Passivity and Thermodynamics Applied to Chemical Process Control

by

Ying Zhao

Department of Chemical Engineering
Norwegian University of Science and Technology

OUTLINE

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What Is *Passivity*?

Passive system: no internal "creation of energy"

Example: RLC circuit

(Khalil, 1996)

Input: u; Output: y; State: $[x_1 \; x_2]^T = [i_L \; V_c]^T$

Energy stored: $V(x) = \dfrac{Lx_1^2 + Cx_2^2}{2}$; Power flow: uy

$$\text{Passive} \Longleftrightarrow \underbrace{\int_0^t u(\tau)y(\tau)d(\tau)}_{\text{the energy absored}} \geq \underbrace{V(x(t)) - V(x(0))}_{\text{increase in the energy stored}}$$

– Equality: the system is *lossless* (no dissipation)

– Strict inequality: the system is *strictly passive*

Definition of Passivity

Dissipative systems (fundamental paper by Willems, 1972*a*)

Consider state space systems

$$\Sigma : \begin{array}{l} \dot{x} = f(x,u) \\ y = h(x,u) \end{array}; \quad x \in X \subset \mathcal{R}^n, \; u \in U \subset \mathcal{R}^m, \; y \in Y \subset \mathcal{R}^p$$

Supply rate w: $U \times Y \to \mathcal{R} \quad (\mathcal{R}^+ = [0, \infty))$

Definition 1 A state space system Σ is **dissipative** w.r.t *supply rate* w if $\exists$ a function $V\colon\ X \to \mathcal{R}^+$, called *storage function*, such that for all $x_0 \in X$, all $t_1 \geq t_0$, and all u

$$\underbrace{V(x(t_1)) \leq V(x(t_0)) + \int_{t_0}^{t_1} w(u(t), y(t))dt}_{\text{dissipation inequality}}$$

Definition of Passivity (Contd.)

Definition 2 A state space system Σ is **passive** if it is dissipative w.r.t. the supply rate $w = u^T y$ (dim. $m = p$).

Dissipation rate d: $X \to \mathcal{R}^+$ (e.g Krstić *et al.*, 1995)

$$V(x(t_1)) = V(x(t_0)) + \int_{t_0}^{t_1} \big(u(t)^T y(t) - d(x(t))\big)\, dt$$

Lossless: $d = 0$

Input strictly passive: $d = \delta \|u\|^2$, $\delta > 0$

Output strictly passive: $d = \epsilon \|y\|^2$, $\epsilon > 0$

*error on p.333, 366, 375 in Willems (1972*a*; 1972*b*) $d = \dot{V} - w$ but required ≥ 0

What Is Its Role in Control?

Passivity approach: an alternative approach to the *stability* analysis of feedback systems

Input-Output Stability (van der Schaft, 1996; Vidyasagar, 1993)

Set L_q: all measurable functions $f : \mathcal{R}^+ \to \mathcal{R}$ and satisfy

$$\int_0^\infty |f(t)|^q dt < \infty \quad q = 1, 2, ...$$

Set L_q^n: all n-tuples $f = [f_1, ..., f_n]^T$, $f_i \in L_q$

L_q-spaces are complete normed linear spaces w.r.t the norms

$$\|f\|_q = \left(\int_0^\infty |f(t)|^q dt \right)^{\frac{1}{q}}$$

Set L_{qe}: all $f : \mathcal{R}^+ \to \mathcal{R}$ such that $f_T \in L_q$,

$$f_T(t) = \begin{cases} f(t), & 0 \le t \le T \\ 0, & t \ge T \end{cases}$$

L_{qe}-space (*extended* L_q-space) is linear but not normed space

Input-output Stability

L_q**-stability:** $G : L_{qe}^m \to L_{qe}^p$ is said to be L_q-*stable* if

$$\mathrm{(input) u \in L_q^m \Rightarrow (output) G(u) \in L_q^p}$$

L_q**-gain:** G is *finite gain* L_q*-stable* ($\gamma_q(G)$) if there exist finite constants γ_q and b_q such that

$$\|(G(u))_T\|_q \leq \gamma_q \|u_T\|_q + b_q, \quad \forall u \in L_{qe}^m, \, \forall T \geq 0$$

Closed-Loop Stability

(van der Schaft, 1996)

$$G_1 : L_{qe}^{m_1} \to L_{qe}^{m_2}; \quad G_2 : L_{qe}^{m_2} \to L_{qe}^{m_1}$$
$$u_1 = e_1 - y_2; \quad u_2 = e_2 + y_1$$
$$y_1 = G_1(u_1); \quad y_2 = G_2(u_2)$$

Small-Gain Theorem Suppose that both G_1 and G_2 have finite L_q-gain given as $\gamma(G_1)$ and $\gamma(G_2)$. Then the *closed-loop system* $\Sigma^f_{G_1,G_2}$ is L_q-stable if

$$\gamma(G_1) \cdot \gamma(G_2) < 1$$

L_2-space: $\|f\|_2^2 =< f, f >$ (inner product)

$$< f, g >= \int_0^\infty f(t)g(t)dt, \quad f, g \in L_2$$

$$< f, g >_T= \int_0^T f_T(t)g_T(t)dt, \quad f, g \in L_{2e}$$

Definition $G : L_{2e}^m \to L_{2e}^p$ is *passive* if $\exists\, \beta$ such that

$$< G(u), u >_T \geq \beta, \qquad \forall u \in L_{2e}^m, \forall T \geq 0$$

G is *strictly input passive* if $\exists\, \beta$ and $\exists\, \delta > 0$ such that

$$< G(u), u >_T \geq \delta \|u_T\|_2^2 + \beta, \qquad \forall u \in L_{2e}^m, \forall T \geq 0$$

G is *strictly output passive* if $\exists\, \beta$ and $\exists\, \epsilon > 0$ such that

$$< G(u), u >_T \geq \epsilon \|(G(u))_T\|_2^2 + \beta, \qquad \forall u \in L_{2e}^m, \forall T \geq 0$$

(van der Schaft, 1996)

Passivity invariance via feedback: the feedback interconnection of passive systems is again passive, i.e.

if G_1 and G_2 are passive, then $\Sigma^f_{G_1,G_2}$ is passive

both strictly output passive $\Rightarrow$ $\Sigma^f_{G_1,G_2}$ strictly output passive

Passivity Theorem (Levine, 1996; Khalil, 1996) If

- both G_1 and G_2 are strictly input passive, OR
- both G_1 and G_2 are strictly output passive, OR
- one passive, the other one input or out strictly passive

Then the well-defined interconnection is finite gain L_2-stable.

*van der Schaft (1996) only claim that if both G_1 and G_2 are strictly output passive then closed-loop system is L_2-stable.

<u>A passive process *via* strictly passive feedback controller can be closed-loop stable.</u>

Lyapunov Stability Linked with Passivity

Consider state space systems

$$\Sigma : \begin{array}{l} \dot{x} = f(x,u) \\ y = h(x,u) \end{array} ; x \in X \subset \mathcal{R}^n, \ ; u \in U \subset \mathcal{R}^m, \ ; y \in Y \subset \mathcal{R}^p$$

- If the system Σ is passive with a positive definite storage function $V(x)$, then the origin of $\dot{x} = f(x,0)$ is stable.

- If the system Σ is output strictly passive and zero-state observable (imply positive definiteness of V), then the origin of $\dot{x} = f(x,0)$ is asymptotically stable.

When a nonlinear system can be *rendered passive* by *feedback*?

Byrnes *et al.* (1991): a nonlinear system, as described by

$$\begin{array}{l} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{array} ; \quad x \in \mathcal{R}^n,\ u \in \mathcal{R}^m,\ y \in \mathcal{R}^p$$

can be rendered *passive* via smooth *state feedback* iff the system has *relative degree 1* and is *weakly minimum phase*.

Ortega (1991): Cascade connection of two Globally asymptotically stable (GAS) nonlinear systems can be made GAS if the first system can be rendered strictly passive for an output which spans the "unstable part" of the vector field of the second system.

Lozano *et al.* (1992): A nonlinear system in cascade with a linear system of relative degree ≥ 1, is globally asymptotically stabilizable at $(0, 0)$ by a smooth state feedback control. (proved by passivity arguments)

Summary

- The concept of passivity plays an important role in stability analysis
- invariance of passivity in relation to feedback interconnection
- close connection with physical properties (e.g. "energy") of the system

Passivity, as physical notion, is a useful analysis tool so that control system analysis and design is more simple, more intuitive, and better understood.

Comparing with "Feedback Linearization" methods:

theoretic methodology, not related with physical properties of system

rely on full state measurement

require perfect model

Application of Passivity-based approach

Motivation primarily from control of mechanical systems, especially robotics.

- Takegaki and Arimoto (1981) (<u>starting point</u> of "passivity-based control"): stabilizing mechanical systems by reshaping the potential energy via feedback and by adding damping.
- Berghuis and Nijmeijer (1993): utilizes the passivity concept to develop combined controller-observer system for <u>robot motion control</u> using position measurement only.
- Egeland and Godhavn (1994): Passivity-based adaptive attitude control of a <u>rigid spacecraft.</u>
- Sira-Ramirez and Ortega (1995): derives passivity-based dynamic feedback controllers for <u>DC power supply.</u>
- Nicklasson (1996): propose new feedback controllers for a large class of <u>electric machines</u> using passivity techniques.
- Ydstie and Alonso (1996): link process thermodynamics with passivity theory of nonlinear control.

Thermodynamics

(Ydstie and Alonso, 1996; Abbott and Ness, 1976)

Thermodynamics: *energy* and its transformations

The first law: Energy is conserved

$$U(x_2) - U(x_1) = \int_1^2 dQ - \int_1^2 dW$$

$U(x) : Z \to \mathcal{R}^+$—internal energy of the system at state x

$\int_1^2 dQ$—heat added to the system from state x_1 to state x_2

$\int_1^2 dw$—work done by the system from state x_1 to state x_2

The second law: Entropy is produced

$$S(x_2) - S(x_1) \geq \int_1^2 \frac{dQ}{T} \quad \text{(Clausius inequality)}$$

$S(x) : Z \to \mathcal{R}^+$—entropy of the system at state x

Links between Passivity and Thermodynamics

Willems (1972a): A thermodynamic system is

- lossless w.r.t supply rate $\dot{Q} - \dot{W}$

$$U(x_2) = U(x_1) + \int_{t_1}^{t_2} \left(\dot{Q}(t) - \dot{W}(t) \right) dt$$

- dissipative w.r.t supply rate $-\frac{\dot{Q}(t)}{T(t)}$

$$-S(x_2) \leq -S(x_1) + \int_{t_1}^{t_2} (-\frac{\dot{Q}(t)}{T(t)}) dt$$

Ydstie and Alonso (1996): However, S can't be used as storage function directly since S is not bounded from above. Novel idea: Combining energy and entropy to form storage function V in dissipation inequality

$$V = U - T_0 S + V_0, \quad T_0 \text{ and } V_0 \text{ are constants}$$

$$\underbrace{V(x(t_1)) \leq V(x(t_0)) + \int_{t_0}^{t_1} w(u(t), y(t)) dt}_{\text{dissipation inequality}}$$

*error on p.S1119 in Alonso and Ydstie (1996) $U_2 - T_2 S_2 \geq U_1 - T_1 S_1$, and $U - TS$ (Helmholtz free energy) was used as storage function.

Links between Passivity and Thermodynamics (contd.)

Main contributions by Ydstie et al:

(Ydstie and Alonso, 1996; Alonso and Ydstie, 1996; Ydstie and Viswanath, 1994)

Action ϕ: Exchanges with environment are called actions on the system (e.g. supplied energy ϕ_U, supplied entropy ϕ_S)

- $\phi_U = \int_1^2 dQ - \int_1^2 dW$ has *conservation* property
- $\phi_S = \int_1^2 \frac{dQ}{T}$ has *Clausius-Planck* property.

$$V = U - T_0 S + V_0; \quad \phi_V = \phi_U - T_0 \phi_S$$

Proposition 1 For every T_0 there exists a constant V_0 so that $V(x) \geq 0$ for all $x \in X$. (Then V is storage function)

Proposition 2 The action ϕ_V satisfies the dissipation inequality.

Proposition 3 The action ϕ_V is passive when the assumption of local equilibrium holds. (exist a local minimum of entropy production)

Incentives for Chemical Process Control

(Stephanopoulos, 1984)

- Disturbance rejection
- Ensuring stability
- Optimizing performance

Associated problems:

- Modeling difficulties, Robustness
- Higher-order dynamic behavior: e.g. Processes with dead time (dead time causes instability) and with inverse response
- Nonlinearity

Applicability of Passivity-based Approach Linked with Thermodynamics to Chemical Process Control

Opening issue: no any previous developments that have successfully linked thermodynamics and passivity theory to chemical process control, except Ydstie *et al*'s work.

Thermodynamics:

- a useful tool for process design and modeling,
- a useful foundation for process control.

Potential advantage:

- Ensure stability
- Enhance robustness
- Cope with nonlinearity
- Less requirement on model accuracy comparing with feedback linearization method

e.g. Distillation column control

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