Optimal Selection of Sensors & Controller Parameters for Economic Optimization of Process Plants

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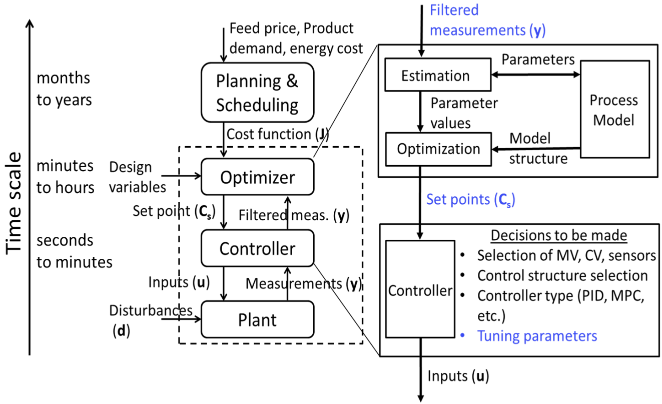
Optimal operation



- Economic performance depends on the structural and parametric decisions
- Focus: Selection of measurements, set point and controller parameters

Optimal operation

Optimizer

$$\min\ J(x,u,d)$$
$$s.t.\ g(x,u,d)=0$$
$$h(x,u,d)\le 0$$

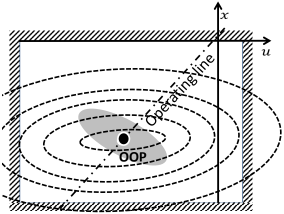
$\{x^*, u^*\}$

Controller

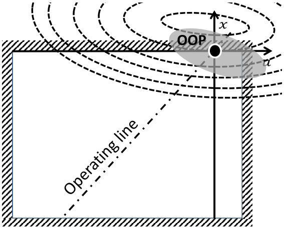
$$\min \sum_{k=1}^{N} (x_k - x^*)^T Q (x_k - x^*) + (u_k - u^*)^T R (u_k - u^*)$$
$$s.t.\ x_{k+1} = A\,x_k + B\,u_k + G\,d_k$$
$$x_{min} \le x_k \le x_{max}$$
$$u_{min} \le u_k \le u_{max}$$

Inputs

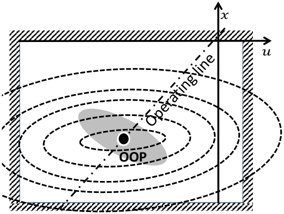
(a) Unconstrained Optimum

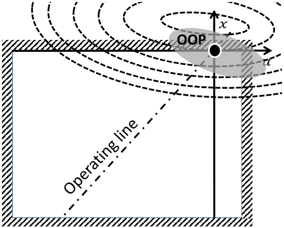


(b) Constrained Optimum



Roadmap of my thesis





Steady state optimizer

↓

If active constraints are perfectly controllable and $n_{uc,dof} = 0$ —yes→ **(Existing)** Design sensor networks using conventional methods such as maximizing precision, etc.

no ↓

If active constraints are perfectly controllable and $n_{uc,dof} > 0$ —yes→ **(Obj. 1)** Design sensor networks for optimal operation

no ↓

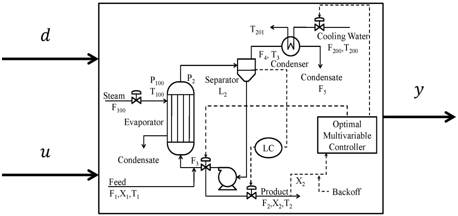
If active constraints are not perfectly controllable and $n_{uc,dof} \geq 0$ →

- Case 1 **(Obj. 2a)**: Disturbance is the only source of uncertainty, determine the optimal set-point and controller parameters
- Case 2 **(Obj. 2b)**: Disturbance and measurement errors are the sources of uncertainty, simultaneously determine the optimal set-point, controller parameters and measurements

$n_{uc,dof}$: number of unconstrained degrees of freedom

(Obj. 1) Structural decision - sensor selection

- **Sensor** : Measuring element in a process plant
- **Sensor Network (SN)** : set of all measured variables of the process
  - large number of combinations
- **Sensor Network Design (SND)** : selection of a sensor network out of all possible combinations



- **Need** : Process monitoring, control and fault diagnosis, etc.

How to design a SN?

Formulate as an optimization problem and solve it to obtain the best network

(Obj. 1) Sensor selection problem

- **Sensor information**
  - Sensor cost
  - Sensor precision / variance
  - Sensor failure probability
- **Network properties**
  - Network cost (sum of the sensor cost)
  - Network precision / variance (minimum MSE)
  - Network reliability (maximize the minimum reliability)
  - Observability (to infer the state of the process)
  - Redundancy (important - sensor failure and data reconciliation)

Representative SND formulation

min Estimation Error

s.t. Network Properties

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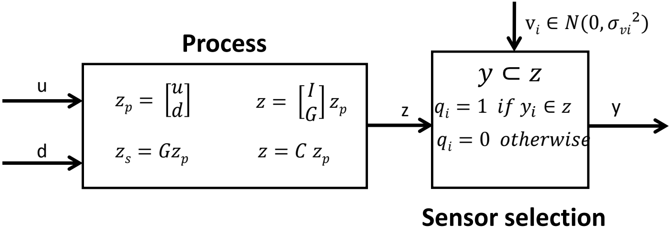
Representative SND formulation

min	Estimation Error
s.t.	Network Properties

Does it ensure optimal operation?
If not, how can it be quantified?

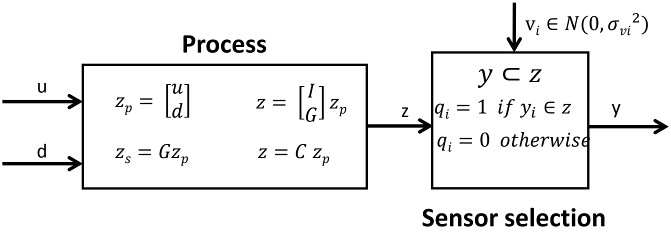
Sensor selection for optimal operation



- **Measurement equation** : $\mathrm{y} = \mathrm{z} + \mathrm{v} = \mathrm{C}\mathrm{z_p} + \mathrm{v},\ \mathrm{v} \in \mathcal{N}(0, \Sigma_\mathrm{v})$
- **Covariance of the estimates** : $\Sigma_\mathrm{z}(\mathrm{q_i}) = \mathrm{C}(\mathrm{C^T Q C})^{-1}\mathrm{C^T}$

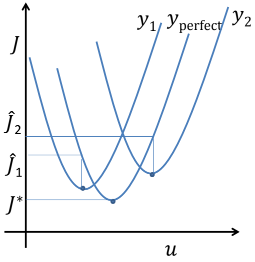
where, $\mathrm{Q} = \begin{bmatrix} \frac{q_1}{{\sigma_{v1}}^2} & & & \\ & \frac{q_2}{{\sigma_{v2}}^2} & & \\ & & \ddots & \\ & & & \frac{q_n}{{\sigma_{vn}}^2} \end{bmatrix}$

Sensor selection for optimal operation



Optimal operation: min J

- Choice of sensor network affect the optimal operation of the plant
- Measurement errors introduce uncertainty and hence $\hat{J} \geq J^*$



Definition

Average loss : Expectation of the difference between optimal values of the cost function due to the presence of measurement error

Mathematically, $\overline{L} = E(\hat{J} - J^*)$

Average loss

$$\overline{L} = \frac{1}{2}\text{Tr}(W\Sigma_z);\ \Sigma_z = C(C^T Q C)^{-1} C^T$$

Weighing matrix

$$W = \begin{bmatrix} J_{ud}^T (J_{uu}^{-1})^T J_{ud} & J_{ud}^T & 0 \\ J_{ud} & J_{uu} & 0 \\ 0 & 0 & 0 \end{bmatrix} = RR^T$$

Recall that Σ_z depends on Sensor Network

Objective : Choose Sensor Network that minimizes $\overline{L} = \frac{1}{2}\text{Tr}(W\Sigma_z)$

Sensor selection for optimal operation

$$\min_{q_i} \frac{1}{2}\mathrm{Tr}(W\Sigma_z)$$

Where,

$$\Sigma_z = C(C^T Q C)^{-1} C^T$$

$$Q = \begin{bmatrix} \frac{q_1}{{\sigma_{v1}}^2} & & & \\ & \frac{q_2}{{\sigma_{v2}}^2} & & \\ & & \ddots & \\ & & & \frac{q_n}{{\sigma_{vn}}^2} \end{bmatrix}$$

$$q_i = \begin{cases} 0, & \text{unmeasured;} \\ 1, & \text{measured.} \end{cases}$$

- Mixed integer nonlinear problem (MINLP)

Sensor selection for optimal operation

MINLP formulation

$$\min_{q_i} \frac{1}{2}\mathrm{Tr}(W\Sigma_z)$$

Where,

$$\Sigma_z = C(C^T Q C)^{-1} C^T$$

$$Q = \begin{bmatrix} \frac{q_1}{{\sigma_{v1}}^2} & & & \\ & \frac{q_2}{{\sigma_{v2}}^2} & & \\ & & \ddots & \\ & & & \frac{q_n}{{\sigma_{vn}}^2} \end{bmatrix}$$

$$q_i = \begin{cases} 0, & \text{unmeasured;} \\ 1, & \text{measured.} \end{cases}$$

Mixed Integer Cone Problem (MICP)

$$\min_{t, q_i, Y} \quad \bar{L} = \frac{1}{2} t$$

$$\textit{s.t.} \quad \mathrm{Tr}(Y) \leq t$$

$$\begin{bmatrix} Y & R^T C \\ (R^T C)^T & (C^T Q C) \end{bmatrix} \succ 0$$

$$q_i \in \{0, 1\}$$

$$Q = \mathrm{diag}\{\frac{q_i}{{\sigma_i}^2}\}$$

Sensor selection for optimal operation

MICP Formulation

$$\min_{t_c, q_i, Y_c} \quad \overline{L}_{cost} = \frac{1}{2} t_c$$

$$s.t. \quad Tr(Y_c) \leq t_c;$$

$$\begin{bmatrix} Y_c & R^T C \\ (R^T C)^T & (C^T Q C) \end{bmatrix} \succ 0;$$

$$q_i \in \{0, 1\}; \quad \sum_{i=1}^{n_z} c_i q_i \leq c^*;$$

$$Q = diag\{\frac{q_i}{{\sigma_i}^2}\}$$

- Formulation is a mixed integer cone program
- Integer relaxation results in a convex problem
- Solved using branch and bound technique
- Globally optimal sensor network
- **Attributes:** redundant network, retrofitting, robust sensor network, etc.

Demonstration: Simple ammonia network

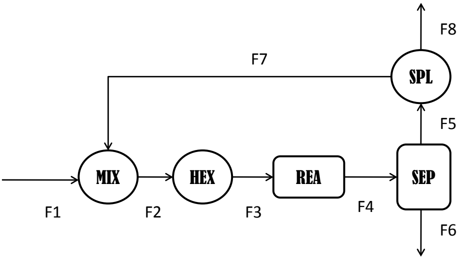


Figure: Simple Ammonia Network

$$J = (F_5 - F_7)^2 + (F_5 + F_1)^2$$

$$u = \begin{bmatrix} F_5 \\ F_7 \end{bmatrix}; d = F_1$$

$$J_{uu} = \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix}; J_{ud} = \begin{bmatrix} 2 \\ 0 \end{bmatrix};$$

Result

Sensor Network	Average Loss [$/h]	Overall Expected Error
F_1, F_2, F_5	5	12
F_1, F_5, F_8	3*	16
F_1, F_7, F_8	4	13
F_2, F_5, F_7	5	11*
F_4, F_5, F_7	5	11
F_5, F_6, F_7	8	14
F_1, F_2, F_8	3	12
F_2, F_7, F_8	3	12

*YALMIP Solution

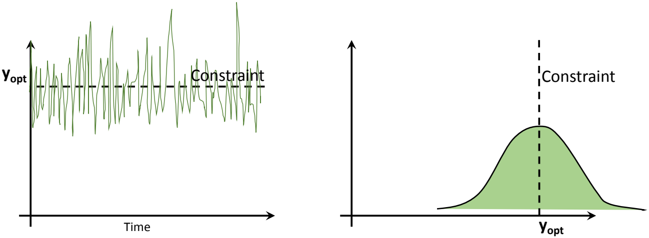
Demonstration: Simple ammonia network

Results

Formulation	Sensor Network	Obs.	Red.	Rob.	Average Loss [$/h]	Overall Error
Average loss	F_1, F_5, F_8	Y	N	N	3	16
Overall error	F_2, F_5, F_7	Y	N	N	5	11
Lexicographic	F_2, F_6, F_8	Y	N	N	3	12
Redundant	F_1, F_2, F_5, F_8	Y	Y	Y	1.75	6.5
Robust optimal (average-case)	F_1, F_2, F_5, F_8	Y	Y	Y	1.75	6.5
Robust optimal (worst-case)	F_1, F_4, F_5, F_8	Y	Y	Y	5	16

Obs. - Observability; Red. - Redundancy; Rob. - Robustness; Y - Yes; N - No

(Obj. 2a) Parametric decisions - set point, controller parameters

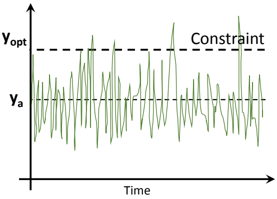


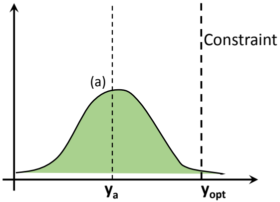
- Constrained cases challenge us on the feasible operation under uncertain conditions.
- Feasibility is ensured by backing-off economically from the active constraints.

Objective

How to determine the profitable set point and controller parameters?

(Obj. 2a) Parametric decisions - set point, controller parameters





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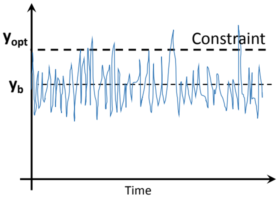

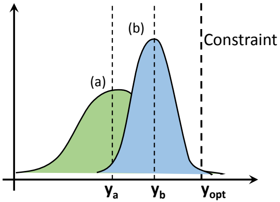

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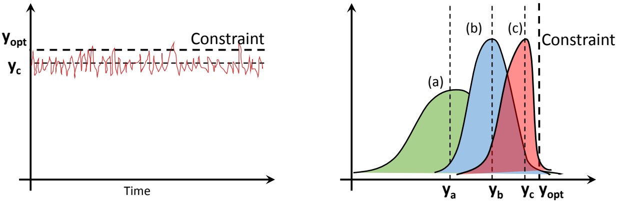

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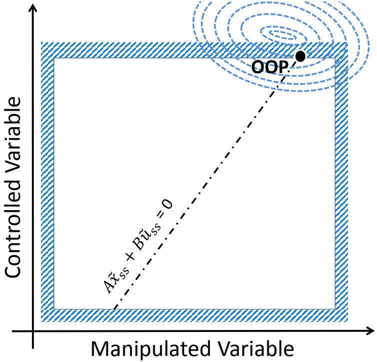
Objective

How to determine the profitable set point and controller parameters?

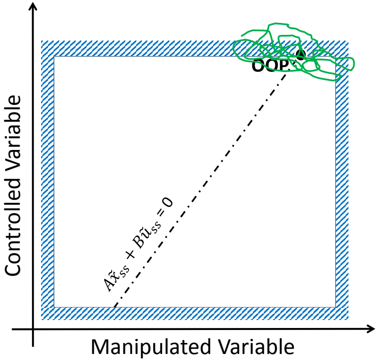
(Obj. 2a) Profitable and dynamically feasible operating point selection



Dynamic Back-off Problem

min	Loss function
s.t.	Process model
	Inequalities
	Dynamic equations
	Controller equations

(Obj. 2a) Profitable and dynamically feasible operating point selection



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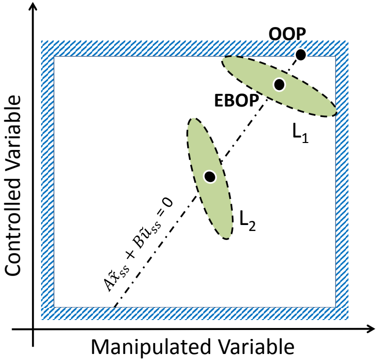
(Obj. 2a) Profitable and dynamically feasible operating point selection



Dynamic Back-off Problem

min	Loss function
s.t.	Process model
	Inequalities
	Dynamic equations
	Controller equations

Stochastic approach

Assumptions

- Disturbance is the only source of uncertainty
- Characterized by GWN process with zero mean and known variance
- Linear multivariable controller with full state feedback ($u = Lx$)
- Process dynamics are represented using a linear state space model

Dynamic process model (State Space Model)

$$\dot{x} = Ax + Bu + Gd; \; z = Z_x x + Z_u u + Z_d d$$
$$\tilde{d}_{min} - \tilde{z}_{ss} \leq z \leq \tilde{d}_{max} - \tilde{z}_{ss}$$

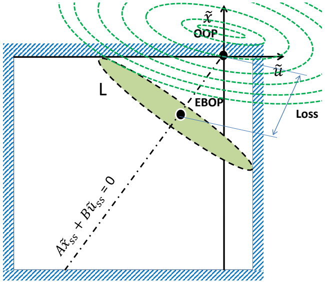
Controller equation

- Full state linear feedback $u = Lx$
- Dynamic operating region is represented as ellipsoids

EBOP selection problem

Optimization formulation

$$\begin{aligned}
\text{min} \quad & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \\
\text{s.t.} \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \\
& (A+BL)\Sigma_x + \Sigma_x (A+BL)^T + G\Sigma_d G^T = 0 \\
& \Sigma_z = (Z_x + Z_u L)\Sigma_x (Z_x + Z_u L)^T + Z_d \Sigma_d {Z_d}^T \\
& P = \Sigma_z^{1/2} \\
& \tilde{z} := \tilde{z}_{ss} + \alpha P z \ \forall \ \|z\|_2 \leq 1 \\
& h_i^T \tilde{z} + q_i \leq 0
\end{aligned}$$



- **Decision variables :** $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, L, $\Sigma_x \succeq 0$, $\Sigma_z \succeq 0$ and $P \succeq 0$
- Infinite dimensional in z , non-linear and non-convex

EBOP selection problem

Optimization formulation

$$\begin{aligned} \min \quad & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \\ s.t. \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \\ & (A+BL)\Sigma_x + \Sigma_x (A+BL)^T + G\Sigma_d G^T = 0 \\ & \Sigma_z = (Z_x + Z_u L)\Sigma_x (Z_x + Z_u L)^T + Z_d \Sigma_d {Z_d}^T \\ & P = \Sigma_z^{1/2} \\ & \tilde{z} := \tilde{z}_{ss} + \alpha P z \ \forall \ \|z\|_2 \leq 1 \\ & h_i^T \tilde{z} + q_i \leq 0 \end{aligned}$$

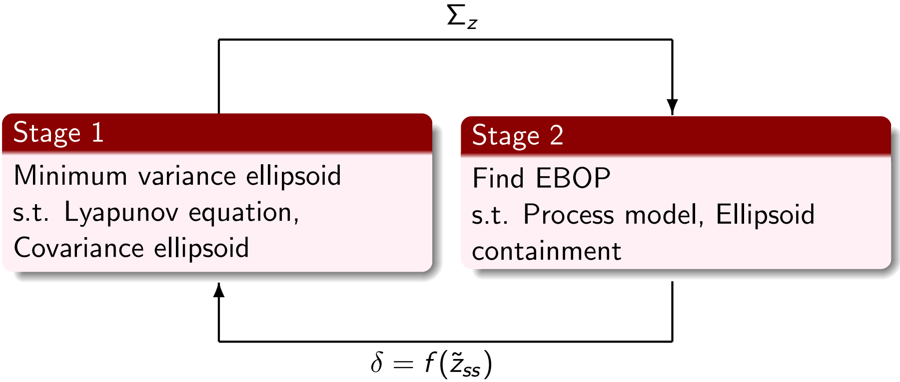
- **Decision variables :** $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, L, $\Sigma_x \succeq 0$, $\Sigma_z \succeq 0$ and $P \succeq 0$
- Infinite dimensional in z , non-linear and non-convex

Relaxed formulation

$$\begin{aligned} \min \quad & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \\ s.t. \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \\ & \tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} \\ & (AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0 \\ & \begin{bmatrix} Z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y \\ (Z_x X + Z_u Y)^T & X \end{bmatrix} \succeq 0 \\ & P = \Sigma_z^{1/2} \\ & \|\alpha P h_i\|_2 + h_i^T \tilde{z}_{ss} + t_i \leq 0, i = 1, \ldots, 2n_z \end{aligned}$$

- **Decision variables :** $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, Y, $X \succeq 0$, $Z \succeq 0$ and $P \succeq 0$
- Finite dimensional but non-linear and non-convex

Solution methodology



Solution methodology

Algorithm

1. Initialize the parameter $\delta_{i,j} = 0$.
2. Find Z by solving the Stage 1 convex problem. If no feasible Z can be found, exit.
3. Compute $P = Z^{1/2}$. Find the BOP ($\tilde{z}_{ss}$) by solving the Stage 2 convex problem.
4. Terminate on convergence. Otherwise, update $\delta_{i,j}$ and proceed to Step 2.

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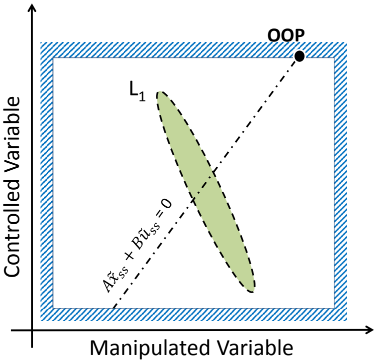

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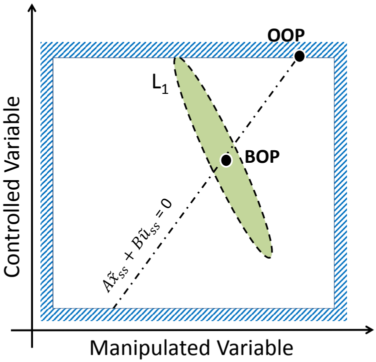

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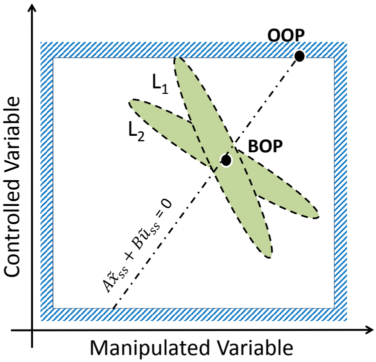

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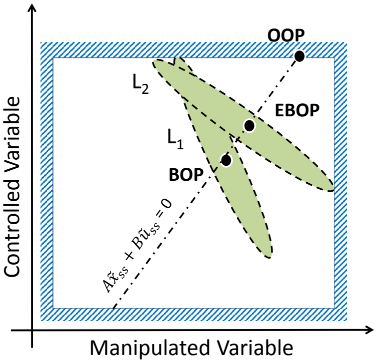
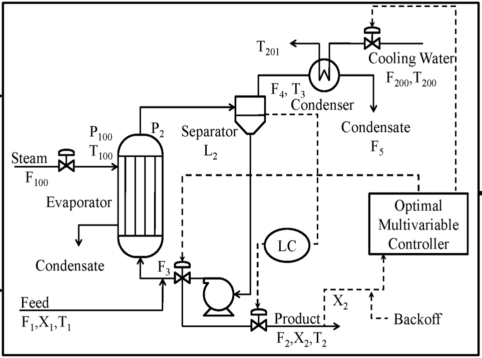


Case study: Evaporator system

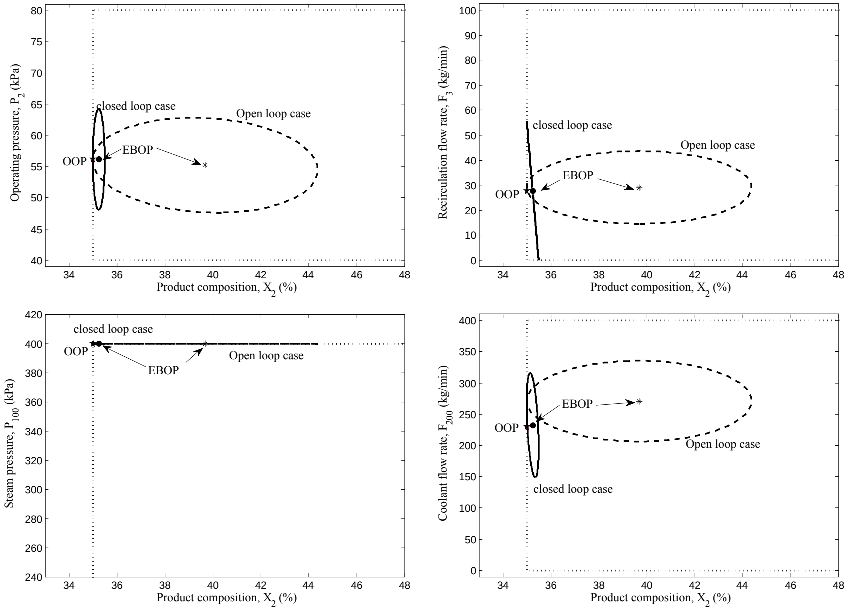
$$d = \begin{bmatrix} F_1 \\ X_1 \end{bmatrix}$$

$$u = \begin{bmatrix} F_3 \\ P_{100} \\ F_{200} \end{bmatrix}$$



$$y = \begin{bmatrix} X_2 \\ P_2 \end{bmatrix}$$

- **Active constraints**: Product composition $X_2 \geq 35\%$
  Steam pressure $P_{100} \leq 400kPa$

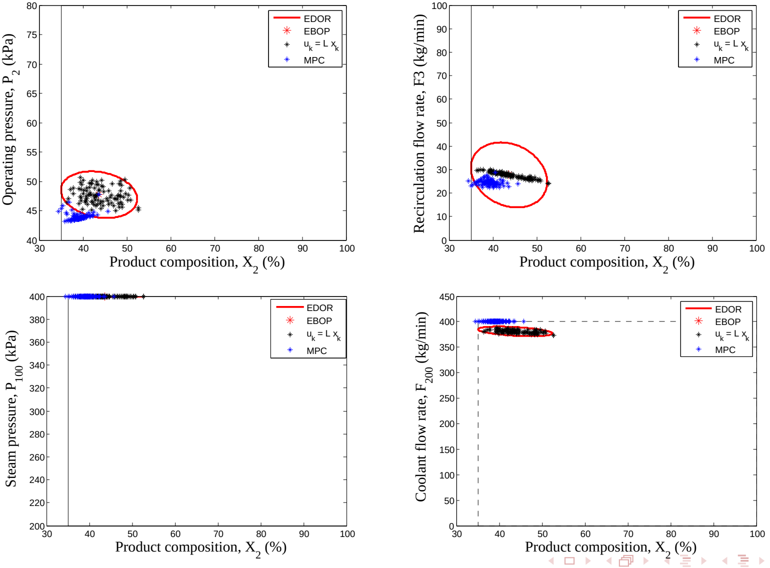

closed loop case
Open loop case
EBOP
OOP
Operating pressure, P2 (kPa)
Product composition, X2 (%)
Recirculation flow rate, F3 (kg/min)
Steam pressure, P100 (kPa)
Coolant flow rate, F200 (kg/min)

Results : Nominal and back-off operation

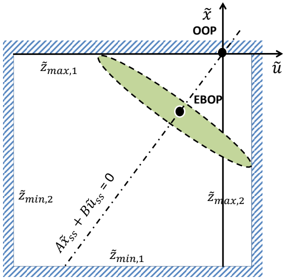
$$u = \begin{bmatrix} -108.5643 & 0.3868 \\ -0.0006 & 0.0002 \\ -123.2216 & 97.3625 \end{bmatrix} x$$

Variables	Units	Nominal value	EBOP solution closed loop (linear)	EBOP solution closed loop (quadratic)	EBOP solution open loop (u =cons)
		States (x)			
X_2	%	35.00	35.41	35.26	39.75
P_2	kPa	56.15	76.53	56.10	55.16
		Inputs (u)			
F_3	kg/min	27.70	35.80	27.78	29.12
P_{100}	kPa	400.00	399.99	400	400
F_{200}	kg/min	230.57	0.01	232.71	271.65
Profit	$\$/h$	693.41	600.12	634.76	-414.92

Economic performance of MPC controller



(Obj. 2b) Simultaneous selection of EBOP, controller and sensors



Objective

To account for measurement error and determine the sensor network that result in minimal operational loss

Assumptions

- Disturbance and measurement errors are the sources of uncertainty
- Characterized by GWN process with zero mean and known variance
- Linear multivariable controller with partial state feedback ($u = L\hat{x}$)
- Process dynamics are represented using a linear state space model
- Kalman filter to estimate the states

Simultaneous selection of EBOP, controller and sensors

Optimization formulation

$$
\begin{aligned}
\min \quad & {J_x}^T \tilde{x}_{ss} + {J_u}^T \tilde{u}_{ss} + \tilde{u}_{ss}^T J_{uu} \tilde{u}_{ss} \\
s.t. \quad & 0 = A\tilde{x}_{ss} + B\tilde{u}_{ss} \\
& \tilde{z}_{ss} = Z_x \tilde{x}_{ss} + Z_u \tilde{u}_{ss} \\
& (AX + BY) + (AX + BY)^T + G\Sigma_d G^T \prec 0 \\
& \begin{bmatrix} Z - Z_d \Sigma_d {Z_d}^T & Z_x X + Z_u Y & 0 \\ (Z_x X + Z_u Y)^T & X & I \\ 0 & I & W \end{bmatrix} \succeq 0 \\
& \begin{bmatrix} C' Q C - A' W - WA & WG \\ (WG)^T & \Sigma_d^{-1} \end{bmatrix} \succeq 0 \\
& P = Z^{1/2} \\
& \begin{bmatrix} -\tau_i - {h_i}^T \tilde{z}_{ss} - t_i & \frac{\alpha}{2} {h_i}^T P \\ (\frac{\alpha}{2} {h_i}^T P)^T & \tau_i I \end{bmatrix} \succeq 0; \tau_i > 0 \\
& Q = diag(\frac{q_i}{\sigma_{v,i}^2})
\end{aligned}
$$

Decision variables: $\tilde{x}_{ss}$, $\tilde{u}_{ss}$, $\tilde{z}_{ss}$, W, Y, $X \succeq 0$, $Z \succeq 0$, $P \succeq 0$ and τ_i

Binary decision variable: q_i

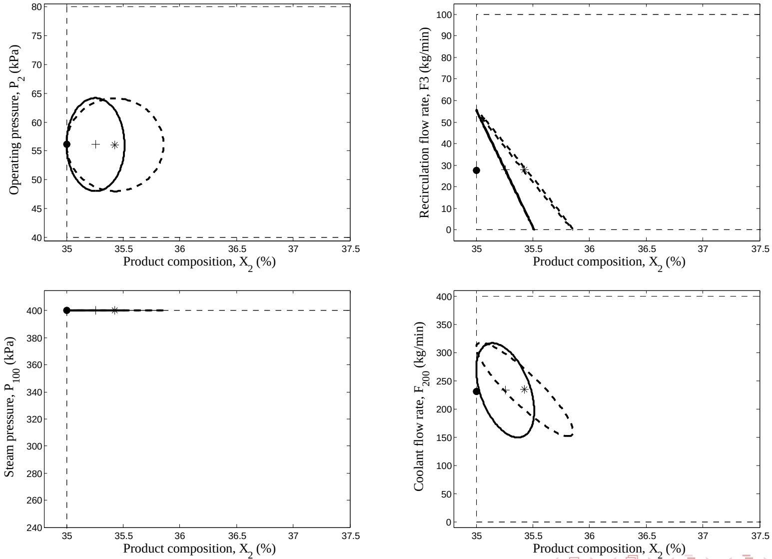
Solved using Branch and Bound technique

Simultaneous selection of EBOP, controller and sensors

Sensor Network	Variance	Loss, $\$/h$
X_2, T_2	$\{0.01, 0.01\}$	98.226*
X_2, P_2	$\{0.01, 0.01\}$	103.62
P_2, T_2	$\{0.01, 0.01\}$	139.06
T_3, P_2	$\{0.01, 0.01\}$	1556.2
X_2, T_2	$\{0.1, 0.1\}$	304.29
T_2, P_2	$\{0.1, 0.1\}$	436.21
X_2, T_2	$\{0.1, 0.01\}$	161.29
X_2, T_3	$\{0.1, 0.01\}$	323.33

*optimal solution obtained using YALMIP

Simultaneous selection of EBOP, controller and sensors



Conclusions

- Selection of sensors, set points, and controller parameters significantly affect the economic performance of the process plant
- When the optimum is unconstrained, sensor selection is important
  - Economics based sensor network design procedure that will be a useful tool for designers
  - Average loss is shown to be weighted error variances. It can be a ready-to-use tool to quickly screen alternatives while retrofitting the existing network.
  - Weighting matrix truly reflects the economic performance of the network.
  - The proposed method will be useful to improve the economic performance for the case when unconstrained degrees of freedom are available.
  - The formulation is casted as an MICP and hence global optimality solution can be obtained.
  - Extended to incorporate the robustness to single sensor failure case.

Conclusions

- When the optimum is constrained, constraint satisfaction is very important
  - Selection of set point and controller parameters often determines the static and dynamic economic performance of the process, respectively.
  - The proposed the economic back-off approach will be an useful scientific tool for operators.
  - A novel two stage solution technique was proposed.
  - Economic performance at dynamic conditions are achieved in the standard MPC framework.
  - Extended to simultaneously determine sensor selection, operating point and controller.
  - The best set of sensors reduces the loss because of measurement errors whereas the optimal controller design reduces the loss in profit due to disturbances.
- The proposed problems involve solving convex optimization problems, which can be solved fairly efficiently

Journal publications

1. M. Nabil and Sridharakumar Narasimhan. Sensor Network Design for Optimal Process Operation Based on Data Reconciliation. *Ind. Eng. Chem. Res.*, **51 (19)**, 6789 – 6797, 2012.
2. M. Nabil, Sridharakumar Narasimhan and Sigurd Skogestad. Profitable and dynamically feasible operating point selection for constrained processes. *Journal of Process Control*, **24 (5)**, 531–541, 2014.

Publications in conference proceedings

1. *Design of Optimal Sensor Network based on Economic Objectives* at the **AIChE Annual Meeting 2010**. Salt Lake City, US, Nov. 7-12, 2010.
2. *Economic back-off selection based on optimal multivariable controller* at the **8th IFAC symposium on Advanced Control of Chemical Processes (ADCHEM 2012)**. Singapore, July 10-13, 2012.
3. *Integrated Sensor Network Design* at the **11th International Symposium on Process Systems Engineering (PSE2012)**. Singapore, July 15-19, 2012.
4. *Optimal selection of sensor network and backed-off operating point based on economics* at the **12th European Control Conference (ECC 13)**. Zurich, Switzerland, July 17-19, 2013.

உவப்பத் தலைக்கூடி உள்ளப் பிரிதல்
அனைத்தே புலவர் தொழில்

-திருக்குறள் (394)

You meet with joy, with pleasant thought you part;
Such is the learned scholar's wonderous art!

- *Translation of Thirukkural (394)*

Thank You

For the process matrix, C and Q (assuming unit variance for all flow variables)

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}; Q = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}$$

the error covariance matrix, Σ_z, can be expressed in terms of sensor network as

$$\Sigma_z = \frac{1}{q_1 q_2 + q_1 q_3 + q_2 q_3} \begin{bmatrix} q_2 + q_3 & q_3 & q_2 \\ q_3 & q_1 + q_3 & -q_1 \\ q_2 & -q_1 & q_1 + q_2 \end{bmatrix}$$

Let us consider the weighting matrix of the form

$$W = \begin{bmatrix} w_{11} & w_{12} & 0 \\ w_{21} & w_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now the sensor network design formulation based on average loss function, after simplification, is given by

$$\overline{L}_{cost} = \frac{w_{11}(q_2 + q_3) + (w_{12} + w_{21})q_3 + w_{22}(q_1 + q_3)}{2(q_1 q_2 + q_1 q_3 + q_2 q_3)}$$