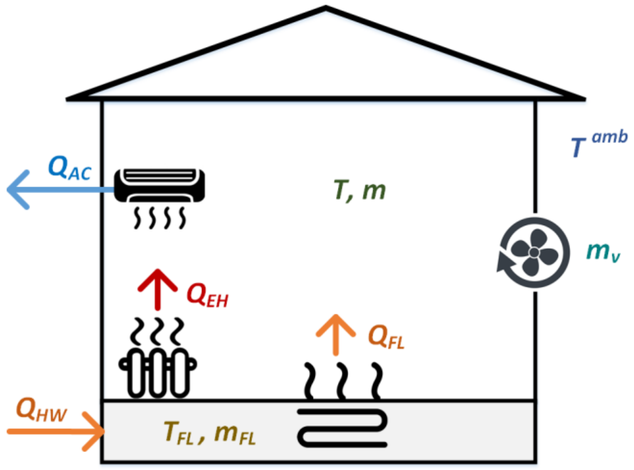
Multiple controllers with different setpoints: Case study



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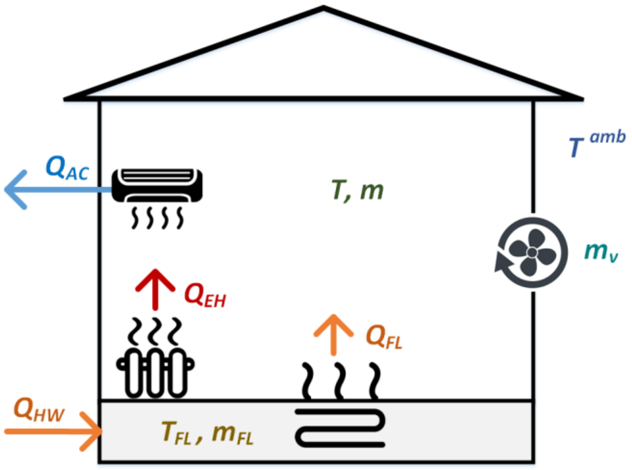
***Inputs* (Q_i) are a linear function of *output* (T)**

$$0 = \alpha(T^{amb} - T) + Q_{HW} + Q_{EH} - Q_{AC} \ [W]$$

Multiple controllers with different setpoints: Room T



Q_{AC} : air conditioning

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Cost: linear for *u* and quadratic for Δ*y*

$$J = \overbrace{\underbrace{p_{AC}Q_{AC}}_{p_1u_1} + \underbrace{p_{HW}Q_{HW}}_{p_2u_2} + \underbrace{p_{EH}Q_{EH}}_{p_3u_3}}^{J_{\text{energy}}} + \underbrace{p_T(T - T^{sp})^2}_{p_y(y-y^{sp})^2} \quad [\$/s]$$

Inputs (Q_i) are a linear function of *output* (T)

$$0 = \alpha(T^{amb} - T) + Q_{HW} + Q_{EH} - Q_{AC} \ [W]$$

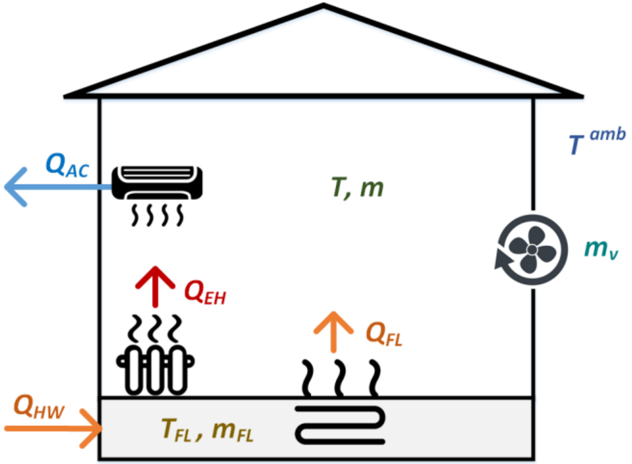
Optimal setpoint deviation minimizing cost

$$\Delta y^{sp,1} = T^{sp}_{AC} - T^{sp} = +\frac{\alpha p_{ac}}{2p_T}$$

$$\Delta y^{sp,2} = T^{sp}_{HW} - T^{sp} = -\frac{\alpha p_{hw}}{2p_T}$$

$$\Delta y^{sp,3} = T^{sp}_{EH} - T^{sp} = -\frac{\alpha p_{el}}{2p_T}$$

Multiple controllers with different setpoints: Room T



Q_{AC} : air conditioning

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Optimal setpoint deviation minimizing cost

$$\Delta y^{sp,1} = T_{AC}^{sp} - T^{sp} = +\frac{\alpha p_{ac}}{2p_T}$$

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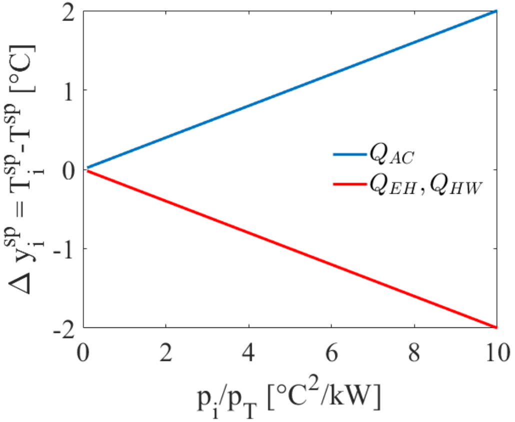
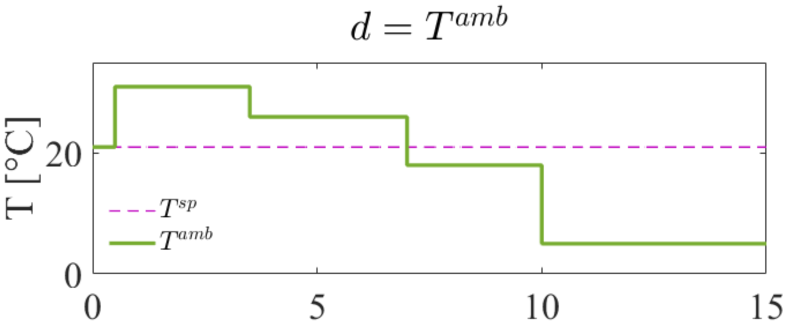
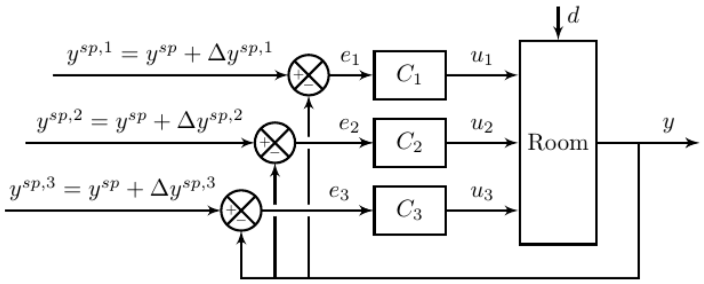
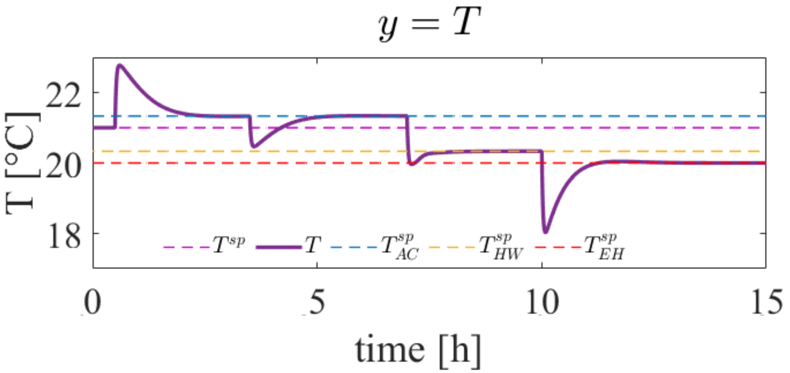
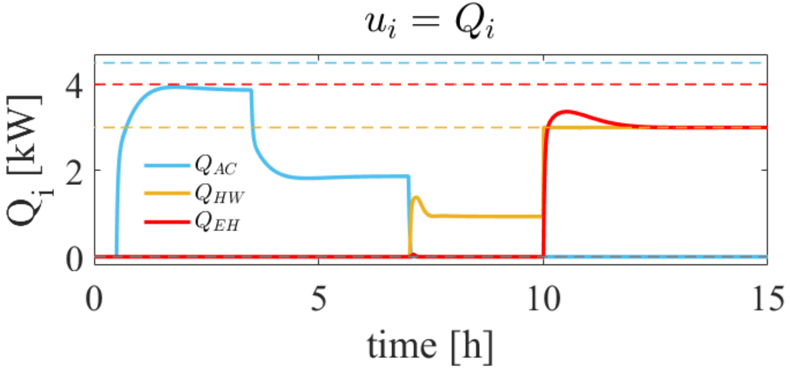


Multiple controllers with different setpoints: Room T

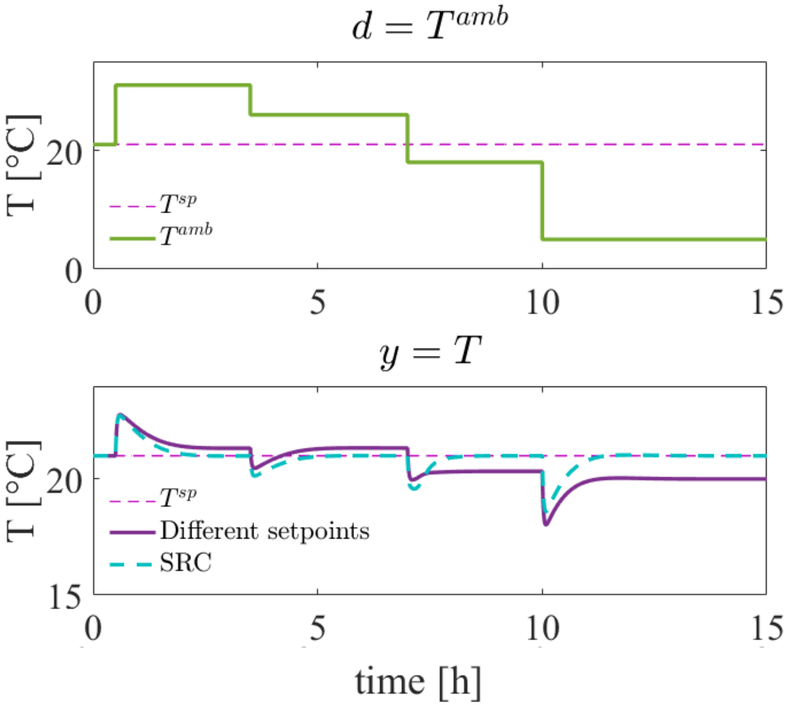


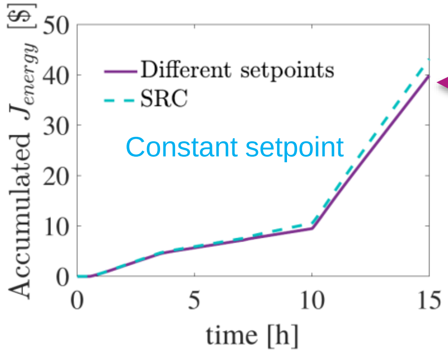




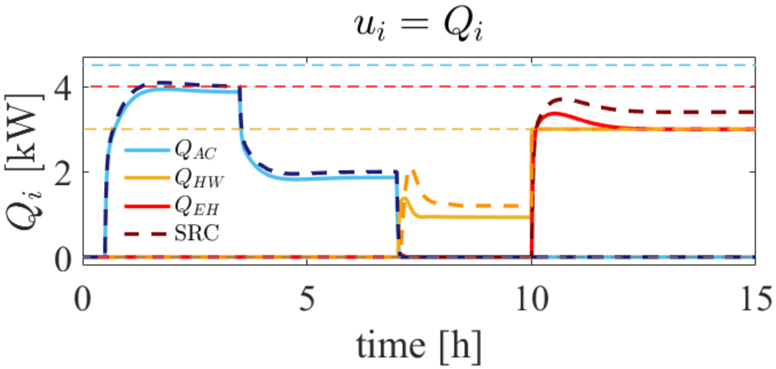


Multiple controllers with different setpoints: Room T





Lower accumulated cost with minimum setpoint deviation



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Final comments

- Steady-state optimal operation may be easily achieved using PID-based control structures
  - Chapters 2,3,4: active constraint switching
  - Chapter 7: optimal setpoints

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- Useful to systematically define control objectives, feasibility and tools
  - Priority list of constraints
  - Control structures available for each type of switch (CV-CV, MV-MV, MV-CV)

Final comments

- Steady-state optimal operation may be easily achieved using PID-based control structures
  - Chapters 2,3,4: active constraint switching
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- Useful to systematically define control objectives, feasibility and tools
  - Priority list of constraints
  - Control structures available for each type of switch (CV-CV, MV-MV, MV-CV)

- Possible to improve performance of PID-based advanced control
  - Chapters 5, 6: design of split range controllers
  - Chapter 8: improved level control

One final comment

- The “gap” between theory and practice can be in both directions

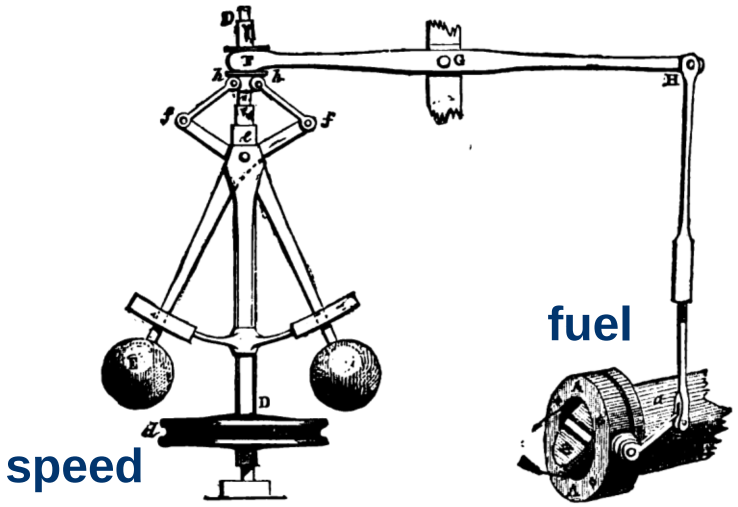

FIG. 4.—*Governor and Throttle-Valve.*

Centrifugal governor used in steam engines in the 1780’s:
Proportionally controls fuel flow to maintain engine speed.

Theoretical investigation started about a century later.

Åström, K. J., & Kumar, P. R. (2014). Control: A perspective. *Automatica*, *50*(1), 3–43.

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Systematic design of advanced control structures

Thank you for your attention!

Norwegian University of Science and Technology