OPTIMAL SELECTION OF SENSORS AND CONTROLLER PARAMETERS FOR ECONOMIC OPTIMIZATION OF PROCESS PLANTS

A THESIS

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THESIS CERTIFICATE

This is to certify that the thesis titled **OPTIMAL SELECTION OF SENSORS AND CONTROLLER PARAMETERS FOR ECONOMIC OPTIMIZATION OF PROCESS PLANTS**, submitted by **M. NABIL**, to the Indian Institute of Technology, Madras, for the award of the degree of **DOCTOR OF PHILOSOPHY**, is a bonafide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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Place: Chennai

Date: 17th September 2014

To my parents

தொட்டனைத் தூறும் மணற்கேணி மாந்தர்க்குக்
கற்றனைத் தூறும் அறிவு

-திருக்குறள் (396)

In sandy soil, when deep you delve, you reach the springs below;
The more you learn, the freer streams of wisdom flow.

- G. U. Pope's Translation of Thirukkural (396)

நுண்ணிய நூல்பல கற்பினும் மற்றுந்தன்
உண்மை யறிவே மிகும்

-திருக்குறள் (373)

In subtle learning manifold though versed man be,
'The wisdom, truly his, will gain supremacy.

- G. U. Pope's Translation of Thirukkural (373)

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ABSTRACT

KEYWORDS: optimal operation; measurement selection; set point selection; dynamic back-off; convex optimization; linear matrix inequality; semi-definite programming; model predictive control.

In a typical chemical plant, the economic performance depends on several structural and parametric decisions of the process. In order to quantify the economic performance, we define the departure or loss function that measures the deviation from the optimal cost, caused because of uncertainties. In this thesis, we particularly focus on making a rational choice in the selection of measurements (structural decision), set points and controller design (parametric decisions) based on the loss function. For this purpose, we classify the relevant problems based on the nominal optimal point which is generally obtained by minimizing the economic cost function subject to steady state model of the process.

Firstly, if the nominal optimal solution is either *unconstrained* or *constrained but perfectly controllable*, and there exists some unconstrained degrees of freedom, then we focus on the measurement selection problem. It deals with determining the sensor network by minimizing the loss function caused because of random measurement errors. The main contributions along these lines include: (a) an analytical expression that quantifies the economic loss caused due to measurement uncertainty is shown to be sum of weighted error variances and an optimization problem that minimizes this loss function is posed as an Mixed Integer Conic Problem (MICP) which can be solved for globally optimal sensor network, (b) an optimization framework for determining the best sensor network that can minimize the average loss and overall error in lexicographic sense is proposed, and (c) an optimization formulation that finds the best set of measurements that are robust to sensor failure situations by ensuring a certain level of estimability of the network is presented.

Secondly, if the nominal optimal point is *constrained but not perfectly controllable*, then we focus on the set point selection problem. It deals with finding a new profitable operating point (i.e., backed-off point) that is also dynamically feasible in the presence of uncertainties. The main contributions along these lines include: (a) assuming disturbances as the only source of uncertainty, we propose a novel two-stage iterative solution algorithm to determine the economically optimal backed-off point when there is no controller and in the presence of linear multivariable controller, (b) assuming measurement errors as an addition source of uncertainty, the methodology is extended to find the best set of measurements that will reduce the amount of back-off caused by measurement errors in addition to controller design, and (c) for the case with disturbances only, the practical implementation of the obtained multivariable controller using Model Predictive Control (MPC) framework is studied by transforming the controller solution into equivalent MPC weights.

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ABBREVIATIONS

BOP	Backed-off Operating Point
CV	Controlled Variables
DR	Data Reconciliation
EBOP	Economic Backed-off Operating Point
EDOR	Expected Dynamic Operating Region
EVO	Engineering Virtual Organization
FSI	Full State Information
LMI	Linear Matrix Inequality
LP	Linear Programming
MICP	Mixed Integer Cone Programming
MILP	Mixed Integer Linear Programming
MINLP	Mixed Integer Non-Linear Programming
MPC	Model Predictive Control
MV	Manipulated Variables
OOP	Optimal Operating Point
PI	Proportional-Integral
PID	Proportional-Integral-Derivative
PSI	Partial State Information
SDP	Semi-Definite Programming
SND	Sensor Network Design
SOC	Second Order Cone

NOTATION

English alphabets

A, A_d	system matrices of a continuous time and discrete time state space model
A_1, A_2	columns of A in the linear process model $Ax = 0$, corresponding to measured and unmeasured variables
A_p, A_s	columns of A in the linear process model $Ax = 0$, corresponding to primary and secondary variables
B, B_d	system matrices of a continuous time and discrete time state space model
C	process matrix
c_i	cost of the sensor for measuring variable z_i
c^*	budget limit for purchasing sensors
d	disturbance variables
G, G_d	system matrices of a continuous time and discrete time state space model
J	negated profit function
k	number of sensors to be selected
L	controller gain matrix in $u = Lx$
$\overline{L}$	operational loss due to measurement errors
ℓ	stage cost in MPC
M	parameter in the lexicographic formulation
N_{min}	minimum number of sensors
n	total number of variables
n_a	number of active constrained variables
n_d	number of disturbance variables
n_e	number of model equations
n_u	number of manipulated variables
$n_{uc,dof}$	number of unconstrained degrees of freedom
n_x	number of state variables
n_y	number of measurements
q_i	binary decision variable denoting the presence or absense of a sensor
Q	$diag(\frac{q_i}{\sigma_i^2})$; objective function weights in MPC
R	objective function weights in MPC
$\mathbb{R}$	set of real numbers
$\mathbb{R}_+$	set of non negative real numbers
$\mathbb{R}_{++}$	set of positive real numbers
$\mathbb{R}^n$	set of real n-vectors
$\mathbb{R}^{m \times n}$	set of real $m \times n$ matrices
SF	set of possible sensors that can fail at a time
$\mathbb{S}^n$	set of symmetric $n \times n$ matrices
$\mathbb{S}^n_+$	set of symmetric positive semidefinite $n \times n$ matrices
$\mathbb{S}^n_{++}$	set of symmetric positive definite $n \times n$ matrices

u	manipulated variables
v_m	error vector
W	weighting matrix in the sensor network design formulation, $W = RR^T$
x	state variables
z	vector of process variables
z_m	measured variables
z_p	primary variables
z_s	secondary variables
z_u	unmeasured variables

Greek alphabets

α	prescribed confidence level
Δu	difference between the BOP value of u and its bound u_{min} or u_{max}
Δx	difference between the BOP value of x and its bound x_{min} or x_{max}
$\delta_{i,j}$	ratio of distance of variable i from its closest bound to the distance of variable j from its closest bound
ϵ_d	error in disturbance variable d
ϵ_u	error in input variable u
ϵ_x	error in state variable x
ϵ_z	error in process variable z
Σ_z	error covariance matrix of the estimates z
Σ_v	diagonal covariance matrix of the error vector v_m
σ_i	variance of the process variable z_i
Φ	total stage cost in standard MPC formulation
Φ_{ebop}	total stage cost in economic back-off based MPC formulation
Φ_{eco}	total stage cost in economic MPC formulation
λ_1, λ_2	objective function weights of the lexicographic formulation

Operators

$\|\|(.)\|\|$	norm of (.)
$diag(.)$	diagonalize (.)
$\mathbf{E}[.]$	expectation of [.]
$g(.)$	process model functions
$h(.)$	process constraint functions

Accents

$\hat{(.)}$	estimate of (.)
$\bar{(.)}$	nominal optimal value of (.)
$\tilde{(.)}$	deviation from the nominal optimal value $\bar{(.)}$

Superscripts and subscripts

t_e, Y_e	internal variables in overall estimation error formulation
t_c, Y_c	internal variables in average loss formulation
x^*	optimal value of x
x_k	value of x at time instant k
x_{max}	maximum value of x
x_{min}	minimum value of x
x_{ss}	steady state values of x

CHAPTER 1

INTRODUCTION

1.1 Introduction

With increasing global competition, it is important to operate a process plant so as to maximize the economic performance. The optimal economic performance depends on understanding the interplay of several decisions that affect the plant operation (Skogestad, 2012; Engell, 2007). Typically, the plant operations are performed in a hierarchical structure based on time scale separations and the simplified structure is shown in Figure 1.1, which consists of an upper scheduling layer, followed by an intermediate optimization layer and the lower control layer with decreasing time scale (Morari *et al.*, 1980). The top planning and scheduling layer dictates the yield requirements based on market research and updates the cost values of raw materials and products for the optimization layer. This is carried out in the order of months to years. Based on the updated cost information, the intermediate optimization layer determines the optimal operating point of the pre-designed process, and it is given as set points to the lower control layer. This is performed in the time scale of minutes to hours. The lower control layer take control actions to operate at the nominal point provided by the optimization layer despite disturbances entering the process. The dynamics are often addressed in this layer within few seconds or minutes.

In general, the design variables are determined even before the plant is commissioned for operations. However, the proposed design might limit the performance of the controller due to unknown disturbances entering the process. Thus, it is often required to ensure operability of the plant at the design stage. In this regard, some of the previous works on the topic of profit control have advocated the necessity of considering economic context of plant controllability at the early stage of design and presented methods to screen alternative designs (Perkins, 1989; Perkins *et al.*, 1989). Though such a study is important to provide design recommendations for new plants, this thesis

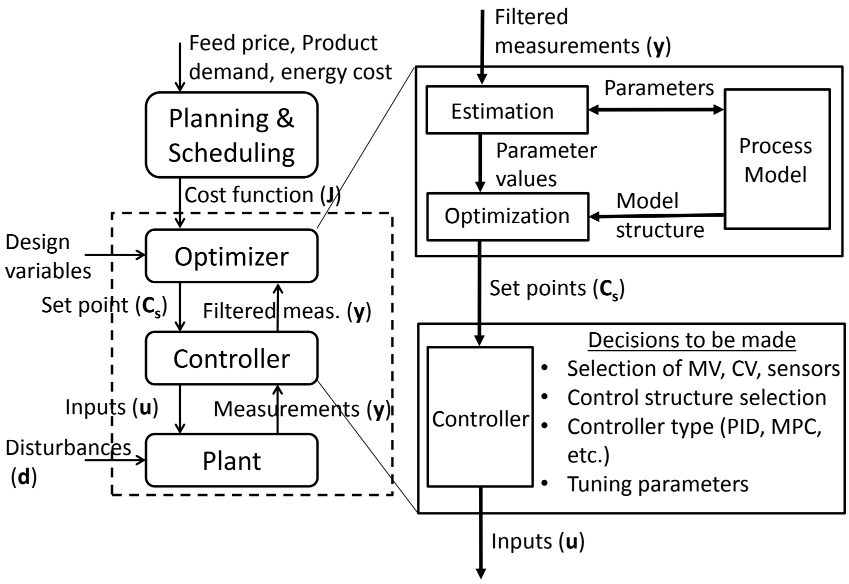

Figure 1.1: Hierarchical structure of plant operations based on time scale separation showing the set of decisions involved in optimization and control layer

focuses on studying the different aspects of achieving optimal operation for a fixed set of design variables in the plant. In other words, the focus of this thesis is to study different decisions that will improve the economic performance of the operating plant. In particular, we study some of the structural and parametric decisions in the optimization and control layer.

In the optimization layer, the performance of the optimizer depends on several structural decisions such as type of model (a simple linear model, an approximate model or a detailed non-linear model), an estimation method (to determine the accurate values of the parameters), etc., (Forbes *et al.*, 1994). This will eventually determine the type of optimization problem we need to solve and hence the performance also depends on the type of solution algorithm we use to solve them (Roberts, 1979). In other words, the parametric decision (i.e., the set point selection) is strongly influenced by the above mentioned structural decisions. Likewise in the control layer, the selection of manipulated variables (MV), controlled variables (CV), type and structure of the controller are crucial structural decisions that will affect the performance of the plant. Yet another structural decision that will affect the optimal operation is the choice of measurements. Therefore, the selection of measured variables, manipulated variables, controlled variables, control structure and controller design, model structure and operating point play a vital role in achieving profitability of a chemical process. However, the focus of this thesis is limited to the selection of operating point (parametric decision) given the model of the plant and structural decisions on the controller. We also focus on the selection of measurements (structural decision) to improve economic benefits.

1.2 Motivation

In this thesis, we address two important problems viz., sensor network design and set point selection along with controller tuning from a profit perspective. Efficient process monitoring, control and fault diagnosis are vital for optimal and safe operation of a chemical process. The success of each of the above activities depends critically on the choice of the sensor network. Although sensor network design problems have been discussed extensively in literature, they are tailored for each activity independently (Chmielewski *et al.*, 2002; Bhushan and Rengaswamy, 2000). Since the individual design problems are incommensurable, the integration of sensor network design is difficult to account for the multi-faceted elements (observability, controllability, redundancy, accuracy, reliability, etc.,). Several years ago, it was identified that sensor networks ought to be designed so that plant performance is optimal from the profit perspective.

Recently, it has been highlighted by the Engineering Virtual Organization (EVO - a consortium of leading US universities, automation industries and the US NSF), that the optimal sensor network design for process plant is an important research issue. Specific research challenges pertaining to the next-generation sensor network design has been listed as (Davis, 2008):

- "Sensor network for plant status - Design sensor networks to improve plant observability and bias free state estimation and control"
- "Network design for sensor/actuator-based control - Develop associated actuator and sensor instrumentation networks for fault-tolerant control that is compatible with other functions such as quality control, production accounting and on-line optimization"

Therefore, it is necessary to define performance metrics based on process economics for designing sensor networks within the integrated optimization and control framework.

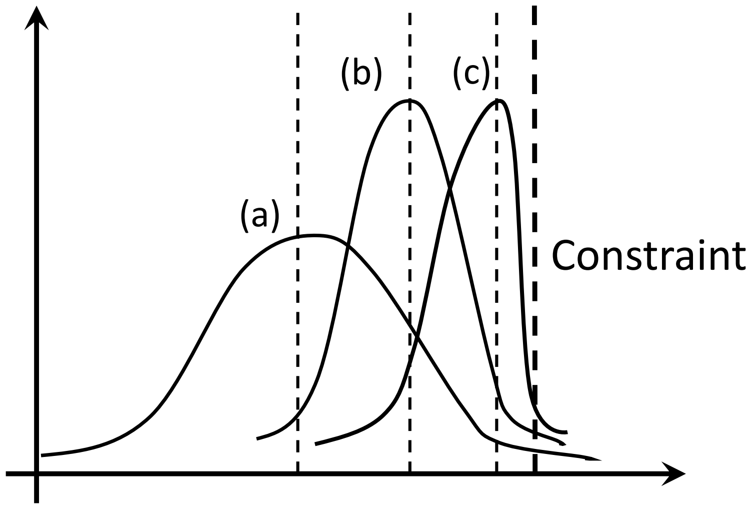

Figure 1.2: Set point selection showing performance of - (a) a badly tuned linear controller, (b) a well tuned linear controller, and (c) a well tuned non-linear controller

In a typical hierarchical decomposition of the plant operation, the set point is selected based on the steady state optimization and the controllers are tuned heuristically. The parametric decisions, such as set point selection and controller tuning, often play a crucial role in the optimal operation of chemical processes. The set point given to the controller determines the static economic performance of the plant whereas controller tuning determines the dynamic economic performance of the plant. Though the optimal performance often occurs at the constraints, it is difficult to operate exactly at the constrained optimal point due to uncertainties. Hence, ensuring feasibility in the presence of disturbance is an essential requirement for safe and efficient operation. In Figure 1.2, the optimal point of the process variable is at the constraints, and hence shows the necessity of set point selection for improving profit while ensuring feasibility. In order to have an acceptably low probability of violating the constraint, the set point for that process variable has to be kept away from the constraints (Maciejowski, 2002). This distance is often called as back-off in control literature. Distribution (a) shows the performance of a badly tuned linear controller resulting in high variability (i.e., higher back-off) and hence the plant operates far away from the optimal point for most of the time. Distribution (b) depicts the performance of the well tuned linear controller having less variability, and hence requiring lesser back-off. Distribution (c) shows the best achievable performance of the well tuned non-linear controller such as Model Predictive Control. Aske (2009) presented the importance of back-off as useful tool for the practitioners and operators. In a recent conference, Modén and Lundh (2013) emphasized the necessity to determine back-off from an industrial perspective and an empirical method to determine back-off was proposed. Hence, the appropriate selection of set point and controller tuning is essential for ensuring operability and improving profitability. Therefore, there is a strong need to devise scientific methods to determine set point and controller tuning for better economic performance.

In this thesis, the relevant decision-making problems will be cast as convex optimization problems, such as semi-definite programs and second-order cone programs. This has received tremendous attention within the research community as an important numerical tool and found a wide range of applications in such diverse fields like traditional convex constrained optimization, systems and control theory, circuit design, and combinatorial optimization, etc. The main advantage of casting such convex problems is that they can solved almost as easily as linear programs using interior-point methods (Boyd and Vandenberghe, 2004).

1.3 Optimal operation

For continuous processes, the foremost step is to determine the optimal steady state operating point for a given design. In general, this is accomplished by solving a nonlinear steady state optimization problem for the nominal values of disturbance vector, $\overline{d}_0 \in \mathbb{R}^{n_d}$. Mathematically, the problem can be expressed as

$$\min_{u_0} \quad J(x_0, u_0, \overline{d}_0) \tag{1.1a}$$

$$s.t. \quad g(x_0, u_0, \overline{d}_0) = 0 \tag{1.1b}$$

$$h(x_0, u_0, \overline{d}_0) \leq 0 \tag{1.1c}$$

where J is the scalar cost function to be minimized (production cost, by-product, etc.) or maximized (profit, productivity, etc.) in terms of state vector, $x_0 \in \mathbb{R}^{n_x}$ and manipulated input vector, $u_0 \in \mathbb{R}^{n_u}$. The equality constraints represent the steady state model of the plant whereas the inequalities define the design constraints, environmental and safety limits, product specifications, etc. The optimal solution is denoted by

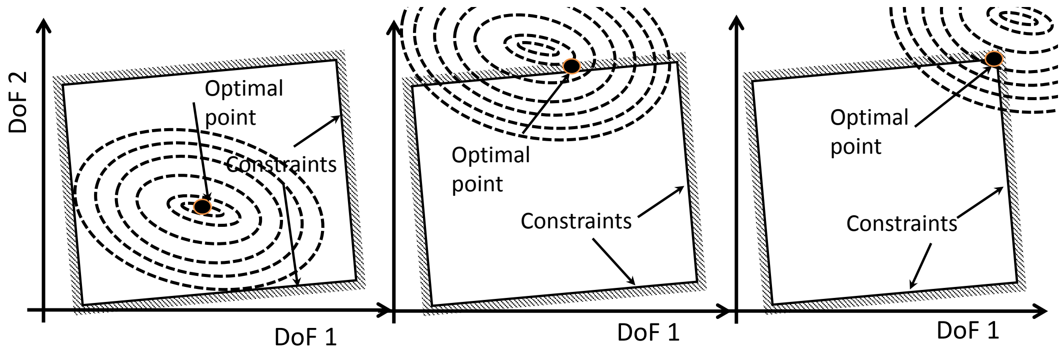

Figure 1.3: Nature of optimal solutions - unconstrained; partially constrained; fully constrained optimum

$\{x_0^*, u_0^*, \overline{d}_0\}$. Typically, the process is set to operate at this optimal steady state point to achieve maximum profit.

Despite the fact that this operating point $\{x_0^*, u_0^*, \overline{d}_0\}$ being most profitable, it may be practically difficult to operate the plant at this point due to the presence of disturbances if the nature of the optimal solution is actively constrained. In order to understand this, first we discuss the different nature of optimal solution and describe the relevant issues associated to it. For this purpose, let us consider the case of quadratic programming problem (J is quadratic; g and h are linear) with two degrees of freedom (i.e., $n_u = 2$). The optimal solution can be either unconstrained (inside the feasible region), partially constrained (at one or more of the constraints but less than the number of degrees of freedom) or fully constrained (at the intersection of constraints) as shown in Figure 1.3. In other words, the number of active constraints at each of the above three cases are zero, less than n_u and equal to n_u, respectively. In general, these are the possible nature of optimal solutions for any optimization problem, except linear programming problems, for which the optimal solution is always at the intersection of constraints.

The relevant issues that are important for each of the three cases to improve profitability are presented below.

1. For the case of unconstrained optimum, we assume that the operating point is far inside the feasible region such that it is very unlikely to violate the constraints. Here, the number of active constraint is zero, i.e., $n_a = 0$. Therefore, the number of unconstrained degrees of freedom available equals the original number of manipulated inputs, $n_{uc,dof} = (n_u - n_a) = n_u > 0$. In this case, the dynamic operation is always feasible (shown as an ellipse in grey scale in the left of Figure 1.4) and hence the conventional PID or Model Predictive Control (MPC) can be used to achieve optimal operation. Further economic benefits can be achieved

by the proper selection of structural decisions. For example, the secondary controlled variables can be selected using the remaining unconstrained inputs based on self-optimizing control principle (Skogestad, 2000). Furthermore, it is important to study the sensor placement problem that will improve the economic performance.

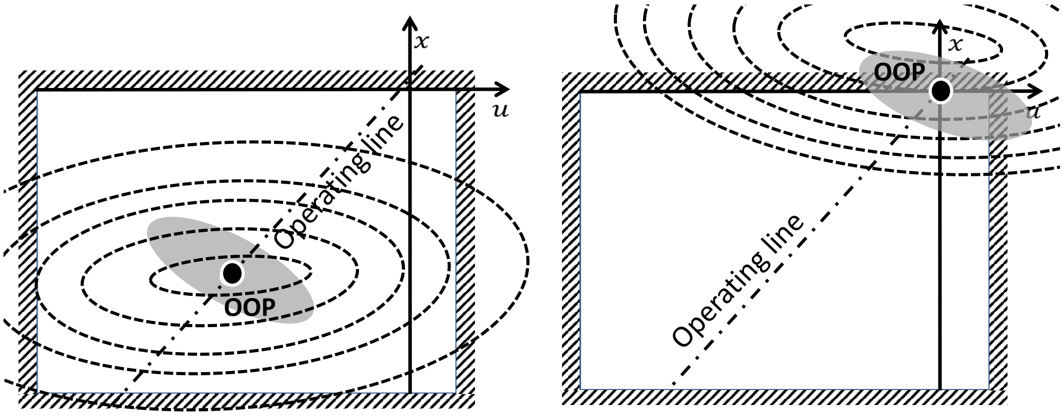

Figure 1.4: Process operation under uncertain conditions in input-output space at unconstrained and constrained optimum

2. For the case of fully constrained optimum, the number of active constraints equals the number of manipulated inputs, $n_a = n_u$. Hence, there are no unconstrained degrees of freedom left for optimal operation (i.e., $n_{uc,dof} = 0$). The operation at this constrained operating point is very difficult, even with the conventional controllers in place, due to the possibility of frequent violation of constraints due to uncertainties (shown as an ellipse in grey scale in the right of Figure 1.4). Therefore, it is necessary to study the profitable yet feasible operating point selection for all possible values of disturbances. Furthermore, selecting measurements and appropriate controller can improve the economic performance of the plant.

3. For the case of partially constrained optimum (general case), some of the constraints are active but the number of active constraints are less than the number of manipulated inputs, $n_a < n_u$ but $n_a > 0$. Hence, there exists some unconstrained degrees of freedom at the optimal operating point, $n_{uc,dof} = (n_u - n_a) > 0$. In this case, there are two possible scenarios:
   (a) If the active constraints shall be controlled at their limiting values, without violating the constraint under uncertain conditions, then the remaining unconstrained degrees of freedom can be used for the selection of measured variables or controlled variables, to improve profitability as in the unconstrained case (see Chapters 3 and 4). On the other hand, if there are no unconstrained degrees of freedom, then we cannot optimize for process economics. In such a case, we can select sensors based on conventional objectives like minimizing the capital cost of sensors, maximizing estimation accuracy or reliability of the sensor network, etc. (see Chapter 2)

   (b) If the active constraints cannot be controlled perfectly under uncertain conditions, then it becomes important to address the issue of determining the profitable operating point, while ensuring feasibility under uncertain conditions (see Chapter 5). Moreover, other structural decisions such as controller design, measurement selection can also be considered (see Chapters 6 and 7). This case is similar to fully constrained case.

In summary, our focus is two-fold: Firstly, we address the measurement selection problem for the case of unconstrained or partially constrained nominal optimum with unconstrained degrees of freedom and secondly, for the case of constrained nominal optimum, we address the problem of set point selection such that it is both profitable and feasible under uncertain conditions.

1.4 Mathematical framework

In this thesis, we cast the decision making process as optimization problems based on strong theoretical foundations. Often, the parametric decisions (say for example, selecting the set point or controller gain) and structural decisions (say for example, a particular variable is being measured or not) are denoted by continuous and discrete decision variables, respectively. In general, these problems are formulated as Mixed Integer Non-Linear Programming (MINLP) problems, for which a candidate optimal solution will guarantee only local optimum. Therefore, in this thesis, we primarily work with conic programming problems which guarantees global optimality. Conic solvers like SeDuMi or SDPT3 can be used under MATLAB to solve Semi-Definite Programming (SDP) problems based on a polynomial-time interior-point method.

1.4.1 Conic programming

The aim of this section is to introduce the reader some of the basic definitions that will be useful in understanding the optimization formulations presented in this thesis. However, it is not meant to be an exhaustive treatment of the topic. For this, the reader is referred to Vandenberghe and Boyd (1996), Alizadeh and Goldfarb (2003) and Anjos and Lasserre (2012).

DEFINITION 1.1 A symmetric matrix $X \in \mathbb{S}^n$ is said to be *positive definite*, denoted by $X \succ 0$, if

$$v^T X v > 0 \quad \text{for all nonzero} \quad v \in \mathbb{R}^n,$$

and *positive semidefinite*, denoted by $X \succeq 0$, if

$$v^T X v \geq 0 \quad \text{for all} \quad v \in \mathbb{R}^n.$$

DEFINITION 1.2 Let two points $x, y \in \mathbb{R}^n$ and $0 \leq \lambda \leq 1$ be given. Then the point

$$z = \lambda x + (1 - \lambda) y$$

is a *convex combination* of the two points x and y.

DEFINITION 1.3 The set $\mathcal{C} \subset \mathbb{R}^n$ is called *convex*, if all convex combinations of any two points $x, y \in \mathcal{C}$ are again in $\mathcal{C}$.

DEFINITION 1.4 The set $\mathbb{K} \subset \mathbb{R}^n$ is a *convex cone* if it is a convex set and for all $x \in \mathbb{K}$ and $\lambda > 0$ we have $\lambda x \in \mathbb{K}$. A cone is called *pointed* if it does not contain any subspace except the origin. A cone is said to be a *proper cone* if it is closed, convex and pointed. The set of non-negative vectors $\mathbb{R}^n_+$ is an example of a proper cone.

DEFINITION 1.5 *Second Order Cone* (SOC) in $\mathbb{R}^{n+1}$ can be expressed as

$$SOC^{n+1} := \{(x, y) \in \mathbb{R}^{n+1} | y \geq \|x\|_2\}$$

The SOC is another example of a proper cone.

DEFINITION 1.6 *Linear Matrix Inequality* (LMI) is an expression of the form,

$$F(x) = F_0 + x_1 F_1 + \cdots + x_m F_m \succeq 0$$

where the matrices $F_i = F_i^T \in \mathbb{S}^n$ are given, and the inequality $F(x) \succeq 0$ means $F(x)$ is positive semidefinite. The set of symmetric positive semidefinite matrices is yet another example of a proper cone. Therefore, the LMI is a convex constraint in the variable $x \in \mathbb{R}^m$. Let X and Y be any symmetric matrices. We also write "$X \succeq Y$" to denote that $X - Y \succeq 0$.

Conic programming is a class of convex optimization problems which minimizes a linear function (or possibly convex quadratic) function over the intersection of an affine set and a proper cone $\mathbb{K}$.

$$\begin{aligned}
\inf_{x} \quad & c^T x && (1.2a)\\
s.t. \quad & a_i^T x = b_i \quad i = 1, \cdots, m && (1.2b)\\
& x \in \mathbb{K} && (1.2c)
\end{aligned}$$

Therefore, *Semi-Definite Programming* (SDP) is a special case of the conic programming problem concerned with optimizing a linear function over the intersection of an affine set and linear matrix inequalities.

$$\begin{aligned}
\min_{x} \quad & c^T x && (1.3a)\\
s.t. \quad & a_i^T x = b_i \quad i = 1, \cdots, m && (1.3b)\\
& F(x) = F_0 + x_1 F_1 + \cdots + x_m F_m \succeq 0 && (1.3c)
\end{aligned}$$

with $F_i = F_i^T \in \mathbb{S}^n, i = 0, \cdots, m.$

1.5 Research objectives

Two important problems under the broad purview of optimal operation are addressed in this thesis:

1. If there exist any unconstrained degrees of freedom (i.e., unconstrained or partially constrained case), $n_{uc,dof} > 0$, we address the following issues with respect to measurement selection problem in the optimization layer:
   (a) What is the rational choice for designing a sensor network? Will such a selection procedure satisfy the typical sensor network properties such as observability, redundancy, estimability, etc.? Does it ensure optimal operation?
   (b) Can such a procedure be adapted to select sensor networks that are robust to sensor failures? If so, how can this be achieved?

2. If there are active constraints with or without unconstrained degrees of freedom (i.e., partially or fully constrained case), we address the following problems relating different structural and parametric decisions in the control layer:
   (a) How does one obtain a profitable yet dynamically feasible operating point?
   (b) Given the controller structure, how does one decide on the controller parameters that will improve profitability while ensuring dynamic feasibility?

(c) How can this performance be obtained using Model Predictive Control (MPC), a widely used controller technique in the process industry.

(d) How does one identify the best set of measurements that will improve profitability?

1.6 Thesis outline

This thesis is organized as follows. Figure 1.5 presents the roadmap of this thesis.

- Chapter 2 presents the brief review of literatures on sensor network design. Next, we briefly describe the data reconciliation problem, followed by the sensor placement problem to obtain better reconciled estimates. In this chapter, we present the Mixed Integer Conic Programming (MICP) formulation for designing sensor networks.

- In Chapter 3, we propose an average loss function based on the notion of operational profit to quantify the performance of sensor network in terms of loss incurred due to measurement errors. An optimization formulation is presented to find a sensor network that minimizes the average loss. Using convex optimization theory, we cast the problem as an MICP, which could be solved for global optimality using existing branch and bound solvers.

- Chapter 4 extends the proposed sensor placement problem based on the average loss function, to design reliable sensor networks that will be robust to sensor failures. This is accomplished by designing redundant sensor networks such that, in case of sensor failures, the resultant subnetwork will be observable. Furthermore, we also modified the optimization formulation to determine the sensor network that minimizes the worst case loss in case of sensor failures.

- Chapter 5 is focused on determining the profitable yet dynamically feasible operating point selection for nominally constrained processes. In this chapter, we survey the relevant literatures and then present a back-off approach to circumvent infeasible operations that could occur due to disturbances entering the process. Since backing off will result in an economic loss, we address the problem by optimizing the economic cost function subject to dynamic feasibility. Here, the amount of back-off is reduced by a suitable controller design. However, the resulting formulation is non-convex and hence we proposed a novel two stage iterative solution technique.

- In Chapter 6, we address the backed-off operating point selection problem using discrete-time formulation for easy implementation using standard MPC framework. For this purpose, the designed linear multivariable controller is converted to equivalent MPC weights and the economic performance of the process is studied. Moreover, relevance to economic MPC is discussed.

- In Chapter 7, we extend the optimization formulation that was addressed for selecting economically backed-off operating point to include sensor selection. Here

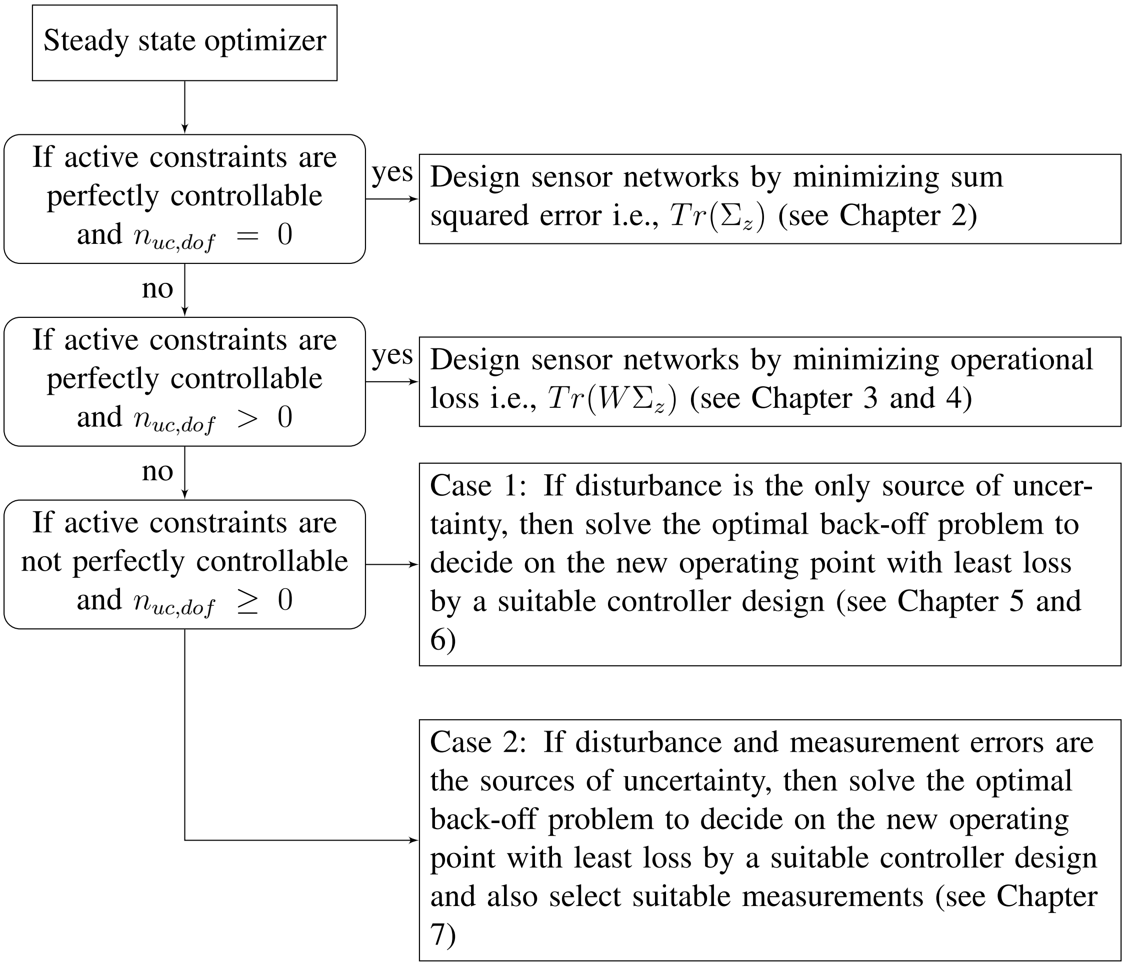

Figure 1.5: Roadmap of this thesis

measurement errors are considered as an additional uncertainty and therefore the choice of measurements is crucial. The sensor selection problem, however, introduces binary decision variables to the optimization formulation. Hence, a branch and bound type solution technique is employed.

- Chapter 8 sums up and recommends suitable extensions to the presented problems.

CHAPTER 2

SENSOR NETWORK DESIGN

In this chapter, we first briefly review the existing literatures on conventional sensor network design procedures. For a particular case of perfectly controllable active constraints with no unconstrained degrees of freedom, the structural decisions such as location of sensors cannot affect the optimal operation. In such a case, the traditional sensor network objectives like minimizing the instrumentation cost of the network, maximizing the reliability of the network or maximizing the estimation accuracy of the network, etc., can be used. However, the optimization formulations will result in MINLPs, in general. Therefore, the primary focus of this chapter is to present an SDP based optimization formulation to design sensor networks that minimizes the estimation error.

2.1 Background

Sensors are the measuring elements of a process plant to infer the state of the process, detect and diagnose faults, give feedback information to the controller, etc. Therefore, the choice of a particular set of sensors plays a crucial role in the optimal operation of a chemical process. The problem of sensor network design (also known as measurement selection) has been widely studied in literature for several decades. However, our literature survey presented here is limited in discussing the developments along the following lines only: (a) First, we survey the different performance measures used in quantifying sensor networks; (b) Next, we discuss the methods developed to handle sensor failure situations, and (c) Finally, we discuss the computational algorithms that have been developed to design sensor networks.

Typically, the problem of sensor network design uses some kind of sensor information such as sensor cost, sensor variability, sensor failure probability, etc. and optimize for some form of the network properties such as precision, reliability, instrumentation cost, etc. while demanding some of the other properties like observability, redundancy, resolvability, etc. being satisfied. In general, sensor network design procedures are carried out to address a specific process activity and the performance measures were defined specific to each activity. From the viewpoint of fault detection and diagnosis, sensors are selected such that the faults are observable and resolvable. Ali (1993) introduced the concept of reliability which uses the available sensor failure probability information to quantify the network performance. In control applications, the problem is usually that of selecting a suitable candidate set of controlled variables (Alstad *et al.*, 2009; Halvorsen *et al.*, 2003). Given the set of measurements, data reconciliation procedures adjust the measurement so as to improve the estimation accuracy. From a data reconciliation perspective, Kretsovalis and Mah (1987) adopted estimation accuracy as a measure to choose sensor networks and also showed that adding redundant measurements improved the estimation accuracy. For efficient process monitoring, a sensor network should be capable of providing precise information about the state of the process in the presence of random and gross errors. Traditional cost based approaches for the design of sensor networks are primarily based on capital cost of the hardware element. Bhushan and Rengaswamy (2002*a*,*b*) formulated an optimization problem that minimizes the capital cost of sensors subject to reliability requirements and vice versa. Specifications on precision, error detectability, resilience can be enforced as constraints to obtain minimum cost networks (Bagajewicz, 1997; Bagajewicz and Sánchez, 2000; Bagajewicz and Cabrera, 2002). Other cost factors such as maintenance cost for sensors have been discussed by Nguyen and Bagajewicz (2009). Also, operational profit based studies are increasingly important in recent years. Profit based metrics are easily decipherable and also of direct use to the end user. From a control viewpoint, Peng and Chmielewski (2005) have developed a simultaneous formulation of sensor selection and minimum backed - off operating point selection by maximizing the operating profit. Likewise, the theory of self-optimizing control addresses the problem of selecting controlled variables that results in a minimum operating loss (Alstad *et al.*, 2009; Halvorsen *et al.*, 2003). Fraleigh *et al.* (2003) have developed expressions for the loss function that quantifies the departure from optimality and have applied them to the sensor selection problem. On the other hand, Narasimhan and Rengaswamy (2007) addressed this problem from a fault diagnostic perspective. Similar attempts have been made in the reconciliation framework by Bagajewicz *et al.* (2005); Mazzour *et al.* (2003). The latter presented an empirical search strategy to arrive at the optimal sensor configuration.

In order to handle sensor failures, it is common to employ a suitable sensor fault detection and diagnostic mechanism. Any such fault diagnostic mechanism (also known as sensor validation technique) usually involves generating residuals from the available set of redundant sensors and analysis the residuals to identify, isolate and eliminate the faulty sensors. For a brief review of various instrument fault diagnosis techniques, the reader is referred to Frank (1990) and Betta and Pietrosanto (1998). All of these sensor validation techniques rely on one desirable property of the sensor network, that is, the reliable set of redundant measurements. In addition, the preventive and corrective maintenance policies that are commonly adopted for the purpose of sensor validation depend on the redundant number of measured variables. Therefore, one of the desirable property of a sensor network to handle sensor failure situations, is redundancy. In this regard, Sánchez and Bagajewicz (2000) studied the selection of optimal number of redundant sensor networks required for employing the corrective maintenance policy in flow networks. Later, Lai *et al.* (2003) addressed the optimal selection of redundant and spare sensors to be used in corrective maintenance policy using genetic algorithms. On the other hand, Nguyen and Bagajewicz (2009) studied the effect of preventive maintenance policies in terms of economic performance of the plant based on stochastic-based accuracy. Some of the previous works that addressed the sensor network design procedure related to sensor fault situations and redundant measurement selection are reviewed here. Bagajewicz and Sánchez (1999) introduced the concept of estimability of a variable to determine the redundant sensor network such that the optimization model of minimizing the capital of sensors satisfy the estimability requirements of each variable. Bhushan *et al.* (2008) addressed the problem of robust design of sensor network for the purpose of fault diagnosis snd proposed to find a sensor network that maximizes the least reliability. However, this reliability is showed to be dual to the error variance problem for the minimum observable case. Therefore, maximizing reliability is equivalent to minimizing error variances (Kotecha *et al.*, 2008).

From a computational viewpoint, there exists several different approaches to design sensor networks. Some of the early works on sensor network design focused on developing algorithms based on graph theory or matrix algebra based methods. Ali and Narasimhan (1993) introduced the concept of maximizing system reliability to design a minimum observable sensor network for linear processes using graph theory based greedy-search algorithms. Similar graph theoretic procedures were later developed for designing redundant sensor networks (Ali and Narasimhan, 1995) and also for bilinear processes (Ali and Narasimhan, 1996). However, the graph theory based methods do not guarantee global optimality. Several algorithms have been developed based on digraph and signed digraph representation of the process model for sensor selection in fault detection and diagnosis framework (Raghuraj *et al.*, 1999; Bhushan and Rengaswamy, 2000). Bagajewicz (1997) developed tree based enumeration algorithm for minimizing the instrumentation cost subject to specifications on precision, error detectability, resilience,etc. However, they are not suitable for large scale systems. Several heuristic approaches such as genetic algorithms and tabu search techniques have also been developed to address the sensor selection problem (Sen *et al.*, 1998; Carnero *et al.*, 2005). Mathematical optimization formulations that minimize the instrumentation cost subject to precision constraints on the variables were put forth by Bagajewicz and Sánchez (2000). These formulations resulted in Mixed Integer Non-Linear Programming (MINLP) problems, which do not ensure global optimality in general. Therefore, Bagajewicz and Cabrera (2002) proposed to transform the problem into Mixed Integer Linear Programming (MILP) problems which however increases the size of the problem thereby demanding higher computational capability. Chmielewski *et al.* (2002) have established the performance specifications as convex LMIs, which explicitly allow for defining binary decision variables without increasing the size of the problem. They proposed a minimal-cost sensor network design formulation subject to convex LMIs which can be solved for global optimality using convex optimization tools (Löfberg, 2004).

To conclude, the existing sensor network design formulations are tailored for each activity independently and hence, there is a strong necessity to define a commensurable performance metric. Secondly, the sensor network we design should be capable of handling actual sensor failure situations. Finally, it is necessary to develop computationally efficient formulations to design sensor networks. In this chapter, we present a computationally efficient SDP approach for designing sensor networks based on minimizing estimation error. In the next chapter, we present the performance metric based on economics and address the problem following the SDP approach presented in the present chapter. In Chapter 4, we present the robust sensor network formulations to handle sensor failure situations.

This chapter is organized as follows: First, we briefly review some of the terminologies used in sensor network design literatures. Next, we present a brief overview of data reconciliation approach presented by Chmielewski *et al.* (2002). Finally, an SDP based sensor network design procedure that minimizes the estimation error is outlined with a simple illustration.

2.2 Preliminaries

In this section, we review some of the basic terminologies used in sensor network design literature.

DEFINITION 2.1 The sensor network is said to be *observable* if there exist atleast one way of estimating all the process variables from the selected measurements using the process model.

DEFINITION 2.2 The sensor network is said to be *minimum observable* if there is exactly one way of estimating all the process variables from the selected measurements using the process model.

DEFINITION 2.3 The sensor network is said to be *redundant* if the network is observable and in addition, some or all of the process variables can be estimated by more than one means.

EXAMPLE 2.1

The purpose of this example is to illustrate some of the qualitative properties of sensor networks. Consider the system with three process units and six streams as depicted in Figure 2.1. The minimum number of independent sensors required for the system to be observable under normal operating conditions is three. If F_1, F_2 and F_4 are measured, then all other variables can be estimated using the model. On the other hand, if we choose to measure F_1, F_2, and F_3 then we cannot estimate F_5 and F_6 as the selected

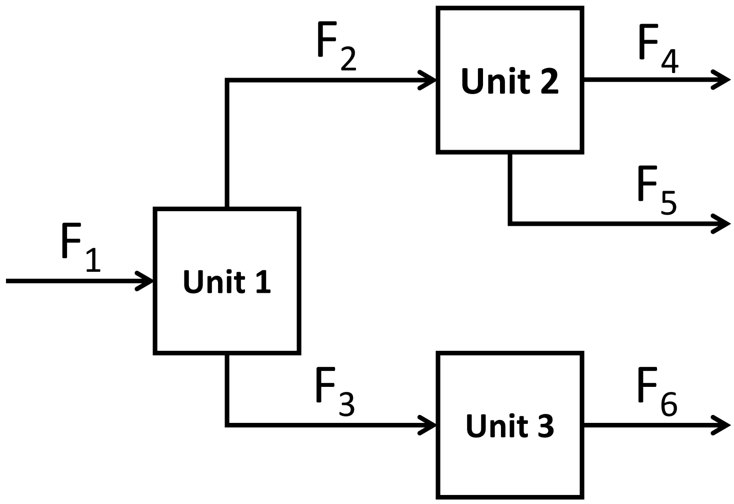

Figure 2.1: Concept of observable and redundant sensor network

set of measurements are not independent. Hence, we say the network $\{F_1, F_2, F_4\}$ is minimum observable and the network $\{F_1, F_2, F_3\}$ is unobservable. The minimum observable sensor network is non-redundant. If we measure F_1, F_2, F_4, and F_5, the sensor network is observable and the variables F_2, F_4, and F_5 can be estimated in two different ways whereas the variable F_1, F_3, and F_6 can be estimated through single means. Now, consider the sensor network $\{F_1, F_2, F_3, F_6\}$. Although the network measures more than the minimum required number of sensors, it is neither observable nor redundant because there is no means to estimate the unmeasured variables F_4 and F_5. Hence, we say the network $\{F_1, F_2, F_4, F_5\}$ is redundant and the network $\{F_1, F_2, F_3, F_6\}$ is non redundant. ∎

2.3 Data Reconciliation (DR)

The sensor network design procedures presented in later sections (or chapters) are based on the data reconciliation framework. Therefore, in this section, we present an overview of the data reconciliation problem. Data reconciliation is a method for improving the accuracy of noisy measurements, and estimating unmeasured quantities, wherever possible, given a model and description of measurement errors. Furthermore, it provides results with improved precision for process economics (mainly accounting), for on-line modeling and optimization, and it is useful for instrument maintenance.

2.3.1 Formulation 1 (Constrained DR problem)

Typically, the linear data reconciliation problem is formulated as follows (Narasimhan and Jordache, 2000): Consider a vector of measurements denoted by y_m of dimension n_y, where n_y denotes the number of measurements. This measurement vector is related to the actual value of the process variable vector, z_m, through the measurement equation:

$$y_m = z_m + v_m \tag{2.1}$$

where the error vector, v_m, is assumed to be normally distributed with zero mean and diagonal covariance matrix, $\Sigma_v = \mathbb{E}[v_m v_m^T]$. Denoting n as the total number of variables and arranging the remaining $(n - n_y)$ unmeasured process variables in vector z_u, we can partition the steady state linearized model (in deviation form) as

$$A_1 z_m + A_2 z_u = 0 \tag{2.2}$$

where the number of rows of A_1 (and A_2) equals the number of model equations (n_e) representing the process, whereas the number of columns of A_1 and A_2 equal the number of measured (n_y) and unmeasured variables $(n - n_y)$, respectively.

The steady state reconciliation problem is formulated to minimize the appropriate least square residual such that the model equations are satisfied. Mathematically, the problem is stated as:

$$\min_{\hat{z}_m, \hat{z}_u} \quad (y_m - \hat{z}_m)^T Q (y_m - \hat{z}_m) \tag{2.3}$$

$$s.t. \qquad A_1 \hat{z}_m + A_2 \hat{z}_u = 0 \tag{2.4}$$

where the weighting matrix $Q = \Sigma_v^{-1} = diag\{\frac{1}{{\sigma_i}^2}\}$ and σ_i^2 is the variance of the measurement i. The optimal solution of the problem, $\hat{z}_m$ and $\hat{z}_u$, is usually called the reconciled value or estimate.

2.3.2 Formulation 2 (Unconstrained DR problem)

An alternate but equivalent formulation has been presented by Chmielewski *et al.* (2002) in which the variables are classified as: primary variables (z_p) and secondary variables (z_s). Denote all possible measurements of interest in the process by z. Then the primary variables, z_p, can be chosen as any subset of process variables (z) that form a minimum observable set. In other words, primary variables are any set of independent variables that form a minimum observable set, which can be either measured or unmeasured whereas the remaining variables form a secondary set (see Example 2.2 for selecting primary and secondary variables). Using this classification of variables, the process model can be expressed as:

$$A_p z_p + A_s z_s = 0 \tag{2.5}$$

As the primary variables are selected such that they form a minimum observable network, the matrix A_s is invertible. Applying block elimination, we have:

$$z_s - B z_p = 0 \tag{2.6}$$

where $B = -A_s^{-1} A_p$. Now, the set of all variables of interest, $z = [z_p^T \ z_s^T]$ is given by

$$z = C z_p \tag{2.7}$$

where the matrix $C = [I; B]$ and I is the identity matrix of size equal to the primary variables. The number of rows of C are equal to the total number of variables, while the number of columns equal the number of primary variables.

EXAMPLE 2.2

The purpose of this example is to illustrate how to choose primary variables and the process matrix C. For the splitter unit shown in Figure 2.2, the set of variables of interest, $z = [F_1 \ F_2 \ F_3]^T$ and the process model is $F_2 = F_1 - F_3$. Thus, to observe the system, at least two independent variables have to be measured. In this particular example, any two variables form an independent set and hence can be chosen as primary variables. Hence, the C matrices for different choice of primary variables are as follows:

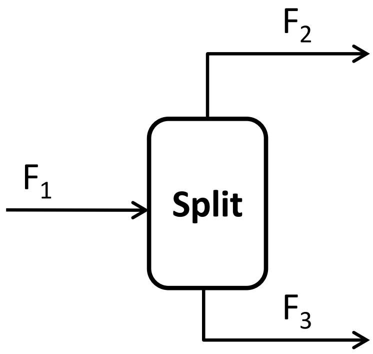

Figure 2.2: Splitter unit

Choice 1: For $z_p = [F_1\ F_2]^T$,

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$$

Choice 2: For $z_p = [F_1\ F_3]^T$,

$$C = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Choice 3: For $z_p = [F_2\ F_3]^T$,

$$C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

It should be noted that the matrix C incorporates the process model inherently in it such that $z = Cz_p$ $\blacksquare$

Now, the measurement equation (2.1) can be re-written in terms of primary variables as

$$y = z + v \tag{2.8}$$

or equivalently,

$$y = Cz_p + v \tag{2.9}$$

where the variable y contains all variables of interest in the process. It is important to note that this measurement equation incorporates the process model implicitly. The data reconciliation problem can be reformulated as

$$\min_{\hat{z}_p} (y - C\hat{z}_p)^T Q(y - C\hat{z}_p) \tag{2.10}$$

where the weighting matrix Q is given by

$$Q = diag\{\frac{q_i}{{\sigma_i}^2}\} \tag{2.11}$$

where q_i is a binary variable (0 or 1) depending on whether the particular variable is unmeasured or measured, respectively. An unmeasured variable ($q_i = 0$) can also be statistically inferred as a sensor with infinite variance. Also, the fact that a particular variable is measured or not is reflected through the weighting matrix Q and hence, this formulation is equivalent to the earlier formulation. However, it is important to note that Q or equivalently q_i's are not decision variables in reconciliation problem. Because the above problem (2.10) is unconstrained in contrast to the constrained one discussed in formulation (2.3), the first order optimality condition yields an analytical solution

$$\hat{z}_p = (C^T QC)^{-1} C^T Qy \tag{2.12}$$

which is the weighted least square solution. Given an estimate of the primary variables $\hat{z}_p$, the estimate of all variables is given by $\hat{z} = C\hat{z}_p$. The error covariance matrix, Σ_z, of estimation error in $\hat{z}$ computed from the above solution is

$$\Sigma_z = C(C^T QC)^{-1} C^T \tag{2.13}$$

Detailed derivation of equation (2.13) is provided in Appendix A. It is important to recall that the objective of data reconciliation is to improve the accuracy of the data (i.e. adjust the data that result in least residual) given the set of measurements. On the other hand, the sensor network design objective is to select those variables (or sensors) that maximizes a performance objective. In this regard, formulation (2.10) is of direct use for designing sensor networks with q_i's as decision variables. Thus, the classification of primary and secondary variables provide an elegant way of defining the sensor network design problem to address the data reconciliation objective.

2.4 Sensor Network Design (SND)

The fundamental problem in optimal sensor network design is to choose a set of important or strategic process variables to be measured. The selection of measured variables is an indispensable task for effective control, monitoring, and safe operation of a chemical process. Of the several hundred variables that exist in a typical chemical process, only a subset of these variables can be measured, because of the nature of the process and the high cost of measuring instruments. From a computational viewpoint, it is a combinatorially difficult problem owing to the large number of variables in the process. For the purpose of efficient process monitoring, a sensor network should be capable of providing precise information about the state of the process in the presence of random and gross errors. Kretsovalis and Mah (1987) adopted estimation accuracy as a measure to choose sensor networks. As an overall measure of estimation accuracy, the sum of squares of estimation error of all variables (which measures the overall inaccuracy) has been used. Other measures such as minimizing the log volume of the covariance of the estimation error (or mean radius) or worst case error variance over all the direction can also be used (Joshi and Boyd, 2009).

2.4.1 Formulation 3 (SND based on overall error)

The typical objective of a sensor network design problem based on reconciliation framework is to choose the set of measurements so as to minimize the overall estimation error of the reconciled estimates i.e., $Tr(\Sigma_z)$ (Narasimhan and Jordache, 2000) where $Tr(.)$ denotes the trace operator. Recalling $Q = diag\{\frac{q_i}{\sigma_i{}^2}\}$ where q_i's are binary variables. Notice that $q_i = 1$ implies the variable z_i is measured and vice-versa. The sensor network design problem can be mathematically stated using expression (2.13) as:

$$\min_{q_i} \quad Tr(C(C^TQC)^{-1}C^T) \tag{2.14}$$

where the invertibility of C^TQC signifies that the system is observable. However for the overall error to be minimum, C^TQC has to be positive definite. Thus, the current formulation inherently considers the observability issue and yields an observable sensor network. It should be noted that the formulation (2.14) is a non-linear integer programming problem.

2.4.2 SDP reformulation

The objective of this subsection is to present the sequence of steps so that the resulting problem is a semidefinite programming problem with relaxed integer constraints, which can be solved to global optimality using existing branch and bound solvers. First, we present here some of the results from convex optimization theory for the sake of convenience,

Fact 01 The epigraph of a function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ is defined as $\mathbf{epi}\ f = \{(x,t)|x \in \mathbf{dom}\ f, f(x) \leq t\}$. A function is convex if and only if its epigraph is a convex set.

Fact 02 If D is positive definite, i.e., $D \succ 0$, then the matrix $S = A - BD^{-1}B^T$ is called the Schur complement of D in the matrix $X = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix}$. Then the condition for positive semi-definiteness of block matrix X is: If $D \succ 0$, then $X \succeq 0$ if and only if $S \succeq 0$

Using the definition of epigraph of the function (see Fact 01), the non-linear integer programming formulation (2.14) can be rewritten using a scalar cost function (t_e) as

$$\min_{t_e, q_i} \quad \overline{L}_{error} = t_e \tag{2.15a}$$

$$s.t. \quad Tr(C(C^TQC)^{-1}C^T) \leq t_e \tag{2.15b}$$

Now, introducing a positive definite (or semidefinite) matrix (Y_e), the last inequality constraint (2.15b) can be written as

$$Tr(Y_e) \leq t_e \tag{2.16a}$$

$$Y_e - C(C^TQC)^{-1}C^T \succ 0 \tag{2.16b}$$

Using Schur complement (see Fact 02), the matrix inequality constraint (2.16b) can be written in LMI form,

$$\begin{bmatrix} Y_e & C \\ C^T & (C^TQC) \end{bmatrix} \succ 0; \tag{2.17}$$

The sub matrix, (C^TQC), in the above LMI has to be positive definite for the LMI to be positive definite. Hence the observability of the sensor network is ensured if the LMI is satisfied. Therefore, the sensor network design problem that finds k sensors that minimizes the overall estimation error is cast as

$$\min_{q_i,t_e,Y_e} \quad \overline{L}_{error} = t_e \tag{2.18a}$$

$$s.t. \quad Tr(Y_e) \leq t_e \tag{2.18b}$$

$$\begin{bmatrix} Y_e & C \\ C^T & (C^TQC) \end{bmatrix} \succ 0 \tag{2.18c}$$

$$q_i \in \{0, 1\} \tag{2.18d}$$

$$Q = diag\{\tfrac{q_i}{\sigma_i{}^2}\} \tag{2.18e}$$

$$\textstyle\sum_{i=1}^{n_z} q_i = k \tag{2.18f}$$

Additionally, we imposed a cardinality constraint (2.18f) to limit the number of sensors being selected. For the system to be observable, we need to choose a value of k greater than or equal to the minimum number of sensors. The minimum number of sensors required for the system to be observable is given by the steady state degrees of freedom of the system, which is defined as the number of variables minus number of equations. The integer restriction of $q_i \in \{0, 1\}$ makes the above formulation non-convex. However, linear relaxation of q_i (i.e., $0 \leq q_i \leq 1$) results in a convex SDP problem. Hence, we can solve the problem to determine globally optimal sensor network using available solvers (Löfberg, 2004). A branch and bound algorithm which uses the internal SDPT3 solver (to solve the SDP with linear relaxation at each branching step) is used to solve the problem.

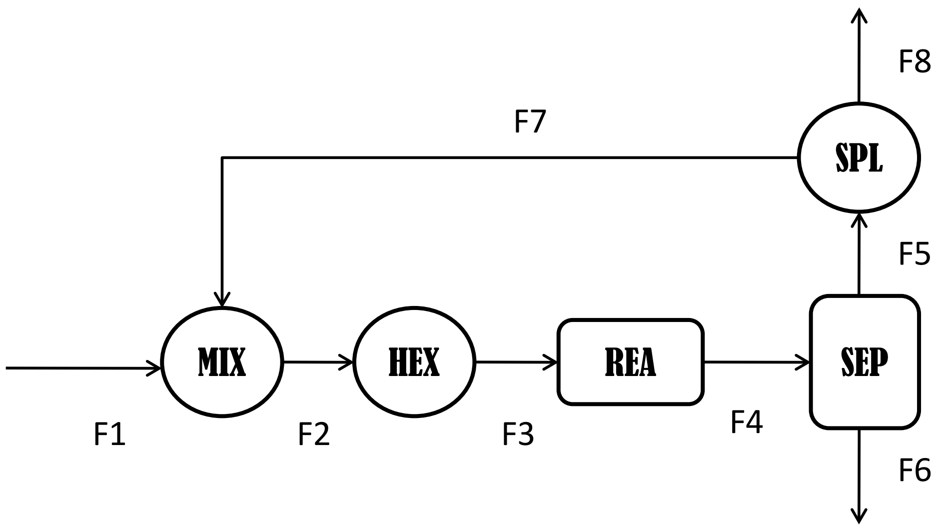

Figure 2.3: Simple ammonia process

2.4.3 Illustration: Simple ammonia process

In this section, we demonstrate the applicability of the presented sensor network design approach to a flow network of an ammonia process described in Narasimhan and Jordache (2000). Figure 2.3 depicts the flow network of the system and the steady state balance equations governing the process are given by

$$F_2 - F_1 - F_7 = 0 \tag{2.19}$$

$$F_3 - F_2 = 0 \tag{2.20}$$

$$F_4 - F_3 = 0 \tag{2.21}$$

$$F_5 + F_6 - F_4 = 0 \tag{2.22}$$

$$F_7 + F_8 - F_5 = 0 \tag{2.23}$$

The minimum number of sensors required for the system to be observable is 3. Therefore, we need to measure at least three independent variables for this system to be observable. Here our objective is to design a minimum observable sensor network. Therefore, we set $k = 3$. For $z_p = [F_2 \; F_5 \; F_8]^T$, the process matrix is given by

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}^T$$

The formulation (2.18) can be solved to global optimality using YALMIP, a freely available toolbox for solving convex and non convex optimization problems in MATLAB (Löfberg, 2004). Assuming the variance of the measured variable to be unity, the sensor network which has a minimal overall error was found to be $\{F_3, F_5, F_7\}$ and the overall error value is 11.

2.5 Summary

In this chapter, we presented an overview of data reconciliation problem to obtain better estimates of process variables. Since the choice of measured variables is critical for reconciliation activity, the sensor network design procedure that uses mean squared error as a measure of estimation accuracy was discussed. Next, an MICP reformulation of the sensor network design problem was set-up which is solved to global optimality using available solvers. However, such a sensor network design procedure cannot guarantee the economic benefit one would achieve by selecting measurements. This will be dealt in the next chapter.

CHAPTER 3

SENSOR NETWORK DESIGN FOR OPTIMAL PROCESS OPERATIONS

Based on the paper published in Industrial & Engineering Chemistry Research

The focus of this chapter is to develop a sensor network design procedure that relates process economics and precision of estimates obtained by data reconciliation. For this purpose, we define an average loss function, which could incorporate the loss in operational profit due to model uncertainties, disturbances and measurement errors. However, the current study focuses on the loss incurred due to measurements corrupted by random Gaussian errors only. The resulting analytical expression that quantifies the loss is shown to be the sum of weighted error variances of the reconciled estimates obtained from reconciliation. Similar to the SDP based sensor network design procedure presented in Chapter 2, we present the formulation that minimizes the average loss function. Demonstrative examples are provided to illustrate the approach.

As discussed in Chapter 1, the optimal operation of a chemical process depends on various structural and parametric decisions. Selection of sensors is an important structural decision for the safe and optimal operation of a chemical plant. The sensor selection procedures for the purpose of data reconciliation typically uses some measure of estimation accuracy such as mean squared error, mean radius or worst case error variance over all direction. However, the resulting sensor network using such performance measures need not select the economically important variables as measurements. This is illustrated using the following Example 3.1.

EXAMPLE 3.1

This example illustrates the need for a new profit based metric to quantify the sensor network. Consider the process unit presented in Example 2.2. Assuming the variance of the measured variable to be unity, the error covariance matrices for different sensor networks are presented below

Case 1: If F_1 and F_2 are measured

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \Sigma_z = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Case 2: If F_1 and F_3 are measured

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \Sigma_z = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Case 3: If F_2 and F_3 are measured

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \Sigma_z = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Clearly all networks have the same overall error (i.e.,$Tr(\Sigma_z) = 4$) which implies that we can choose any sensor network. However, the individual error variance of a variable is different for different choice of a sensor network. On the other hand, say F_3 is the product stream, then we need to measure or estimate this economically important variable more accurately than others. In other words, we need to incorporate this economic information in the selection process. Therefore, the gist of this work is to choose a weighting matrix reflecting the economic importance of the variables of interest for the sensor network to be economically optimal. ∎

In this chapter, the "average loss" is formally defined and an analytical expression for the same is obtained and the importance of the formulation is exemplified using a simple process unit. Following that, the problem is reformulated by rewriting the constraints as LMIs for which solvers are currently available. Finally, the proposed formulation is illustrated using case studies.

3.1 Sensor network design for optimal operation

We make the following assumptions in formulating the problem:

- Measurement error is the only source of uncertainty and it is characterized by a zero mean Gaussian white noise process with known variance.
- Active constraints, if any, are perfectly controlled and measured directly.
- There exists some unconstrained degrees of freedom.

3.1.1 Problem formulation

The key idea behind deciding the weighting matrix based on process economics is that any deviation from optimal operation results in a loss. The nominal operating point is determined by solving a nonlinear optimization problem (which maximizes the operating profit of the plant) subject to the set of equality and inequality constraints. The equality constraints are typically model equations while inequality constraints include safety, design, thermodynamic feasibility and product specification. The cost function J describes the negative operational profit of the process accounting for product value, raw material cost and utility cost. It is assumed that the active constraints (if any) are perfectly controlled. Let us denote the set of manipulated variables by $u \in \mathbb{R}^{n_u}$ and exogenous disturbances by $d \in \mathbb{R}^{n_d}$ and the set of all variables of interest by $z \in \mathbb{R}^{n_z}$ (i.e., $z = [d\ u\ x]^T$) where x stands for internal variables and $n_z = n_u + n_d + n_x$. The equality constraints and active constraints are linearized and the corresponding optimization problem in the reduced space can be written in terms of the remaining unconstrained degrees of freedom:

$$\min_{u} \quad J(u,d) \tag{3.1}$$

Rewriting all the variables in deviation form and expanding the profit function about the optimal point, we obtain an approximation for the profit function that is accurate upto second order (Alstad *et al.*, 2009),

$$J = \frac{1}{2}u^T J_{uu} u + u^T J_{ud} d + \frac{1}{2} d^T J_{dd} d + u^T J_u + d^T J_d \tag{3.2}$$

Assuming the nominal point is optimal, $J_u = 0$. From the first order optimality condition, the optimal values of u and the associated cost function is found to be

$$u^* = -J_{uu}^{-1} J_{ud} d \tag{3.3}$$

$$J^* = -\frac{1}{2} d^T J_{ud}^T (J_{uu}^{-1})^T J_{ud} d + \frac{1}{2} d^T J_{dd} d + d^T J_d \tag{3.4}$$

However, in reality, equation (3.3) cannot be satisfied exactly due to uncertainty in the model and measurements. Here, we assume that the only source of uncertainty is random errors affecting the measurements. When all variables z are observable, the reconciled estimates, $\hat{u}$ and $\hat{d}$ will satisfy the following:

$$\hat{u} = -J_{uu}^{-1} J_{ud} \hat{d} \tag{3.5}$$

In this work, we assume that a suitable controller is available for implementing the optimal strategy and to control active constraints. Now, we can equivalently express the above equation as

$$u^* + \varepsilon_u = -J_{uu}^{-1} J_{ud} (d + \varepsilon_d) \tag{3.6}$$

where ε_d and ε_u are the errors in the estimate of d and u^* respectively. The resulting achieved cost, $\hat{J}$ can be expressed as follows:

$$\begin{aligned}
\hat{J} = & -\frac{1}{2} d^T J_{ud}^T (J_{uu}^{-1})^T J_{ud} d + \frac{1}{2} d^T J_{ud}^T (J_{uu}^{-1})^T J_{ud} \varepsilon_d + \frac{1}{2} d^T J_{ud}^T (J_{uu}^{-1})^T J_{uu} \varepsilon_u \\
& -\frac{1}{2} \varepsilon_d^T J_{ud}^T (J_{uu}^{-1})^T J_{ud} d + \frac{1}{2} \varepsilon_d^T J_{ud}^T (J_{uu}^{-1})^T J_{ud} \varepsilon_d + \frac{1}{2} \varepsilon_d^T J_{ud}^T (J_{uu}^{-1})^T J_{uu} \varepsilon_u \\
& -\frac{1}{2} \varepsilon_u^T J_{ud} d + \frac{1}{2} \varepsilon_u^T J_{ud} \varepsilon_d + \frac{1}{2} \varepsilon_u^T J_{uu} \varepsilon_u + \frac{1}{2} d^T J_{dd} d + d^T J_d
\end{aligned} \tag{3.7}$$

where $\hat{J} \geq J^*$ and hence any implementation strategy results in a loss. Due to the randomness in measurement error, we express the term average loss as the statistical expectation of the deviation of the achieved cost from the optimal cost. Mathematically, the average loss, $\overline{L}_{cost}$ is defined as

$$\overline{L}_{cost} = \mathbb{E}(\hat{J} - J^*) \tag{3.8}$$

Now, assuming ϵ_u is uncorrelated with d,

$$\overline{L}_{cost} = \mathbb{E}(\tfrac{1}{2}\varepsilon_d^T J_{ud}^T (J_{uu}^{-1})^T J_{ud}\varepsilon_d + \tfrac{1}{2}\varepsilon_d^T J_{ud}^T (J_{uu}^{-1})^T J_{uu}\varepsilon_u + \tfrac{1}{2}\varepsilon_u^T J_{ud}\varepsilon_d + \tfrac{1}{2}\varepsilon_u^T J_{uu}\varepsilon_u) \quad (3.9)$$

$$= \tfrac{1}{2}Tr(W\mathbb{E}([\varepsilon_d^T \varepsilon_u^T \varepsilon_x^T][\varepsilon_d \varepsilon_u \varepsilon_x])) \quad (3.10)$$

where, the weighting matrix W is symmetric and is given by

$$W = \left[\begin{array}{cc|c} \overbrace{J_{ud}^T (J_{uu}^{-1})^T J_{ud}}^{n_d \mathrm{x} n_d} & \overbrace{J_{ud}^T}^{n_d \mathrm{x} n_u} & \overbrace{\mathbf{0}}^{(n_d+n_u)\mathrm{x} n_x} \\ J_{ud} & \underbrace{J_{uu}}_{n_u \mathrm{x} n_u} & \vdots \\ \hline \mathbf{0} & \cdots & \underbrace{\mathbf{0}}_{n_x \mathrm{x} n_x} \end{array}\right] \quad (3.11)$$

And, noting the error vector, $\varepsilon_z = [\varepsilon_d \varepsilon_u \varepsilon_x]$ and recalling the estimation error covariance matrix, $\Sigma_z = \mathbb{E}(\varepsilon_z {\varepsilon_z}^T) = C(C^T QC)^{-1}C^T$ as given in equation (2.13), we obtain the following expression for average loss:

$$\overline{L}_{cost} = \frac{1}{2}Tr(W\Sigma_z) \quad (3.12)$$

Hence, the average loss is shown to be the weighted sum of error variances (and covariances) of individual measurements, where the weights reflect the economic importance of each individual measurement and their interactions. Therefore, our objective is to obtain a sensor network that minimizes this average loss. In other words, find $Q(= diag\{\frac{q_i}{{\sigma_i}^2}\})$ that results in minimum average loss. The resulting formulation is mathematically stated as

$$\min_{q_i} \quad \overline{L}_{cost} = \frac{1}{2}Tr(WC(C^T QC)^{-1}C^T) \quad (3.13)$$

Remarks

1. The most important attribute of the weighted formulation is that it inherently allows for the variance of the different variables to be compared. In other words, the choice of scaling required for handling different physical variables (such as temperature and pressure) are taken care by the formulation rigorously.
2. The weighting matrix W is independent of J_{dd} and J_d as they do not get affected while implementing u^* and they cancel each other in computing $\hat{J} - J^*$. There-

fore, it is sufficient to obtain only J_{uu} and J_{ud} and the positive definiteness of J_{uu} (i.e., $J_{uu} \succ 0$) assures that the optimal solution is minimum. It is not necessary to obtain an explicit reduced quadratic cost function. J_{uu} and J_{ud} can be obtained numerically perturbing the inputs and disturbances around the nominal optimal operating point.

3. If J_{uu} is not invertible (i.e., $J_{uu} \succeq 0$), then it means that the columns (or rows) of J_{uu} are linearly dependent. In other words, it implies that there are multiple solutions for the optimal input and we may fix some inputs (depending on the rank of J_{uu}) and allow the other inputs to be determined. Mathematically, this can be achieved by applying singular value decomposition analysis. We identify a set of inputs that are economically unimportant based on the zero singular values and solve the optimization problem for the reduced Hessian. This in turn conditions the Hessian.

4. The loss is clearly a function of how accurately the inputs u and disturbances d are estimated. However, this does not imply that other variables are unimportant as the variances of estimates of u and d are themselves a function of the sensor network and the model, i.e., $trace(W\Sigma_z)$ is important. This is demonstrated in Example 3.3.

5. The analytical expression (3.13) is an important result of our formulation as it could be readily used to choose new measurements for retrofitting an existing network. This can be done by setting $q_i = 1$ for the available measurements and requiring the formulation to find the additional set of measurements from the remaining variables.

EXAMPLE 3.2

Recall the splitter unit considered in Example 3.1, if $d = [F_1]$ and $u = [F_3]$, then $z = [F_1\ F_3\ F_2]^T$. Let $z_p = [F_1\ F_3]$, then C matrix and Q (for the sensor network $\{F_1, F_2\}$) are

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}; Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Assuming the cost function $J = (F_3 - F_1)^2$, then $J_{uu} = 2$ and $J_{ud} = -2$ results in

$$W = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \Sigma_z = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Hence the average loss, $\overline{L}_{cost} = \frac{1}{2}Tr(W\Sigma_z) = 1$. For sensor networks $\{F_1, F_3\}$ and $\{F_2, F_3\}$, the average loss are 2 and 1 respectively. It is clear that the network $\{F_1, F_3\}$

is not economically optimal. This simple unit is presented to illustrate the basic idea.

■

EXAMPLE 3.3

Recall the splitter unit considered in Example 3.2, if $d = [F_1]$ and $u = [F_3]$, then $z = [F_1\ F_3\ F_2]^T$. Let $z_p = [F_1\ F_3]$, then C matrix and Q (assuming unit variance for all flow variables) are

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}; Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}$$

The error covariance matrix, Σ_z, can be expressed in terms of sensor network using (2.13) as

$$\Sigma_z = \frac{1}{q_1q_2 + q_1q_3 + q_2q_3} \begin{bmatrix} q_2 + q_3 & q_3 & q_2 \\ q_3 & q_1 + q_3 & -q_1 \\ q_2 & -q_1 & q_1 + q_2 \end{bmatrix}$$

Let us consider the weighting matrix of the form

$$W = \begin{bmatrix} w_{11} & w_{12} & 0 \\ w_{21} & w_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now the sensor network design formulation (3.13) based on average loss function, after simplification, is given by

$$\overline{L}_{cost} = \frac{w_{11}(q_2 + q_3) + (w_{12} + w_{21})q_3 + w_{22}(q_1 + q_3)}{2(q_1q_2 + q_1q_3 + q_2q_3)}$$

Clearly the average loss function, $\overline{L}_{cost}$, is a function of covariance estimates of all the process variables, which are, indeed, functions of sensor network we choose among all process variables of interest. It is important to note that the covariance estimates are not just the functions of manipulated (F_3) and disturbance variables (F_1) only. ■

3.1.2 Mixed Integer Cone Program (MICP)

The weighted formulation (3.13) is a non-linear integer programming problem. However, we can pose the problem with integer relaxations as a semidefinite programming problem similar to the one discussed in Section 2.4.2. The resulting MICP can be solved using available software to obtain globally optimal solutions.

First, we factorize the weighting matrix as $W = RR^T$, where R is a positive semi-definite square root of W to induce symmetry in the matrix inequality. Invoking the property that trace operator is invariant under cyclic permutations (i.e., $Tr(ABD) = Tr(BDA) = Tr(DAB) \neq Tr(ADB)$), the average loss can be expressed as (Refer equation 3.13)

$$\overline{L} = \frac{1}{2}Tr(WC(C^TQC)^{-1}C^T) \quad (3.14)$$

$$= \frac{1}{2}Tr(RR^TC(C^TQC)^{-1}C^T) \quad (3.15)$$

$$= \frac{1}{2}Tr(R^TC(C^TQC)^{-1}C^TR) \quad (3.16)$$

In the overall error formulation, we selected 'k' sensors such that k is any value greater than N_{min} where N_{min} is the number of degrees of freedom of the system considered required for observability. Thus the sum of all q_i is set to k,

$$\sum_{i=1}^{n_z} q_i = k \quad (3.17)$$

The above constraint allows one to find a redundant sensor network if you set $k > N_{min}$. It is well known that redundancy in measurements improves the quality of the reconciled estimates. Thus, selecting more measurements improves the accuracy of the estimates which in turn reduces the average loss as a result of error reduction. However this might lead to a situation where all measurements are selected if you set $k \geq N_{min}$. On the other hand, it is more meaningful to impose a constraint on the available capital cost. Thus, the capital cost constraints to account for the available resource limit (c^*)

can be incorporated as follows:

$$\sum_{i=1}^{n_z} c_i q_i \leq c^* \tag{3.18}$$

where c_i is the individual sensor cost.

Now, let us recall the original overall error formulation (2.14) and compare it with the above average loss formulation (3.14). The mathematical structure of both the problems are the same. Therefore, we can employ the same convex optimization results to reformulate the problem as

$$\min_{t_c, q_i, Y_c} \quad \overline{L}_{cost} = \frac{1}{2} t_c \tag{3.19a}$$

$$s.t. \quad Tr(Y_c) \leq t_c \tag{3.19b}$$

$$\begin{bmatrix} Y_c & R^T C \\ (R^T C)^T & (C^T Q C) \end{bmatrix} \succ 0 \tag{3.19c}$$

$$q_i \in \{0, 1\} \tag{3.19d}$$

$$Q = diag\{\frac{q_i}{\sigma_i{}^2}\} \tag{3.19e}$$

$$\sum_{i=1}^{n_z} c_i q_i \leq c^* \tag{3.19f}$$

Now the problem is to determine the binary variable q_i that minimizes the average loss. As the problem is a mixed integer cone program, it can be solved using a branch and bound procedure. It is clear that the constraint (3.19c) is linear in Q which in itself is linear in the binary decision variables q_i. Hence, the linear relaxation of the integer constraints is a convex problem. Hence, at each stage of the branch and bound algorithm, a convex problem is solved and this guarantees global optimality, which is not possible in general MINLP problems. The original problem can be solved to global optimality using YALMIP, a freely available software for solving convex optimization problems developed by Löfberg (2004). For some large scale systems, as the number of variables are quite large, the branch and bound method might be computationally demanding. In such cases, we need specialized algorithms to speed up the solution.

3.1.3 Illustration 1: Simple ammonia process

Problem description. Let us revisit the flow network of a simplified ammonia process discussed in Section 2.4.3 and the system is shown in Figure 2.3. In this section, our objective is to demonstrate the average loss formulation of the sensor network design problem presented in this chapter.

Degree of freedom analysis. The number of degrees of freedom for the system is three (N_{min} = 3) of which F_1 is considered to be the disturbance while F_5 and F_7 are considered to be the manipulated variables.

Process matrix. For $z_p = [F_2\ F_5\ F_7]^T$, the process matrix is given by

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T \tag{3.20}$$

Weighting matrix. Let us assume the cost function to be $J = (F_5 - F_7)^2 + (F_5 + F_1)^2$ and the Hessian matrices obtained are the following

$$J_{uu} = \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix}; J_{ud} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \tag{3.21}$$

The resulting weighting matrix is

$$W = \left[\begin{array}{ccc|c} 2 & 2 & 0 & \overbrace{\mathbf{0}}^{3\text{x}5} \\ 2 & 4 & -2 & \vdots \\ 0 & -2 & 2 & \\ \hline \mathbf{0} & \cdots & & \mathbf{0} \end{array}\right] \tag{3.22}$$

Results. To make a fair comparison, we only consider minimum observable sensor networks (i.e., it is assumed that the cost of the individual sensors are same and the capital cost is available for selecting only the minimum number of sensors). The average loss computed through enumeration for 32 minimum observable sensor networks of the simplified ammonia process using the above proposed formulation are compared with the overall error formulation as studied by Narasimhan and Jordache (2000). Equal sensor cost and unit variance are assumed in this example to explain the contribution of the operating profit, and the results obtained are listed in Table 3.1. For the cost function considered above, the first term represents the importance of F_8 while the other term signify the combined importance of F_1 and F_5. Hence it is trivial that the network $\{F_1, F_5, F_8\}$ is economically optimal since it is observable. Therefore, to select other combinations, it is tempting to choose the network that contains F_8 as a variable since it has direct effect. However, because of the propagation of errors in estimating the other variables (i.e. indirect effect), the average loss could still be higher. For instance, consider the network $\{F_6, F_7, F_8\}$ which incur the loss of \$ 7/h while the other network $\{F_1, F_5, F_6\}$ is better as the loss is \$ 4/h. It is important to note that there are multiple economically optimal solutions. In summary, the optimal sensor network obtained using our formulation is based on the combined effect of process profit and the error propagation as explained by weighting and error covariance matrix, respectively.

Table 3.1: Comparison of average loss and overall error of a simple ammonia process

Sensor Network	Average loss (\$/h)	Overall Error$^+$	Sensor Network	Average loss (\$/h)	Overall Error$^+$
F_1, F_2, F_5	5	12	F_1, F_2, F_6	5	11
F_1, F_2, F_8	3	12	F_2, F_5, F_7	5	11*
F_2, F_5, F_8	3	12	F_2, F_6, F_7	9	12
F_2, F_6, F_8	3	12	F_2, F_7, F_8	3	12
F_1, F_3, F_5	5	12	F_1, F_3, F_6	5	11
F_1, F_3, F_8	3	12	F_3, F_5, F_7	5	11
F_3, F_5, F_8	3	12	F_3, F_6, F_7	9	12
F_3, F_6, F_8	3	12	F_3, F_7, F_8	3	12
F_1, F_4, F_5	5	12	F_1, F_4, F_6	5	11
F_1, F_4, F_8	3	12	F_4, F_5, F_7	5	11
F_4, F_5, F_8	3	12	F_4, F_6, F_7	9	12
F_4, F_6, F_8	3	12	F_4, F_7, F_8	3	12
F_1, F_5, F_6	4	14	F_1, F_5, F_7	4	14
F_1, F_5, F_8	3*	16	F_1, F_6, F_7	8	14
F_1, F_7, F_8	4	13	F_5, F_6, F_7	8	14
F_5, F_6, F_8	4	13	F_6, F_7, F_8	7	16

* indicates *YALMIP* branch and bound solution obtained using **sdpt3** solver
+ results from Narasimhan and Jordache (2000)

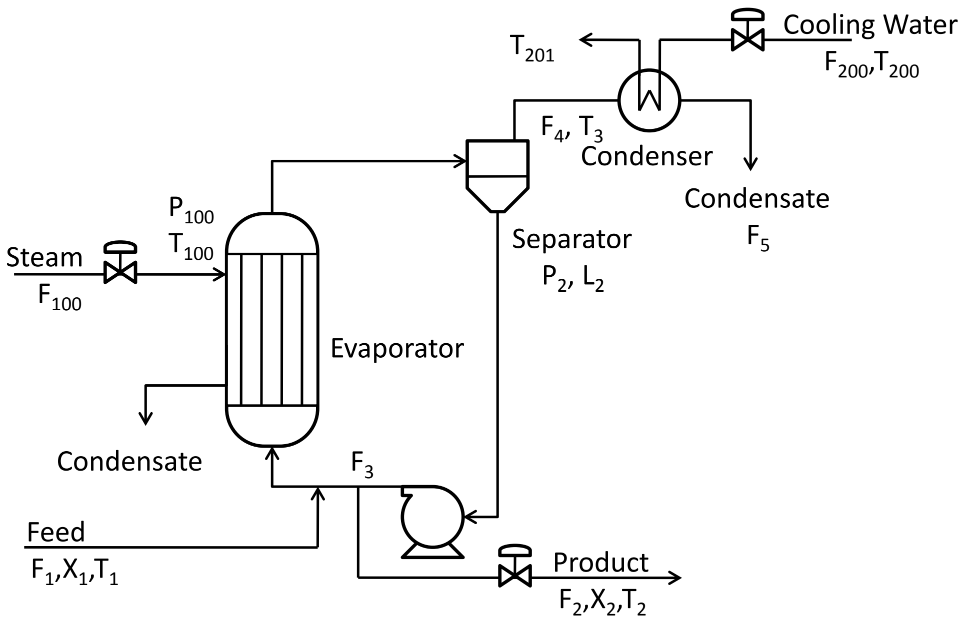

Figure 3.1: Evaporator system

3.1.4 Illustration 2: Evaporation process

Problem description. The optimal sensor network design procedure described in this chapter is applied to the realistic evaporation process of Newell and Lee (1989). The forced-circulation evaporator system is depicted in Figure 3.1, where the concentration of the feed stream is increased by evaporating the solvent through a vertical heat exchanger with circulated liquor. The dynamic process model governing the evaporation process is presented below. Here the solvent is water and the solute is nonvolatile. The process liquid is assumed to always exists at its boiling point and to be perfectly mixed. Hence, the liquid and vapor temperature equations presented below are obtained by linearization of the saturated liquid line for water about the steady state values. The dynamics of the energy balance is assumed to be very fast. Also, the dynamics within the steam heater jacket and within the condenser are assumed to be very fast. For the present study, we consider only steady state equations and hence, the derivative terms of the first three equations are set to zero.

$$
\begin{aligned}
20\frac{dL_2}{dt} &= F_1 - F_4 - F_2 && \text{(3.23a)}\\
20\frac{dX_2}{dt} &= F_1X_1 - F_2X_2 && \text{(3.23b)}\\
4\frac{dP_2}{dt} &= F_4 - F_5 && \text{(3.23c)}\\
T_2 &= 0.5616P_2 + 0.3126X_2 + 48.43 && \text{(3.23d)}\\
T_3 &= 0.507P_2 + 55 && \text{(3.23e)}\\
F_4 &= \frac{Q_{100} - 0.07F_1(T_2 - T_1)}{38.5} && \text{(3.23f)}\\
T_{100} &= 0.1538P_{100} + 90 && \text{(3.23g)}\\
Q_{100} &= 0.16(F_1 + F_3)(T_{100} - T_2) && \text{(3.23h)}\\
F_{100} &= \frac{Q_{100}}{36.6} && \text{(3.23i)}\\
Q_{200} &= \frac{0.9576F_{200}(T_3 - T_{200})}{0.14F_{200} + 6.84} && \text{(3.23j)}\\
T_{201} &= T_{200} + \frac{13.68(T_3 - T_{200})}{0.14F_{200} + 6.84} && \text{(3.23k)}\\
F_5 &= \frac{Q_{200}}{38.5} && \text{(3.23l)}
\end{aligned}
$$

The economic objective is to maximize the operational profit [$/h], formulated as a minimization problem of the negative profit given in Kariwala *et al.* (2008)

$$J = 600F_{100} + 0.6F_{200} + 1.009(F_2 + F_3) + 0.2F_1 - 4800F_2 \qquad (3.24)$$

The first three terms of equation (3.24) are utility costs relating to steam, water, and pumping. The fourth term is the raw material cost, whereas the last term is the product value. The process has the following constraints related to product specification, safety, and design limits:

$$X_2 \geq 35 + 0.5\% \tag{3.25}$$

$$40\ kPa \leq P_2 \leq 80\ kPa \tag{3.26}$$

$$P_{100} \leq 400\ kPa \tag{3.27}$$

$$0\ kg/min \leq F_{200} \leq 400\ kg/min \tag{3.28}$$

$$0\ kg/min \leq F_1 \leq 20\ kg/min \tag{3.29}$$

$$0\ kg/min \leq F_3 \leq 100\ kg/min \tag{3.30}$$

The nominal values are obtained by solving the above nonlinear optimization problem presented above and are presented in Table 3.2. The optimal cost is $J = -\$582.23/h$.

Table 3.2: Variables and their nominal optimal values of the evaporation process

Variables	Description	Nominal value
F_1	feed flow rate	9.469 kg/min
F_2	product flow rate	1.334 kg/min
F_3	circulating flow rate	24.72 kg/min
F_4	vapor flow rate	8.135 kg/min
F_5	condensate flow rate	8.135 kg/min
F_{100}	steam flow rate	9.434 kg/min
F_{200}	cooling water flow rate	217.8 kg/min
T_1	feed temperature	40.00 °C
T_2	product temperature	88.40 °C
T_3	vapor temperature	81.07 °C
T_{100}	steam temperature	151.5 °C
T_{200}	inlet temperature of cooling water	25.00 °C
T_{201}	outlet temperature of cooling water	45.55 °C
P_2	operating pressure	51.41 kPa
P_{100}	steam pressure	400.0 kPa
Q_{100}	heat duty	345.3 kW
Q_{200}	condenser duty	313.2 kW
X_1	feed composition	5.000 %
X_2	product composition	35.50 %

Degree of freedom analysis (Kariwala *et al.*, 2008). The process model has seven degrees of freedom. Note that a 0.5% backoff has been enforced on X_2 to ensure that the variable remains feasible for all possible disturbances. At the optimal point, there are two active constraints in the whole range of disturbance: $X_2 = 35.5\%$ and

$P_{100} = 400\ kPa$. The second active constraint fixes T_{100} from the model equation. Assuming the active constraints are enforced, the degrees of freedom becomes five (N_{min} = 5). Three of them are disturbances,(i.e., $d = [X_1\ T_1\ T_{200}]^T$). The case with $X_1 = 5\%, T_1 = 40°\text{C}$, and $T_{200} = 25°\text{C}$ is taken as the nominal operating point. The allowable disturbance set corresponds to 5% variation in X_1 and 20% variation in T_1 and T_{200} of their nominal values. The manipulated inputs are, $u = [F_{200}\ F_1]^T$. The other variables of interest are collected in vector, $x = [F_2\ F_3\ F_4\ F_5\ F_{100}\ T_2\ T_3\ T_{201}\ P_2\ Q_{100}\ Q_{200}]^T$ and hence all variables of interest, $z = [d\ u\ x]^T$.

Process matrix. For $z_p = [F_2\ F_3\ F_{100}\ F_{200}\ T_{201}]^T$, the process matrix obtained by linearization around the nominal operating point is

$$C = \begin{bmatrix} 2.8950 & -0.3261 & 1.1817 & 0.0092 & 0.1956 \\ 47.1210 & 42.0090 & -207.4600 & -1.1371 & -24.0980 \\ -1.5598 & -1.5598 & 5.6527 & 0.1385 & 1.9359 \\ 0 & 0 & 0 & 1 & 0 \\ 1.6175 & 0.6175 & -2.2379 & -0.0175 & -0.3705 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0.6175 & 0.6175 & -2.2379 & -0.0175 & -0.3705 \\ 0.6175 & 0.6175 & -2.2379 & -0.0175 & -0.3705 \\ 0 & 0 & 1 & 0 & 0 \\ 2.9861 & 2.9861 & -10.822 & -0.0323 & -0.6840 \\ 2.6958 & 2.6958 & -9.7698 & -0.0291 & -0.6175 \\ 0 & 0 & 0 & 0 & 1 \\ 5.3172 & 5.3172 & -19.2700 & -0.0575 & -1.2179 \\ 0 & 0 & 36.600 & 0 & 0 \\ 23.7740 & 23.7740 & -86.1570 & -0.6731 & -14.264 \end{bmatrix}^T \tag{3.31}$$

Weighting matrix. The following Hessian matrices are obtained numerically at the nominally optimal operating point (Kariwala *et al.*, 2008):

$$J_{uu} = \begin{bmatrix} 0.006 & -0.133 \\ -0.133 & 16.737 \end{bmatrix}; J_{ud} = \begin{bmatrix} 0.023 & 0 & -0.001 \\ -158.373 & -1.161 & 1.484 \end{bmatrix}$$

and the corresponding weighting matrix is

$$W = \left[\begin{array}{ccccc|c} 1856.4 & 13.652 & -17.194 & 0.023 & -158.37 & \overbrace{\mathbf{0}}^{5x14} \\ 13.652 & 0.100 & -0.126 & 0 & -1.161 & \\ -17.194 & -0.126 & 0.159 & -0.001 & 1.484 & \vdots \\ 0.023 & 0 & -0.001 & 0.006 & -0.133 & \\ -158.37 & -1.161 & 1.484 & -0.133 & 16.373 & \\ \hline \mathbf{0} & & \cdots & & & \mathbf{0} \end{array}\right]$$

Results. In the initial analysis, we consider only minimum observable sensor networks. The results obtained by enumeration shows that there are 76 minimum observable networks out of 252 possible combinations of the available set $\{P_2, T_2, T_3, F_2, F_{100}, T_{201}, F_3, F_5, F_{200}, F_1\}$. The implementation or measurement error for flow and pressure measurements are taken to be $\pm 2\%$ and $\pm 2.5\%$ of the nominal values, respectively. For temperature, the measurement noise is considered to be $\pm 1°\mathrm{C}$. The objective of this study is to highlight the importance of the weighting matrix W. Hence, for purpose of illustration, we do not impose any capital cost constraints. However, this is not a limitation of the formulation and is imposed subsequently. In contrast to the ammonia case study, the variables in this example are different physical quantities (pressure, temperature, heat duty, flow rate and composition). Therefore, it is meaningless to use $Tr(\Sigma_z)$ unless the variables are expressed as dimensionless quantities. This requires some scaling strategies like normalizing the variable in the range 0 - 1 using the maximum possible value. The maximum possible values used here are the maximum of the nominal values of the similar variable (i.e., for flow rate $F_{max} = 217.7$). Clearly, if the aim is to compare incommensurate quantities, different scaling strategies are possible. However, our formulation described in this work handles this issue rigorously because of our choice of the economically relevant weighting matrix. The overall error (in terms of normalized variables) and average loss obtained for some representative observable networks are presented in Table 3.3. The average loss and overall error of the economically optimal network $\{F_2, F_3, F_{100}, F_{200}, T_{201}\}$ are \$ 10.28/h and 353.8, respectively. The overall error of this network is considerably higher than the network $\{F_2, F_3, F_5, F_{200}, T_2\}$, which has the least overall error of 61.97. The former network has the larger error because of the propagation of error variances in estimating the unmeasured variables. However, this error propagation is shown to be economically unimportant. On the other hand, the low overall error networks, $\{F_2, F_3, F_5, F_{200}, T_2\}$ and $\{F_2, F_5, F_{100}, F_{200}, T_2\}$ also incur comparably lesser loss. Although the last network $\{F_1, F_5, F_{100}, F_{200}, T_3\}$ has lower overall error of 66.58, it is unfavorable economically ($\overline{L}_{cost}$ = \$ 601.6/h). In summary, the choice of minimizing the overall error is unjustified in terms of profit, hence underpinning our introductory argument that the economic importance should be considered in the sensor selection process.

Table 3.3: Comparison of average loss and overall error of the evaporation process for some representative sensor networks

Sensor Network	Overall Error	Average loss (\$/h)
$F_2, F_3, F_{100}, F_{200}, T_{201}$	353.8	10.28*
$F_2, F_3, F_5, F_{200}, P_2$	90.25	11.81
$F_1, F_3, F_5, F_{100}, F_{200}$	87.15	601.1
$F_2, F_3, F_{100}, F_{200}, P_2$	965.6	27.90
$F_3, F_5, F_{100}, F_{200}, P_2$	137.3	8041
$F_2, F_3, F_5, F_{200}, T_2$	61.97*	13.40
$F_2, F_5, F_{100}, F_{200}, T_2$	64.50	12.38
$F_1, F_5, F_{100}, F_{200}, T_3$	66.58	601.6

* indicates *YALMIP* branch and bound solution obtained using **sdpt3** solver

Furthermore, the formulation is also useful for retrofitting. The exiting network could be defined by setting $q_i = 1$ to represent that 'i'th sensor is present. This does not alter the formulation in any form. The best sensor that should be added to the existing sensor network are reported in Table 3.4, and the corresponding loss is also presented. For instance, consider the minimum overall error network, $\{F_2, F_3, F_5, F_{200}, T_2\}$, which incur the loss of \$ 13.40/h , the formulation finds that the best sensor that should be added is F_{200}, which reduces the operational loss by \$ 3.55/h.

3.1.5 Computational issues

In order to study the computational issues of the proposed formulation, we present the number of nodes explored by YALMIP branch and bound solver in Table 3.5 for three different systems (System 1 - Ammonia process; System 2 - Evaporation process; System 3 - Simulated system). In all three systems, we consider only minimum observable

Table 3.4: Best additional sensor for a given network of the evaporation process

Sensor Network	New Sensor	Average loss ($/h)
$F_2, F_3, F_{100}, F_{200}, T_{201}$	F_5	9.57
$F_2, F_3, F_5, F_{200}, P_2$	F_{100}	9.49
$F_1, F_3, F_5, F_{100}, F_{200}$	F_2	12.18
$F_2, F_3, F_{100}, F_{200}, P_2$	F_{100}	9.49
$F_3, F_5, F_{100}, F_{200}, P_2$	F_2	9.49
$F_2, F_3, F_5, F_{200}, T_2$	F_{100}	9.85
$F_2, F_5, F_{100}, F_{200}, T_2$	F_3	9.85
$F_1, F_5, F_{100}, F_{200}, T_3$	F_2	12.36

network. The first two systems have been discussed extensively elsewhere in this chapter. The third system is a synthetic example of a moderately sized problem with 28 variables. The cost function is randomly generated such that J_{uu} is positive definite and well-conditioned. Out of 2×10^7 (approx.) possibilities, we obtained the solution in approximately 10^4 node openings which is reasonable. However, for a large scale system, we need specialized techniques to speed up the algorithm. Efficient algorithms for branch and bound method are being developed (Kariwala and Cao, 2010; Menon *et al.*, 2013).

Table 3.5: Computational efficiency of the proposed MICP approach

	System 1	System 2	System 3
No. of variables	8	16	28
Minimum no. of sensors required for observability	3	5	17
Total no. of combinations	$\binom{8}{3}$ = 56	$\binom{16}{5}$ = 4368	$\binom{28}{17}$ = 21474180
No. of nodes explored	14	16	10837

3.2 Lexicographic optimization

The optimization formulations proposed in this thesis previously are useful for designing a sensor network that optimizes any one selected performance measure. The average loss formulation presented in this chapter will give the best set of sensors from an operational cost viewpoint. Such a network may have a higher overall estimation error. Similarly, a sensor network designed using a least overall estimation error framework may result in a higher average loss. However, in practice, we need to simultaneously optimize for both the performance measures. To ameliorate this shortcoming, Bagajewicz and Sánchez (2000) presented a minimum cost model for the design of reliable sensor networks. This was accomplished by minimizing the sensor cost while ensuring reliability of some or all variables above a pre-specified threshold values. They also demonstrated the cases wherein sensor network design procedure was carried out with pre-specified threshold values for other measures such as error variances and estimability. However, satisfying threshold values do not optimize for the performance measures.

Alternately, one could rank or prioritize the objectives and solve for them in an orderly way. In this approach, the objective with the highest priority is optimized first. With the optimal value of the first objective being set as an equality constraint, the objective with the second highest priority is optimized. Now, with the optimal function values of the first two objectives being set as equality constraints, the next prioritized objective is optimized and so on. In other words, the multiobjective problem is solved in lexicographic sense. Equivalently, this problem could be cast as a one-step optimization problem by combining all the objective functions into a single objective function with the proper choice of weights for each of the objective based on the assumed priority level (Sherali, 1982). Following similar approach, Bhushan and Rengaswamy (2002*a*) presented a way of combining the objective of maximizing reliability with (minimizing) sensor cost. Later, Bhushan *et al.* (2008) presented the sensor selection problem that were not only reliable and cost optimal but also robust to uncertainties in the underlying model and probability of sensor failure data. They combined the objectives to solve the problem in lexicographic sense using the algorithm proposed by (Sherali, 1982) to obtain the corresponding weights for each of the objective function.

In this section, our focus is to address the sensor network design problem that will minimize both the average loss and overall error in the lexicographic sense. Here we treat average loss as the primary objective and overall estimation error as our secondary objective. Now, the optimization formulation can be expressed as

$$\min_{q_i,t_c,Y_c,t_e,Y_e} \quad \overline{L} = \frac{1}{2}\lambda_1 t_c + \lambda_2 t_e \tag{3.32a}$$

$$s.t. \quad Tr(Y_c) \leq t_c \tag{3.32b}$$

$$\begin{bmatrix} Y_c & R^T C \\ (R^T C)^T & (C^T Q C) \end{bmatrix} \succ 0 \tag{3.32c}$$

$$Tr(Y_e) \leq t_e \tag{3.32d}$$

$$\begin{bmatrix} Y_e & C \\ C^T & (C^T Q C) \end{bmatrix} \succ 0 \tag{3.32e}$$

$$q_i \in \{0, 1\} \tag{3.32f}$$

$$Q = diag\{\frac{q_i}{{\sigma_i}^2}\} \tag{3.32g}$$

$$\sum_{i=1}^{n_z} c_i q_i \leq c^* \tag{3.32h}$$

The resulting formulation preserves the mixed integer cone programming formulation presented in the previous section. The constraints (3.32b)-(3.32c) correspond to the minimum loss framework whereas the constraints (3.32d)-(3.32e) correspond to minimum error framework.

The most crucial step in this formulation is selecting the appropriate weights, λ_1 and λ_2. In this work, we select the weights using Algorithm 2 of Sherali (1982). Recall that our primary objective is minimizing average loss, therefore, we set the value of $\lambda_1 = (1 + M)$ and $\lambda_2 = 1$, where M is the maximum possible value among all feasible solutions of the minimum error formulation and $M > 0$. For the case of two different objectives, $f_1(q)$ and $f_2(q)$, the appropriate choice of a single parameter will render the lexicographic solution and the value of M used here can be interpreted as the maximum possible relative change in the value of primary cost function for a minimal change in the secondary cost function value. According to Algorithm 2 of Sherali (1982), the value of M is chosen as the upper bound on loss value. The idea behind such a choice is that $\lambda_1 f_1(q) > \lambda_2 f_2(q)$ for any solution q. However, if there is one feasible solution q' such that $f_1(q') > f_2(q')$, then one can choose the value of M to be $f_1(q')$ because minimization of $(1 + f_1(q'))f_1(q) > f_2(q)$ if $f_1(q) > 1$ and $f_2(q) > 1$

for any q. The conditions, $f_1(q) > 1$ and $f_2(q) > 1$, can be easily tested by selecting all the sensors and evaluating the analytical expressions available for $f_1(q) = Tr(\Sigma_z(q))$ and $f_2(q) = Tr(W\Sigma_z(q))$. If this is true, then the subset of q will also satisfy the above condition. On the other hand, if the feasible solution q' satisfies $f_2(q') > f_1(q')$, then one can choose the value of M to be $f_2(q')$ following similar arguments. In summary, if a feasible solution has been determined, then the value of M can be taken as $max(f_1(q'), f_2(q'))$. Now, it is important to notice that the set of primary variables is one of the feasible solution. Therefore, we could approximate the value of M to be the overall error obtained using primary variables as candidate measurements assuming the conditions presented above are valid. This framework could be easily modified to consider overall estimation error as primary objective and average loss as secondary objective.

3.2.1 Illustration: Simple ammonia process

In this section, our objective is to demonstrate the lexicographic approach of designing sensor network to minimize both the average loss and overall error. For this purpose, let us revisit the flow network of a simplified ammonia process discussed in subsection 3.1.3 and the system is depicted in Figure 2.3. For the purpose of demonstration, we only consider minimum observable sensor networks. In Table 3.1, we presented the results obtained by our formulation and also those obtained by explicit enumeration of all observable solutions. Notice that the minimum average loss network was found to be $\{F_1, F_5, F_8\}$ but it has the highest overall error. On the other hand, there are 12 other economically optimal sensor networks with better overall error value than the sensor network $\{F_1, F_5, F_8\}$. Therefore, this necessitates the importance of minimizing overall error along with the average loss function. Hence, we propose to use the lexicographic approach of solving multi-objective optimization problems. The value of M is set to be 11 based on the chosen primary variables . The lexicographic solution was found to be $\{F_2, F_6, F_8\}$ which incurs an average loss of \$ 3 /h and the overall error is 12. This overall error value is better than that of the sensor network $\{F_1, F_5, F_8\}$ which had the error value of 16.

3.3 Summary

This chapter addressed the issue of obtaining the economically optimal sensor network in the presence of uncertainty caused by measurement error. The procedure relates process economics and estimation accuracy by determining the loss caused due to uncertainty in measurements. The analytical solution for the loss function showing the economic importance has been achieved by the proper choice of a weighting matrix. It is proper in the sense that cost function has been defined using economics of the process. This analysis helps the design engineer to quickly evaluate between the alternatives and choose the optimal network. The solution strategy is also presented to convert the MINLP into a mixed integer cone program which can be solved to global optimality. Finally, we discussed the lexicographic optimization framework to obtain the minimum loss sensor network that also minimizes the overall error of the network.

CHAPTER 4

ROBUST OPTIMAL SENSOR NETWORK DESIGN

Partly based on the paper presented in 11th International Symposium on Process Systems Engineering

In this chapter, we extend our sensor network design procedure that addressed data reconciliation, to handle sensor faults scenarios. It is assumed here that only one sensor can fail at a time. Sensor failure situations can be handled by increasing the redundancy of the network. Therefore, the primary aim of this chapter is to determine a redundant set of sensors that will retain observability of the system in case of sensor failures. In other words, the resulting network should be robust to sensor failures. To address this, we propose a scenario based optimization formulation that finds a robust optimal sensor network by minimizing the average loss function subject to satisfying observability conditions for each sensor failure scenarios. Although such a robust sensor network will result in an observable sub-network in case of sensor failure, the sub-network need not be optimal. Therefore, we also extend the formulation to determine the robust optimal sensor network by minimizing the worst-case loss function.

4.1 Introduction

To achieve effective monitoring, control and fault diagnosis of chemical processes, reliable measurements of process variables must be available. Sensor networks designed for the purpose of process monitoring should provide a good estimate of process variables. In this regard, the data reconciliation framework allows us to determine the sensor network that will provide precise estimates of the process variables. In the previous chapter, we intended to find a sensor network based on monetary value such that the operational loss caused due to measurement errors was minimized. However, sensors are prone to faults like bias, drift or complete failure, etc. This will eventually degrade the performance, demand process shutdown or even cause fatal accidents. Therefore, the sensor network we design should be robust enough to handle sensor failure situations.

The desirable property of a sensor network, to handle actual sensor failure situations, is redundancy. This could be achieved either by adding more sensors to measure the same variables (hardware redundancy) or by measuring additional variables such that there are multiple ways of estimating the process variables using process model (analytical or spatial redundancy). Since adding multiple sensors is costly, our focus is to exploit analytical redundancy to obtain a redundant sensor network. Some of the previous works that addressed the sensor network design procedure related to sensor fault situations and redundant measurement selection were reviewed in Chapter 2. Recall that a sensor network design procedure based on maximizing system reliability will maximizes the probability of estimating process variables, when sensors are likely to fail. Though such a design may presumably reduce the sensor fault occurrence, it will not be useful in case of actual sensor faults, because of its inability to obtain the estimate of unmeasured variables. However, this metric is not readily deciphered by the designers, as discussed earlier. Hence, our current focus is to design an economically optimal sensor network that can handle sensor failure situations.

This chapter is organized as follows: First, we enforce the property of system redundancy to design a reliable or robust sensor network that is capable of observing all the process variables even in case of sensor failures. This system redundancy property is utilized within the framework of minimum loss formulation presented in the previous chapters. Next, we present the modified optimization formulation that addresses the problem of robust optimal sensor network design by minimizing the worst case loss in case of sensor failures. Finally, illustrations are provided to demonstrate the approach.

4.2 Robust optimal sensor network design: Average loss formulation

The sensor network we design should be capable of providing a good estimate of the unmeasured variables, should be able to detect and identify faults, and also be robust to uncertainties such as process faults and sensor faults, etc. The choice of a redundant sensor network plays a vital role in identifying instrument faults, in aiding us to observe the process variables under sensor fault conditions by exploiting analytical redundancy, and also improving the estimates through data reconciliation. It is important to note that analytical redundancy (that is, the use of redundancy through the use of models) cannot be increased unless new instrumentation is added. Therefore, the objective of this section is to make a clever choice on the set of redundant measurements that are not only useful in improving the estimates but also help us to estimate all the unmeasured variables and measured variables for which the sensors have failed. In other words, we focus on designing the sensor network that will improve the analytical redundancy.

Bagajewicz (1997) defined robustness as the ability of a sensor network to detect gross errors (error detectability), provide results at a certain level of precision in the presence of gross errors (availability), and minimize corruption of data by undetected gross errors (resilience). In this work, we assume a suitable fault detection and maintenance policy is in place to identify sensor faults and take corrective action. Also, we assume that the data reconciliation is then performed, eliminating the faulty measurements. Hence, our scope is to design a suitable set of redundant measurements that possesses a certain level of analytical redundancy, to be useful in case of sensor failures.

DEFINITION 4.1 The sensor network is said to be *robust* if the network is redundant and in addition, all the process variables should be estimable by 'more than k' different means to account for k sensors failing at a time.

EXAMPLE 4.1

The purpose of this example is to illustrate the idea of robust sensor network to single sensor failure situations. For the case of single sensor failing at a time, there should be atleast two ways of estimating all the variables. Consider the system with three process units and six streams as depicted in Figure 4.1.

The minimum number of independent sensors required for the system to be observable under normal operating conditions is three. Recall from Example 2.1 that the sensor network $\{F_1, F_2, F_4\}$ is observable and the network $\{F_1, F_2, F_3\}$ is unobservable. The minimum observable sensor network is non-redundant. For a non-redundant sen-

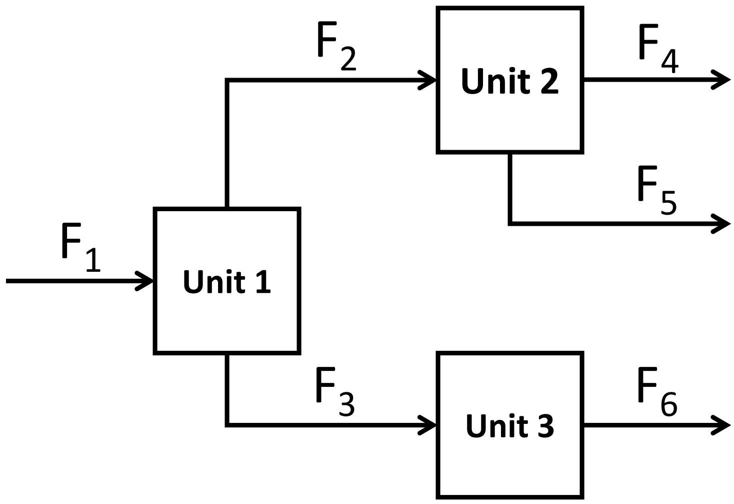

Figure 4.1: Concept of robust sensor network

sor network, even if one sensor fails, some flow variables cannot be estimated. Hence, the minimum observable sensor network is not useful in case of sensor failures. Also recall that the sensor network $\{F_1, F_2, F_4, F_5\}$ is redundant whereas the sensor network $\{F_1, F_2, F_3, F_6\}$ is non redundant. Though the sensor network $\{F_1, F_2, F_4, F_5\}$ is redundant, the variable F_3 and F_6 becomes unobservable if the sensor measuring F_1 fails. Hence, it is clear not all redundant networks are useful in case of sensor failures. Now consider the sensor network $\{F_1, F_3, F_4, F_5\}$. For this network, all the process variables can be estimated by more than one means, therefore the sensor network is robust to single sensor failures. In other words, all the sub-networks of the sensor network $\{F_1, F_3, F_4, F_5\}$ (i.e., $\{F_1, F_4, F_5\}$, $\{F_1, F_3, F_5\}$, $\{F_1, F_3, F_4\}$, $\{F_1, F_3, F_4\}$ and $\{F_3, F_4, F_5\}$) are observable in case of single sensor failure. In summary, the sensor network $\{F_1, F_2, F_4, F_5\}$ is redundant but not robust whereas the sensor network $\{F_1, F_3, F_4, F_5\}$ is robust. However, both the sensor networks will improve the estimation accuracy. ∎

In the previous chapter, we presented the sensor network design procedure for selecting redundant measurements, by directly specifying the number of sensors or by the specifying the budget limit on the total available capital cost for purchasing sensors. However, from the example just presented, it is clear that such a redundant sensor network may not be robust. Notice that a robust sensor network is one for which all possible sub-networks are observable after eliminating the failed sensors. It is assumed here that only one sensor can fail at a time. In order to obtain a robust sensor network, a new set of observability conditions has to be imposed, that would yield a redundant sensor network of degree one (i.e., if any one of the sensor fails, the system is still observable). For the system to be observable, it is mentioned earlier that C^TQC has to be positive definite. Hence, with our weighted variance formulation, to account for a single sensor failure situation, the following 'n' (number of potential variables available for measurement) additional constraints have to be imposed such that C^TQC is positive definite for all failure scenarios:

$$C^T \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{q_2}{\sigma_2^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{q_3}{\sigma_3^2} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \frac{q_n}{\sigma_n^2} \end{bmatrix} C \succ 0, \text{ if first sensor fails}$$

$$C^T \begin{bmatrix} \frac{q_1}{\sigma_1^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{q_3}{\sigma_3^2} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \frac{q_n}{\sigma_n^2} \end{bmatrix} C \succ 0, \text{ if second sensor fails and so on.}$$

These matrix inequality constraints ensure that the system is observable if that sensor fails. In other words, the formulation finds a sensor network with one degree of system redundancy. The robust optimal sensor network design formulation that minimizes the operational loss due to measurement errors can be mathematically expressed as

$$\min_{q_i} \quad \overline{L}_{robust} = \frac{1}{2} Tr(R^T C (C^T Q C)^{-1} C^T R) \tag{4.1}$$

$$s.t. \quad (C^T Q_j C) \succ 0 \quad \forall j \in SF \tag{4.2}$$

$$Q_j = diag\{\frac{q_i}{{\sigma_i}^2}\} \quad \forall i = 1, \ldots, n \tag{4.3}$$

$$q_i \in \{0, 1\} \tag{4.4}$$

$$\sum_{i=1}^{n_z} c_i q_i \leq c^* \tag{4.5}$$

where SF denotes the set of possible sensors that can fail at a time. In case of a single sensor failing at a time, $SF = [\{q_1 = 0\}, \{q_2 = 0\}, \cdots, \{q_n = 0\}]$. For two sensors failing at a time, $SF = [\{q_1 = 0, q_2 = 0\}, \{q_1 = 0, q_3 = 0\}, \cdots]$. For the case of multiple sensor failures, $\begin{pmatrix} n \\ r \end{pmatrix}$ additional constraints have to be imposed where 'r' denotes the number of sensors that can fail at a time [1]. In such a case, the formulation (4.1) along with the suitable set of constraints would yield a degree 'r' redundant sensor network. However, multiple sensors failing at a time is less common in practice and hence not discussed further. Using the convex optimization results presented in the previous chapters, the sensor network design formulation for determining the robust optimal sensor network can be expressed as

$$\min_{q_i, t_c, Y_c} \quad \overline{L}_{robust} = \frac{1}{2} t_c \tag{4.6a}$$

$$s.t. \quad Tr(Y_c) \leq t_c \tag{4.6b}$$

$$\begin{bmatrix} Y_c & R^T C \\ (R^T C)^T & (C^T Q C) \end{bmatrix} \succ 0 \tag{4.6c}$$

$$(C^T Q_j C) \succ 0 \quad \forall j \in SF \tag{4.6d}$$

$$Q_j = diag\{\frac{q_i}{{\sigma_i}^2}\} \quad \forall i = 1, \ldots, n \tag{4.6e}$$

$$q_i \in \{0, 1\} \tag{4.6f}$$

$$\sum_{i=1}^{n_z} c_i q_i \leq c^* \tag{4.6g}$$

Notice that the new set of constraints added to incorporate robustness are LMIs. Hence the above constraints are convex if the integer constraints are relaxed. Therefore, the mixed integer cone programming formulation of the sensor selection problem presented previously is preserved, and hence can be solved using a branch and bound technique to obtain a globally optimal sensor network. In this work, the problem is solved using YALMIP, a freely available software for solving convex optimization problems (Löfberg, 2004).

Remark

[1] $\begin{pmatrix} n \\ r \end{pmatrix}$ denotes the number of combinations and is given by the formula $\frac{n!}{r!(n-r)!}$

The term robust sensor network defined in this section is related to the concept of estimability of the variables presented in Bagajewicz and Sánchez (1999). For the case of single sensor failure scenario, the robust sensor network is equivalent to demanding the estimability of all the variable being strictly greater than one or setting the inequality as greater than or equal to two. However, the concept of estimability requires one to generate the possible cutsets of a graph a priori and in addition, the overall optimization formulation resulted in an MINLP which does not guarantee global optimality. On the other hand, in our formulation we state the sensor failure scenarios as LMIs and hence it can be solved to global optimality efficiently.

4.2.1 Illustration: Evaporation process

Consider the forced circulation evaporation process discussed in Section 3.1.4. Recall that the process model has five degrees of freedom assuming the active constraints are enforced. Therefore, at least five variables have to be measured for the system to be observable for data reconciliation. However, if any of the sensors fail then the system is always unobservable. Hence, a redundant sensor network of at least degree one is required to handle a single sensor failing at a time.

First, we consider the sensor selection problem given the budget limit for purchasing sensors. In order to consider the effect of capital cost constraint, the cost of flow, temperature and pressure sensors are assumed to be \$ 100 , \$ 150 and \$ 200 respectively. This constraint limits the number of sensors to be selected based on the available capital cost c^*. The economically optimal networks for different available capital cost are presented in Table 4.1. For the available cost of \$ 500 , it is trivial to see that only flow sensors could be selected as they form a minimum observable network which cause the average loss of \$ 12.2/h. With extra \$ 50 of available resource, we obtain a different optimal network $\{F_2, F_3, F_{100}, F_{200}, T_{201}\}$ which corresponds to the loss of \$ 10.28/h. It is important to recall that this network is also the optimal minimum observable network with equal sensor costs. With further increase in available capital cost, we obtain redundant networks, as redundant measurements improve the estimation accuracy and hence reduces the operational loss due to measurement uncertainty.

Now we compare the results of sensor network design formulation for two cases: without and with sensor failure conditions. The corresponding economically optimal

Table 4.1: Redundant sensor selection of the evaporation process given different budget limits

Available resource, c^* ($)	Sensor Network	Sensor cost ($)	Average loss ($/h)
500	$F_2, F_3, F_5, F_{100}, F_{200}$	500	12.2
550	$F_2, F_3, F_{100}, F_{200}, T_{201}$	550	10.28
600	$F_2, F_3, F_{100}, F_{200}, T_{201}$	550	10.28
650	$F_2, F_3, F_5, F_{100}, F_{200}, T_{201}$	650	9.57
700	$F_2, F_3, F_5, F_{100}, F_{200}, P_2$	700	9.48
750	$F_2, F_3, F_5, F_{100}, F_{200}, P_2$	700	9.48
800	$F_2, F_3, F_5, F_{100}, F_{200}, T_2, T_{201}$	800	9.10

sensor networks obtained are tabulated in Table 4.1 and Table 4.2, respectively. Up to the available resource of $ 600, only five sensors could be selected as shown in Table 4.1 and hence there is no feasible sensor network available to account for sensor faults. With $ 650 of available resource, the economically optimal network obtained for both the cases are the same. However, for the available resource of $ 700, the optimal network is different for the two cases considered. This implies that, although the sensor network $\{F_2, F_3, F_5, F_{100}, F_{200}, P_2\}$ results in a lower average loss than the network $\{F_2, F_3, F_5, F_{100}, F_{200}, T_{201}\}$, the latter can handle sensor failure situations while the former cannot. From the result obtained for the budget of $ 750, it can be inferred that the available resource is utilized to exploit redundancy. Indeed, the average loss is the same for both cases. It is also important to observe that increasing the number of measurements reduces operational loss.

Table 4.2: Robust optimal sensor networks of the evaporation process given different budget limits

Available resources, c^* ($)	Sensor Network	Sensor cost ($)	Average loss ($/h)
upto 600	Infeasible	-	-
650	$F_2, F_3, F_5, F_{100}, F_{200}, T_{201}$	650	9.57
700	$F_2, F_3, F_5, F_{100}, F_{200}, T_{201}$	650	9.57
750	$F_1, F_2, F_3, F_5, F_{100}, F_{200}, T_{201}$	750	9.48
800	$F_2, F_3, F_5, F_{100}, F_{200}, T_2, T_{201}$	800	9.10

4.3 Robust optimal sensor network design: Worst-case average loss formulation

In the previous section, we presented the sensor network design formulation that minimized the average loss function for selecting the redundant measurements that are robust to handle sensor failures. This was accomplished by specifying the observability conditions for each sensor failure situation. Although such a robust sensor network will result in an observable sub-network in case of sensor failure, the sub-network need not be optimal. Therefore, the current objective is to determine the robust sensor network that minimizes the average loss function among all observable sub-networks. In other words, our focus is on determining the redundant sensor network that minimizes the maximum average loss in case of sensor failure situations. It is important to recall that the term average loss signifies the monetary loss one would incur because of the presence of measurement errors. Also notice that the value of average loss represents the loss in profit due to measurement errors only and does not account for the actual loss we incur in the event sensor failure, however, the sensor network will still be observable in case of sensor failures. Hence we define the term worst-case average loss as the maximum possible monetary loss due to measurement errors in the event of any sensor sensor failing at a time of the designed sensor network. Therefore, in order to reduce the average loss we incur in case of sensor failure, we need to determine the sensor network by minimizing the worst-case average loss function.

Mathematically, the problem could be stated as:

$$\min_{q_i \in \{0,1\}} \max_{j \in SF} \quad L = \frac{1}{2} Tr(R^T C (C^T Q C)^{-1} C^T R) \tag{4.7}$$

where $Q = diag\{\frac{q_i}{\sigma_i{}^2}\}$ and SF denotes the set of possible sensors that can fail at a time. In case of a single sensor failing at a time, $SF = [\{q_1 = 0\}, \{q_2 = 0\}, \cdots, \{q_n = 0\}]$. For two sensors failing at a time, $SF = [\{q_1 = 0, q_2 = 0\}, \{q_1 = 0, q_3 = 0\}, \cdots]$. In other words, SF is a set that contains all possible sensor failure scenarios. The min-max formulation is a non-linear integer programming problem. However, expressing the above problem in epigraph form for all possible scenarios, we obtain

$$\min_{q_i, t_c, Y_{c,j}} \quad \overline{L}_{robust,wc} = \frac{1}{2} t_c \tag{4.8a}$$

$$s.t. \quad Tr(Y_{c,j}) \leq t_c \tag{4.8b}$$

$$\begin{bmatrix} Y_{c,j} & R^T C \\ (R^T C)^T & (C^T Q_j C) \end{bmatrix} \succ 0 \quad \forall j \in SF \tag{4.8c}$$

$$Q_j = diag\{\frac{q_i}{{\sigma_i}^2}\} \quad \forall i = 1, \ldots, n \tag{4.8d}$$

$$q_i \in \{0, 1\} \tag{4.8e}$$

$$\sum_{i=1}^{n_z} c_i q_i \leq c^* \tag{4.8f}$$

where matrix variables, $Y_{c,j}$'s are the internal variables created during convexification for each of the sensor failure scenarios. The final formulation is a mixed integer conic problem. The linear relaxation of the integer constraints result in a convex formulation, hence preserving convexity of our original formulation. This can be readily solved to global optimality using branch and bound algorithms.

4.3.1 Illustration: Evaporation process

Let us revisit the forced circulation evaporation process presented previously in the context of robust optimal sensor selection by minimizing average loss function. Recall that for the system to be observable, we need to measure at least five variables. Further recall that to handle sensor failure events, we need to identify the sensor network of at least system redundancy of degree one. Now, we illustrate the sensor selection problem that minimizes the worst case average loss function.

Let us assume the cost of flow, temperature and pressure sensors to be \$ 100, \$ 150 and \$ 200, respectively, to show the effect of available monetary resource. The robust optimal sensor networks obtained for different budget limits are tabulated in Table 4.3. Up to the available resource of \$ 600, only five sensors could be selected as shown in Table 4.2 and hence there is no feasible sensor network available to account for sensor faults. With \$ 650 of available capital cost, the robust optimal network $\{F_1, F_2, F_3, F_{100}, F_{200}, T_{201}\}$ incur a loss of \$ 1571.94/h under worst-case scenario. With an additional \$ 100 of budget, we could reduce the worst-case loss to \$ 492.30/h. Addition of F_5 to the existing network reduces the worst-case loss by a factor of 3. However, further increase in capital cost does not cause any significant reduction in worst-case loss, even if we measure all possible measurements. It is to be noted that the worst-case loss denotes the maximum possible loss we might incur if any of the sensors fail at a particular time. Also, all the sub-networks of measurements will be observable and hence robust to sensor failure.

Table 4.3: Optimal worst-case average loss value of the robust sensor network of the evaporation process given different budget limits

Budget limit, c^* (\$)	Sensor Network	Sensor cost (\$)	Worst-case loss (\$/h)
Upto 600	Infeasible	-	-
650	$F_1, F_2, F_3, F_{100}, F_{200}, T_{201}$	650	1571.94
700	$F_1, F_2, F_3, F_{100}, F_{200}, T_{201}$	650	1571.94
750	$F_1, F_2, F_3, F_5, F_{100}, F_{200}, T_{201}$	750	492.30
800	$F_1, F_2, F_3, F_5, F_{100}, F_{200}, T_{201}$	750	492.30
850	$F_1, F_2, F_3, F_5, F_{100}, F_{200}, T_{201}$	750	492.30
900	$F_1, F_2, F_3, F_5, F_{100}, F_{200}, T_2, T_{201}$	900	486.98
950	$F_1, F_2, F_3, F_5, F_{100}, F_{200}, P_2, T_{201}$	950	485.39
1250	$all\ posssible\ measurement$	1250	484.10

4.4 Summary

An SDP based sensor network design procedure to handle sensor failures in the framework of data reconciliation, based on an economic cost function, was presented. In this regard, we presented two formulations: First, we proposed to select sensors that satisfy only the observability requirements in the event of sensor failure; Next, we extended the formulation to find the set of sensors that minimize the worst-case average loss in the case of sensor failure. Mathematically, both the formulations result in an MICP, wherein relaxing the integer constraints yields a convex formulation. Therefore, a globally optimal solution can be found using a branch and bound approach. In summary, the redundant sensor network that is observable and also incurs the minimal average loss among all possible sensor failures, can be found using the proposed framework. The approach was successfully demonstrated using the evaporator system.

CHAPTER 5

PROFITABLE AND DYNAMICALLY FEASIBLE OPERATING POINT SELECTION FOR CONSTRAINED PROCESSES

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The operating point of a typical chemical process is determined by solving a non-linear optimization problem where the objective is to minimize an economic cost subject to constraints. Often, some or all of the constraints at the optimal solution are active, i.e., the solution is constrained. Though it is profitable to operate at the constrained optimal point, it might lead to infeasible operation due to uncertainties. Hence, industries try to operate the plant close to the optimal point by "backing-off" to achieve the desired economic benefits. Therefore, the primary focus of this chapter is to present an optimization formulation for solving the dynamic back-off problem based on an economic cost function. In this regard, we work within a stochastic framework that ensures feasible dynamic operating region within the prescribed confidence limit. In this work, we aim to reduce the economic loss due to the back-off by simultaneously solving for the operating point and a compatible controller that ensures feasibility. Since the resulting formulation is non-linear and non-convex, we propose a novel two-stage iterative solution procedure such that a convex problem is solved at each step in the iteration. Finally, the proposed approach is demonstrated using case studies.

5.1 Introduction

Profitability is the major concern of the plant operator and one approach to achieve this is to operate the plant at the optimal point obtained by solving a (typically nonlinear) steady state optimization problem. The optimizer minimizes a suitable cost function subject to equality and inequality constraints. Often, the solution of the optimizer is constrained at some of the inequalities, that is, there are several active constraints. Typically, it is assumed that these active constraints should be controlled at their limiting values to achieve economic benefits. However, the presence of uncertainties in the form of measurement noise, modeling error, parametric uncertainties and disturbances might cause constraint violations. Therefore, it is important to find an operating point close to the active constraints such that the plant remains feasible for the expected range of uncertainties. Thus, the focus of our work is to propose an optimization formulation that obtains the best trading-off between feasibility and profitability.

Optimal process operations depend on process design and safety thresholds, etc. These constraints define the feasible operating window to the optimizer. To ensure feasible operation under uncertain conditions, it may be necessary to "back-off" from the active constraints which however results in loss of achievable profit. Hence, the optimizer minimizes a loss function for backing-off from the active constraints. The term "back-off" is defined as,

$$\begin{aligned} Back-off \quad &= |Actual\ steady\ state\ operating\ point \\ &\quad -Nominally\ optimal\ steady\ state\ operating\ point| \end{aligned} \tag{5.1}$$

Based on the notion of back-off, Narraway *et al.* (1991) presented a method to assess the economic performance of the plant in the presence of disturbances. To ensure feasibility, the maximum amplitude of the disturbance for a certain range of frequency was used to determine the necessary back-off and alternate designs were evaluated. They assume the set of measurements are perfectly controlled and controllability is tested after obtaining the solution. Later, Narraway and Perkins (1993) extended their frequency response based method of estimating the closed loop constraint back off on the assumption of perfect control hypothesis to select the optimal set of measurements and manipulated inputs. This was accomplished by introducing the binary decision variable into the bounds of all possible measurements and manipulations. Also, the method was extended for the case of realistic PI controllers. Although the formulation is an MILP, the dimension of the problem is very high owing to the number of frequencies considered for each of the constraints. To solve this, a solution algorithm was presented where the obtained solution is compared with the open loop (without control) solution to quantify the profitability that would achieved by the controller and the controller with less benefits are eliminated (Heath *et al.*, 2000). All of the above methods were developed to handle single disturbance only.

To address the case of multiple disturbances, Bahri *et al.* (1996) addressed the back off problem for control of active constraints in the regulatory layer by solving the open loop problem. Figueroa *et al.* (1996) extended the above approach to the closed-loop case where the figure of merit "maximum percentage recovery" is defined to choose between alternative control configurations. In summary, disturbance is the only source of uncertainty considered in evaluating the different control structures. However, in some cases measurement noise and control error also play a significant role.

Disturbances are typically categorized based on the time scale or frequency of occurrence as fast or high-frequency disturbance and slow or low-frequency disturbance. The lower regulatory layer generally handles the fast disturbances whereas the slow disturbances are handled by the steady state optimizer. The objective of the optimization layer is to provide set points to the control layer. These set points depend on the set of design variables and measurements selected for estimating the model parameters. And, the choice of measurements have a profound impact in the steady state economics. In this regard, de Hennin *et al.* (1994) presented a method for estimating the likely economic benefit that could be achieved by implementing a steady state optimizer. The cost of instrumentation is also included in addition to the operational cost to determine the best optimal measurements.

Alternatively, Govatsmark and Skogestad (2005) presented the weighted-cost formulation to determine a robust set-point by enforcing a finite set of disturbance values in contrast to the maximum expected values of the disturbance vector. However, the determined set-points might be too conservative and they are indeed affected by the structural decisions such as control structure, type of controller, choice of manipulated, controlled and measured variables, etc. Loeblein and Perkins (1998) proposed a measure of average deviation from optimum that allows the estimation of economic value of different online optimization structures. In addition to measurement selection, their work addressed the impact of model uncertainty on the economics of the optimizer.

To analyze this issue, the authors considered a simple model, approximate model and rigorous model and concluded that approximate model is appropriate for on-line optimization. Later, Loeblein and Perkins (1999*a*,*b*) extended their method of average deviation from optimum to analyze the dynamic economics of regulatory layer which is assumed to be implemented using MPC framework. However, fixed control structures are assumed to rank between the alternatives. Several other authors have addressed the role of process economics on control structure selection (Bahri *et al.*, 1995, 1996; Kookos and Perkins, 2002; Kookos, 2005; Young *et al.*, 1996).

Peng *et al.* (2005) proposed a stochastic formulation for the determination of back-off points based on the notion of expected dynamic operating region. The basic idea in their approach is that the simultaneous selection of controller and back off point will find a optimal controller that minimizes the variability of the active constrained variables. Since the disturbances are assumed to be stochastic, the dynamic operation is defined in terms of variance. Extensions of the method to discrete time and partial state information case do not alter the formulation. Despite this, the final form of the optimization problem contains a set of reverse convex constraints which make the problem difficult to solve. Therefore, a branch and bound type algorithm was proposed. Sensor selection for control purposes are addressed in this framework (Peng and Chmielewski, 2005). Chmielewski and Manthanwar (2004) have found that the obtained optimal multivariable feedback controller can be used to tune the objective function weights of the MPC controller.

In this work, we propose a stochastic formulation of the dynamic back-off problem that ensures feasible operation for the prescribed confidence limit. Following Peng *et al.* (2005), the dynamic operating region is defined for the given disturbances which follow from the closed loop covariance analysis of the state space model of the process. The loss function, is a measure of departure from optimality and we develop a theoretically and conceptually sound loss function. Controller selection also plays a crucial role in shaping the dynamic operating region while the size of the region is characterized by the prescribed confidence limit and variance of the disturbance considered. Thus, consideration of the controller gain as a decision variable is important in determining the optimal operating point which minimizes the loss in profit. Therefore, the focus of our work is to propose an optimization formulation that determines the economic backed-off operating point by finding at the same time a suitable controller gain.

The current formulation contains an explicit representation of the ellipsoid to describe the system dynamics and can handle partially constrained cases. The formulation presents a back-off term as slack variable in terms of the respective variances. Furthermore, a novel solution methodology has been presented to solve the non-linear non-convex problem.

This chapter is organized as follows. In the next section, we define the problem and present a development of stochastic formulation and convex relaxations of the constraints. Next, a solution algorithm has been developed. Finally, illustrations are provided to demonstrate the approach.

5.2 Formulation of dynamic back-off problem

The objective of this section is to present an optimization formulation that determines the most profitable steady state operating point given that the plant has to remain feasible for the expected set of disturbances affecting the process. Hence, the optimization formulation should also include differential constraints that characterize the dynamic operating region of the plant. The feasibility becomes an important issue while operating the plant at the constrained optimal point. Therefore, we need to solve a dynamic back-off problem.

5.2.1 Optimization formulation

We start by determining the Optimal Operating Point (OOP) at steady state by minimizing the economic cost (the negative of the operating profit) $J(x_0, u_0, \overline{d}_0)$ where x_0, u_0 and $\overline{d}_0$ denote the states, manipulated inputs and nominal value of disturbances. Thus, the steady state optimizer solves the non-linear steady state optimization problem of the form,

$$\min_{u_0} \quad J(x_0, u_0, \overline{d}_0) \tag{5.2a}$$

$$s.t. \quad g(x_0, u_0, \overline{d}_0) = 0 \tag{5.2b}$$

$$h(x_0, u_0, \overline{d}_0) \leq 0 \tag{5.2c}$$

At OOP, the states and manipulated inputs are denoted as x_0^* and u_0^*, respectively. At OOP, there are three possible cases: unconstrained optimum (no active constraints), partially constrained (the number of active constraints is less than the number of manipulated inputs) and fully constrained (the number of active constraints equals the number of manipulated inputs). Peng *et al.* (2005) have addressed the problem for fully constrained case and the back-off from the linearized optimal solution is determined. In the present work, the focus is on the more general partially constrained case. In contrast to the fully constrained case where a linear approximation of the cost function around the optimal point is valid, the partially constrained case requires one to include a quadratic penalty for the inputs to account for the unconstrained degrees of freedom.

As mentioned previously, operating at OOP is usually not possible because of disturbances leading to infeasible operation. Therefore it is necessary to back off from the OOP. We introduce the deviation variables with respect to the nominally optimal point:

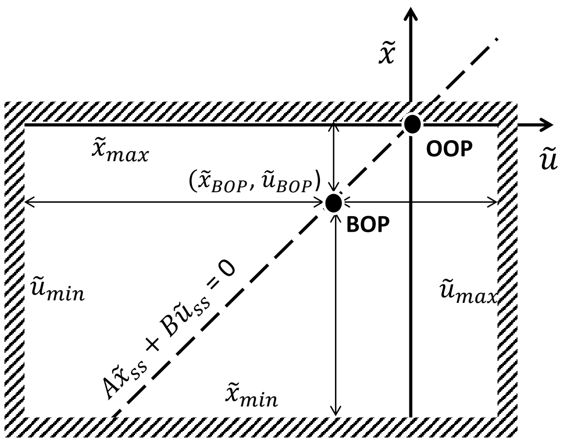

Figure 5.1: Feasible region: dynamic (box) and steady state (dashed line).

$\tilde{x} = x_0 - x_0^*$, $\tilde{u} = u_0 - u_0^*$, and $\tilde{d} = d_0 - \overline{d}_0$. In the deviation variable space, the optimal operating point is the origin as shown in Figure 5.1. Now, linearizing the steady state process models (5.2b) yield,

$$A\tilde{x}_{ss} + B\tilde{u}_{ss} = 0 \tag{5.3}$$

where A and B are the partial derivative of g evaluated at $(x_0^*, u_0^*, \overline{d}_0)$. Equation (5.3) defines the set of feasible back-off operating points $(\tilde{x}_{ss}, \tilde{u}_{ss})$. This is shown as the dashed line in Figure 5.1 for a single input and single output system. Now, the inequality performance limits (5.2c) are linearized around $(x_0^*, u_0^*, \overline{d}_0)$ and writing in bounded form by defining a new variable z_0 as:

$$z_0 = Z_x x_0 + Z_u u_0 + Z_d \overline{d}_0 \tag{5.4}$$

$$z_{min} \leq z_0 \leq z_{max} \tag{5.5}$$

where Z_x, Z_u and Z_d are the partial derivative of h evaluated at $(x_0^*, u_0^*, \overline{d}_0)$. Re-writing in terms of deviation variables, we get

$$\tilde{z} = Z_x \tilde{x} + Z_u \tilde{u} + Z_d \tilde{d} \tag{5.6}$$

$$\tilde{z}_{min} \leq \tilde{z} \leq \tilde{z}_{max} \tag{5.7}$$

where $\tilde{z}_{min} = z_{min} - Z_x x_0^* - Z_u u_0^* - Z_d \overline{d}_0$ and $\tilde{z}_{max} = z_{max} - Z_x x_0^* - Z_u u_0^* - Z_d \overline{d}_0$. It is important to note that, $\tilde{d} = 0$ at steady state.

In order to formulate the dynamic back-off problem, we need to define the system dynamics around the back-off point which has to be determined such that the economic loss is minimum. We assume that disturbances are rejected by the linear multivariable controller and full information about the state is available. Now, the dynamic model and the performance constraints is rewritten in terms of the new deviation variables around the Backed-off Operating Point (BOP) ($\tilde{x}_{ss}$,$\tilde{u}_{ss}$,$\tilde{d}$) and is given by

$$\dot{x} = Ax + Bu + Gd \tag{5.8}$$

$$z = Z_x x + Z_u u + Z_d d \tag{5.9}$$

$$\tilde{z}_{min} - \tilde{z}_{ss} \leq z \leq \tilde{z}_{max} - \tilde{z}_{ss} \tag{5.10}$$

where $x = \tilde{x} - \tilde{x}_{ss}$, $u = \tilde{u} - \tilde{u}_{ss}$ and $d = d_0 - \overline{d}_0$. The above set of equations define the dynamic operating region around the BOP.

The optimal operating point determined using (5.2) is the maximum achievable profit. As mentioned previously, we need to back-off from this optimal point to ensure dynamic feasibility. Hence, we need to define the loss function that minimizes the loss in achievable profit due to backing-off from the non-linear constrained optimal point. Therefore, we propose a linear approximation of the cost plus a quadratic penalty term to account for input usage,

$$ {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \tag{5.11} $$

where J_x, J_u and J_{uu} are the partial derivatives of J evaluated at $(x_0^*, u_0^*, \overline{d}_0)$. This is contrary to the linear cost function proposed by Peng *et al.* (2005), where the optimal steady state operating point is the result of the linearized model and not the non-linear optimal solution. This quadratic term forces the backed-off point to be closer to the non-linear optimal solution. It is also important to note that the cost function considers only the steady state effect on economics to determine the dynamically feasible steady state operating point. Now, we can pose the dynamic back-off problem for linear systems as

$$ \begin{aligned} min \quad & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} && (5.12) \\ s.t. \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} && (5.13) \\ & \dot{x} = Ax + Bu + Gd \quad \forall d \in \mathcal{D}, \forall x(0) \in \mathcal{X} && (5.14) \\ & z = Z_x x + Z_u u + Z_d d && (5.15) \\ & \tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} && (5.16) \\ & \tilde{z}_{min} - \tilde{z}_{ss} \leq z \leq \tilde{z}_{max} - \tilde{z}_{ss} && (5.17) \\ & u = Lx && (5.18) \end{aligned} $$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, L, x and u are the decision variables. $\mathcal{D}$ and $\mathcal{X}$ are the admissible sets of disturbances and initial values. This problem finds the new operating point that is close to the optimal operating point, in the economic sense, with the help of linear controller design as a part of the formulation. It is important to note that the new operating point would be different from the optimal steady state to ensure feasibility caused by some disturbances. The formulation is still semi-infinite dimensional due to differential constraints and non-linear due to the inclusion of controller design. Therefore, in the next section, we present a stochastic framework for addressing the dynamic back-off problem.

5.2.2 Stochastic framework

In this section, we develop a stochastic formulation that ensures feasible operation modulo, a prescribed confidence limit, i.e., the probability that the constraints are satisfied is greater than or equal to the confidence limit (Peng *et al.*, 2005). We make the following assumptions in formulating the problem:

- Disturbances are the only source of uncertainty considered and they are characterized by Gaussian white noise process with zero mean and known variances.
- A linear multi-variable controller with full state information ($u = Lx$) is available for feedback.
- A linear state space model to describe the dynamic operation of the system is given.

The differential equations that define the dynamic operating region can be expressed using the closed loop covariance analysis of the state space model of the process. Under the above-mentioned assumptions, the dynamic operating region can be expressed as ellipsoids with the BOP as center and the size and orientation determined by the covariance. Therefore, the current objective is to formulate the optimization problem that aims at determining the center of the ellipsoid (back-off operating point) and also orient the ellipsoid (i.e., finding a suitable controller) such that the dynamic operating region remains feasible for the given confidence limit while minimizing the loss in profit.

Following Peng *et al.* (2005), we develop closed loop covariance expressions that describe the Expected Dynamic Operating Region (EDOR). In this framework, the EDOR is a region such that the probability that the system is confined to the EDOR is greater than the prescribed confidence limits. This covariance matrix depends on the process dynamics, controller and also on the set of measurement. Assuming full state information and linear feedback, $u = Lx$, the closed-loop steady state covariance matrix of the state vector ($\Sigma_x := \lim_{t \to \infty} \mathbf{E}[x(t)^T x(t)]$) is given by the Lyapunov equation

$$(A + BL)\Sigma_x + \Sigma_x(A + BL)^T + G\Sigma_d G^T = 0 \tag{5.19}$$

where Σ_x is the symmetric positive semi-definite solution to the Lyapunov equation. Correspondingly, the covariance of the output signal z is given by

$$\Sigma_z = (Z_x + Z_u L)\Sigma_x(Z_x + Z_u L)^T + Z_d \Sigma_d {Z_d}^T \tag{5.20}$$

Given the center, $\tilde{z}_{ss}$, and the covariance $\Sigma_z = P^2$, the ellipsoidal EDOR is expressed as

$$\mathcal{E} = \{\tilde{z}_{ss} + \alpha P z \mid \|z\|_2 \leq 1\} \tag{5.21}$$

where P is the positive square root of Σ_z and α depends on the confidence limit (e.g., for a confidence limit of 95%, $\alpha = 2$). It is important to note that $\tilde{z} = \tilde{z}_{ss} + \alpha P z$. Therefore, we describe the dynamic feasibility as finding the ellipsoid within the performance bounds which is given by

$$\mathcal{E} = \{(\tilde{z}_{min} \leq \tilde{z}_{ss} + \alpha P z \leq \tilde{z}_{max}) \mid \|z\|_2 \leq 1\} \tag{5.22}$$

This representation ensures that the whole ellipsoid should lie within the performance bounds defined by (5.7). These bounds can be written as $h_i^T \tilde{z} + t_i \leq 0; i = 1, \ldots, m$ where h_i's, t_i's are the respective rows and elements of the matrix $H = [I; -I]$ and vector $t = [\tilde{z}_{max}; -\tilde{z}_{min}]$. Thus, the problem can be restated as finding the center of the ellipsoid close to the optimal operating point such that the ellipsoid is contained within performance bounds. Thus, we write the Economic Backed-off Operating Point

(EBOP) selection problem as

$$\begin{aligned}
min \quad & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} && (5.23a)\\
s.t. \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} && (5.23b)\\
& (A+BL)\Sigma_x + \Sigma_x(A+BL)^T + G\Sigma_d G^T = 0 && (5.23c)\\
& \Sigma_z = (Z_x + Z_u L)\Sigma_x (Z_x + Z_u L)^T + Z_d \Sigma_d {Z_d}^T && (5.23d)\\
& P = \Sigma_z^{1/2} && (5.23e)\\
& \tilde{z} := \tilde{z}_{ss} + \alpha P z \; \forall \; \|z\|_2 \leq 1 && (5.23f)\\
& h_i^T \tilde{z} + t_i \leq 0; i = 1, \ldots, m && (5.23g)
\end{aligned}$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, L, $\Sigma_x \succeq 0$, $\Sigma_z \succeq 0$ and $P \succeq 0$ are the decision variables. There are specifically two factors that make the above optimization problem challenging. First, equations (5.23c) - (5.23e) are non-linear in the decision variables. Second, the formulation is infinite-dimensional due to the explicit description of the ellipsoid (5.23f). In other words, we need to find the ellipsoid centered at the BOP for an infinite set of z. Hence, we present convex relaxations of the constraints in the next section.

5.2.3 Convex relaxations

Convex optimization tools are highly useful in transforming "difficult-to-solve" non-linear constraints into solvable LMI forms (Boyd and Vandenberghe, 2004). First, we present some facts used in this work, from convex optimization and control theory.

Fact 01 [Schur complement (Boyd and Vandenberghe, 2004)]. If C is positive-definite, i.e., $C \succ 0$, then the matrix $S = A - BC^{-1}B^T$ is called the Schur complement of C in the matrix $X = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$. Then the condition for positive semi-definiteness of block X is: If $C \succ 0$, then $X \succeq 0$ if and only if $S \succeq 0$.

Fact 02 [S - procedure (Boyd and Vandenberghe, 2004)]. The implication

$$x^T F_1 x + 2g_1^T x + h_1 \leq 0 \Rightarrow x^T F_2 x + 2g_2^T x + h_2 \leq 0,$$

where $F_i \in \mathbf{S}^n$,$g_i \in \mathbf{R}^n$,$h_i \in \mathbf{R}$, holds if and only if there exists a τ such that

$$\tau \geq 0; \begin{bmatrix} F_2 & g_2 \\ g_2^T & h_2 \end{bmatrix} \preceq \tau \begin{bmatrix} F_1 & g_1 \\ g_1^T & h_1 \end{bmatrix},$$

provided there exists a point $\hat{x}$ with $\hat{x}^T F_1 \hat{x} + 2g_1^T \hat{x} + h_1 < 0$.

Theorem 5.2.1 *(Peng* et al., *2005)* $\exists$ *stabilizing* L, $\Sigma_x \succeq 0$ *s.t.* $(A+BL)\Sigma_x + \Sigma_x(A+BL)^T + G\Sigma_d G^T = 0$ *and* $\Sigma_z = (Z_x + Z_u L)\Sigma_x (Z_x + Z_u L)^T + Z_d \Sigma_d {Z_d}^T$ *if and only if* $\exists\, Y$,$X \succ 0$ *and* $Z \succ 0$ *s.t.*
$(AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0,$
$\begin{bmatrix} Z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succ 0.$
For proof of the above theorem, the reader is referred to Chmielewski and co-workers (Peng et al., *2005).* ∎

Corollary 5.2.2 *The controller gain is recovered as* $L = YX^{-1}$ *and the covariance matrix of the states is* $\Sigma_x = X$.

Theorem 5.2.3 *(Boyd and Vandenberghe, 2004) Equations* (5.23f) - (5.23g) *can be expressed as a set of second order cone constraints of the form* $\|\alpha P h_i\|_2 + h_i^T \tilde{z}_{ss} + t_i \leq 0$.

Proof *. Recall that the feasibility condition,* $h_i^T \tilde{z} + t_i \leq 0\ \forall \alpha P z + \tilde{z}_{ss} |\ \|z\|_2 \leq 1$. *This can be rewritten as*

$$\sup_{\|z\|_2 \leq 1} h_i^T (\alpha P z + \tilde{z}_{ss}) + t_i \leq 0, i = 1, \ldots, m$$

$$\iff \sup_{\|z\|_2 \leq 1} (h_i^T \alpha P z) + h_i^T \tilde{z}_{ss} + t_i \leq 0, i = 1, \ldots, m$$

$$\iff \|\alpha P h_i\|_2 + h_i^T \tilde{z}_{ss} + t_i \leq 0, i = 1, \ldots, m \quad \blacksquare$$

Let us consider the covariance constraint (5.23d) of the output z

$$\Sigma_z = (Z_x + Z_u L)\Sigma_x \Sigma_x^{-1} \Sigma_x (Z_x + Z_u L)^T + Z_d \Sigma_d {Z_d}^T \tag{5.24}$$

Taking Σ_x within brackets, we get

$$\Sigma_z = (Z_x\Sigma_x + Z_uL\Sigma_x)\Sigma_x^{-1}(Z_x\Sigma_x + Z_uL\Sigma_x)^T + Z_d\Sigma_d{Z_d}^T \tag{5.25}$$

This form allows one to write it as an LMI using change of variables and Schur complement (see Fact 01). Next, let us consider the ellipsoidal constraint (5.23f) and the output bounds defined by the polytopic constraint (5.23g). As mentioned previously, these two constraints make the EBOP selection problem as an infinite dimensional one. However, we can represent them using finite number of second order cone constraints using Theorem 5.2.3

$$\|\alpha Ph_i\|_2 + h_i^T\tilde{z}_{ss} + t_i \leq 0, i = 1, \ldots, m \tag{5.26}$$

Now the EBOP selection problem is reformulated in terms of LMI constraints as :

$$
\begin{aligned}
min \quad & {J_x}^T\tilde{x}_{ss} + {J_u}^T\tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu}\tilde{u}_{ss} && \text{(5.27a)}\\
s.t. \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} && \text{(5.27b)}\\
& \tilde{z}_{ss} = Z_x\tilde{x}_{ss} + Z_u\tilde{u}_{ss} && \text{(5.27c)}\\
& (AX + BY) + (AX + BY)^T + G\Sigma_dG^T \prec 0 && \text{(5.27d)}\\
& \begin{bmatrix} Z - Z_d\Sigma_d{Z_d}^T & Z_xX + Z_uY \\ (Z_xX + Z_uY)^T & X \end{bmatrix} \succeq 0 && \text{(5.27e)}\\
& P = Z^{1/2} && \text{(5.27f)}\\
& \|\alpha Ph_i\|_2 + h_i^T\tilde{z}_{ss} + t_i \leq 0, i = 1, \ldots, 2n_z && \text{(5.27g)}
\end{aligned}
$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, Y, $X \succeq 0$, $Z \succeq 0$ and $P \succeq 0$ are the decision variables. The objective function and all the constraints in the above formulation (5.27) except (5.27f) are convex. Thus, the formulated minimum back off operating point selection problem is a non-linear and non-convex program. However, this problem is solved using the solution methodology developed in Section 5.3.

Remarks

1. The formulation presented in Peng *et al.* (2005) differs from our formulation in many ways: (1) there is no explicit ellipsoidal constraints, (2) the dynamic

feasibility of the ellipsoid is ensured by the reverse convex constraints and, (3) a branch and bound type of algorithm was proposed.

2. Note that this cost function considers only the steady state effect on economics. Since the disturbances are assumed to be Gaussian and zero mean, this implies that the cost accounts only for the nominal steady state value of disturbances.

3. The linear terms in the cost function could be interpreted as the sum of the product of back-off variables and their Lagrange multipliers.

4. By direct comparison of (5.27g) with the robust LP with random constraints (Boyd and Vandenberghe, 2004),

$$\Phi^{-1}(\eta)\|Ph_i\|_2 + h_i^T \tilde{z}_{ss} + t_i \leq 0 \tag{5.28}$$

   we can choose the parameter α using the inverse of the cumulative distribution function $\Phi^{-1}(\eta)$ where η denotes the probability level of a particular constraint being satisfied.

5. The term $\|\alpha Ph_i\|_2$ denotes the amount of required back-off. Hence, given the controller design, we can directly compute the back-off from the covariance estimates.

6. An equivalent LMI representation of the second order cone constraints (5.27g) is given by S-procedure (see Fact 02),

$$\begin{bmatrix} -\tau_i - h_i{}^T \tilde{z}_{ss} - t_i & \frac{\alpha}{2} h_i{}^T P \\ (\frac{\alpha}{2} h_i{}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \tau_i > 0;\ i = 1 \cdots 2n_z \tag{5.29}$$

7. Hard and soft constraints could be handled within the proposed formulation by selecting different values α for each of the constraints. Higher value of α is chosen for a hard constraint which signifies that probability of violating that constraint should be less. On the other hand, lower values of α are chosen for soft constraints to achieve the appropriate tolerance level.

5.3 Solution methodology

The main challenge in obtaining solution to the proposed formulation is the non-linearity in Z. In our formulation, the objective was to orient the ellipsoid (i.e., controller gain, L) such that the center of the ellipsoid is close to optimal operating point (i.e., EBOP, $\tilde{z}_{ss}$). In this section, we present a solution technique to solve the proposed formulation using the geometrical inference of the solution space. In this regard, we develop a two-stage iterative procedure where a convex problem is solved in each stage.

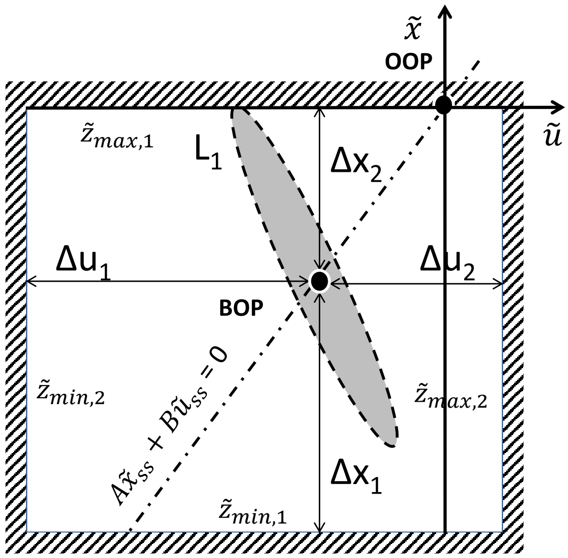

Figure 5.2: Non-optimal controller design (solution from Stage 1)

The basic idea of the solution strategy is illustrated in Figure 5.2 where we first determine a feasible covariance ellipsoid Z_1 that describe the dynamic operating region for the given confidence limit (say 95%). Next, we determine the backed-off operating point for the computed Z_1. However, the solution obtained may not be economically optimal as no cost information is included in stage 1. In other words, the backed-off operating point depends critically on the computed Z_1 (solution from stage 1). It can be seen from Figure 5.3 that choosing a different covariance ellipsoid Z_2 leads to a better economically backed-off operating point. It should also be noted that at the economic back-off point, the dynamic operating region touches the manipulated input constraint and the active constraint (controlled variable). This illustrates the fact that the dynamic back-off required is due to imperfect control caused by the input constraints for the assumed disturbance magnitude. Hence, the covariance ellipsoid Z_1 is approached toward Z_2 on subsequent iterations by creating lower bounds on the individual variances based on the available manipulated inputs.

5.3.1 Stage 1

In the first stage, we find the smallest (in terms of trace) feasible ellipsoid Z that describes the dynamic operating region for the considered disturbance magnitude. In other words, we have designed a controller ($L = YX^{-1}$) that result in a minimum variance.

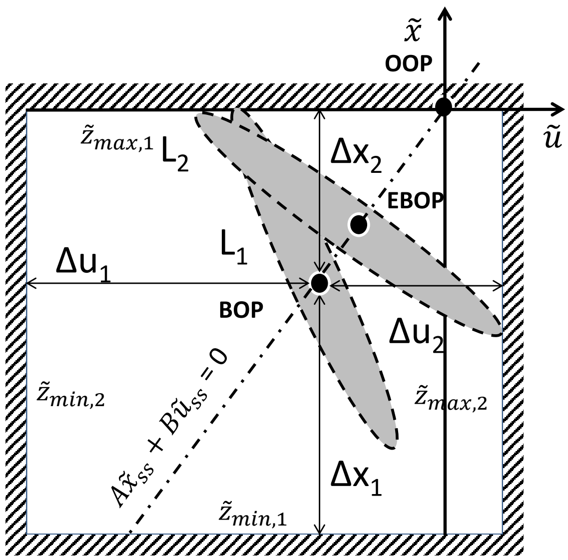

Figure 5.3: Optimal controller design (after convergence)

At the first stage, we impose the following constraints on the individual variances to determine the Z (and hence L) that ensures feasibility in the second stage,

$$\sigma_{z,i}^2 < \frac{1}{4\alpha^2}\left(\tilde{z}_{max,i} - \tilde{z}_{min,i}\right)^2; i = 1 \cdots n_z \tag{5.30}$$

where $\sigma_{z,i}^2$ is the variance of the i^{th} component of z, viz., z_i. For the given confidence interval (assume 95%), $2\sigma_i$ should be within the performance bounds. This enables us to determine the feasible ellipsoid. Additionally, we define the following constraints with respect to variance of the j^{th} variable $\sigma_{z,j}^2$,

$$\sigma_{z,i}^2 > \frac{\delta_{i,j}^2}{\alpha^2}\sigma_{z,j}^2; i = 1, j-1, j+1, n_z \tag{5.31}$$

where the iterative parameters $\delta_{i,j}^2$ are chosen such that the BOP selected in stage 2 is used to select the new minimum variance ellipsoid that forces the BOP close to OOP. The parameter $\delta_{i,j}$ is defined as

$$\delta_{i,j} = \frac{distance\ of\ variable\ i\ from\ its\ closest\ bound}{distance\ of\ variable\ j\ from\ its\ closest\ bound} \tag{5.32}$$

The δ for the case shown in Figure 5.2 is given by

$$\delta_{i,j} = \frac{min(\Delta u_1, \Delta u_2)}{min(\Delta x_1, \Delta x_2)} \tag{5.33}$$

Physically, the solution tries to exploit the available manipulated input space to be utilized to find the economic back-off point and the optimal multi-variable controller. Hence, we solve the following problem to find the dynamic operating region:

$$\min_{X\succeq 0, Z\succeq 0, Y} \quad Tr(Z) \tag{5.34a}$$

$$s.t. \quad (AX+BY)+(AX+BY)^T+G\Sigma_d G^T \prec 0 \tag{5.34b}$$

$$\begin{bmatrix} Z - Z_d\Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 \tag{5.34c}$$

$$\sigma_{z,i}^2 < \frac{1}{4\alpha^2}\left(\tilde{z}_{max,i} - \tilde{z}_{min,i}\right)^2; i = 1\cdots n_z \tag{5.34d}$$

$$\sigma_{z,i}^2 > \frac{\delta_{i,j}^2}{\alpha^2}\sigma_{z,j}^2; i = 1, j-1, j+1, n_z \tag{5.34e}$$

The solution of stage 1 results in a feasible covariance ellipsoid Z_1. The upper bound on the individual variances ensure that Z_1 is feasible in the second stage. If the solution from stage 1 is infeasible, then the solution to the original problem is infeasible. The parameter δ is used to create lower bounds on the individual variances such that the economically optimal ellipsoid is approached on subsequent iterations. The parameter δ is initialized to zero during the start of the algorithm which defines that the individual variances should be non-negative. Hence, on solving the first stage problem, we obtain Z and letting $P = Z^{1/2}$, a second optimization problem is solved to obtain the back-off point. This would yield an approximation to the economic back-off point.

5.3.2 Stage 2

$$\min_{\tilde{x}_{ss}, \tilde{u}_{ss}, \tilde{z}_{ss}} \quad {J_x}^T\tilde{x}_{ss} + {J_u}^T\tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu}\tilde{u}_{ss} \tag{5.35a}$$

$$s.t. \quad A\tilde{x}_{ss} + B\tilde{u}_{ss} = 0 \tag{5.35b}$$

$$\tilde{z}_{ss} = Z_x\tilde{x}_{ss} + Z_u\tilde{u}_{ss} \tag{5.35c}$$

$$\|\alpha P h_i\|_2 + h_i^T\tilde{z}_{ss} + t_i \leq 0, i = 1,\ldots,2n_z \tag{5.35d}$$

In the second stage, we determine a backed-off operating point ($\tilde{z}_{ss}$) that is close to the optimal point for the predetermined ellipsoid (solution from the first stage). However, the proximity to the economically optimal point depends on the orientation of the co-

Table 5.1: Algorithm for selecting economic back-off operating point

1	Initialize the parameter $\delta_{i,j} = 0$.
2	Find Z by solving the Stage 1 convex problem (5.34). If no feasible Z can be found, exit.
3	Compute $P = Z^{1/2}$. Find the BOP ($\tilde{z}_{ss}$) by solving the Stage 2 convex problem (5.35).
4	Terminate on convergence. Otherwise, update $\delta_{i,j}$ using (5.32) and proceed to Step 2.

variance ellipsoid. As we have written the inequalities as box constraints, the surface of the ellipsoid should touch the box at optimality. Hence, we need to re-orient the ellipsoid such that dynamic operating region touches the box constraint. This is accomplished by creating lower bounds for the individual variances using the parameter δ. The δ's are updated based on the newly found backed-off point. This information is used to recompute Z (and hence L) in the first stage. This process is iterated until convergence. It should be noted that P is not a decision variable since Z is known from the first stage. Now, it can be easily recognized that both stages contains only convex constraints, which could be easily solved using CVX, a package for specifying and solving convex programs (Grant and Boyd, 2011). Initializing $\delta_{i,j}$ to zero and given two successive iterates, $\tilde{z}_{ss}^{iter-1}$ and $\tilde{z}_{ss}^{iter}$ this process is iterated until the convergence criteria $\|\tilde{z}_{ss}^{iter} - \tilde{z}_{ss}^{iter\text{-}1}\|_2 \leq \epsilon$ is satisfied where ϵ being the prescribed tolerance limit. The solution algorithm is presented in Table 5.1.

5.4 Illustrations

5.4.1 Mass spring damper system

The purpose of this example is to illustrate the proposed backed-off operating point selection algorithm in a single-input two-output system.

Description. Consider the mass-spring-damper system depicted in Figure 5.4. Let r denote the mass position, v the velocity, g the gravitational force, f the manipulated input force, and w a disturbance force. The system dynamics are described by linear

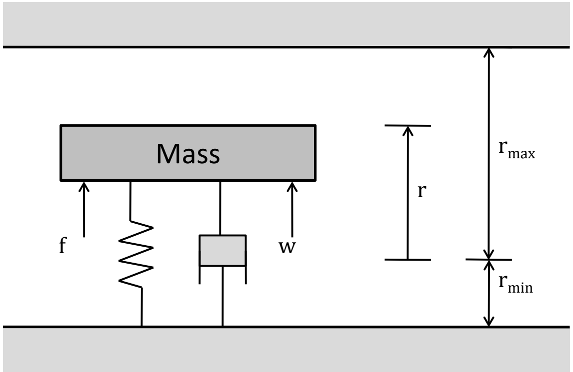

Figure 5.4: Mass spring damper system

differential equations (Peng *et al.*, 2005):

$$\frac{dr}{dt} = v \tag{5.36}$$

$$\frac{dv}{dt} = -3r - 2v - g + f + w \tag{5.37}$$

We will further assume that the system is constrained by the following inequalities $r_{min} \leq r \leq r_{max}$ and $f_{min} \leq f \leq f_{max}$. Hence, the signal matrices are given by $Z_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$; $Z_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $Z_d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

BOPs. The economic objective is to bring the mass as close as possible to the upper bound on position. Thus, it can be easily realized that the OOP is constrained at the mass position, $r^* = r_{max}$, $v^* = 0$ and $f^* = 3r_{max} + g$ (assuming $f_{max} \geq 3r_{max} + g$). Rewriting in deviation form, the system matrices are $A = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and the corresponding BOPs which define the steady state feasible points are $\tilde{v}_{ss} = 0$, $\tilde{f}_{ss} = 3\tilde{r}_{ss}$. The dynamic feasible region is defined by box constraints: $\tilde{r}_{min} \leq \tilde{r} \leq \tilde{r}_{max}$ and $\tilde{f}_{min} \leq \tilde{f} \leq \tilde{f}_{max}$.

Results. If $r_{min} = -1$, $r_{max} = 1$, $f_{min} = 0$, $f_{max} = 15$, $g = 9.8$ and $\Sigma_w = 10$, the OOP is $r^* = 1$, $v^* = 0$ and $f^* = 12.8$ (since $f_{max} = 15 \geq 3r_{max} + g = 12.8$). The data presented here are the base case values (Case A). For the current system, we have

Table 5.2: EBOP values for change in constraint polytope of mass spring damper system

Case	constraint	(r^*, f^*)	L
A	f_{max} = 15	(0.64,11.72)	[-6.4319 -2.1066]
B	f_{max} = 18	(0.83,12.30)	[-22.883 -5.0544]
C	f_{min} = 9.5	(0.36,10.90)	[-1.6327 -0.6952]

assumed a confidence level of 63% (i.e., $\alpha = 1$). The economic backed-off operating point determined is $(r_{EBOP} = 0.64, f_{EBOP} = 11.72)$ which results in a loss of 0.36. The multi-variable controller ($u = Lx$) designed to operated feasibly at the economic backed-off operating point is $L = [-6.4319 - 2.1066]$. The results obtained here are in agreement with the results presented in Peng *et al.* (2005). The impact of change in the constraint polytope is shown in Figure 5.5 by increasing the f_{max} to 18 N (Case B) and f_{min} to 9.5 N (Case C). The results are tabulated in Table 5.2. We see that increasing the upper limit in input force reduces the necessary back-off because this extra input force is used to compensate for the disturbances and hence pushes the mass position close to the optimal point. Whereas increasing the lower bound requires more back-off as it reduces the available dynamic feasible region. Hence, increasing the dynamic feasible region on the input will result in keeping the mass close to the true optimal point.

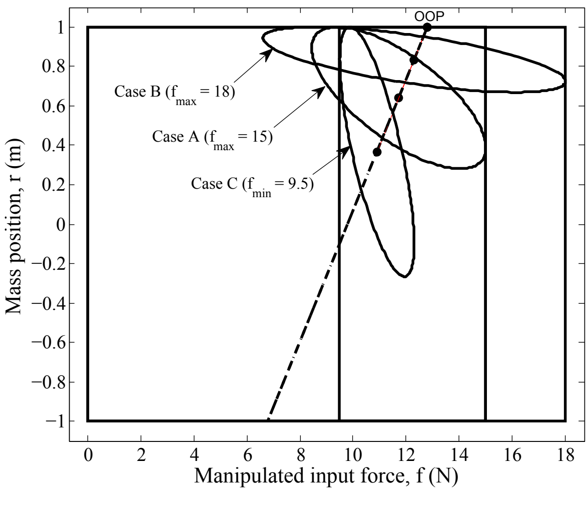

Figure 5.5: Impact of change in constraint for mass spring damper system

5.4.2 Preheating furnace reactor system

This example illustrates the proposed back-off approach in a multi-input multi-output system which is fully constrained at the nominal optimal point.

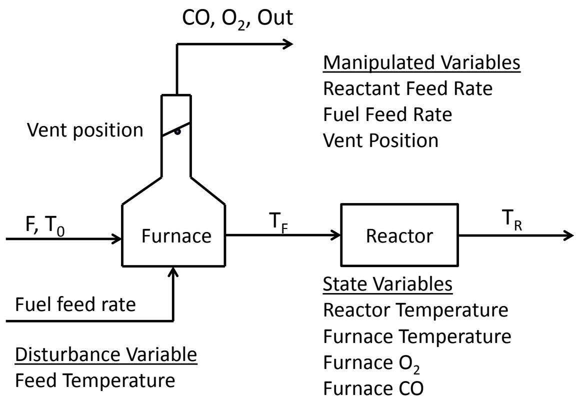

Figure 5.6: Preheating furnace reactor system

Description. Consider the preheating furnace reactor system shown in Figure 5.6. The system matrices are given by (Peng *et al.*, 2005)

$$A = \begin{bmatrix} -8000 & 0 & 0 & 0 \\ 2000 & -1500 & 0 & 0 \\ 0 & 0 & -5000 & 0 \\ 0 & 0 & 0 & -5000 \end{bmatrix};$$

$$B = \begin{bmatrix} -75 & 75000 & 0 \\ -25 & 0 & 0 \\ 0 & -8500 & 8.5 * 10^5 \\ 0 & 0 & -5 * 10^7 \end{bmatrix} \text{ and } G = \begin{bmatrix} 10000 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where states 1 and 2 correspond to the temperature of the reactor and furnace, T_R and T_F, respectively, and states 3 and 4 correspond to the O_2 and CO concentrations in the furnace, respectively. The manipulated inputs are the changes in the feed flow rate (F_R), fuel flow rate (F_F) and furnace vent position (V_P). Feed temperature, T_0 is assumed as the disturbance input with mean zero and variance $\Sigma_d = (0.13975)^2$. Feasibility is defined by the following state constraints

$$\begin{bmatrix} 355 \\ 495 \\ 3 \\ 70 \end{bmatrix} \leq \begin{bmatrix} T_F \\ T_R \\ C_{O_2} \\ C_{CO} \end{bmatrix} \leq \begin{bmatrix} 395 \\ 505 \\ 5 \\ 130 \end{bmatrix}$$

and input constraints

$$\begin{bmatrix} 9900 \\ 8 \\ 0.09 \end{bmatrix} \leq \begin{bmatrix} F_R \\ F_F \\ V_P \end{bmatrix} \leq \begin{bmatrix} 10100 \\ 12 \\ 0.11 \end{bmatrix}$$

Nominal point. The nominally optimal operating point (OOP) obtained (Peng *et al.*, 2005) is $x^* = [372\ 495\ 4.79\ 70]$ and $u^* = [10100\ 9.83\ 0.103]$. At this point, the active constraints are at the lower limit of CO concentration and furnace temperature and at the upper limit of feed flow rate. In this case, the number of active constraints equal the number of manipulated inputs. Therefore, the system is fully constrained at the optimal point. Hence, the first order approximation of the cost would be suffice for further analysis. The linearized negative profit function (in \$/h) is $J_x = [0\,0\,0\,0.01]^T$; $J_u = [-10\ 30\ 0]^T$. Next, the performance signal z is defined by the matrices, $Z_x = [I_{4\times4}|0_{4\times3}]^T$; $Z_u = [0_{4\times3}|I_{3\times3}]^T$; $Z_d = [0]$ and the bound constraints written in the form of ${h_i}^T\tilde{z}_{ss} + t_i \leq 0$ are obtained from the rows of the matrix H and elements of vector t, $H = [I_{7\times7}| - I_{7\times7}]^T$; $t = [-23\ -10\ -0.21\ -60\ 0\ -2.17\ -0.007\ -17\ 0\ -1.79\ 0\ -200\ -1.83\ -0.013]^T$.

Results. The economically optimal operation of the preheating furnace reactor system can be achieved if we control the active constraints (i.e., furnace temperature and CO concentration) and keep the feed flow rate at its upper limit. For the assumed disturbance variances, there is no feasible backed off operating point in the open loop case (without the controller). However, with the help of controller design as a part of the formulation, we find the economic backed off operating point for the system as tabulated in Table 5.3. At the economic backed off point, the input constraint on feed flow rate is still at its bound which means that the economic value of this input is very high relative to other inputs and hence other inputs are used to achieve profitability. The

Table 5.3: Nominal values and EBOP solution of the preheater furnace reactor system

Variables	Description	Units	Nominal values	EBOP (closed loop)
	States (x)			
T_R	reactor temperature	°C	495	496.45
T_F	furnace temperature	°C	372	373.09
C_{O_2}	O_2 concentration	ppm	4.79	4.2517
C_{CO}	CO concentration	ppm	70	90.083
	Inputs (u)			
F_R	feed flow rate	bbl/day	10100	10100
F_F	fuel flow rate	bbl/day	9.83	9.9458
V_P	furnace vent position	%	0.103	0.10099

dynamic operating region along with the economic back-off point for the assumed confidence level is shown in Figures 5.7 - 5.12. We can see that, in order to ensure dynamic feasibility, the furnace temperature and CO concentration are backed-off from the active constraints whereas feed flow rate requires no back-off. However, increasing the disturbance magnitude may demand the feed flow rate to be backed-off. The optimal multivariable controller gain L designed using our approach is given by

$$L = \begin{bmatrix} 0.001 & 0.008 & 0.010 & 0.000 \\ -0.538 & -4.038 & 5.608 & 0.099 \\ -0.001 & -0.013 & -33.498 & 1.249 \end{bmatrix}$$

It is important to note from the first row of the L matrix that the feed flow rate is hardly adjusted under dynamic conditions. In other words, the feed flow rate should be kept at its limiting value to achieve optimality. Therefore, other inputs (fuel flow rate and vent position) are manipulated to ensure feasible operation under dynamic conditions. The lost profit for operating the system at the economic backed-off operating point is $3.93 per day.

5.4.3 Evaporation process

In this example, we illustrate the backed-off operating point selection problem in a partially constrained system, that is, when there exists some unconstrained degrees of freedom at the nominal optimal point. Further, the economic impact of controller design is addressed.

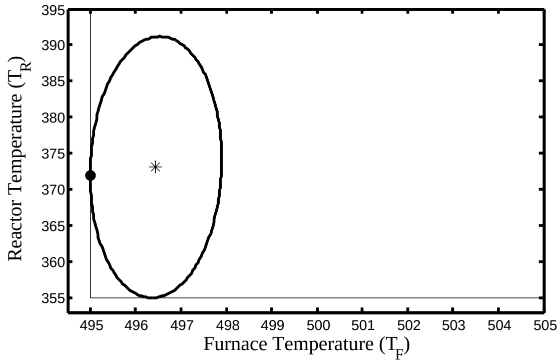

Figure 5.7: Furnace temperature vs reactor temperature

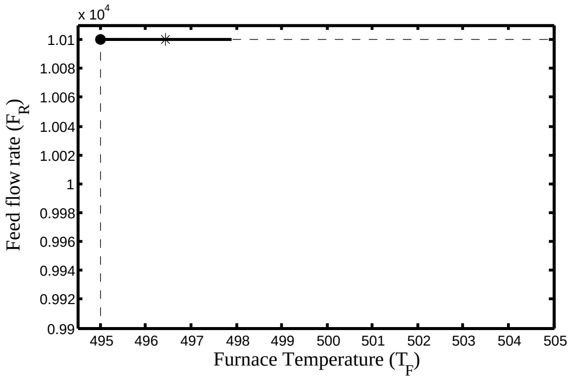

Figure 5.8: Furnace temperature vs feed flow rate

Description. The forced-circulation evaporator system is depicted in Figure 5.13, where the concentration of the feed stream is increased by evaporating the solvent through a vertical heat exchanger with circulated liquor (Newell and Lee, 1989). The overhead vapor is condensed by the use of process heat exchanger. The separator level is assumed to be perfectly controlled using the exit product flow rate F_2 which also eliminates the integrating nature of the state. The economic objective is to maximize the operational profit [$/h], formulated as a minimization problem of the negative profit (Kariwala *et al.*, 2008). The first three terms of (5.38) are utility costs relating to steam, coolant and pumping, respectively. The fourth term is the raw material cost, whereas the last term is the product value.

$$J = 600F_{100} + 0.6F_{200} + 1.009(F_2 + F_3) + 0.2F_1 - 4800F_2 \tag{5.38}$$

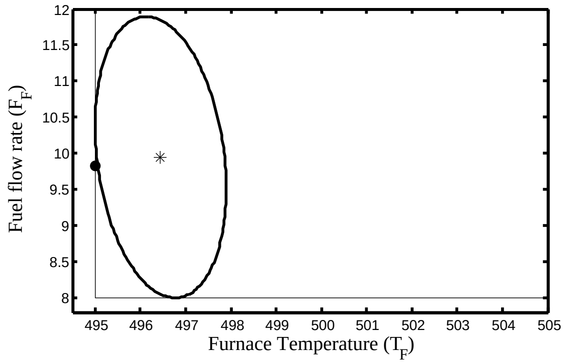

Figure 5.9: Furnace temperature vs fuel flow rate

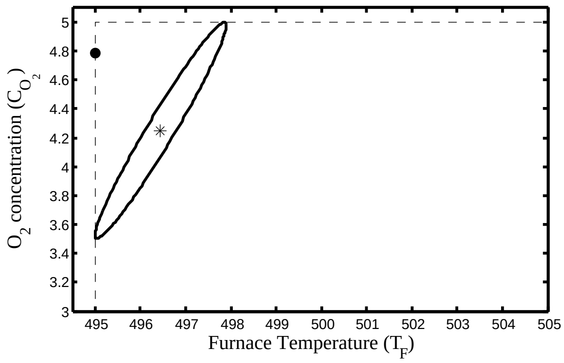

Figure 5.10: Furnace temperature vs O_2 concentration

The process has the following constraints related to product specification, safety, and design limits:

$$X_2 \geq 35\% \tag{5.39}$$

$$40\ kPa \leq P_2 \leq 80\ kPa \tag{5.40}$$

$$P_{100}\ \leq 400\ kPa \tag{5.41}$$

$$0\ kg/min \leq F_{200} \leq 400\ kg/min \tag{5.42}$$

$$0\ kg/min \leq F_1 \leq 20\ kg/min \tag{5.43}$$

$$0\ kg/min \leq F_3 \leq 100\ kg/min \tag{5.44}$$

Nominal operating point. The nominal steady state values are obtained by solving a non-linear optimization problem for the nominal values of disturbances and the profit is found to be $J = \$693.41/h$ and the nominal values are shown in Table 5.4. At the nominal optimal point, there are two active constraints: product composition, $X_2 = 35\%$ and steam pressure, $P_{100} =\ 400\ kPa$. The corresponding Lagrange multipliers

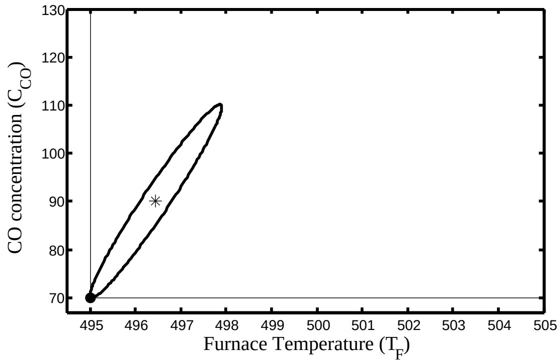

Figure 5.11: Furnace temperature vs CO concentration

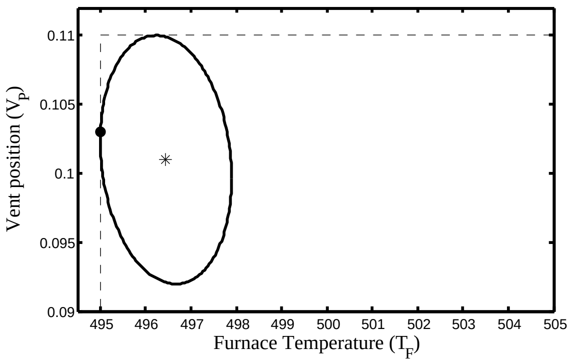

Figure 5.12: Furnace temperature vs vent position

are 229.36 $\$/\%\ h$ and -0.096685 $\$/kPa\ h$, respectively.

Degree of freedom analysis. The process model has seven degrees of freedom. Inlet conditions of the feed (flow rate, composition, temperature) and inlet temperature of the condenser are considered as disturbances (i.e., $d = [F_1\ X_1\ T_1\ T_{200}]^T$). There are three manipulated inputs, $u = [F_3\ P_{100}\ F_{200}]^T$. The disturbance range is assumed to be 10% variation of the nominal value (i.e., $\Sigma_d = diag([1\ 0.25\ 16\ 6.25])^2$) and the set of active constraints do not change in the whole range of disturbances . It is important to note that there is one unconstrained degrees of freedom.

Linearized steady state model. A linear approximation of the process model at the nominal optimum yields,

$$A = \begin{bmatrix} -0.16709 & -0.17185 \\ -0.013665 & -0.043132 \end{bmatrix};$$

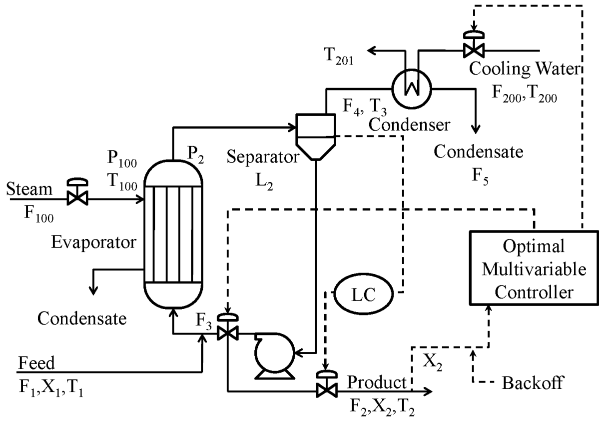

Figure 5.13: Evaporator system

Table 5.4: Variables and nominal optimal values of the evaporation process

Variables	Description	Nominal value
	States (x)	
X_2	product composition	35.00 %
P_2	operating pressure	56.15 kPa
	Inputs (u)	
F_3	recirculating flow rate	27.70 kg/min
P_{100}	steam pressure	400 kPa
F_{200}	cooling water flow rate	230.57 kg/min
	Disturbances (d)	
F_1	feed flow rate	10.00 kg/min
X_1	feed composition	5.00 %
	Dependent variables	
F_2	product flow rate	1.43 kg/min
F_4	vapor flow rate	8.57 kg/min
F_5	condensate flow rate	8.57 kg/min
F_{100}	steam flow rate	9.99 kg/min
Q_{100}	heat duty	365.63 kW
Q_{200}	condenser duty	330.00 kW

$$B = \begin{bmatrix} 0.44083 & 0.04217 & 0 \\ 0.062976 & 0.0060243 & -0.0016249 \end{bmatrix};$$

$$G = \begin{bmatrix} -1.2211 & 0.5 & 0.031818 & 0 \\ 0.039837 & 0 & 0.0045455 & 0.03665 \end{bmatrix}$$

The output z are defined by the matrices,

$$Z_x = [I_{2\times2}|0_{2\times3}]^T; Z_u = [0_{3\times2}|I_{3\times3}]^T; Z_d = [0_{4\times5}]^T$$

and the bound constraints written in the form of ${h_i}^T \tilde{z}_{ss} + t_i \leq 0$ are obtained from

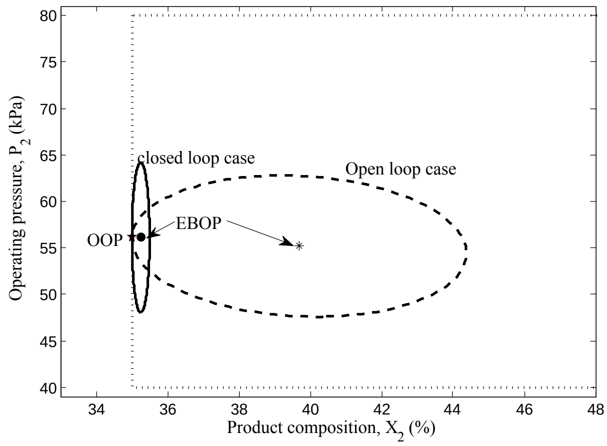

Figure 5.14: Product composition vs operating pressure. a) Open loop case: F_3 and F_{200} are constant. b) Closed loop case: F_3 and F_{200} are used for control of X_2

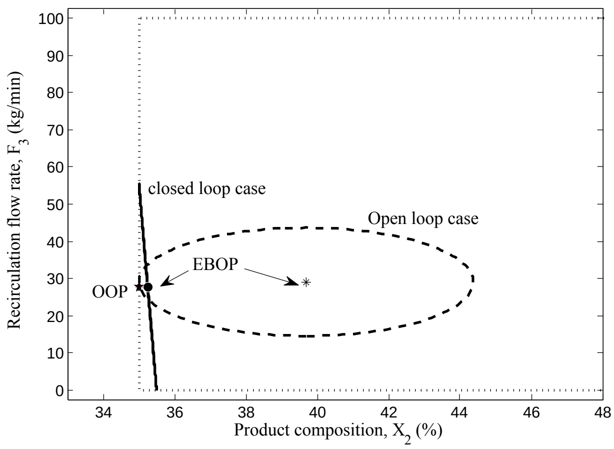

Figure 5.15: Product composition vs recirculation flow rate

the rows of the matrix H and elements of vector t, $H = [I_{5\times5}| - I_{5\times5}]^T$; $t = [-5\ -23.849\ -72.299\ 0\ -169.43\ 0\ -16.151\ -27.701\ -200\ -230.57]^T$. The linearized negative profit function is

$$J_x = [-293.23\ -526.8]^T; J_u = [1368.9\ 130.85\ 0.6]^T$$

As the input P_{100} is constrained, the quadratic penalty is included only for the unconstrained inputs and the numerical perturbation of inputs F_3 and F_{200} yield,

$$J_{uu} = \begin{bmatrix} 4.4953 & 0.00010226 \\ 0.00010226 & 0.0052699 \end{bmatrix}$$

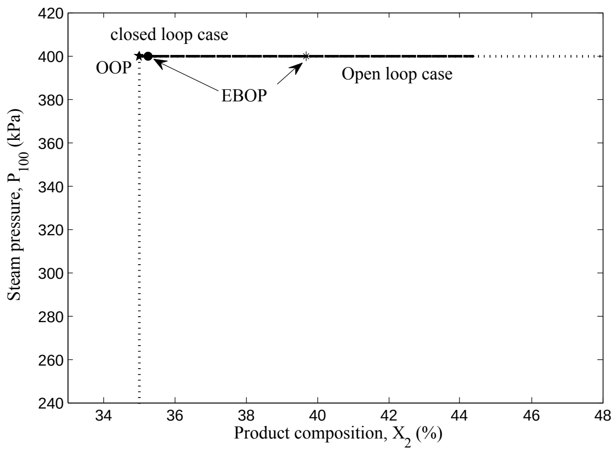

Figure 5.16: Product composition vs steam pressure

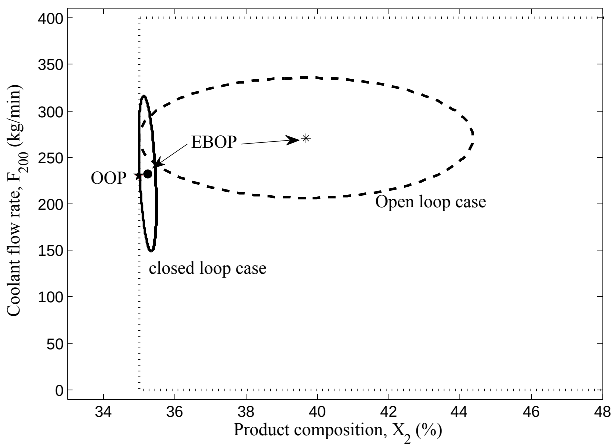

Figure 5.17: Product composition vs coolant flow rate

Table 5.5: Nominal values and EBOP solution of the evaporator system

Variables	Units	Nominal values	EBOP (quadratic cost) closed loop (proposed)	EBOP (quadratic cost) open loop ($u = 0$)	EBOP (linear cost) closed loop
		States			
X_2	$\%$	35.00	35.26	39.75	35.41
P_2	kPa	56.15	56.10	55.16	76.53
		Inputs			
F_3	kg/min	27.70	27.78	29.12	35.80
P_{100}	kPa	400.00	400	400	399.99
F_{200}	kg/min	230.57	232.71	271.65	0.01
Profit	$\$/h$	693.41	634.76	-414.92	600.12

Results. For the case of full state information, the amount of back off required to remain feasible for a 10% variation in the nominal disturbances is tabulated in Table 5.5. It is to be noted that the amount of back-off for steam pressure (P_{100}) is zero as expected as it is a input variable. However, the assumed disturbances have significant effect on product exit composition, X_2. The EBOP solution and EDOR for the open loop and closed loop case are shown as ellipses in Figures 5.14 - 5.17. The loss obtained for operating the evaporator at this backed off operating point is $\$58.65/h$ which corresponds to the achievable profit of $\$634.76/h$. In other words, the loss we incur to ensure feasible operation with 95% confidence interval is $\$58.65/h$. Indeed, the back-off estimated is the best possible lower bound for the product composition to ensure feasibility because of the simultaneous consideration of controller in the formulation. This could be inferred from Table 5.5 by comparing the closed loop solution with the open loop solution. The multivariable feedback controller ($u = Lx$) to be implemented to operate the system profitably is

$$L = \begin{bmatrix} -108.5643 & 0.3868 \\ -0.0606 & 0.0002 \\ -123.2216 & 97.3625 \end{bmatrix}$$

Without the controller (open loop case), the amount of back off required is higher and the process would incur a loss of $\$414.92/h$. Note that the optimal controller is using both F_3 and F_{200} to control the product composition with the aim of minimizing the overall cost. The corresponding state feedback gain could be used to determine the appropriate objective function weights using the inverse optimality results of Chmielewski and Manthanwar (2004) and could then be implemented using model predictive control. The back off operating point determined above is given as set point to the control system. It is important to note that without the quadratic term, the EBOP solution obtained by solving formulation (5.27) is $[x^T u^T] = [35.41\ 76.53\ 35.80\ 399.99\ 0.01]$. Note that for instance, F_{200} is changed from 230.57 to 0.01 kg/min, which is unrealistic. This corresponds to the lower left corner in Figure 5.17. Hence, the quadratic term in the cost function is important in the partially constrained case to get a meaningful solution.

Computational issues. In order to study the computational complexity of the algorithm, we present the number of iterations and total CPU time taken by the proposed two-stage iterative solution scheme in Table 5.6. The algorithm is implemented on a

Windows machine with Intel(R)Core(TM)i7 processor @ 3.4GHz CPU and 4 GB of RAM. At each iterating step, two convex problems are solved. The size of the problem (number of states and inputs) and the feasible input space (as defined by the bounds) determines the fastness of the algorithm. From the results of mass spring damper system presented in Table 5.6, it can be inferred that for all three cases the time taken per iteration are almost the same. This could be attributed to the size of the problem solved at each iteration being the same. The increase in size increases the time taken per iteration, as seen from the results of pre-heater furnace reactor system and evaporator system. On the other hand, the number of iterations depends on the size of the feasible input space as the solution progresses along the iteration by redefining the bounds on the variance of inputs.

Table 5.6: Computational efficiency of the proposed EBOP algorithm

Case study	CPU time (s)	No. of iterations	Time/iter (s)
Mass spring damper system - Case A 2 states and 1 input	4.0747	8	0.5093
Mass spring damper system - Case B 2 states and 1 input	7.0932	14	0.5066
Mass spring damper system - Case C 2 states and 1 input	6.4460	14	0.4604
Pre-heater reactor furnace system 4 states and 3 inputs	6.6480	6	1.1080
Evaporator system 2 states and 3 inputs	5.7975	9	0.6441

5.5 Summary

A stochastic formulation to compute the most profitable and feasible operating point for Gaussian white noise type disturbances has been presented. A two-stage iterative algorithm has been proposed to solve the dynamic back-off problem. Several case studies here demonstrate the generality of the formulation (i.e., applicability to both fully constrained and partially constrained cases). In particular, the evaporator system demonstrated the need for including quadratic approximations of the cost function in partially constrained systems to achieve meaningful economic backed-off point. Since the controller is a decision variable in the formulation, the most economical operating point is determined which, in fact, gives the best possible lower bound of the achievable profit. The formulation can be extended to include measurement noise as an additional source of uncertainty.

CHAPTER 6

ECONOMIC PERFORMANCE OF MODEL PREDICTIVE CONTROL FOR CONSTRAINED PROCESSES

In this chapter, we address the economic performance of MPC while operating at economically backed-off operating point. For this purpose, we revisit the economic back-off selection problem presented in Chapter 5, in discrete time framework. Recall that the operation at constrained optimal point will result in infeasibility, and hence the selection of an economically optimal yet dynamically feasible operating point is vital for efficient operation. The basic idea in set point selection based on economic back-off is that the dynamic operating region should have the least variability in the active constrained variables while ensuring feasibility of other variables. In other words, the dynamic operating region is oriented by proper design of a linear multivariable controller such that the variability in active constrained variables are as low as possible. This controller design can be transformed into an equivalent objective function weights of the MPC controller. Demonstrative case studies are presented to illustrate the economic performance of the MPC controller at economic back-off point.

6.1 Introduction

In a typical chemical process, the plant-wide control is carried out in a two-layer hierarchial setting: the upper real time optimization layer determines the operating point by minimizing an economic cost function subject to steady state model of the process; whereas the lower control layer steers the plant to the computed optimal point despite disturbances entering the process (Aske *et al.*, 2008). Model Predictive Control (MPC) is one of the successfully employed techniques in the control layer because of its ability to handle constraints, its direct applicability to multivariable systems and its potential benefit in improving the economic performance of the plant. In the standard MPC problem, the process is regulated at the optimal steady state point compensating for the undesirable disturbances and hence minimizes the cost function that measures the deviation from the steady state point. This framework relies on two fundamental assumptions: Firstly, the operating point lies in the interior of the feasible region to guarantee feasibility of the control problem and secondly, only steady state effect on economics is considered to be important and hence the economic benefit one could achieve during transients is ignored. However, optimizing for process economics under steady state conditions may tend to operate the process plant at the boundary of the feasible region. Therefore, the focus of this chapter is to present the MPC framework for nominally constrained processes and discuss the notion of profitability during transient operation.

Maximizing profitability of the plant at steady state may require one to operate at the constrained optimal point. For the case of active state or output constraints at the optimal point, uncertainties such as disturbances, modelling errors and measurement errors might cause frequent violation of the constraints. Consequently, it leads to exceeding environmental limits, producing inferior quality products and at times result in unsafe process conditions. In other words, the set point decided on the basis of steady state model of the process may not be robust enough to handle dynamic conditions caused by the uncertain process environments. As a result, it is difficult to guarantee feasibility of the control problem. In order to address the infeasibility issue, Scokaert and Rawlings (1999) have developed two approaches that reduces both the magnitude and duration of constraint violations. In the first approach (optimized minimal time), the performance is Pareto optimal, that is, the magnitude of constraint violation is minimal and consistent with the faster return of the state. In the second approach (soft constrained MPC), the use of sum of linear and quadratic penalty terms at each time step guarantees the control algorithm to have a solution, however, even when the state constraints can be satisfied, the soft-constraint law may not enforce the state constraints and unnecessary violations may still result. On the other hand, in addition to the problem of infeasible operation, the input constraints being active at the optimal point might result in unreachable target values for some set of disturbances. In this regard, Rao and Rawlings (1999) have demonstrated for the case of constant step-like disturbances, the possibility of unreachable target values due to input saturation and infeasible operations. Therefore, they proposed a modified MPC algorithm which involves updating the reachable target values before determining the optimal input sequence at every time step. The target values are computed by solving a quadratic programming problem using exact penalties for the estimated values of states and disturbances. The problem of target calculation, to handle model uncertainties, is casted as a second order cone programming problem. In summary, although regulating the plant at the active constraints will be profitable, it poses difficulty in operation due to constraint violations. To circumvent this infeasibility issue, we presented the back-off approach in Chapter 5.

To gain economic advantage during transients, very recently there is increasing interest in Economic Model Predictive Control, by coupling the two-layer structure into a single layer that directly optimizes the economic objective at each time step. Unlike the quadratic cost function of the standard MPC problem, the economic cost function can be non-linear and non-convex, in general. Therefore, the existing MPC properties are no longer valid. Hence, to establish the closed loop properties (such as convergence and nominal asymptotic stability) of the economic MPC problem, Rawlings *et al.* (2008) proposed a terminal constraint MPC formulation, in which the system is driven to optimal steady state point at the end of the horizon. The receding horizon implementation of the terminal constraint MPC is demonstrated for the case of unreachable set points due to input constraints. The asymptotic stability result was also presented for terminal penalty MPC by introducing rotated stage cost. Later, Diehl *et al.* (2011) presented a Lyapunov function for the economic MPC problems with terminal constraints that satisfy strong duality assumption of the steady state problem to establish asymptotic stability of the closed loop system. Similar Lyapunov based stability analysis for the terminal penalty formulation was established by imposing a region constraint on the terminal state (Amrit *et al.*, 2011). Further, they showed that strict dissipativity is sufficient for guaranteeing asymptotic stability of the closed loop system. In Angeli *et al.* (2012), the average control performance of the eMPC was shown to outperform the optimal steady state economic performance.

In this chapter, we work within the MPC framework for the case of active state or input constraints. For this purpose, we adapt the economic back-off approach presented in the previous chapter, to determine the most profitable operating point. The benefit of this approach is that the optimal multivariable controller, that can achieve the profitability without constraint violations, comes as part of the solution. This optimal multivariable controllers can be tailored to be used in MPC framework using inverse optimality results. Finally, illustrations are provided to demonstrate the performance of MPC using the proposed approach.

6.2 Standard MPC vs Economic MPC

Consider the discrete-time, constrained dynamic system of the form

$$s_{k+1} = f(s_k, m_k, p_k) \tag{6.1a}$$

$$v_k = h(s_k, m_k, p_k) \tag{6.1b}$$

$$v_{min} \leq v_k \leq v_{max} \tag{6.1c}$$

with state variables $s_k \in \mathbb{R}^{n_x}$, manipulated inputs $m_k \in \mathbb{R}^{n_u}$ and disturbances $p_k \in \mathbb{R}^{n_d}$. The input and output constraints are accounted in vector, v_k. In general, the process plants are operated at the steady state point that will yield maximum profit. Typically this operating point is determined by solving a non-linear optimization problem for the nominal values of disturbances ($\overline{p}_0$) with negated profit function, J as objective and the vector valued functions, f and h, denote the steady state model of the process and performance bounds on the variables, respectively. Mathematically, the steady state optimization problem can be expressed as

$$\min_{m_s} \quad J(s_s, m_s, \overline{p}_0) \tag{6.2a}$$

$$s.t. \quad s_s = f(s_s, m_s, \overline{p}_0) \tag{6.2b}$$

$$v_s = h(s_s, m_s, \overline{p}_0) \tag{6.2c}$$

$$v_{min} \leq v_s \leq v_{max} \tag{6.2d}$$

The optimal solution is denoted by $\{s_s^*, m_s^*\}$. It can be either unconstrained or constrained. However, if some of the bound constraints are active at the optimal solution, then there might be violation of constraints for some other values of disturbance variables. If the manipulated inputs are constrained, they typically correspond to physical limitations. On the other hand, if the state variables are constrained, they correspond to safety and performance degradations. Therefore, it is difficult to operate at this nominally constrained optimal point without performance degradation or infeasibility issues.

In continuous processes, the plant has to be regulated at the economically optimal steady state point despite disturbances for achieving maximum profit. MPC can be successfully deployed for this purpose. The general philosophy of MPC is to utilize the dynamic plant model to predict the transients and determine the optimal input sequence that minimizes the departure from the optimal steady state point. The MPC controller, however, implements only the first of the input sequence at the current time instant. In the next time instant, the current measurements give feedback information to the controller and solve for the next set of optimal input sequence in a receding horizon fashion (Maciejowski, 2002; Qin and Badgwell, 2003; Rawlings and Mayne, 2009).

Let us define deviation variables with respect to the nominally optimal point: $\tilde{x}_k = s_k - s_s^*$, $\tilde{u}_k = m_k - m_s^*$, and $\tilde{d}_k = p_k - \overline{p}_0$. Linearizing around the nominal point, the dynamic process model can be expressed as

$$\tilde{x}_{k+1} = A_d\tilde{x}_k + B_d\tilde{u}_k + G_d\tilde{d}_k \tag{6.3}$$

where A_d, B_d and G_d are the partial derivative of f evaluated at $(s_s^*, m_s^*, \overline{p}_0)$. Similar development on the output vector v_k (i.e., denoting $\tilde{z}_k = v_k - v_s^*$) yields

$$\tilde{z}_k = Z_x\tilde{x}_k + Z_u\tilde{u}_k + Z_d\tilde{d}_k \tag{6.4a}$$

$$\tilde{z}_{min} \leq \tilde{z} \leq \tilde{z}_{max} \tag{6.4b}$$

where Z_x, Z_u and Z_d are the partial derivative of h evaluated at $(s_s^*, m_s^*, \overline{p}_0)$. In a standard MPC formulation, the objective function is a measure of the weighted sum of squares of deviation of the dynamic values of the state and input, (s_k, m_k), from the optimal steady state point, (s_s^*, m_s^*). Therefore, in terms of deviation variables, the plant has to be regulated at the origin. This objective can be mathematically expressed as:

$$\ell(\tilde{x}_k, \tilde{u}_k) = \tilde{x}_k^T Q \tilde{x}_k + \tilde{u}_k^T R \tilde{u}_k \tag{6.5}$$

with $Q \succ 0$ and $R \succ 0$. For the control horizon of N moves, we can denote the overall stage cost as $\Phi(\tilde{\mathbf{x}}, \tilde{\mathbf{u}}) = \sum_{k=0}^{N-1} \ell(\tilde{x}_k, \tilde{u}_k)$. The MPC regulation problem with terminal constraints can be now stated as:

$$\min_{\tilde{\mathbf{u}}} \quad \Phi(\tilde{\mathbf{x}}, \tilde{\mathbf{u}}) \tag{6.6a}$$

$$s.t. \quad \tilde{x}_{k+1} = A_d \tilde{x}_k + B_d \tilde{u}_k + G_d \tilde{d}_k, \quad k = 0 \quad to \quad N-1 \tag{6.6b}$$

$$d_{k+i} = d_k, \quad \forall k + i > k \tag{6.6c}$$

$$\tilde{z}_k = Z_x \tilde{x}_k + Z_u \tilde{u}_k + Z_d \tilde{d}_k \tag{6.6d}$$

$$\tilde{z}_{min} \leq \tilde{z}_k \leq \tilde{z}_{max} \tag{6.6e}$$

$$\tilde{x}_N = 0 \tag{6.6f}$$

The terminal constraint on the state variables enforces stability of the MPC problem. Alternately, the terminal penalty can be included in the cost function such that $\Phi_N(\tilde{x}_N) = \tilde{x}_N^T P \tilde{x}_N$, where P is the terminal penalty matrix. The resulting optimal input sequence that takes the state from $\tilde{x}_0$ to origin, is given by

$$\tilde{\mathbf{u}}^* = \{\tilde{u}_0^*, \tilde{u}_1^*, \ldots, \tilde{u}_{N-1}^*\} \tag{6.7}$$

and the corresponding state sequence is given by

$$\tilde{\mathbf{x}}^* = \{\tilde{x}_1^*, \tilde{x}_2^*, \ldots, \tilde{x}_N^*\} \tag{6.8}$$

However, the MPC control law implements only the first input move of the optimal input sequence, and it can be expressed as

$$\tilde{u}_{mpc} = \tilde{u}_0^*(x) \tag{6.9}$$

We assume that the measurement of the state variables are available accurately. Given the current values of state variables as feedback information to the controller, the optimization problem is resolved to find the new set of optimal input sequence. At each time step, the MPC controller requires one to solve a quadratic programming problem for which efficient algorithms like active set or interior point methods are available (Nocedal and Wright, 2006). Notice that Q and R are the tuning parameters in the controller. Choosing large values of Q in comparison to R, will allow for large control actions and quickly drive the state to the optimal steady state point. Conversely, choosing large values of R relative to Q will penalize for large control action and slow down the rate at which the state approaches the optimal operating point. Hence, appropriate selection of the tuning parameters is necessary for better control.

Alternately, one can choose the operational cost of the plant defined in (6.2a) as MPC stage cost. This will directly optimize for the dynamic economic performance rather than regulating the plant at the optimal steady state point. However, to ensure stability and convergence, terminal constraints on the state variables can be set as the optimal operating point. Taking the stage cost as $\ell(\tilde{x}_k, \tilde{u}_k) = J(\tilde{x}_k, \tilde{u}_k)$, the economic MPC formulation with terminal constraints can be expressed as:

$$\min_{\tilde{\mathbf{u}}} \quad \Phi_{eco}(\tilde{\mathbf{x}}, \tilde{\mathbf{u}}) = \sum_{k=0}^{N-1} J(\tilde{x}_k, \tilde{u}_k) \tag{6.10a}$$

$$s.t. \quad \tilde{x}_{k+1} = A_d\tilde{x}_k + B_d\tilde{u}_k + G_d\tilde{d}_k, \quad k = 0 \quad to \quad N-1 \tag{6.10b}$$

$$d_{k+i} = d_k, \quad \forall k+i > k \tag{6.10c}$$

$$\tilde{z}_k = Z_x\tilde{x}_k + Z_u\tilde{u}_k + Z_d\tilde{d}_k \tag{6.10d}$$

$$\tilde{z}_{min} \leq \tilde{z}_k \leq \tilde{z}_{max} \tag{6.10e}$$

$$\tilde{x}_N = 0 \tag{6.10f}$$

The direct use of an economic objective function as MPC cost enables one to determine the economically optimal input sequence at each time step. In other words, economic MPC couples the two-layer hierarchical setting of the traditional plant-wide control structure into a single layer. Further, it can also be interpreted as optimizing the input sequence for the current value of disturbance, rather than for the nominal value of disturbance as in standard MPC problem. Although one can achieve economic advantage during transients, the stability and robustness properties are still required for the general non-linear form of the economic cost. Some of the recent works, along these lines, established that dissipativity is sufficient for guaranteeing asymptotic stability of the closed loop system (Angeli *et al.*, 2012; Rawlings *et al.*, 2012). For a general non-convex economic objective, the regularization terms are often added to achieve dissipativity. However, it was advocated in Biegler (2013) that the plant operation is easier to monitor and manage at steady state conditions. Further, stability and robustness properties can be easily analyzed under steady state assumptions.

6.3 Target selection using economic back-off approach

In this section, we present an algorithm to determine the most profitable operating point (i.e., economic back-off point) in the presence of random Gaussian white noise disturbances for optimally constrained, discrete-time processes. First, we present the optimization formulation involving conic constraints to obtain the economic backed-off point. The resulting formulation contains a non-linear matrix equality, which makes the resulting optimization problem as non-convex. Therefore, finally we present the iterative solution procedure to obtain the economic back-off point.

6.3.1 Problem formulation

The objective of this subsection is to present the optimization formulation that finds the economically optimal backed-off operating point by a suitable controller design. Typically, this requires one to solve a dynamic optimization problem involving differential constraints (Figueroa *et al.*, 1996; Mohideen *et al.*, 1996). Owing to the infinite-dimensional nature of the problem and higher computational cost, we propose to present an alternate Lyapunov based stochastic approach. For this purpose, we make the following assumptions:

- Disturbances are the only source of uncertainty considered and they are characterized by Gaussian white noise process with zero mean and known variances.
- The set of active constraints do not change over the assumed disturbance magnitude.
- The dynamic behavior of the system is described by a linear, discrete-time, state space model.

- Complete information about the states are available at any instant of time.
- A linear multi-variable controller with full state information ($u_k = L_d x_k$) is available for feedback.

Following the definition of deviation variables as presented in previous section and assuming zero mean Gaussian white noise type of disturbance vector, we have $\tilde{d}_{ss} = 0$. Now, the steady state process model can be expressed as

$$\tilde{x}_{ss} = A_d \tilde{x}_{ss} + B_d \tilde{u}_{ss} \tag{6.11}$$

Similar development on the output vector $\tilde{z}$ yields

$$\tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} \tag{6.12a}$$

$$\tilde{z}_{min} \leq \tilde{z}_{ss} \leq \tilde{z}_{max} \tag{6.12b}$$

Recall that the selection of backed-off operating point should result in dynamic feasibility and hence defining the new deviation variables around the back-off point to define the dynamics: $x = \tilde{x} - \tilde{x}_{ss}$, $u = \tilde{u} - \tilde{u}_{ss}$, we can write the following discrete-time linear state space model and the performance constraints as

$$x_{k+1} = A_d x_k + B_d u_k + G_d d_k \tag{6.13}$$

$$z_k = Z_x x_k + Z_u u_k + Z_d d_k \tag{6.14}$$

$$\tilde{z}_{min} - \tilde{z}_{ss} \leq z_k \leq \tilde{z}_{max} - \tilde{z}_{ss} \tag{6.15}$$

Our objective is to determine the backed-off operating point that minimizes the loss in operational profit while ensuring the feasible process dynamics. For this purpose, we assume a linear multivariable controller of the form, $u_k = L_d x_k$. The system dynamics around the back-off point can be described using closed loop steady state covariance matrix of the state vector ($\Sigma_x := \mathbf{E}[x_k^T x_k]$), which is a symmetric positive semi-definite solution to the Lyapunov equation,

$$\Sigma_x = (A_d + B_d L_d)\Sigma_x (A_d + B_d L_d)^T + G_d \Sigma_d G_d^T \tag{6.16}$$

where Σ_d is the diagonal matrix denoting the variances of the disturbance variables.

Similar development of the signal z_k yields

$$\Sigma_z = (Z_x + Z_u L_d)\Sigma_x(Z_x + Z_u L_d)^T + Z_d \Sigma_d {Z_d}^T \tag{6.17}$$

It is important to notice that the covariance, Σ_z can be geometrically inferred as ellipsoids that approximates the dynamic operating region with backed-off operating point as center. Denoting $\Sigma_z = P^2$, the ellipsoidal description of the operating region can be expressed as

$$\mathcal{E} = \{\tilde{z}_{ss} + \alpha P z \mid \|z\|_2 \leq 1\} \tag{6.18}$$

where P is the positive square root of Σ_z and α depends on the confidence limit (e.g., for a confidence limit of 95%, $\alpha = 2$). Notice that $\tilde{z} = \tilde{z}_{ss} + \alpha P z$. Therefore, we enforce dynamic feasibility by finding the ellipsoid within the performance bounds which is given by

$$\mathcal{E} = \{(\tilde{z}_{min} \leq \tilde{z}_{ss} + \alpha P z \leq \tilde{z}_{max}) \mid \|z\|_2 \leq 1\} \tag{6.19}$$

This representation ensures that the whole ellipsoid should lie within the performance bounds. Thus, the problem can be restated as finding the center of the ellipsoid close to the optimal operating point such that the ellipsoid is contained within performance bounds.

Theorem 6.3.1 *(Peng* et al.*, 2005)* $\exists$ *stabilizing* L_d, $X \succeq 0$ *s.t.* $\Sigma_x = (A_d+B_d L_d)\Sigma_x(A_d+B_d L_d)^T + G_d \Sigma_d G_d^T$ *and* $\Sigma_z = (Z_x + Z_u L_d)\Sigma_x(Z_x + Z_u L_d)^T + Z_d \Sigma_d {Z_d}^T$ *if and only if* $\exists\, Y$, $X \succ 0$ *and* $Z \succ 0$ *s.t.*

$$\begin{bmatrix} X - G_d \Sigma_d G_d^T & (A_d X + B_d Y) \\ (A_d X + B_d Y)^T & X \end{bmatrix} \succ 0,$$

$$\begin{bmatrix} Z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succ 0.$$

For proof of the above theorem, the reader is referred to (Peng et al.*, 2005).* ∎

Application of Theorem 5.2.3 recasts the infinite dimensional dynamic feasibility constraints as finite dimensional second order cone constraints which can be solved very efficiently using existing tools (Löfberg, 2004). Application of Theorem 6.3.1 ensures that the stabilizing multivariable controller for the process can be found. Hence, the final formulation of the discrete time economic back-off selection problem can be expressed as:

$$
\begin{aligned}
\min \quad & J_x{}^T \tilde{x}_{ss} + J_u{}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} && \text{(6.20a)}\\
s.t. \quad & \tilde{x}_{ss} = A_d \tilde{x}_{ss} + B_d \tilde{u}_{ss} && \text{(6.20b)}\\
& \tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} && \text{(6.20c)}\\
& \begin{bmatrix} X - G_d \Sigma_d G_d^T & (A_d X + B_d Y) \\ (A_d X + B_d Y)^T & X \end{bmatrix} \succ 0 && \text{(6.20d)}\\
& \begin{bmatrix} Z - Z_d \Sigma_d Z_d{}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 && \text{(6.20e)}\\
& P = Z^{1/2} && \text{(6.20f)}\\
& \|\alpha P h_i\|_2 + h_i^T \tilde{z}_{ss} + t_i \leq 0, i = 1, \ldots, 2n_z && \text{(6.20g)}
\end{aligned}
$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, $Y = L_d X^{-1}$, $X = \Sigma_x \succeq 0$, $Z = \Sigma_z \succeq 0$ and $P \succeq 0$ are the decision variables.

6.3.2 Solution methodology

The proposed optimization formulation for economic back-off selection is non-convex because of the non-linearity in Z. As discussed previously, the main challenge is to orient the ellipsoid (i.e., controller gain, L_d) such that the center of the ellipsoid is close to optimal operating point (i.e., EBOP, $\tilde{z}_{ss}$). In this regard, we recently proposed a two stage iterative algorithm to determine the economically backed-off operating point in continuous time framework. Here, we adapt the solution technique to discrete time framework. The basic concept of this two stage approach is shown in Figure 6.1 where we first find the feasible covariance ellipsoid Z_1 (or equivalently L_1) and progressively decrease the variances of the active constrained variables by creating relative bounds for other variables. For this purpose, we define the parameter δ as

$$
\delta_{i,j} = \frac{distance\ of\ variable\ i\ from\ its\ closest\ bound}{distance\ of\ variable\ j\ from\ its\ closest\ bound} \tag{6.21}
$$

The δ for the case shown in Figure 6.1 is given by

$$\delta_{i,j} = \frac{min(\Delta u_1, \Delta u_2)}{min(\Delta x_1, \Delta x_2)} \tag{6.22}$$

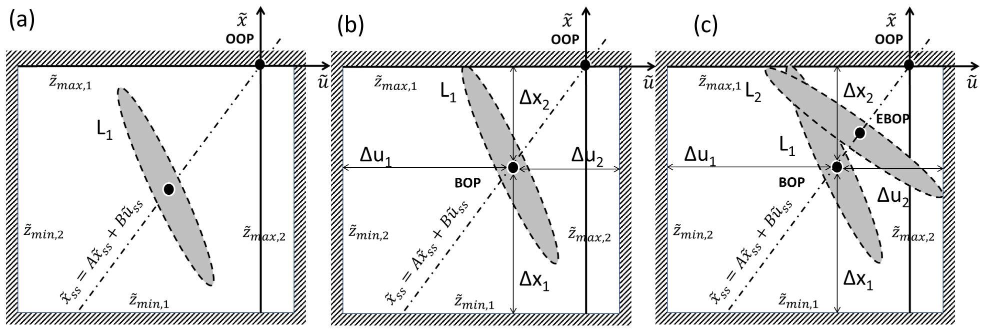

Figure 6.1: Two stage iterative solution approach for economic back-off selection: (a) stage 1 solution; (b) stage 2 solution; (c) converged stage 2 solution

The detailed algorithm is presented in Table 6.1. In the first stage, we determine a feasible covariance ellipsoid Z_1 that describe the dynamic operating region for the given confidence limit (say 95%). Although our objective is to minimize the variability in the active constrained variables to achieve profitability, directly minimizing for the respective variances might cause infeasibility since the operating point (i.e., the center of the ellipsoid) will be selected only in the second stage. Therefore, the better choice would be to minimize the variances of all the variables and progressively decrease the variability in active constrained variables. For this purpose, we choose the first stage cost function as trace of the ellipsoid. In the second stage, we determine the backed-off operating point for the computed Z_1. However, the solution obtained may not be economically optimal as no cost information is included in stage 1. In other words, the backed-off operating point depends critically on the computed Z_1 (solution from stage 1). From Figure 6.1, we can see that choosing a different covariance ellipsoid Z_2 leads to a better economically backed-off operating point. Notice that at the economic back-off point, the dynamic operating region touches the manipulated input constraint and the active constraint (controlled variable). From the stage 2 solution, we can update the parameter δ based on the obtained back-off point (i.e., center of the ellipsoid) to create lower bounds for the individual variances. This will result in reorienting the ellipsoid

Table 6.1: Algorithm for selecting economic back-off operating point for discrete time process

1	Initialize the parameter $\delta_{i,j} = 0$.
2	Find Z by solving the following convex problem (Stage 1), $$\begin{aligned} \min_{X\succeq 0, Z\succeq 0, Y} \quad & Tr(Z) \\ s.t. \quad & \begin{bmatrix} X - G_d\Sigma_d G_d^T & (A_dX + B_dY) \\ (A_dX + B_dY)^T & X \end{bmatrix} \succ 0 \\ & \begin{bmatrix} Z - Z_d\Sigma_d {Z_d}^T & Z_xX + Z_uY \\ (Z_xX + Z_uY)^T & X \end{bmatrix} \succeq 0 \\ & \sigma_{z,i}^2 < \frac{1}{4\alpha^2}\left(\tilde{z}_{max,i} - \tilde{z}_{min,i}\right)^2; i = 1 \cdots n_z \\ & \sigma_{z,i}^2 > \frac{\delta_{i,j}^2}{\alpha^2}\sigma_{z,j}^2; i = 1, j-1, j+1, n_z \end{aligned}$$ If no feasible Z can be found, exit.
3	Compute $P = Z^{1/2}$. Find the BOP ($\tilde{z}_{ss}$) by solving the following convex problem (Stage 2), $$\begin{aligned} \min_{\tilde{x}_{ss}, \tilde{u}_{ss}, \tilde{z}_{ss}} \quad & {J_x}^T\tilde{x}_{ss} + {J_u}^T\tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu}\tilde{u}_{ss} \\ s.t. \quad & \tilde{x}_{ss} = A_d\tilde{x}_{ss} + B_d\tilde{u}_{ss} \\ & \tilde{z}_{ss} = Z_x\tilde{x}_{ss} + Z_u\tilde{u}_{ss} \\ & \|\alpha P h_i\|_2 + h_i^T\tilde{z}_{ss} + t_i \leq 0, i = 1, \ldots, 2n_z \end{aligned}$$
4	Terminate on convergence. Otherwise, update $\delta_{i,j}$ using (6.21) from the current value of BOP and proceed to Step 2.

(i.e., designing a better controller) that has reduced variability in active constrained variables on subsequent iterations. This procedure is iterated until convergence. Notice that both the stage requires one to solve convex problems, which could be easily solved using CVX, a package for specifying and solving convex programs (Grant and Boyd, 2011).

6.4 MPC regulation at economic back-off point

In the previous section, we presented a method to determine the economical and dynamically feasible operating point along with the optimal linear multivariable controller. Though there exist different controllers that define the dynamic operating region, the one that gives the minimal loss in profit, due to backing-off, is determined. Therefore, the aim of this section is to utilize this economically backed-off operating point as set point in the MPC controller. Recall that the backed-off operating point is both economical and dynamically feasible in the presence of the obtained multivariable controller. In other words, the economic back-off point is statistically guaranteed to be profitable and feasible within the expected dynamic operating region, only if we employ the deviced multivariable controller. Therefore, the present challenge is on utilizing the designed multivariable controller within the MPC framework. To this end, we invoke the following inverse optimality result that finds the equivalent objective function weights of the MPC controller given the multivariable controller.

Theorem 6.4.1 *(Chmielewski and Manthanwar, 2004) If* $\exists\, P \succ 0$ *and* $R \succ 0$ *s.t.*

$$\begin{bmatrix} P - A_d^T P A_d + L_d^T (R + B_d^T P B_d) L_d & -L_d^T (R + B_d^T P B_d) - A_d^T P B_d \\ -(R + B_d^T P B_d) L_d - B_d^T P A_d & R \end{bmatrix} \succ 0$$

then $Q \hat{=} P - A_d^T P A_d + L_d^T (R + B_d^T P B_d) L_d$ *and* $M \hat{=} - L_d^T (R + B_d^T P B_d) - A_d^T P B_d$ *will be s.t.*

$$\begin{bmatrix} Q & M \\ M^T & R \end{bmatrix} \succ 0$$

and P *and* L_d *will satisfy* $P = A_d^T P A_d + Q - (M + A_d^T P B_d)(R + B_d^T P B_d)^{-1}(M + A_d^T P B_d)^T$ *and* $L_d = -(R + B_d^T P B_d)^{-1}(M + A_d^T P B_d)^T$. *For proof of the above* *theorem, the reader is referred to (Chmielewski and Manthanwar, 2004).* ∎

Given the system matrices, $\{A_d, B_d\}$ and the controller gain matrix, L_d, application of Theorem 6.4.1 will yield the objective function weights of the MPC controller (Ahmed and Chmielewski, 2013). Notice that the multivarible controller obtained from the proposed economic back-off formulation can be inferred as the LQR controller satisfying the constraints within the prescribed probability. In other words, the operation at the economic back-off point with the obtained multivariable controller will result in the (unconstrained) LQR solution in statistical sense. Therefore, the obtained multivariable controller is identical to the unconstrained MPC solution with infinite horizon. Therefore, the objective function weights will result in the operating region identical to that of the multivariable controller. In this work, we utilize this objective function weights with finite horizon approximation in the standard MPC framework. Denoting the objective function as the deviation from the economically backed-off operating point, we can define

$$\ell_{ebop}(x_k, u_k) = x_k^T Q x_k + x_k^T M u_k + u_k^T R u_k \tag{6.23}$$

The economically optimal MPC regulation framework can now be casted as:

$$\min_{\mathbf{u}} \quad \Phi_{ebop}(\mathbf{x}, \mathbf{u}) = \sum_{k=0}^{N-1} \ell_{ebop}(x_k, u_k) \tag{6.24a}$$

$$s.t. \quad x_{k+1} = A_d x_k + B_d u_k + G_d d_k, \quad k = 0 \quad \text{to} \quad N-1 \tag{6.24b}$$

$$z_k = Z_x x_k + Z_u u_k + Z_d d_k, \quad d_k \sim \mathcal{N}(0, \Sigma_d) \tag{6.24c}$$

$$z_{min} \le z_k \le z_{max} \tag{6.24d}$$

$$x_N = \tilde{x}_{ss} \tag{6.24e}$$

This approach is compatible with the standard MPC framework, where a quadratic programming problem is solved at each time step. The main benefit of this approach is that the profitable dynamic operating region is naturally accomplished by this choice of tuning parameters because the controller is designed by minimizing the variability of the active constrained variables. Hence, the dynamic economic benefits can also be attained within this framework.

6.5 Illustrations

6.5.1 Mass spring damper system

The purpose of this example is to illustrate the performance of MPC, regulated at the proposed backed-off operating point. Let us revisit the mass-spring-damper system discussed in Chapter 5. Recall that the economic objective is to bring the mass as close as possible to the upper bound on position. The optimal operating point is constrained at the mass position, $r^* = r_{max}$, $v^* = 0$ and $f^* = 3r_{max} + g$ (assuming $f_{max} \geq 3r_{max} + g$). Rewriting in deviation form, the discrete-time system matrices obtained with the sampling time of 0.5 min, are given by

$$A_d = \begin{bmatrix} 0.7397 & 0.2786 \\ -0.8359 & 0.1825 \end{bmatrix}; B_d = \begin{bmatrix} 0.0868 \\ 0.2786 \end{bmatrix}; G_d = \begin{bmatrix} 0.0868 \\ 0.2786 \end{bmatrix}$$

The dynamic feasible region is defined by box constraints: $\tilde{r}_{min} \leq \tilde{r} \leq \tilde{r}_{max}$ and $\tilde{f}_{min} \leq \tilde{f} \leq \tilde{f}_{max}$. Hence, the signal matrices are given by

$$Z_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; Z_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; Z_d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Results. If $r_{min} = -1$, $r_{max} = 1$, $f_{min} = 0$, $f_{max} = 15$, $g = 9.8$ and $\Sigma_w = 2.5$, the OOP is $r^* = 1$,$v^* = 0$ and $f^* = 12.8$ (since $f_{max} = 15 \geq 3r_{max} + g = 12.8$). For the current system, we have assumed a confidence level of 95% (i.e. $\alpha = 2$). The economic backed-off operating point determined is $(r_{EBOP} = 0.667, f_{EBOP} = 11.8)$ which results in a loss of 0.333. The multi-variable controller ($u_k = L_d x_k$) designed to operated feasibly at the economic backed-off operating point is $L_d = [-4.449, -2.0429]$. For the purpose of illustration, we generated the disturbance vector as 100 samples of the Gaussian white noise sequence with the variance of 2.5. Figure 6.2 shows the scatter plot of the resulting multivariable controller. The ellipse represents the theoretically expected dynamic operating region for the assumed disturbance magnitude. The simulated mass position values lies more or less within the expected dynamic operating region. Application of Theorem 6.4.1 yields the following

$$Q = \begin{bmatrix} 3.9489 & 0.9961 \\ 0.9961 & 0.6957 \end{bmatrix}; R = \begin{bmatrix} 0.1272 \end{bmatrix}; M = \begin{bmatrix} 0.617 \\ 0.2629 \end{bmatrix}$$

The scatter plot showing the performance of MPC controller, obtained using the above tuning parameters, is shown in Figure 6.3. Figures 6.4 - 6.5 are the time series plot showing the performance of the multivariable controller and MPC controller, respectively.

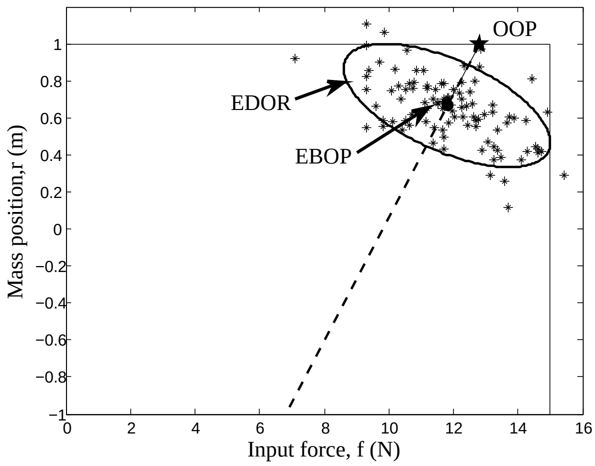

Figure 6.2: Scatter plot of a mass spring damper system showing the performance of multivariable controller, $u_k = L_d x_k$

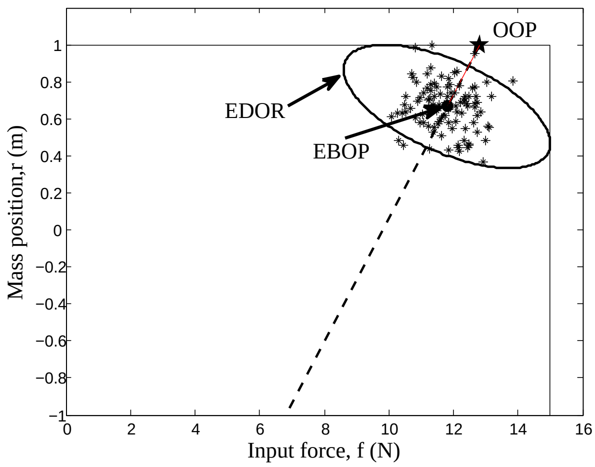

Figure 6.3: Scatter plot of a mass spring damper system showing the performance of MPC controller

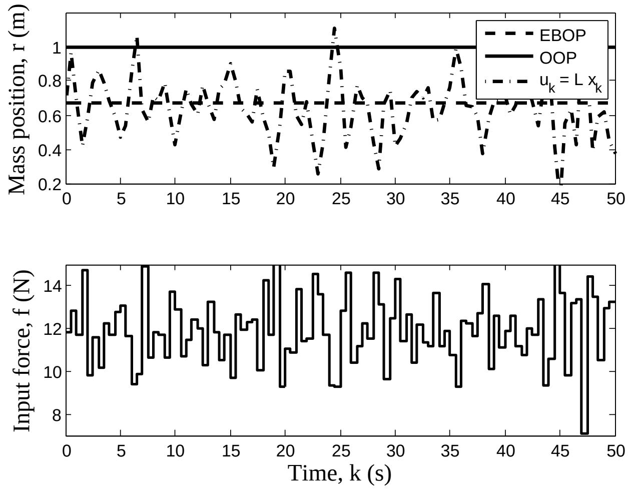

Figure 6.4: Time series plot of a mass spring damper system showing the performance of multivariable controller, $u_k = L_d x_k$

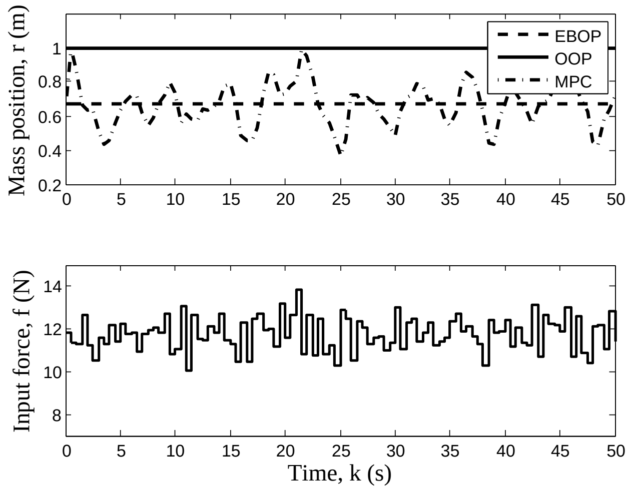

Figure 6.5: Time series plot a mass spring damper system showing the performance of MPC controller

6.5.2 Evaporation process

In this example, we illustrate the economic performance of MPC at the economic backed-off operating point. Let us revisit the forced-circulation evaporator system discussed in Chapter 5. Recall that the economic objective is to maximize the operational profit [$/h], formulated as a minimization problem of the negative profit. The process model has seven degrees of freedom. Inlet conditions of the feed (flow rate, composition, temperature) and inlet temperature of the condenser are considered as disturbances (i.e., $d = [F_1\ X_1\ T_1\ T_{200}]^T$). There are three manipulated inputs, $u = [F_3\ P_{100}\ F_{200}]^T$. The disturbance range is assumed to be 10% variation of the

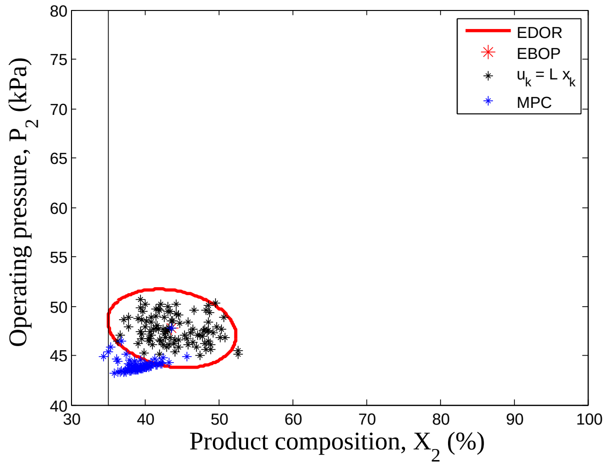

Figure 6.6: Scatter plot showing the EDOR between product composition vs operating pressure

Table 6.2: Nominal values and backed-off operating point of the evaporation process

Variables	Units	Nominal value	EBOP solution
		States	
X_2	$\%$	35.00	43.69
P_2	kPa	56.15	47.72
		Inputs	
F_3	kg/min	27.70	27.71
P_{100}	kPa	400.00	399.98
F_{200}	kg/min	230.57	381.04
Profit	$\$/h$	693.41	130.09

nominal value (i.e., $\Sigma_d = diag([1\ 0.25\ 16\ 6.25])^2$) and the set of active constraints do not change in the whole range of disturbances . It is important to note that there is one unconstrained degrees of freedom.

Linearized steady state model. A linear approximation of the discrete-time process model, obtained with the sampling time of 5 min, at the nominal optimum is given below

$$A_d = \begin{bmatrix} 0.4495 & -0.5214 \\ -0.0415 & 0.8256 \end{bmatrix}; B_d = \begin{bmatrix} 1.4101 & 0.1350 & 0.0025 \\ 0.2313 & 0.0221 & -0.0074 \end{bmatrix};$$

$$G_d = \begin{bmatrix} -4.2387 & 1.7101 & 0.1019 & -0.0566 \\ 0.3304 & -0.0614 & 0.0167 & 0.1661 \end{bmatrix}$$

Results. The amount of back off required in discrete time framework, for 10% variation

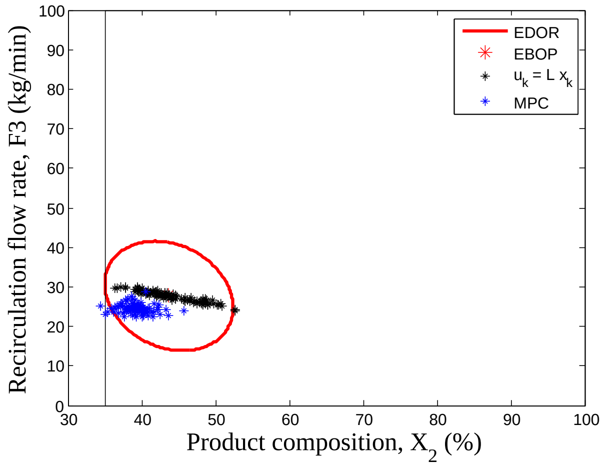

Figure 6.7: Scatter plot showing the EDOR between product composition vs recirculation flow rate

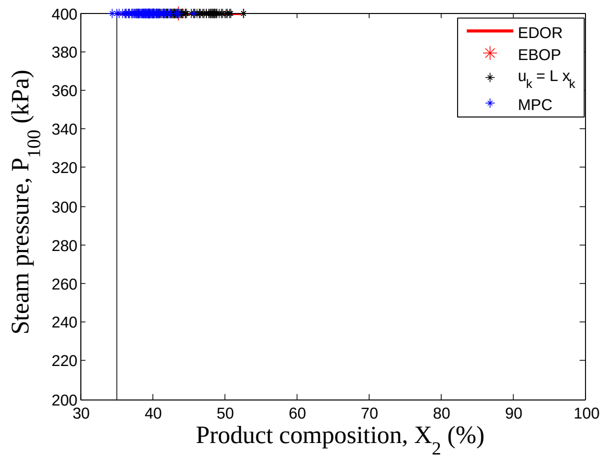

Figure 6.8: Scatter plot showing the EDOR between product composition vs steam pressure

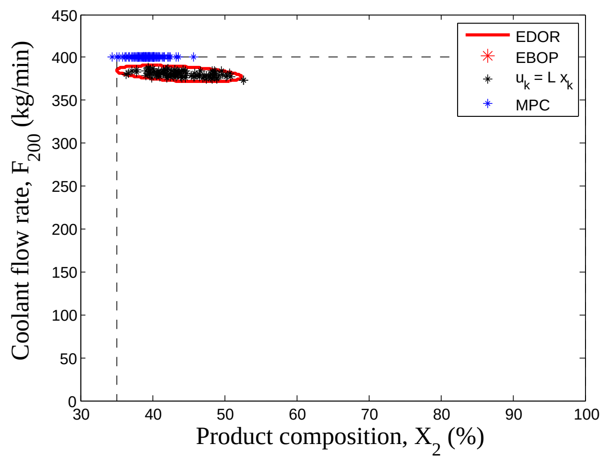

Figure 6.9: Scatter plot showing the EDOR between product composition vs coolant flow rate

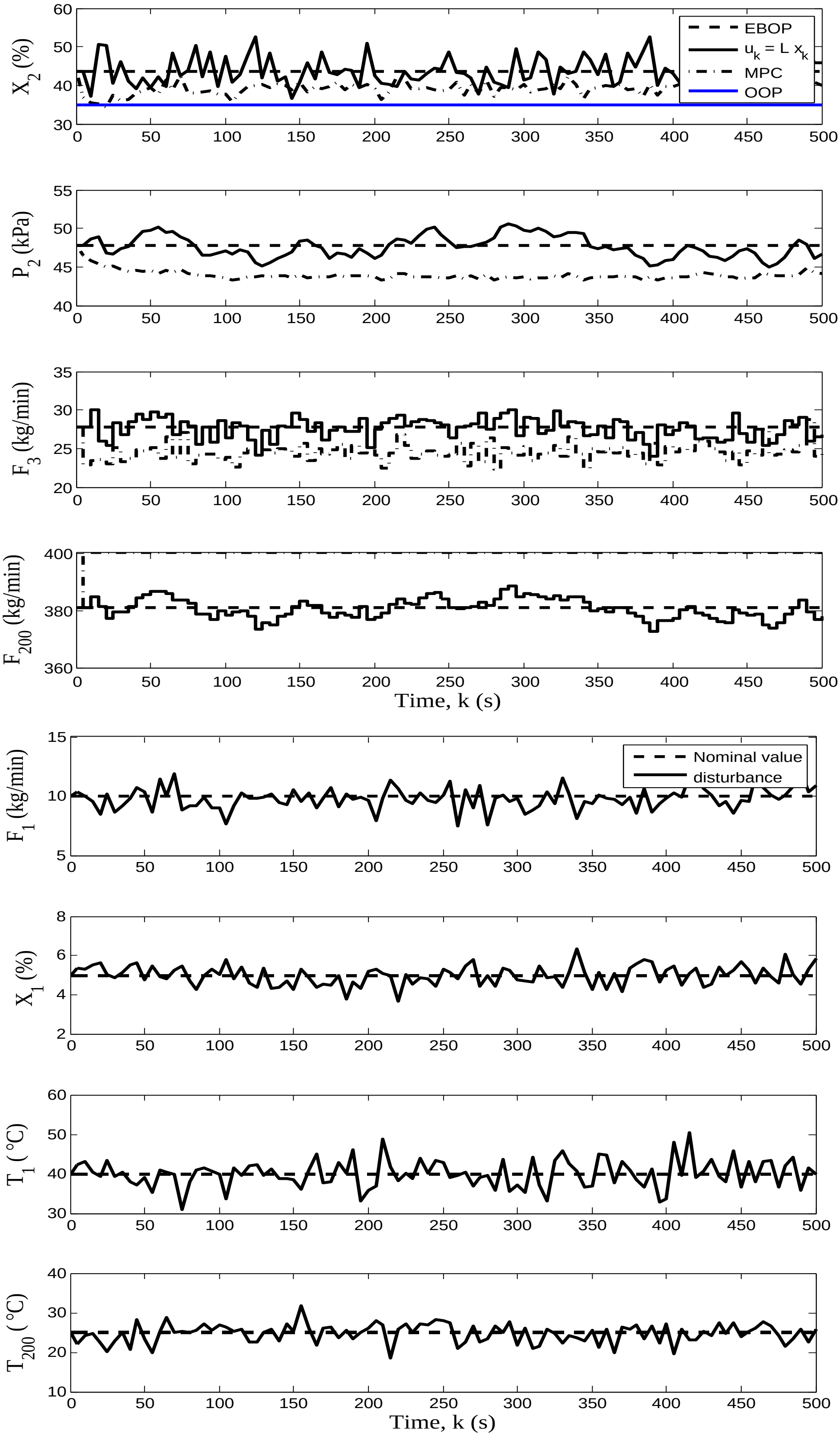

Figure 6.10: Time-series plot of the evaporation process showing the controller performances at the economic back-off point

in nominal values of disturbances, is tabulated in Table 6.2. The amount of achievable profit at this economic back-off point is \$ 130/h. Notice that the steam pressure (P_{100}) requires no back-off for the assumed disturbance magnitude and can be set at fully open valve position. The multivariable feedback controller ($u_k = L_d x_k$) designed to operate the system profitably is

$$L_d = \begin{bmatrix} -0.3128 & 0.32224 \\ 0.0000 & 0.0000 \\ -0.27626 & 2.1848 \end{bmatrix}$$

Application of Theorem 6.4.1 results in the following tuning matrices,

$$Q = \begin{bmatrix} 1.8455 & -0.26981 \\ -0.26981 & 2.9259 \end{bmatrix}; R = \begin{bmatrix} 3.1648 & -0.0004 & -0.4441 \\ -0.0004 & 3.4408 & -0.0021 \\ -0.4441 & -0.0021 & 0.6318 \end{bmatrix};$$

$$M = \begin{bmatrix} 0.87607 & 0.00012 & 0.034883 \\ -0.11931 & -0.00179 & -1.2314 \end{bmatrix}$$

Simulation was performed by generating 100 samples of disturbance variables, obtained from zero mean Gaussian white noise sequences with variances of Σ_d. Scatter comparing the performances of the resulting multivariable controller with MPC controller is shown in Figures 6.6 - 6.9. The theoretically expected dynamic operating regions are shown as ellipses. Recall that the center of the ellipse denotes the economic back-off point. From Figure 6.9, we can observe that the theoretically required back-off value for the coolant flow rate is necessary in case of multivariable controller. However, the MPC controller always keeps the coolant flow rate at its maximum value. Further, it should be noted that the MPC controller operates closer to the active constrained variable (i.e., product composition) than the multivariable controller. Hence, the economic performance of the MPC controller at the economic back-off point is better than the multivariable controller. This can also be seen from the time series plot presented in Figure 6.10.

6.6 Summary

In this chapter, we presented the MPC framework that addresses the economic performance of the constrained process in the standard MPC framework. To accomplish this, we obtained the optimal multivariable controller using the economic back-off approach and transformed it into an equivalent MPC weighting matrices using inverse optimality results. The proposed approach is successfully demonstrated using a mass spring damper system and an evaporator system.

CHAPTER 7

SIMULTANEOUS SELECTION OF BACKED-OFF OPERATING POINT, CONTROLLER AND MEASUREMENTS BASED ON ECONOMICS

Based on the paper presented in 12th European Control Conference

This chapter discusses the simultaneous selection of measurements and economic backed off operating point when the nominal optimal operating point is constrained. However, operation at this point becomes infeasible due to uncertainties. In Chapter 5, we proposed an optimization formulation that determines the economic backed-off point to ensure feasibility assuming accurate measurement of the states are available and disturbance as the only source of uncertainty. Here, we extend the formulation to partial state information case and also determine the optimal set of measurements for economical operation. The formulation also finds a suitable multivariable controller to achieve economic benefits. The final formulation is a mixed integer non-linear program. Hence, we propose to use a branch and bound type solution such that a two stage iterative problem is solved at each branching step. Finally, the proposed approach is demonstrated in an evaporator system.

7.1 Introduction

Optimal operating point of a chemical process is determined using a non-linear optimizer and it is often constrained. However, process plants are typically operated at the more conservative operating point to ensure safe operation of the plant. Owing to the developments in control theory, the process plants could be operated more aggressively and closer to the constraints to increase profitability while ensuring safe operation. Therefore, the notion of back-off is highly useful in determining the dynamically feasible and profitable operating point. In Chapter 5, we presented an optimization formulation to determine the economic backed-off operating point such that feasibility is ensured under dynamic conditions of the plant. Also, the back-off point selection problem was presented based on a continuous-time model. We assumed disturbance as the only source of uncertainty and it is characterized by Gaussian white noise process with zero mean and known variance. Furthermore, we assumed full state feedback ($u = Lx$). In the current study, we consider partial state information case ($u = L\hat{x}$) which considers measurement error as an additional source of uncertainty. Thus, the loss we incur in backing off from the active constraints consists of two components: First, the loss due to disturbances which could be partially recovered by a suitable controller design and second, the loss due to measurement error which could be partially recovered using the state estimator. The performance of the state estimator depends critically on the chosen sensors. Hence, the problem of measurement selection is an important task to achieve optimal operation. Therefore, the current study focuses on addressing the issue of simultaneous selection of measurements and economic backed off operating point.

In the next section, we formulate the economic back-off selection problem for a partial state information case. Next, convex relaxations of the constraints are presented and a solution methodology is proposed. Finally, the proposed formulation is exemplified using an evaporator system.

7.2 Problem formulation

In this section, we develop an optimization formulation that determines the optimal steady state (backed-off) operating point such that the process dynamics remain feasible under uncertain conditions for the prescribed confidence limit. Also, we need to determine the sensor network that results in a minimum economic loss.

7.2.1 Economic back-off

Consider the state variables $x_0 \in \mathbb{R}^{n_x}$, manipulated inputs $u_0 \in \mathbb{R}^{n_u}$ and disturbances $d_0 \in \mathbb{R}^{n_d}$. As discussed in previous chapters, the economically optimal point is determined by solving a non-linear optimization problem which minimizes the negated profit function J subject to the process model g and performance bounds h,

$$\begin{aligned} \min_{u_0} \quad & J(x_0, u_0, \bar{d}_0) && \text{(7.1a)} \\ s.t. \quad & g(x_0, u_0, \bar{d}_0) = 0 && \text{(7.1b)} \\ & h(x_0, u_0, \bar{d}_0) \leq 0 && \text{(7.1c)} \end{aligned}$$

This is a steady state optimization problem solved for the nominal values of the disturbance variables, $\bar{d}_0$. And, the optimal values of the states and manipulated inputs are denoted by x_0^* and u_0^*. However, if some of the bound constraints are active then there might be violation of constraints for some values of disturbance variables. Therefore, we need to ensure dynamic feasibility for all possible disturbances. One possible solution is to move the optimal operating point (back-off) inside the feasible region such that the system dynamics are feasible and the economic loss due to backing off is minimum. Thus, back off is defined as in (5.1) and we present the final formulation of the economic back-off selection problem presented in Chapter 5

$$\begin{aligned} \min \quad & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} && \text{(7.2a)} \\ s.t. \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} && \text{(7.2b)} \\ & \tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} && \text{(7.2c)} \\ & (AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0 && \text{(7.2d)} \\ & \begin{bmatrix} Z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 && \text{(7.2e)} \\ & P = Z^{1/2} && \text{(7.2f)} \\ & \begin{bmatrix} -\tau_i - {h_i}^T \tilde{z}_{ss} - t_i & \frac{\alpha}{2} {h_i}^T P \\ (\frac{\alpha}{2} {h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; && \text{(7.2g)} \end{aligned}$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, $\tau_i > 0$, $Y = LX^{-1}$, $X = \Sigma_x \succeq 0$, $Z = \Sigma_z \succeq 0$ and $P \succeq 0$ are the decision variables. It is important to note in the above formulation that we assumed full state information. In other words, state variables are measured accurately. However, measurements contain errors and this uncertainty contributes to a further loss. Therefore, we need to find a set of measurements that minimizes the operational loss.

Hence, the focus of this article is to extend the formulation to a partial state information case and also find the optimal sensor network from the set of possible measurements.

7.2.2 Sensor placement

Consider the measurement vector $y = Cx + v$ where the measurement error vector, v is a zero mean, normally distributed variables with diagonal covariance matrix $\Sigma_v (= \mathbb{E}[vv^T])$. It is well known that Kalman filter is an optimal state estimator. Thus, we use the Kalman filter to estimate the states from the set of available measurements. Also, we assume the measurement errors are independent and uncorrelated which represents a diagonal Σ_v. Let us denote $Q = \Sigma_v^{-1} = diag(\frac{q_i}{\sigma_{v,i}^2})$ where q_i is a binary variable (0 or 1) denoting that the particular variable is unmeasured or measured respectively. And, $\sigma_{v,i}^2$ is the corresponding variance. It is important to note that an unmeasured variable ($q_i = 0$) can also be statistically inferred as a sensor with infinite variance. This definition of Q helps us to address the sensor placement problem with q_i as decision variables.

In order to describe the system dynamics for the partial state information case with disturbance variances Σ_d and variance of the measurement noise Σ_v, the steady state covariance of the signal z is given by

$$\Sigma_z = (Z_x + Z_u L)(\Sigma_x - \Sigma_e)(Z_x + Z_u L)^T + Z_x \Sigma_e Z_x^T + Z_w \Sigma_d Z_w^T \qquad (7.3)$$

where Σ_x and Σ_e are the positive semi definite solutions to

$$A\Sigma_x + \Sigma_x A^T + BL(\Sigma_x - \Sigma_e) + (\Sigma_x - \Sigma_e)B^T L^T + G\Sigma_d G^T = 0 \qquad (7.4)$$

and

$$A\Sigma_e + \Sigma_e A^T - \Sigma_e C^T Q C \Sigma_e + G\Sigma_d G^T = 0 \qquad (7.5)$$

Theorem 7.2.1 $\exists$ *stabilizing* L, $\Sigma_x \succeq 0$, $\Sigma_e \succeq 0$ *and* Σ_z *s.t.* $A\Sigma_x + \Sigma_x A^T + BL(\Sigma_x - \Sigma_e) + (\Sigma_x - \Sigma_e)B^T L^T + G\Sigma_d G^T = 0$, $A\Sigma_e + \Sigma_e A^T - \Sigma_e C^T Q C \Sigma_e + G\Sigma_d G^T = 0$, $\Sigma_z =$

$(Z_x+Z_uL)(\Sigma_x-\Sigma_e)(Z_x+Z_uL)^T+Z_x\Sigma_eZ_x^T+Z_w\Sigma_wZ_w^T$ *and* $\sigma_{z,i}^2 \leq \overline{z}_i^2$, $i = 1,\dots n_z$, *if and only if* $\exists\, Y$, $X \succ 0$, $W \succ 0$ *and* $\sigma_{z,i}^2$ *s.t.* $(AX+BY)+(AX+BY)^T+G\Sigma_dG^T \prec 0$, $\begin{bmatrix} C'QC - A'W - WA & WG \\ (WG)^T & \Sigma_d^{-1} \end{bmatrix} \succ 0$, $\begin{bmatrix} Z - Z_d\Sigma_d{Z_d}^T & Z_xX + Z_uY & 0 \\ (Z_xX + Z_uY)^T & X & I \\ 0 & I & W \end{bmatrix} \succ 0$ *and* $z_i \leq \overline{z}_i^2$. *For proof, the reader is referred to the original article of Chmielewski and Manthanwar (2004).* ∎

Now the simultaneous economic back-off and measurement selection problem is reformulated in terms of LMI constraints as :

$$\begin{aligned}
min \quad & {J_x}^T\tilde{x}_{ss} + {J_u}^T\tilde{u}_{ss} + \tilde{u}_{ss}^TJ_{uu}\tilde{u}_{ss} && \text{(7.6a)}\\
s.t. \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} && \text{(7.6b)}\\
& \tilde{z}_{ss} = Z_x\tilde{x}_{ss} + Z_u\tilde{u}_{ss} && \text{(7.6c)}\\
& (AX + BY) + (AX + BY)^T + G\Sigma_dG^T \prec 0 && \text{(7.6d)}\\
& \begin{bmatrix} Z - Z_d\Sigma_d{Z_d}^T & Z_xX + Z_uY & 0 \\ (Z_xX + Z_uY)^T & X & I \\ 0 & I & W \end{bmatrix} \succeq 0 && \text{(7.6e)}\\
& \begin{bmatrix} C'QC - A'W - WA & WG \\ (WG)^T & \Sigma_d^{-1} \end{bmatrix} \succeq 0 && \text{(7.6f)}\\
& P = Z^{1/2} && \text{(7.6g)}\\
& \begin{bmatrix} -\tau_i - {h_i}^T\tilde{z}_{ss} - t_i & \frac{\alpha}{2}{h_i}^TP \\ (\frac{\alpha}{2}{h_i}^TP)^T & \tau_iI \end{bmatrix} \succeq 0; \tau_i > 0 && \text{(7.6h)}
\end{aligned}$$

where $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, W, Y, $X \succeq 0$, $Z \succeq 0$, $P \succeq 0$ and τ_i are the continuous decision variables. Also, recall $Q = diag(\frac{q_i}{\sigma_v^2})$ where q_i is a binary decision variable. Hence, the final formulation is an MINLP problem. It is important to note that the last constraint (7.6h) in based on the explicit ellipsoid representation of the dynamics (covariance) constrained by the polytope. This helps us to find the feasible backed off operating point such that the system dynamics defined by the LMI constraints (7.6d) - (7.6f) are satisfied. The non-linearity (and hence non convexity) in the formulation is due to (7.6g). Therefore, we need a specialized solution technique to solve this non-convex problem which will be addressed in the next section.

In the above formulation, quadratic term for inputs denotes the economic penalty for backing off the inputs from the nominal optimal value. In other words, it penalizes the excess use of the available unconstrained degrees of freedom. And, it is important to include this term in the cost function to get meaningful back-off points when there exists some unconstrained degrees of freedom. This situation arises when the number of manipulated inputs is greater than the number of active constraints. Thus, we need second order information on inputs ($J_{uu} \succeq 0$) which can be obtained numerically by perturbing the unconstrained inputs. Note that this cost function considers only the steady state effect on economics. Since the disturbances are assumed to be Gaussian and zero mean, this implies that the cost accounts only for the nominal steady state value of disturbances. Furthermore, we design an optimal stabilizable controller such that the back-off point selected is close to the optimal operating point.

7.3 Solution methodology

The formulation of simultaneous selection of economic back-off and measurements results in a mixed integer non-linear program. The integer decision variables is a result of sensor placement problem. First, let us consider the relaxed problem where the binary decision variables are considered to be continuous in the range 0 - 1. Now, the problem is still non convex due to the non linearity in $P = Z^{1/2}$. In this regard, we presented a simple two stage iterative procedure for a full state information case that reduce the variability of the economically important (i.e., active constrained) variables by progressively increasing the variability of the economically unimportant variables at each iteration. At each stage of the iteration, we solve a convex problem. In this work, we adapt the solution technique to handle the partial state information case with relaxed integer constraints.

The basic idea in the two stage approach is to first determine the feasible dynamic operating region (solution of stage 1) and then determine the back-off point (solution of stage 2) corresponding to the dynamic region as discussed in Chapter 5. From this solution, we can determine the departure from the true optimal point by defining the parameter $\delta_{i,j}$, as in (5.32), which is used to create bounds for the individual variances. At the start of the algorithm, this parameter δ is initialized to zero and are updated on further iterations.

7.3.1 Stage 1

In the first stage, our objective is to determine the smallest (in terms of trace) feasible ellipsoid Z and a suitable multivariable controller L.

$$
\begin{aligned}
\min_{X \succeq 0, \Sigma_z \succeq 0, Y} \quad & Tr(Z) \\
\text{s.t.} \quad & (AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0 \\
& \begin{bmatrix} Z - Z_d \Sigma_d Z_d{}^T & Z_x X + Z_u Y & 0 \\ (Z_x X + Z_u Y)^T & X & I \\ 0 & I & W \end{bmatrix} \succeq 0 \\
& \begin{bmatrix} C^T Q C - A^T W - WA & WG \\ (WG)^T & \Sigma_d^{-1} \end{bmatrix} \succeq 0 \\
& \sigma_{z,i}^2 < \frac{1}{4\alpha^2} \left(\tilde{z}_{max,i} - \tilde{z}_{min,i}\right)^2; i = 1 \cdots n_z \\
& \sigma_{z,i}^2 > \frac{\delta_{i,j}^2}{\alpha^2} \sigma_{z,j}^2; i = 1, j - 1, j + 1, n_z
\end{aligned}
$$

The output of the stage 1 after first iteration is a feasible closed loop operating region. Since no economic information is used in the objective function, the resulting controller and output covariance matrix might not be economically optimal. If the solution is infeasible, then there is no feasible solution to the original problem for the assumed uncertainty. Note that the integer variables in Q are relaxed and hence the sub problem is a semi-definite program which is known to be convex and could be solved for global optimality.

7.3.2 Stage 2

In the second stage, the covariance ellipsoid Z is used to determine the closest possible back-off point ($\tilde{z}_{ss}$) to the OOP ($x_0^*, u_0^*, \overline{d}_0$) such that the dynamics lie in the feasible space. To achieve this, we first compute $P = Z^{1/2}$ which is used to determine the center of the ellipsoid such that the ellipsoid is within the constraints polytope.

$$
\begin{aligned}
\min_{\tilde{x}_{ss},\tilde{u}_{ss},\tilde{z}_{ss}} \quad & {J_x}^T\tilde{x}_{ss} + {J_u}^T\tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu}\tilde{u}_{ss} \\
\text{s.t.} \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \\
& \tilde{z}_{ss} = Z_x\tilde{x}_{ss} + Z_u\tilde{u}_{ss} \\
& \begin{bmatrix} -\tau_i - {h_i}^T\tilde{z}_{ss} - t_i & \frac{\alpha}{2}{h_i}^T P \\ (\frac{\alpha}{2}{h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \\
& \tau_i \succeq 0; \; i = 1, \cdots 2n_z
\end{aligned}
$$

This sub problem is a convex program. The back-off point obtained at the first iteration might not be economically optimal because of non-optimal Z. However, this BOP is used to create bounds and update the parameter δ and resolve Stage 1. It is to be noted that P is not a decision variable since Z is known from first stage.

In general, the result of the above iterative procedure might result in non integer solutions to the binary variables q_i. Hence, we can use the traditional branch and bound type of algorithms to solve for integer variables where the two stage iterative procedure described above is used at each branching step. The proposed solution scheme could be implemented using YALMIP, a freely available software for solving semi-definite problems (Löfberg, 2004).

7.4 Illustration: Evaporation process

The proposed approach for simultaneously selecting the back-off operating point and measurements is applied to the evaporation process of Newell and Lee (1989) as described in previous chapters. For complete details on the nominal optimal point, linearized model and cost function, the reader is referred to subsection 5.4.3 of Chapter 5. The C matrix in the measurement model with possible measurements (i.e., $y = [X_2 \; P_2 \; T_2 \; T_3]^T$) is given by

$$
C^T = \begin{bmatrix} 1 & 0 & 0.5616 & 0 \\ 0 & 1 & 0.3126 & 0.507 \end{bmatrix}
$$

and the measurement error is considered to be $\Sigma_v = diag([0.01 \; 0.01 \; 0.01 \; 0.01])^2$.

Results. The amount of necessary back off to remain feasible for 10% variation in the

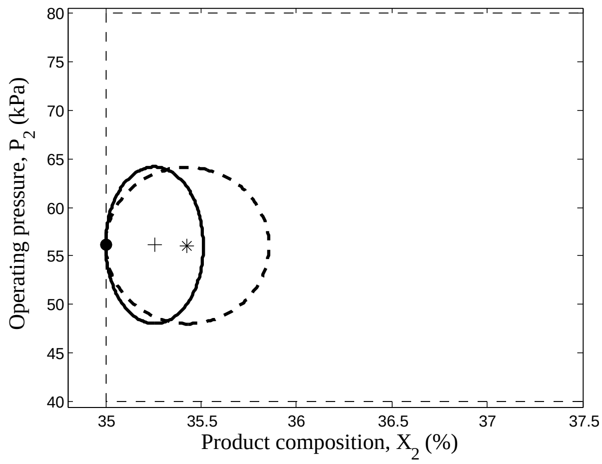

Figure 7.1: Product composition vs operating pressure. a) Continuous line : FSI case b) Dashed line : PSI case

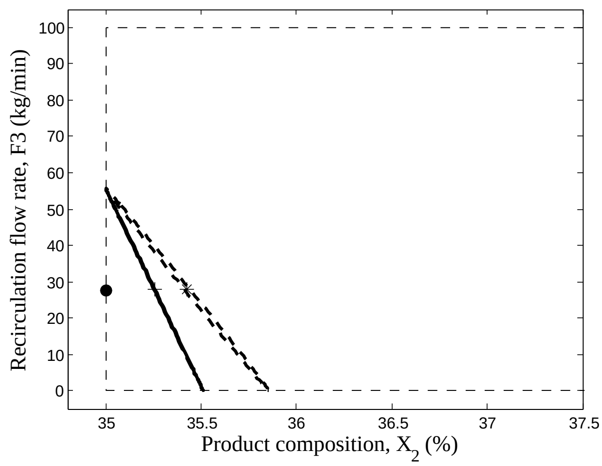

Figure 7.2: Product composition vs recirculation flow rate

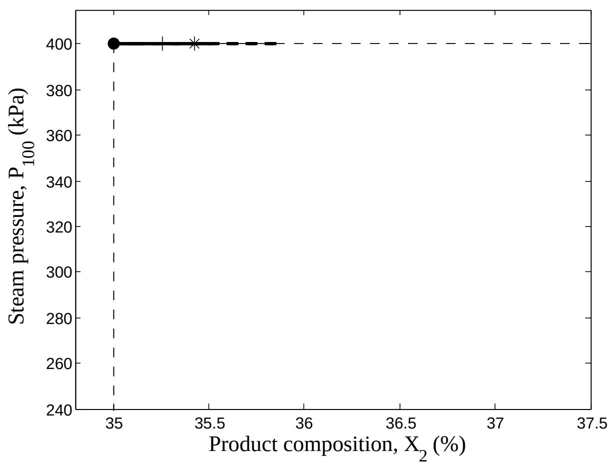

Figure 7.3: Product composition vs steam pressure

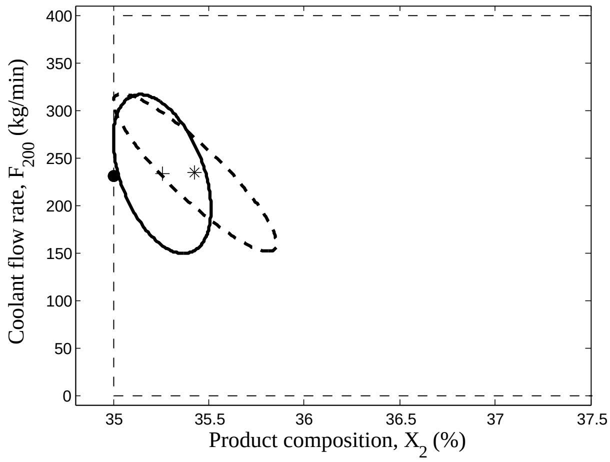

Figure 7.4: Product composition vs coolant flow rate

Table 7.1: Comparison of backed-off solutions of evaporation process for FSI and PSI cases

Variables	Units	Nominal value	Closed loop back-off	
			FSI case (7.2)	PSI case(7.6)
		States		
X_2	%	35.00	35.26	35.428
P_2	kPa	56.15	56.10	56.067
		Inputs		
F_3	kg/min	27.70	27.78	27.833
P_{100}	kPa	400.00	400	400
F_{200}	kg/min	230.57	232.71	234.22
Profit	$\$/h$	693.41	634.76	595.18

Table 7.2: Sensor network design of evaporation process operated at backed-off operating point

Sensor network	Variances	Loss, $\$/h$
X_2, T_2	{0.01,0.01}	98.226*
X_2, P_2	{0.01,0.01}	103.62
X_2, T_3	{0.01,0.01}	103.63
P_2, T_2	{0.01,0.01}	139.06
T_3, T_2	{0.01,0.01}	140.08
T_3, P_2	{0.01,0.01}	1556.2
X_2, T_2	{0.1,0.1}	304.29
X_2, P_2	{0.1,0.1}	324.17
T_2, P_2	{0.1,0.1}	436.21
X_2, T_2	{0.1,0.01}	161.29
X_2, P_2	{0.1,0.01}	321.68
X_2, T_3	{0.1,0.01}	323.33

*optimal solution obtained using YALMIP (Löfberg, 2004) nominal disturbances (Full state information case) is tabulated in Table 7.1. Also, the economic back-off required for the partial state information case is tabulated. Since steam pressure (P_{100}) is a input variable and constrained at the optimal solution, it can be set at its optimal value without backing off. This could be easily recognized from zero back-off in Table 7.1. On the other hand, product exit composition X_2 requires significant back-off for the assumed disturbances. It is important to note that the lagrange multiplier for X_2 is very high (has a value of 229.36 $\$/\%\ h$) and hence even a small variation in product composition will result in a very high loss. The dynamic operating region for the Full State Information (FSI) and Partial State Information (PSI) cases are shown as ellipses in Figures 7.1 - 7.4. The center of the ellipse denotes economic back-off solution. For PSI case, the loss obtained for operating the evaporator at this backed off operating point is $\$98.226/h$ which corresponds to the achievable profit of $\$595.18/h$. In other words, the loss we incur to ensure feasible operation with 95% confidence interval is $\$98.226/h$. The multivariable feedback controller ($u = L\hat{x}$) to be implemented to operate the system profitably is

$$L = \begin{bmatrix} -64.97 & 0.7556 \\ -0.0585 & 0.0007 \\ -171.4 & 28.45 \end{bmatrix} \tag{7.7}$$

This controller gain could be used to find the objective function weights of MPC using the inverse optimality results (Chmielewski and Manthanwar, 2004). Table 7.2 gives the loss for different set of measurements obtained by enumeration and the minimum loss network obtained using YALMIP (Löfberg, 2004). From the enumerated list, it can be inferred that we need to measure product composition more precisely to minimize the loss. The sensor network $\{X_2, T_2\}$ is obtained by solving the relaxed problem. Since the solution to this problem resulted in integer solution to the binary variables, branch and bound technique is not used in this case. However in the absence of concentration measurements, we need precise measurements of $\{P_2, T_2\}$ which however results in an additional loss of $\$40.838/h$. This loss in the absence of concentration measurement could be attributed to the error in estimating the concentration variable. This feature illustrates the importance of sensor network design for optimal operation.

7.5 Summary

In this chapter, we addressed the economic back-off operating point selection problem for partial state information case where both disturbances and measurement errors are considered as uncertainties. The formulation also yields a multivariable controller which when implemented to operate the evaporation process at the determined economic back-off operating point will ensure feasible and profitable operation. Furthermore, we obtained the optimal set of measurements from the formulation that result in minimal loss.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

In this thesis, we demonstrated that the selection of sensors, set points, and controller parameters significantly affect the economic performance of the process plant. It is important to understand the nature of optimal solution (constrained or unconstrained), available unconstrained degrees of freedom, and disturbance charactersitics, while making the structural and parametric decisions to improve profitability.

The sensor network obtained using the conventional design procedures leads to poor economic performance as demonstrated in ammonia and evaporator case studies. Therefore, the proposed average loss based sensor network design formulation will be a useful tool for designers, as the loss function directly quantifies the potential economic benefit one would achieve using the resulting sensor network. In addition, the explicit availability of analytical expression for the average loss function, can be a ready-to-use tool while retrofitting the existing network, to quickly screen alternatives. This profit based formulation is an important step towards obtaining sensor networks using commensurable metrics, within the integrated optimization and control framework. The proposed average loss formulation can be used to select sensors that will improve the economic performance, only if we have some unconstraint degrees of freedom to optimize. Therefore, if there are no unconstrained degrees of freedom, measurement selection does not affect optimal operation critically, and we can select measurements using conventional sensor network formulations. Also, the average loss based sensor network design formulation proposed in Chapter 3 can be seen as the generalization of the one presented in Chapter 2, where identical weighting is given to all process variables of interest. More importantly, the optimization formulations of the sensor selection problem resulting from integer relaxation are convex. Therefore, the main benefits of MICP approach is two-fold: (1) efficient solvers for solving MICP problems are currently available, and (2) the resulting optimal solution is globally optimal. Since the overall estimation error of the sensor network should also be as minimal as possible for better control, the lexicographic approach presents the elegant way to combine both the objectives. Furthermore, the problem was extended to find the sensor network that was robust to sensor failures while retaining the above mentioned benefits. This was accomplished by selecting redundant measurements such that all sub-networks were observable. In all the relevant problems studied, the original MICP nature of the sensor selection problem is preserved, and hence guaranteeing globally optimal solutions.

In the set point selection problem, we considered the case when the nominal steady state solution is at the intersection of constraints. Since the constrained steady state optimal point results in operational difficulties, back-off approach is often identified as an important tool for practitioners to avoid infeasibility issues. Also, the back-off directly quantifies the static economic performance one would achieve if operated at the backed-off point. Therefore, the proposed the economic back-off approach will be an useful scientific tool for operators to find the new operating point in the presence of white noise type disturbances. For this purpose, a novel two-stage iterative solution procedure was proposed.Additionally, the proposed method determines the best linear multivariable controller to be employed to achieve the dynamic economic performance. In other words, the potential economic benefit one could achieve by reducing the level of back-off with the help of linear multivariable controller was discussed. At the constrained optimal point, if there exists some unconstrained degrees of freedom, then it is important to use quadratic approximation of the cost to obtain meaningful back-off values. Further, we extended the economic back-off approach by additionally considering the presence of measurements errors. In this case, the level of back-off is due to the contribution of both the uncertainties. Therefore, to conclude, the optimal controller design can reduce the level of back-off caused because of disturbances and a suitable sensor network selection can reduce the level of back-off contribution caused by measurement errors. Finally, the actual economic performance of the designed controller was compared with an equivalent MPC controller.

8.2 Recommendations

The following list briefly describes some of the possible research directions.

- A systematic procedure on how to make the rational choice on the parametric and structural decisions in the context of optimal operation was presented. This framework should be extended to include non-linear models, model uncertainties, etc.

- The use of branch and bound technique in the measurement selection problem limits the applicability of the proposed MICP approach to moderately sized integer decision variables. Therefore, a computationally efficient technique for large scale systems will be of great use in making the optimal decision. One probable approach could be to find tight SDP or SOC relaxations of the integer restrictions instead of the linear relaxations used here. This will result in better bound values that could possibly require lesser number of branching and hence can substantially decrease the computational burden for large scale systems. Another approach could be to solve the integer relaxed problem by adding an appropriate penalty term to the cost function for not being closer to the integer values. In this regard, the sum of the weighted ℓ_1 norm could be a possible penalty function.

- The MICP approach presented for sensor selection is based on the linear (or linearized) model of the process. The approach can be extended for bilinear systems, which typically has linear models for mass balances (involving flow variables only) and bilinear models for species balances (involving bilinear terms of flow and composition variables) and energy balances in some cases (involving bilinear terms of flow and temperature variables). In this case, the sensor selection problem can be solved sequentially in two steps: first, decide on the flow measurements using the linear model involving flow variables only and second, assuming all flows are estimable which makes the species and energy balances linear, use the proposed MICP approach to decide on the remaining number of sensors for obtaining better estimate of the temperature or composition variables. This decoupling enables one to solve two sensor selection problems of smaller size. However, such an approach may not result in globally optimal sensor network though the sub-problems are solved to global optimality.

- The network properties such as observability, redundancy and estimability are incorporated in the current MICP approach. However, there exist no SDP or SOC representation for reliability of the sensor network. Obtaining such a representation will allow the reliability based sensor selection problem to be casted within the framework of conic optimization.

- The sensor selection presented in this thesis for sensor failure situations assumes equal sensor failure probability for all the sensors. Extension to unequal sensor failure probability case will be more useful and general.

- Since sensor failures are considered as different scenarios, the number of LMIs to be added also increase by a factor of n in case of single sensor failure. For multiple sensor failures, the number of LMIs to be added will grow exponentially, therefore a better approximation is required even for a moderately sized problem. One approach could be to consider each sensor failure as different scenarios and fit an ellipsoidal representation that contains all the scenarios and solve the robust version of the problem with ellipsoidal uncertainty. This might result in computationally tractable robust formulation. Otherwise, scenario optimization approach based on radomization of the observability constraints might be possible.

- For constrained processes which give rise to feasibility issues in the presence of disturbances, the optimal back-off approach was presented for Gaussian white noise type disturbances. A systematic approach for a more general class of disturbances will be useful.

- In this work, we presented a novel two-stage solution technique for solving a non-convex problem which arises in back-off studies. Theoretical properties of the proposed algorithm need to be investigated and its applicability to a wide class of non-convex optimization problems can be studied.

- Design variables play a significant role in the optimal operation. Therefore, optimal design studies that could result in flexible operation under uncertain conditions for nominally constrained processes is a possible research direction.

- The proposed economic back-off approach is based on fixed set of active constraints. However, in some cases the set of active constraints might change depending on the disturbance magnitude. New methods have to be developed for achieving profitable and flexible operation in the face of changing active constraints.

APPENDIX A

DERIVATION OF Equation 2.13

Recall that the process model has been expressed in terms of the primary variables z_p. The true state variables of the process z are related to true primary variables as follows:

$$z = Cz_p \tag{A.1}$$

and the measurement equation is now expressed as:

$$y = Cz_p + v \tag{A.2}$$

The reconciled estimates $\hat{z}_p$ and $\hat{z}$ satisfy the following:

$$\hat{z}_p = (C^TQC)^{-1}C^TQy \tag{A.3}$$

$$\hat{z} = C\hat{z}_p \tag{A.4}$$

Let us derive the expressions for the expected values:

$$\mathbb{E}(y) = \mathbb{E}(Cz_p + v) = Cz_p \tag{A.5}$$

$$\mathbb{E}(yy^T) = \mathbb{E}(Cz_p + v)(Cz_p + v)^T = Cz_pz_p^TC^T + Q^{-1} \tag{A.6}$$

as v is zero mean with covariance Q^{-1}

$$\mathbb{E}(\hat{z}) = C(C^TQC)^{-1}C^TQ\mathbb{E}(y) = C(C^TQC)^{-1}C^TQCz_p = Cz_p \tag{A.7}$$

The error covariance of the reconciled estimates is defined as:

$$\Sigma_z = \mathbb{E}((z - \hat{z})(z - \hat{z})^T) = zz^T - 2z\mathbb{E}(\hat{z}^T) + \mathbb{E}(\hat{z}\hat{z}^T) \tag{A.8}$$

The first two terms in the above is simply $-Cz_pz_p^TC^T$. Let us evaluate the last term:

$$\begin{aligned}
\mathbb{E}(\hat{z}\hat{z}^T) &= \mathbb{E}((C(C^TQC)^{-1}C^TQy)(C(C^TQC)^{-1}C^TQy)^T) && \text{(A.9)}\\
&= C(C^TQC)^{-1}C^TQ\mathbb{E}(yy^T)QC(C^TQC)^{-1}C^T && \text{(A.10)}\\
&= C(C^TQC)^{-1}C^TQ(Cz_pz_p^TC^T + Q^{-1})QC(C^TQC)^{-1}C^T && \text{(A.11)}\\
&= Cz_pz_p^TC^T + C(C^TQC)^{-1}C^T && \text{(A.12)}
\end{aligned}$$

Thus, the error covariance of the estimates is:

$$\begin{aligned}
\Sigma_z &= -Cz_pz_p^TC^T + Cz_pz_p^TC^T + C(C^TQC)^{-1}C^T && \text{(A.13)}\\
&= C(C^TQC)^{-1}C^T && \text{(A.14)}
\end{aligned}$$

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LIST OF PAPERS BASED ON THESIS

Journal publications

1. M. Nabil and Sridharakumar Narasimhan. Sensor Network Design for Optimal Process Operation Based on Data Reconciliation. *Ind. Eng. Chem. Res.*, **51 (19)**, 6789 – 6797, 2012.
2. M. Nabil, Sridharakumar Narasimhan and Sigurd Skogestad. Profitable and dynamically feasible operating point selection for constrained processes. *Journal of Process Control*, **24 (5)**, 531–541, 2014.
3. M. Nabil, Sridharakumar Narasimhan and Sigurd Skogestad. Economic performance of Model Predictive Control for constrained processes. Manuscript under preparation.

Publications in conference proceedings

1. Presented titled *Design of Optimal Sensor Network based on Economic Objectives* at the **AIChE Annual Meeting 2010**. Salt Lake City, US, Nov. 7-12, 2010.
2. Presented titled *Economic back-off selection based on optimal multivariable controller* at the **8th IFAC symposium on Advanced Control of Chemical Processes (ADCHEM 2012)**. Singapore, July 10-13, 2012.
3. Presented titled *Integrated Sensor Network Design* at the **11th International Symposium on Process Systems Engineering (PSE2012)**. Singapore, July 15-19, 2012.
4. Presented titled *Optimal selection of sensor network and backed-off operating point based on economics* at the **12th European Control Conference (ECC 13)**. Zurich, Switzerland, July 17-19, 2013.