INTEGRATED DESIGN AND CONTROL OF HEAT EXCHANGER NETWORKS

by

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ABSTRACT

Energy recovery through heat exchangers networks is an important part of most chemical processes. The energy integration introduces interactions, and may make the process more difficult to control and operate. Dynamic behaviour and possible control problems are addressed in this thesis. Moreover, it is discussed how the gained understanding and insight about control and operation may be taken into account during control configuration design and network design.

It is shown that simple heat exchanger networks may have both monovariable and multivariable right-half-plane zeros. The right-half-plane zeros are caused by parallel, opposing effects and may put a severe limitation on the achievable control performance. The parallel effects may be due to stream splitting, but may also occur for no-split designs.

Interactions in heat exchanger networks are analysed, and two-ways interactions may occur for networks with heat load loops involving only process heat exchangers or networks where one of the matches has two controlled outputs. For the latter case a simple analytic expression for the steady-state relative gain array has been derived. The expression shows that the preffered pairing for decentralized control only depends on the thermal effectiveness of the match with the two outputs.

Controllability of heat exchanger networks depends heavily on the control configuration or bypass locations. Usually manipulating single direct effect bypasses are preferred, but energy or steady-state disturbance rejection considerations may favour bypasses on upstream exchangers, multi-bypasses or split fractions.

Operability of heat exchanger networks depends on stream and exchanger parameters, but also on structure. It is explained that control of networks where remixed split streams are controlled outputs and networks with matches where both outlets are controlled outputs may be difficult. Operational considerations also favor no-split designs without inner matches with downstream process heat exchangers on both sides. These results are formulted as operability heuristics, and it is explained in detail how the heuristics may be taken into account during heat exchanger network synthesis with the pinch design method and mathematical programming. Typically, heat exchanger network synthesis have flat optimas, there is a large number of designs with near-minimum cost. So although the heuristics may seem restrictive near-minimum cost designs fulfilling all the operability heuristics usually exist.

Existing methods for automated synthesis of flexible networks are based on problem decomposition where targets are derived ahead of the design. It is known that this may give topology traps for synthesis problems for one nominal operating point, and it is illustrated that simultaneous optimization of structure, area and energy may be even more important for flexibility problems.

Contents

Chapter 1

Introduction

1.1 Background

The industrial nations have largely built their wealth on the chemical process industries which include a variety of plants and processes. One of the few common features of these processes is that energy recovery is an important issue. Energy is generally recovered through process heat exchangers where heat is transferred from hot to cold process streams in order to save both hot and cold utility.

In the 60's and 70's this energy integration was done in an ad hoc manner. Initiated by the discovery of the pinch (Hohmann, 1971; Linnhoff and Flower, 1978), and with the energy crisis in the 70's as the driving force, systematic and powerful methods for energy integration have been developed (e.g., Linnhoff *et al.*, 1982; Floudas *et al.*, 1986; Trivedi *et al.*, 1988). Pinch technology has become a major research area for both chemical and mechanical engineers. The basic idea that heat cannot be transferred across pinch without an energy penalty has also been successfully applied to analysis and synthesis of utility systems, heat and power systems and separation systems. Due to remarkable energy savings both on new plants and retrofit projects (Linnhoff and Turner, 1981; Linnhoff and Vredeveld, 1984) the importance of pinch technology is now generally accepted in the industry.

Gundersen and Næss (1988) give a comprehensive review of heat exchanger network (HEN) synthesis. They cite over 200 papers, but novel results have already made the review somewhat outdated.

The review by Gundersen and Næss (1988) include some papers on analysis and synthesis of flexibility or multi-period problems, but no work on dynamics and control. Heat exchanger networks seems to be almost non-existent problem within the control community, although control of single heat exchangers are considered in many text-books (e.g., Shinskey, 1979; Balchen and Mumme, 1988) and technical articles (e.g., Alsop and Edgar, 1989; Khambanonda *et al.*, 1990).

1.2 Motivation

The lack of research on control of HENs is the main motivation for this work together with a general interest in computer-aided chemical engineering and a particular interest in energy integration. Issues concerning dynamics, control, operation and design of HENs are addressed in this thesis. The research topics should be interesting both for academia and industry due to the following facts about chemical processes:

1. Chemical processes are energy-demanding. Energy costs represent a substantial part of the operating costs for most chemical processes.

2. The heat exchanger network is expensive. Chemical processes often include a large number heat exchangers, and the total cost of all the heat exchangers (i.e., the HEN) may represent an appreciable part of the capital for new or modified plants.

3. Heat exchanger network control is important for the operation of the plant. During design the HEN is usually considered to be of minor importance for the overall plant economics compared to the reactor and separation system (e.g., Douglas, 1988). Streams exiting the HEN are usually fed to reactors or separation equipment, temperature variations may seriously affect operating costs through off-spec distillation products, catalyst deactivation or even melt-down of nuclear reactors.

4. Control valves are expensive. The cost of the control system for temperature and heat load control in the HEN may be considerable. The cost of a control valve may be larger than the the cost of a heat exchanger, and auxiliary equipment like connecting pipes, bypasses, manual valves, measurements may also be considerable.

5. Most controllers are simple decentralized controllers (e.g. Yamamoto and Hashimoto, 1991). For such controllers the control algorithm is simple and the controller tuning is straightforward unless there are strong interactions whereas the control configuration may be very important for the performance. There is a lack of systematic methods for control configuration design (Downs and Vogel, 1993)

6. Process design may seriously affect process control. Recently, there has been increased interest in the interactions between design and control of chemical processes. Process design may put irreparable limitations on the controllability or achievable control performance. Development of controller-independent controllability measures have improved the possibilities of addressing control in the early design phase.

Stream	$T_s[^oC]$	$T_t[^oC]$	$w[kW/^oC]$
$H1$	150	60	20
$H2$	90	60	80
$C1$	20	125	25
$C2$	25	100	30

Table 1.1: Stream data for Example 1 from Linnhoff and Hindmarsh (1983)

1.3 Basic concepts and definitions

1.3.1 Pinch and Heat Recovery Level

The heat exchanger network (HEN) problems considered here consist of a set of N^h *hot streams* that are to be cooled from *supply temperatures* to *target temperatures* and a set of N^c *cold streams* that are to be heated from supply temperature to target temperatures. The HEN that fulfills these heating and cooling requirements consists of a set of N_U heat exchange units or matches that may be process exchangers between hot and cold process streams (N_{hx}), or *utility exchangers* (N_{ux}), a common word for heaters and coolers:

$$N_U = N_{hx} + N_{ux} \tag{1.1}$$

HEN problems may be divided into *pinch problems*, where both hot and cold utility is required, and *threshold problems* with no heating or no cooling requirements. The pinch point may be defined as the point where the hot and cold *composite curves* are closest together, and is best explained through an example. The example include two hot and two cold streams, and the (constant) supply temperatures T_s, target temperatures T_t and heat capacity flowrates w are shown in Table 1.1. The four process streams may be visualized in a heat load/temperature diagram, see Fig. 1.3a. The hot streams may be added together to a hot composite curve and the cold streams to a cold composite curve as shown in Fig. 1.3b. The minimum vertical difference between the curves is the heat recovery approach temperature (HRAT) and defines the pinch point. Importantly, HRAT determines the minimum hot and cold utility consumption. In the example HRAT is selected to be 20^0C, which gives $Q^h_{min} = 1075kW$ and $Q^c_{min} = 400kW$.

Even though a specific HEN problem is a pinch problem with a specified heat recovery it may become a threshold problem if HRAT is reduced. For our example Q^c_{min} drops to zero if HRAT is reduced to 12.72^0C. The maximum heat recovery level, which corresponds to minimum utility consumption, is determined by minimum temperature driving forces at process pinch, i.e., $HRAT = 0$. HENs designs achieving the heat recovery corresponding to a prespecified HRAT are, somewhat misleading, denoted maximum energy recovery (MER) designs.

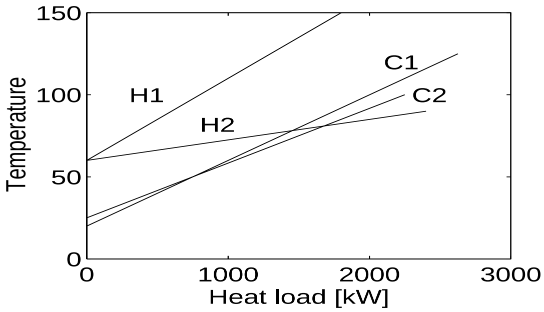

Figure 1.1: a) Process streams in a heat load/temperature diagram

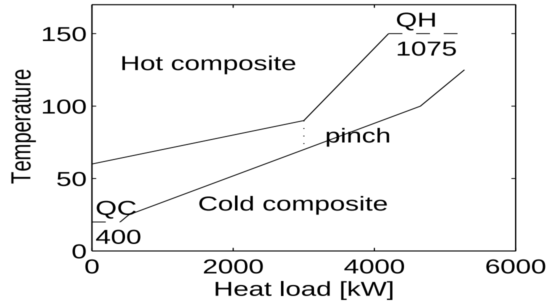

Figure 1.2: b) Composite curves defining the pinch

Figure 1.3: Minimum utility consumption is determined by the stream data and the heat recovery level (HRAT)

1.3.2 Heat load loops and Degrees of Freedom

The minimum number of units required to fulfill all heating and cooling requirements is given by Eulers formula:

$$N_{U,min} = N^h + N^c + N_{util} - 1 \tag{1.2}$$

where N_{util} is the number of utilities and it is assumed that the HEN is connected, i.e., only one subset. For pinch problems with single utilities:

$$N_{U,min} = N^h + N^c + 1$$

The minimum number of matches for such HENs is given by:

$$N_{hx,min} = N^h + N^c - 1 \tag{1.3}$$

i.e. minimum number of unit pinch designs with one subset have one heater and one cooler. Minimum number of unit designs often have low heat recovery, and it may be cost-effective to increase the number of units to reduce the utility consumption. Increasing the number of units introduces *heat load loops*.

At steady-state HENs have one degree of freedom per unit, i.e., the heat load. Some of the degrees of freedom are fixed to fulfill control objectives, and the number of degrees of freedom (N_{dof}) is simply:

$$N_{dof} = N_U - N_y$$

where N_U is the number of units and N_y is the number of controlled outputs. Usually the controlled outputs are the target temperatures of the streams, which yields:

$$N_{dof} = N_U - N^h - N^c = N_{U,min} + N_L - N^h - N^c = 1 + N_L$$

where N_L is the number of independent heat load loops. This shows that even pinch designs with minimum number of units have one degree of freedom, which in synthesis may be considered as the heat recovery level determined by HRAT. However, during operation, this degree of freedom may be exploited to minimize utility consumption or resetting a control loop to its ideal resting value.

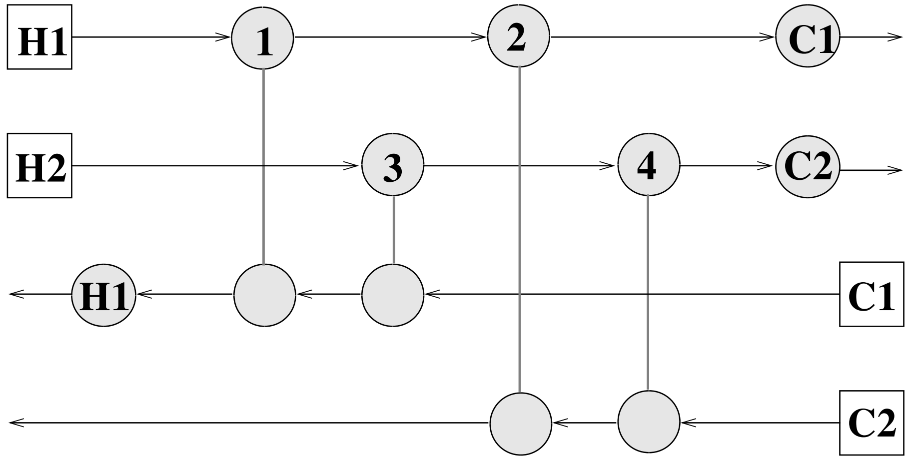

Figure 1.4: HEN with no loops according to the control definition used in this paper, but two (independent) loops according to the synthesis definition introduced by Linnhoff *et al.* (1982).

1.3.3 Downstream paths and loops

Linnhoff and Kotjabasakis (1986) introduced the term downstream paths to describe how disturbances propagate cocurrent along process streams and across matches in HENs. With control terminology one may simply say that a downstream path between an input and output in HENs exists if the input affects the output so that the gain is structurally not zero. Parallel downstream paths between an input (bypass or split fraction) and an output exist if both branches have a downstream path to the output. A loop exists if there is a natural feedback in the HEN, i.e., a stream temperature variation affect itself. Our definition of loop is the one normally used when addressing control problems, i.e., cause-effect loops. Note that the loop definition by Linnhoff *et al.* (1982) commonly used in HEN synthesis papers, i.e., heat load loops, is much broader. For example, consider Fig. 1.4 where according to our definition this network has no loops. However, according to the synthesis definition this HEN has three loops, the first loop includes all 4 matches, the second loop includes matches 1 and 3 plus the coolers C1 and C2, and the third loop matches 2 and 4 plus the coolers. Of the three loops only two are independent. The number of independent loops may be computed or checked by comparing the number of units (in this case 7) with the global minimum (in this case 5).

1.3.4 Categories of outputs, matches and splits

The key results in this thesis are based on structural properties of certain HENs. To describe these properties we have divided the outputs, matches and stream splits into categories.

- Utility-controlled outputs. In HENs utility exchangers are often the final units on the streams, and the target temperatures downstream utility exchangers are usually controlled by manipulating the heat load on the utility exchanger. Controlled outputs downstream utility exchangers are denoted utility-controlled outputs.

- Bypass-controlled outputs. The remaining outputs must be controlled by manipulating heat loads on matches which is normally done by adjusting a bypass stream. Thus, controlled outputs *not* downstream a utility exchanger are denoted bypass-controlled outputs.

- Final splits. Some streams may be split to enable parallel heat exchange. If the parallel heat exchangers are the final units on the stream, the split is denoted a final split.

- Inner matches. Matches where both the hot and the cold streams continue to other matches are denoted inner matches. In Fig. 1.4 match 3 is an inner match.

- Double output matches. Matches where both the hot outlet and the cold outlet are controlled outputs are denoted double output matches. In Fig. 1.4 there are no double output matches, but match 2 would become a double output match if cooler $C1$ was removed since it is assumed that all target temperatures are controlled outputs.

- Combining matches. For multivariable problems (or subproblems) with two inputs and two outputs there are four downstream paths (cause-effects) from inputs to outputs. If all four downstream paths traverse the same match, this match is denoted a combining match. The simplest example of combining matches are double output matches where bypasses on two upstream matches are used to control the two outputs on the double output match. If it is still assumed that cooler $C1$ in Fig. 1.4 is removed, match 2 become a combining match if bypasses on matches 1 and 4 are used as the manipulated inputs.

We will explain how final splits, inner matches, double output matches and combining matches may yield control problems in terms of singularities, inverse responses or interactions.

1.3.5 Controllability, Flexibility and Operability

The research on pinch technology is largely aimed at design, i.e. towards the synthesis of cost-optimal heat exchanger networks. A possible disadvantage of integrating the process is that it may become more difficult to operate. This is rather obvious since control loop interaction is a well-known control problem. However, the processes of the 60's and 70's were already largely integrated, and Linnhoff and Turner, (1981) reported that applying the pinch design method for energy integration made the plants more operable. The reasons may be that the process is decomposed in a hot (above pinch) and a cold (below pinch) temperature region, and that exchangers with small temperature driving forces are avoided. Still, operational issues of pinch technology have largely been neglected, especially the controllability. Operability of HENs may be considered to include the eight factors shown in Fig. 1.5. Note that flexibility include both feasibility and utility consumption. The whole purpose of the HEN is to save energy, so the ability to reject static variations (i.e. feasibility) with excessive utility consumption is not very useful. Controllability is defined as the achievable control

1.4 Heat exchanger network problems

The four main problems discussed in this thesis may be defined as:

1. Optimal operation. For a HEN with given structure, exchanger areas and bypasses, and a given steady-state operating point (supply temperatures, heat capacity flowrates, heat transfer coefficients and target temperatures), find the set of manipulated inputs (bypass fractions and possible split fractions) that minimizes energy cost. This problem is studied in chapter 5 and 9. The optimal operation problem may be extended to include dynamic variations (controllability) by estimating the expected dynamic disturbances, and specifying allowed dynamic deviations for the target temperatures. This will make it necessary to restrict the range for the manipulated inputs (bypass and split fractions). One may estimate the upper and lower bounds for the manipulated inputs from dynamic simulation or from a set of simplifications. This problem is formulated as an open-loop optimization. One may also consider the regulatory control loops for the target temperatures closed, and optimize the utility consumption with the remaining manipulated inputs. This corresponds to a hierarchical control structure.

2. Control configuration design. For a HEN with given structure, exchanger areas, bypasses, control valves and measurements, disturbances with expected deviations, allowed target temperature deviations, find the control configuration that minimizes energy costs. A practical subproblem is to disregard energy costs and determine the control configuration that minimizes the target temperature deviations. Alternatively, one could also consider the cost of bypasses and control valves.

3. Area optimization. For a HEN with given structure and a given steady-state operating point (supply temperatures, heat capacity flowrates, heat transfer coefficients and target temperatures), find the set of manipulated inputs (areas and possible split fractions) that minimizes exchanger and energy cost. The area optimization problem may be extended to include dynamic variations as described for the optimal operation problem, and/or the costs of bypasses and control valves. One may also optimize exchanger areas for a set of steady-state operating points, i.e., for flexibility.

4. Network synthesis. For a HEN problem with a given steady-state operating point (supply temperatures, heat capacity flowrates, heat transfer coefficients and target temperatures), find the set of manipulated inputs (structure, areas and possible split fractions) that minimizes exchanger and energy cost. This is the conventional HEN synthesis problem for a nominal operating point. This problem may be extended to include dynamic variations and/or the cost of bypasses and control valves and/or flexibility as the area optimization problem.

The tasks start from the control/operation issues (problem 1) where the plant (HEN with installed areas and bypass lines) is given and move towards the design issues (problems 2-4) where the plant is to be decided. Problem 4 including dynamic and static variations and costs of bypasses and control valves is the appropriate integrated control and design problem. Because this problem is too difficult to solve directly the other problems are defined and studied to understand the subtasks.

	Optimal operation	Control configuration design	Area optimization	Network synthesis
Main task	Select bypass fraction values	Select bypasses	Select exchanger areas	Select structure

Table 1.2: Main tasks for the four HEN operation and design problems

	Optimal operation	Control configuration design	Area optimization	Network synthesis
Energy cost	$+$	$\pm$	$+$	$+$
Bypass/control valve cost	$-$	$\pm$	$\pm$	$\pm$
Dynamic variations	$\pm$	$+$	$\pm$	$\pm$
Static variations	$\pm$	$\pm$	$\pm$	$\pm$

Table 1.3: Cost factors and stream parameter variations that may or may not be taken into consideration in the four HEN operation and design problems

1.5 Thesis overview

This thesis consists of eight separate articles on HENs that may be read independently. Still, the order of the chapters was quite self-evident (with the possible exception of chapter nine). In the first four articles various operational aspects are discussed. The last half of the thesis discuss how operable HENs may be designed.

Ch. 2: Dynamic models

In order to assess controllability a dynamic model is required. The main objective in this work is to determine what model features that is required to accurately assess controllability of HENs. We have chosen to use a lumped multicell model of the heat exchangers, and several cells must be used to describe the dead-time in the exchangers. Surprisingly, it is found that wall capacitance is required even for liquid heat exchangers, whereas flow configuration has small effect on the dynamic behaviour. At steady-state the logarithmic mean is not defined for heat exchangers with equal heat capacity flowrates, and in steady-state optimization and design the logarithmic mean is often approximated with expressions that may may be used for the entire parameter range (e.g., Paterson, 1988; Chen, 1991). For dynamic simulations these approximations are not so helpful due to possible temperature crossover during transients. Finally, the flow dependence of the heat transfer coefficients should be included as it will *increase* the gains from bypass manipulations, and may have an appreciable effect for control.

Ch. 3: Dynamic behaviour and control limitations

The main objective of this article is to identify and explain dynamic characteristics of HENs. The emphasis is on behaviour or phenomena that may give control limitations.

In the first part we discuss how inlet temperature, flowrate variations and bypass manipulations propagate in HENs. The fundamental difference between temperature and flowrate disturbances is pointed out. Temperature disturbances are dampened and slowed down when traversing heat exchangers whereas flowrate disturbances are immediate over series of heat exchangers. A simple, but important result is that changes in bypass streams will propagate from the manipulated match as two temperature disturbances with opposing effects.

HENs are (open-loop) stable, but other main control problems like singularities, right-half-plane zeros, time delays, input constraints and interactions may occur. A comprehensive discussion on these problems is presented in the second part of this chapter. A key result for the thesis is to explain how parallel downstream paths may occur, and that these parallel downstream paths give opposing effects that may give parametric singularity or a right-half-plane zero.

Ch. 4: Control configuration design

The gained insight about dynamic behaviour and control problems is used together with controllability measures to select control configuration. For HENs the outputs are usually the stream target temperatures, and temperatures downstream utility exchangers are usually controlled by manipulating the utility exchanger heat load. Thus, the main issues discussed in this paper are bypass placement and how multi-bypasses and splitters compare to single bypasses.

Ch. 5: Optimal operation

In this paper steady-state optimal operation is discussed. Optimal operation may be defined as feasible operation with minimum utility consumption. It is shown that for some HEN structures, the optimal input combination for energy is independent of problem parameters. This property makes it possible to determine the optimal combination by hand (without a numerical optimization. It may also be used to determine whether the manipulations for utility consumption coincide with the preferred manipulations for target temperature deviations.

Ch. 6: Control considerations in design

Based on the gained insight about control and operation, it is discussed how network structure and heat load or area distribution affect control. Controllability depends heavily on control configuration and problem parameters, but some HENs tend to give operability problems even with the optimal control configuration. The main point of this chapter is to formulate heuristics that describes common features of HENs that are easy to control and operate.

Ch. 7: Operability considerations during synthesis with the pinch design method

It is explained how the operability heuristics may be taken into account during synthesis with the pinch design method, which is the most common synthesis method both in industry and academia. The pinch design method consists of three stages; targeting, synthesis and evolution; and the heuristics may be applied in the last two. It is discussed whether the resulting designs are easier to control and operate, easier to identify or more expensive than designs derived with the conventional pinch design method.

Ch. 8: Operability considerations in synthesis with mathematical programming

One of the most recent and promising methods for automated synthesis of HENs using mathematical programming is the stage-wise approach suggested by Yee *et al.*, (1990). An important advantage of this approach is that all constraints are linear. In this paper we show how the operability heuristics may be formulated as logic statements and linear constraints by including a new set of binary variables. It is discussed whether the resulting extended model yields designs that are easier to control and operate, easier to identify (in terms of convergence properties and execution times), or more expensive than designs derived with the original model.

Ch. 9: Effect of flexibility requirements on design

This paper may be divided in two. In the first part it is shown how existing methods for automated design of flexible or multi-period HEN problems, give suboptimal designs because the heat recovery level is the same for all operating points. In the latter part the effect of including flexibility requirements on nominal optimal or near-optimal designs is considered. It may be concluded that flexibility may be expensive and favor simple designs with few matches and low total area.

Final conclusions and some suggestions for further work are presented in chapter 10.

Presentations

Preliminary versions of some of the papers in this thesis have been presented at international chemical engineering conferences (see also references):

- 1991 American Institute of Chemical Engineers (AIChE) Annual Meeting. Parts of chapters 3 and 4.

- 1992 European Symposium of Computer Aided Process Engineering, (ESCAPE-1). Parts of chapters 3 and 4.

- 1992 American Institute of Chemical Engineers (AIChE) Spring National Meeting. First part of chapter 9. This presentation was also held at the 1992 Nordic Process Control Workshop (NPC-IV).

- 1992 American Institute of Chemical Engineers (AIChE) Annual Meeting. Second part of chapter 9.
- 1993 European Symposium of Computer Aided Process Engineering-3 (ESCAPE-3). Chapter 2.
- 1994 Process Systems Engineering (PSE-94). Chapter 5.

Altogether, all results from three articles (Ch. 2, 5 and 9) and the majority of the results from two articles (Ch. 3 and 4) have been presented, whereas the results in chapters 6, 7 and 8 are unpublished.

Some ideas and results from this study has also been included in two conference presentations by Erik A. Wolff (Wolff *et al.*, 1991; Wolff *et al.*, 1992).

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Chapter 2

Dynamic Models for Heat Exchangers and Heat Exchanger Networks

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Abstract

Dynamic models are needed to assess controllability of heat exchangers and heat exchanger networks. A simple model is desirable, but all important model features must be included. We discriminate between important and less important model features by order of magnitude argumentation, comparison of controllability measures and dynamic simulations. Important model features for single heat exchangers are number of compartments in the lumped model (model order), wall capacitance and fluid compressibility, whereas flow configuration and temperature driving force have small effect on the dynamics. The most important model features for heat exchanger *networks* are structure, residence time and model order of the bypasses and the connecting pipes. Simpler models may fail to identify inherent control limitations as zeros in the right half plane.

2.1 Introduction

The steady-state energy balance for heat exchangers is

$$w^h(T_i^h - T_o^h) = w^c(T_o^c - T_i^c) = UA\Delta T_{hx} \tag{2.1}$$

where superscript h means hot side, c cold side; $w = FM_w c_p$ heat capacity flowrate and $U = h^h * h^c/(h^h + h^c)$ overall heat transfer coefficient. The overall temperature driving force of the heat exchanger, ΔT_{hx}, depends on the flow configuration. By defining the static heat exchanger effectiveness $P^h = (T_i^h - T_o^h)/(T_i^h - T_i^c)$ and heat capacity flow ratio $R^h = w^h/w^c = (T_o^c - T_i^c)/(T_i^h - T_o^h)$, the steady-state transfer function between inlet and outlet temperatures may be expressed as (Kern, 1950; appendix 1):

$$\begin{bmatrix} T_o^h \\ T_o^c \end{bmatrix} = \begin{bmatrix} 1-P^h & P^h \\ P^c & 1-P^c \end{bmatrix} \begin{bmatrix} T_i^h \\ T_i^c \end{bmatrix} \tag{2.2}$$

where P is a function of flow configuration, number of transfer units ($N_{TU} = (UA)/w$) and R only. By definition P^h and $P^c = R^h P^h$ are physically bounded between zero and unity, and they are usually below 0.8. R and N_{TU} are often between 0.2 and 5 except for reboilers and condensers.

We will use a dynamic model for controllability assessment, and it is known that model assumptions may be important. In particular, the model must identify limitations of the achievable control performance. Important control limitations are 1) time delays, 2) right half plane (RHP) zeros, 3) input constraints and 4) sensitivity to model uncertainty. A simple model is desirable, and the main objective of this paper is to discriminate between appropriate and less appropriate model assumptions.

The following model features for single heat exchangers will be addressed: 1) heat transfer coefficients, 2) model order, 3) temperature driving force, 4) wall capacitance, 5) flow configuration and 6) fluid compressibility. For HENs we consider 1) heat transfer coefficients (again), 2) bypass placement, 3) pipe residence time and 4) model order for the pipes. We have chosen to use lumped heat exchanger models, where each fluid is modelled as mixed tanks in series, because of mathematical simplicity as well as physical resemblance to shell-and-tube exchangers with baffles. Lumped models also match experimental frequency response data well. Furthermore, when a lumped compartment or cell model is used, distributed model behaviour may be achieved by using a large number of cells or using the logarithmic mean temperature difference as the temperature driving force (e.g., Reimann, 1986; Rinard and Nieto, 1990). The latter represents a hybrid between a lumped and a distributed model.

2.2 Dynamic multicell heat exchanger model

The lumped compartment or "multicell" model of a heat exchanger is shown in 2.1. In addition to ideal mixing tank assumptions we assume negligible heat loss, constant heat capacity, and that exchanger area A and volume V are equally distributed over the N cells. For liquid exchangers fluid densities are assumed constant and pressure drop

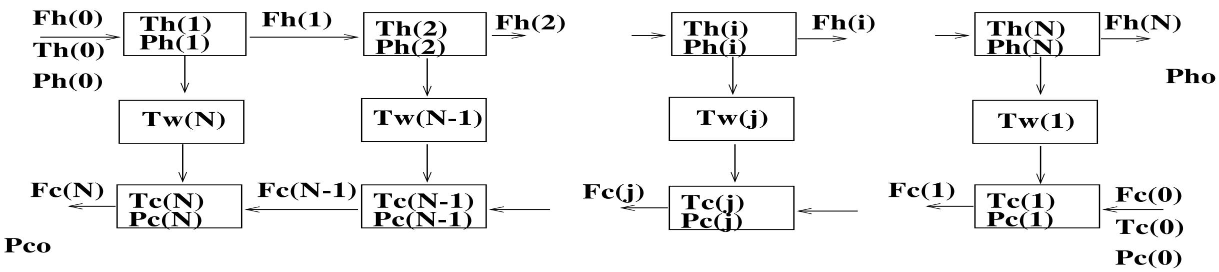

Figure 2.1: Cell model of heat exchanger where fluid temperatures, wall temperatures and fluid pressures are state variables and flowrates are computed from pressure drops.

neglected. For gas exchangers densities are computed from ideal gas law and flowrates from pressure drop. For brevity, we only give the resulting differential equations for liquid exchangers including wall capacitance. A complete derivation can be found in appendix 2.

$$\frac{dT^h(i)}{dt} = (T^h(i-1) - T^h(i) - \frac{h^h A}{w^h N}\Delta T^h(i))\frac{F^h M_w^h N}{\rho^h V^h} \tag{2.3}$$

$$\frac{dT^w(j)}{dt} = (h^h \Delta T^{wh}(j) - h^c \Delta T^{wc}(j))\frac{A}{\rho^w c_p^w V^w} \tag{2.4}$$

$$\frac{dT^c(j)}{dt} = (T^c(j-1) - T^c(j) + \frac{h^c A}{w^c N}\Delta T^c(j))\frac{F^c M_w^c N}{\rho^c V^c} \tag{2.5}$$

where all symbols are explained in the nomenclature section.

The dynamic characteristics for heat exchangers are determined by flow configuration and time constants related to three holdups of energy: 1) hot side, 2) wall, 3) cold side, and may be extracted from the dynamic model as (respectively):

$$\tau_F^h = \frac{V^h \rho^h}{F^h M_w^h} \tag{2.6}$$

$$\tau_k^w = \frac{(V^w \rho^w c_p^w)}{(h^h + h^c)A} \tag{2.7}$$

$$\tau_F^c = \frac{V^c \rho^c}{F^c M_w^c} \tag{2.8}$$

where τ_F is the residence time. Typical values are $0.5 < \tau_F < 60$ seconds and $0.1 < \tau_k = 30$ seconds. Differences in heat transfer coefficient of single- and multiphase fluids, density of gases and liquids and area density (m^2/m^3) of different heat exchangers are the main reasons for the large ranges. In most of our examples we use a liquid heat exchanger with equal heat capacity flowrates ($R = 1$) and heat transfer coefficients ($h^h = h^c$) where the time constants are $\tau_F^h = \tau_F^c = 32$ seconds and $\tau_k = 14$ seconds.

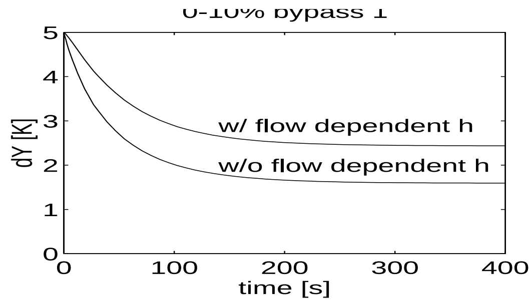

Figure 2.2: a) Effect of flow dependent heat transfer coefficients.

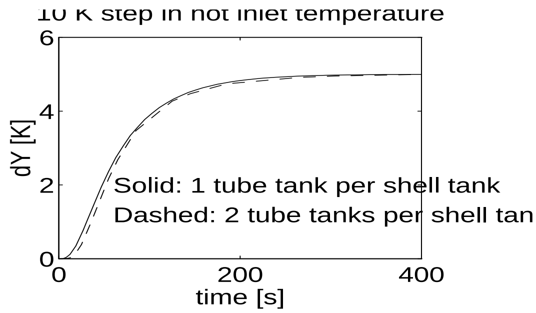

Figure 2.3: b) Effect of 2 tube tanks per shell tank.

Figure 2.4: Model features of 1-1 exchanger modelled with 6 cells and thermal effectiveness $P = 0.5$.

2.3 Model Features of Single Heat Exchangers

2.3.1 Heat transfer coefficients

Usually heat transfer coefficients are based on a distributed heat exchanger model (i.e., U_{lm}). Heat transfer coefficients used in lumped models (U) must be increased to give the same overall effectiveness, larger increases are needed for fewer cells. By combining effectiveness-expressions for one single mixing tank (Stevens *et al.*, 1957) with equations for series of countercurrent heat exchangers (Domingos, 1969), the necessary heat transfer coefficient may be computed from the number of lumped cells. For equal heat capacity flowrates the relationship is simple:

$$N = \frac{N_{TU,lm}}{1 - U_{lm}/U} \qquad N_{TU,lm} = \frac{U_{lm}A}{w} \tag{2.9}$$

where N is the number of cells in the lumped heat exchanger model. When different model features yield different temperature driving forces, we adjust heat transfer coefficients to get the same static thermal effectiveness. From steady-state considerations the heat transfer coefficient dependence on flowrate should be included because the effects are significant. A comparison for a typical countercurrent heat exchanger ($R = 1$ and $N_{TU} = 1$) is shown in Fig. 2.4a. Including flow dependent film coefficients will also affect dynamics by increasing the speed of response to flowrate variations. The effect is illustrated in Table 2.1 where the asymptotic phase shifts for different model features are compared. Note that the phase shift from cold flowrate is reduced when flow-dependent heat transfer coefficients are introduced. This makes it more desirable to include wall capacity. The table will be further commented in the following discussion of other model features.

2.3.2 Number of mixing tanks (model order)

During simplified controllability analysis of HENs it is often implicitly or explicitly assumed a one-cell heat exchanger model (Georgiou and Floudas, 1989; Reeves *et al.*,

Transfer function to T_o^h from:	T_i^h	T_i^c	F^h	F^c
Lumped; h const.; w/o wall cap.; incompr.	$-N\pi/2$	$-\pi$	$-\pi/2$	$-\pi$
Lumped; h flow-dep.; w/o wall cap.; incompr.	$-N\pi/2$	$-\pi$	$-\pi/2$	$-\pi/2$
Hybrid; h const; w/o wall cap.; incompr.	$-N\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi$
Hybrid; h flow-dep.; w/o wall cap.; incompr.	$-N\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$
Lumped; h flow-dep.; w/ wall cap.; incompr.	$-N\pi/2$	$-3\pi/2$	$-\pi/2$	$-\pi$

Table 2.1: Different model assumptions and model features give different asymptotic phase shifts of the frequency response.

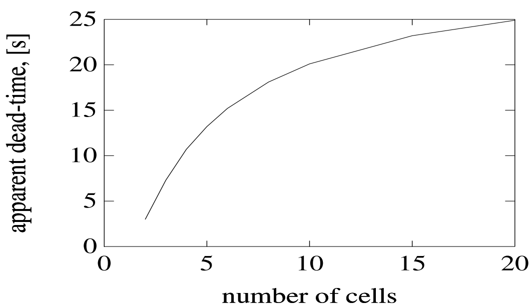

Figure 2.7: a) Apparent dead-time from hot inlet to hot outlet temperature as function of number of cells.

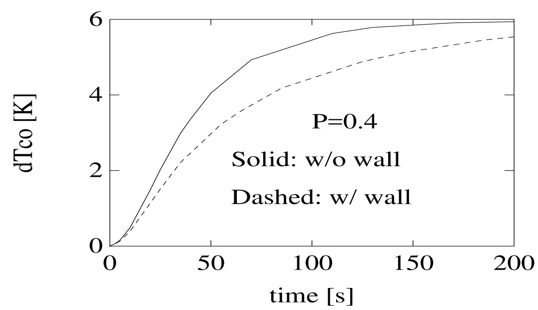

Figure 2.8: b) Cold temperature response to step in hot inlet temperature with and without wall capacitance.

Figure 2.9: 1-1 exchanger with thermal effectiveness P = 0.5.

1991; Huang and Fan, 1992; Daoutidis and Kravaris, 1992). Such models will fail to predict the apparent dead-time of countercurrent heat exchangers and must be rejected from dynamic considerations. Moreover, steady-state arguments may also be used to reject the *pure lumped* one-cell model. Consider a heat exchanger with equal heat capacity flowrates ($R = 1$) where it is assumed that the inlet temperatures are manipulated inputs (for example by manipulating bypasses of upstream heat exchangers on each stream) and the outlet temperatures are the controlled outputs. Even with infinite heat transfer coefficients, the thermal effectiveness P cannot exceed 0.5, and the transfer matrix (Eq. 2.2) has full rank for all parameter combinations. For two or more cells the effectiveness may become 0.5 which makes the system singular. This occurs when the outlet temperatures become equal (Reimann, 1986; Mathisen *et al.*, 1994*).

From Eq. 2.9 it is clear that the number of cells N must be greater than the number of transfer units N_{TU} (U will approach infinity as N approaches $N_{TU,lm}$). This simple steady-state consideration seems to be overlooked by previous authors (e.g., Papastratos *et al.*, 1992). However, to be able to predict the apparent dead-time with good accuracy even more cells are usually necessary, see Fig. 2.9a. For shell and tube heat exchangers, the number of cells is usually recommended to be one above the

*corresponds to chapter 3 of this thesis

	Min. from steady-state	Min. from dynamics	Max.
Pure lumped model	N_{TU}	2	$(N_B+1)N_P$
Hybrid model	1	3	$(N_B+1)N_P$

Table 2.2: Recommended number of cells. The maximum is computed from the number of baffles N_B and number of tube passes N_P, and assumes a shell-and-tube heat exchanger.

number of baffles (N_B), which seems intuitively attractive. It may be argued that the number of mixing tanks should be larger on the tube side than the shell side due to less back-mixing, but for typical number of cells ($N > 6$) this was found to have small effect on the apparent dead-time. A comparison for a typical countercurrent heat exchanger ($R = 1$ and $N_{TU} = 1$) is shown in Fig. 2.4b. Furthermore, at the early design stage, discrimination between the tube and the shell side is usually not made.

2.3.3 Temperature driving force (hybrid or pure lumped model)

Using the logarithmic mean temperature difference (LMTD) as the temperature driving force of the cells has been suggested by several researchers (e.g., Reimann, 1986; Rinard and Nieto, 1992; Papastratos *et al.*, 1992). Such a hybrid model is attractive because heat transfer coefficients are often based on a distributed model and only one cell is necessary to match any steady-state thermal effectiveness. For the hybrid model, the driving force of a mixing tank is computed from inlet temperatures as well as the tank or outlet temperatures. The response of hybrid models will therefore be faster than for pure lumped models with the same number of cells, and from dynamic considerations it may be recommended to use more cells with the hybrid model than with the pure lumped model. Our recommendations are summarized in Table 2.2. A comparison of the overall phase shift of the lumped and the hybrid model is given in Table 2.1. Note that the phase shift from cold inlet temperature and flowrate is reduced when using a hybrid model, which is a disadvantage and favors the pure lumped model compared to the hybrid model.

Because LMTD is undefined when the temperature differences on each side are equal, it may be advantageous to use the Paterson-approximation of LMTD (Paterson, 1984):

$$\Delta T_{lm} \approx \frac{1}{3}(\Delta T_1 + \Delta T_2)/2 + \frac{2}{3}\sqrt{\Delta T_1 \Delta T_2} \tag{2.10}$$

where ΔT_1 and ΔT_2 are the temperature differences on each end of the exchanger. Chen (1987) suggested another approximation of the logarithmic mean:

$$\Delta T_{lm} \approx ((\Delta T_1 + \Delta T_2)/2)(\Delta T_1 \Delta T_2))^{1/3} \tag{2.11}$$

These very simple functions of the arithmetic and geometric means are good approximations of the logarithmic mean over a wide range of the two temperature differences, and they are well defined at steady-state. During a transient, however, the temperature differences may have opposite signs, and this will discontinue an on-going simulation. So for dynamic simulations, the Paterson and Chen approximations are not as helpful as for steady-state simulation or optimization. The possibility of temperature cross-over during dynamic transients also favors the pure lumped model compared to the hybrid model.

2.3.4 Wall capacitance

The time constants for energy holdup of the fluids and the wall are related:

$$\tau_k^w = \frac{(V^w \rho^w c_p^w)}{(h^h + h^c)A} = C \frac{V^w \rho^w c_p^w}{V^c \rho^c c_p^c} \frac{\tau_F^c}{N_{TU}} \qquad C \varepsilon \, [0.25, 1] \tag{2.12}$$

Appreciable effect of neglecting the wall capacitance is expected for large wall capacitance ratios $((V^w \rho^w c_p^w)/(V^c \rho^c c_p^c) > 1$. For liquid heat exchangers the wall capacitance ratio may well be larger than one, and this indicates that the commonly used assumption that wall capacitance may be neglected for liquid exchangers is not valid. For gas exchangers the ratio is at least an order of magnitude larger due to the lower fluid density. Thus, wall capacitance dominate the dynamics of gas exchangers, and this is in accordance with previous results. Note that the time constants for energy holdup are related through the number of transfer units N_{TU}, revealing a close connection between steady-state and dynamic behaviour of heat exchangers.

A numerical comparison with and without wall capacitance is shown in Fig. 2.9b. The time simulation confirms that there is a considerable delay in the response when the wall capacitance is included. Additional time simulations with larger number of transfer units N_{TU} show smaller differences in the response. This is also as expected from Eq. 2.4 as convection becomes more important to the overall dynamics.

2.3.5 Flow configuration

Countercurrent 1-1 heat exchangers with one tube and one shell pass are almost invariably assumed during conceptual design. Due to mechanical, maintenance or pressure drop considerations heat exchangers with two tube passes per shell pass (1-2 exchangers) are more common in practice. Thus, we will compare dynamic behaviour of 1-1 and 1-2 exchangers, and have selected heat transfer coefficients and the number of transfer units to give the same steady-state thermal effectiveness. Interestingly, the dynamic response from 1-2 exchangers may become different from 1-1 exchangers because of the short-cut via the opposite stream, see Fig. 2.12a, and that this appears as a dip in the phase response (Wolff *et al.*, 1991). A comparison for a typical liquid exchanger is shown in Fig. 2.12b. The temperature response for 1-1 and 1-2 exchangers with the first tube pass countercurrent is similar. The short-cut path including conduction from cold to hot and back is relatively slow, which agrees with our conclusion that wall capacitance is important for the overall dynamics. The importance of the short-cut will increase with increasing heat transfer coefficients, and will be more important for reboilers and condensers and less important for gas exchangers.

For high N_{TU} exchangers, the steady-state temperature profile of 1-2 exchangers may not be monotone due to internal temperature crossover. For illustration, consider our example exchanger with thermal effectiveness 0.6. The response to a 20% step

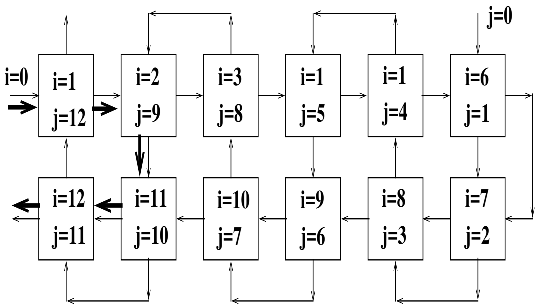

Figure 2.10: a) Cell model of 1-2 heat exchanger showing the possible short-cut from inlet to outlet temperature of tube fluid.

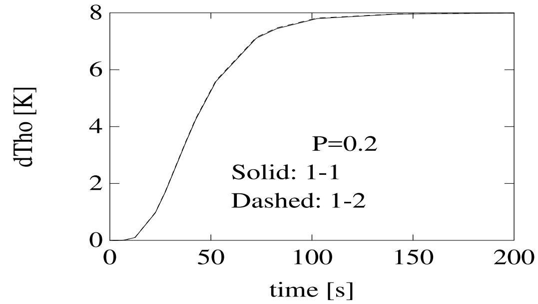

Figure 2.11: b) Step response of 1-1 and 1-2 heat exchangers modelled as 12 lumped cells.

Figure 2.12: Comparison of 1-1 and 1-2 heat exchangers

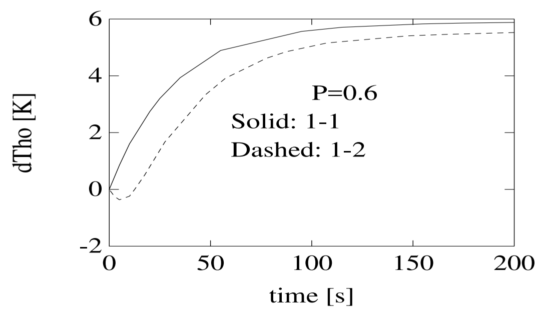

Figure 2.13: a) Hot temperature response to a 20% hot flowrate increase. 1-1 and 1-2 heat exchangers modelled as 12 lumped cells.

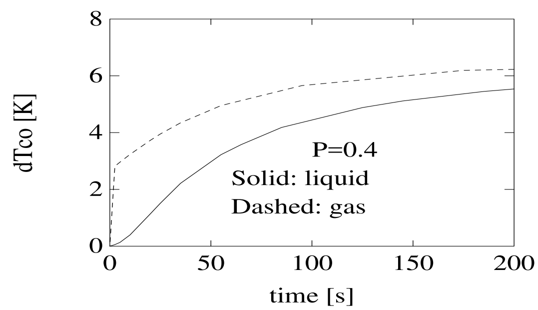

Figure 2.14: b) Cold temperature response to 10 K cold inlet temperature increase. 1-1 liquid and gas exchangers modelled as 3 lumped cells with wall capacitance.

Figure 2.15: dynamic Response of single heat exchangers

increase in hot flowrate is inverse, see Fig. 2.15a. This inverse response will usually not occur in practice because it requires very a high number of transfer units, and it may be concluded that 1-1 and 1-2 exchangers have similar dynamics. Thus, discrimination between single and multiple tube pass exchangers is not critical for controllability assessment as long as steady-state values are accurate.

2.3.6 Fluid compressibility

Dynamics of gas and liquid exchanger have important differences which are mainly due to two factors concerning the density. Firstly, fluid density of gases is much lower than for liquids, which gives higher volumetric flowrates and shorter residence times. Secondly, the heat and mass balances are coupled due to the compressibility. An inlet temperature change of gas exchangers will change the flowrate, and therefore have a much faster effect on the outlet temperature on the same side than liquid exchangers. An inlet pressure change of gas exchangers will not have an immediate effect on the flow throughout the exchanger as for liquid exchangers due to the compressibility. Typical responses from gas and liquid exchangers are shown in Fig. 2.15b. The initial response for gases are much faster than for liquids, and remain faster when heat exchangers with equal residence times are compared. The gas exchanger response is fast because the inlet temperature increase reduces the cold flowrate. The approach to steady-state is rather slow, mainly due to the considerable wall capacitance. One may conclude that distinction between incompressible and compressible fluids is important for controllability assessment.

2.4 Model Features of Heat Exchanger Networks

In its simplest form a dynamic model of a HEN consists only of heat exchanger units and algebraic calculations for the splitters and mixers. We want to investigate the possible effect of pipe-layout, valves and measurements, too.

2.4.1 Heat transfer coefficients

If one fits HENS to steady-state heat transfer data as was proposed for single heat exchanger, one will have to adjust the coefficients for each exchanger separately. During conceptual design this is clearly unacceptable, one should then let the intermediate temperatures vary, and only match the outlet temperatures. Interestingly, only matching the outlet temperatures tends to distribute the driving forces more equally among the exchangers in the network, and thus remove a weakness of conceptual designs based on logarithmic temperature driving forces. Heggs (1985) and Kafarov *et al.* (1988) have pointed out that one of the exchangers in optimized HENs often has a thermal effectiveness ($P \approx 0.9$) that is impossible to achieve using conventional shell-and-tube heat exchangers.

2.4.2 Bypass placement

Bypasses are installed to be able to manipulate heat exchanger duties so that disturbances in terms of inlet temperatures and flowrates may be rejected. Clearly, the selection of the heat exchangers to be bypassed may be important for control. Based on steady-state considerations it does not matter whether the hot or the cold side of a given heat exchanger is bypassed, but these bypass placements are different dynamically. This difference may in some cases become very important for controllability, e.g., the HEN in Fig. 2.20a. The stream properties are equal for all the streams except for the flowrate of stream $H2$ which is 10% higher than for the other streams. The heat exchanger parameters are equal for all the exchangers and pipe residence times are neglected. For this system, the steady-state is positive, but close to zero due to the competing effects. Bypassing the cold side of exchanger 2 gives a negative initial response, whereas bypassing the hot side give a positive initial response, see Fig. 2.20b.

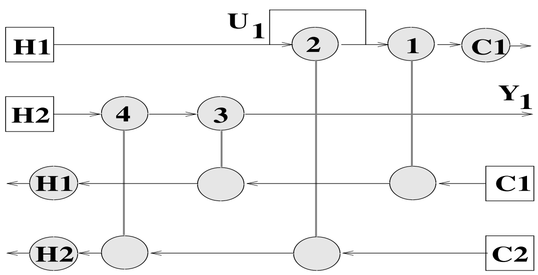

Figure 2.16: a) Heat exchanger network where RHP-zero may exist.

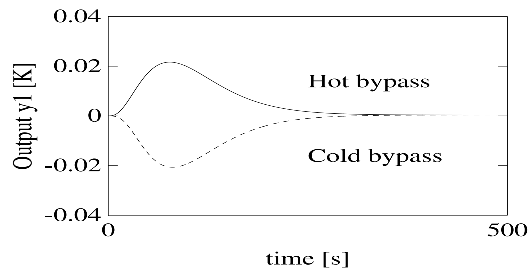

Figure 2.17: b) Response from hot and cold bypass of match 2 in Fig. 10.

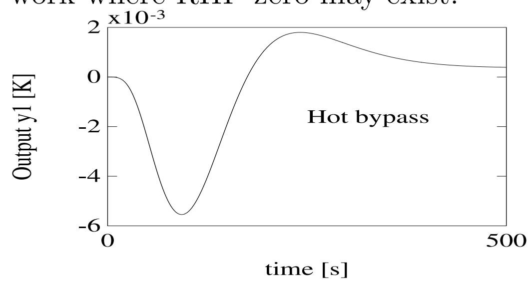

Figure 2.18: c) Inverse response and RHP zero due to pipe residence time with bypass on cold side of exchanger 2.

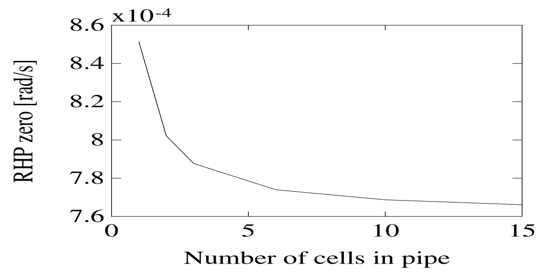

Figure 2.19: d) RHP zero depends on pipe model order.

Figure 2.20: Model features for HENs.

This is a somewhat extreme example, but because wall capacity is important, the bypass side becomes important for less extreme examples, too.

2.4.3 Pipe residence times

Pipe residence times should be included because they are essential for correct prediction of the apparent dead-time, and the dead-time in the pipes may exceed the dead-time in the exchangers. This will typically occur at low plant loads where bypass fractions often are high. Less obvious is the fact that depending on the pipe residence time RHP zeros may or may not occur. Consider again the example in Fig. 2.20a and assume that the residence time in the pipe connecting exchangers 1 and 3 is of same order of magnitude as the residence times in the heat exchanger. This may give an inverse response due to a RHP zero when the hot side of exchanger 2 is bypassed, see Fig. 2.20c.

When bypass fractions are used as manipulated inputs, it is recommended to bypass the final heat exchangers of the controlled streams to get fast responses (Mathisen and Skogestad, 1994*). Such bypass placements make the process between manipulated inputs and controlled outputs independent of pipe residence times for incompressible fluids. However, because controllability assessment also depend on the disturbances entering at the network inlet, pipe residence times should always be included in the dynamic model of HENs.

*corresponds to chapter 4 of this thesis

2.4.4 Pipe model order

Pipe model order and pipe residence time determines the apparent dead-time in the pipes. The model order of the pipes should reflect the degree of back-mixing, which depends on pipe surface roughness, number and type of bends, mixing layout, valves and measurement devices. It is difficult to give general recommendations, but our experience is that three mixing tanks gives a good prediction of the apparent dead-time. Sometimes the difference may be unimportant for controllability assessment. In Fig. 2.20d we have shown how the RHP zero of the example discussed above depends on the number of mixing tanks used to model the connecting pipe. Note that the RHP zero is just below $0.001 rad/s$ in all cases, which is bad.

2.5 Discussion

2.5.1 Model type

Lumped models are preferred to empirical models to get a rational transfer function and to be able to simulate different flow configurations. Furthermore, low-order heat exchanger models may become "ill-consistent" in closed-loop (Jacobsen and Skogestad, 1993). Thus, we used lumped compartment or cell heat exchanger models in this paper. However, these high-order models yield little insight into the physical background for the interesting dynamic characteristics such as the dominant time constant or apparent delay. Then semi- empirical low-order models yield more information. The step response of passive systems is often close to a first or second order lags with delay, and empirical low-order models to existing exchangers may be very accurate. In order to yield insight the model parameters should be a function of stream and exchanger data only. There are two simple semi-empirical heat exchanger models that may be recommended:

1. Thal-Larsen's model (Thal-Larsen, 1960) made known by Buckley (1964).

2. Ma *et al.*'s model (Ma *et al.*, 1992)

Both models include the hot and cold fluid residence times as key parameters.

Thal-Larsen uses the average of the fluid residence times to estimate both the dominating time constant and the apparent time delay from inlet temperature to the outlet temperature on the other side (i.e., the cross-response).

$$\tau_1 = \frac{\tau^h + \tau^c}{2} \tag{2.13}$$

$$\tau_2 = \tau_1/4 \tag{2.14}$$

Deadtime is equal to the smallest time constant (τ_2).

The response through a series of heat exchangers (i.e., self-responses) is approximated with a time delay of the total residence time in the exchangers and the connecting pipes. This approximation is found to be too simple (Brambilla and Nardini, 1972).

The reason is probably that the opposite side fluid affects the dynamic response in addition to the steady-state response. Furthermore, wall capacitance, which is neglected, may also affect the dynamics.

Ma *et al.*, (1992) present one model that neglects wall capacitance and another model that includes it. The model that includes it is (for the cross-response) a second order model with time constants:

$$\tau_1 = \frac{1}{(1/\tau^{wh'} + 1/\tau^{wc'})} \tag{2.15}$$

$$\tau_2 = \frac{\tau_1}{\tau^{wh'}} \frac{K(\tau^h + \tau^c)}{N_{TU}^{c'}(1 \pm \exp(-(N_{TU}^{c'} + N_{TU}^{h'})} \tag{2.16}$$

where K is the steady-state gain; $N_{TU}^{c'} = (h^c A)/w^c)$ and $N_{TU}^{h'} = (h^h A)/w^h)$ are the one-side or "pseudo" number of heat transfer units; $\tau^{wh'} = (\rho^w c_p^w V^w)/(h^h A)$ and $\tau^{wc'} = (\rho^w c_p^w V^w)/(h^c A)$ are the one-side or "pseudo" time constant for energy holdup in the wall (wall capacitance) for the hot and cold sides; and $\tau^h = (\rho^h c_p^h V^h)/w^h$ and $\tau^c = (\rho^h c_p^h V^h)$ are the residence times for the hot and cold fluids.

2.5.2 Pressure drop

The flowrate of gas exchangers are computed from the pressure drop. The friction factors are adjusted to match specified steady-state flowrates (in similar fashion as heat transfer coefficients are adjusted to match specified temperatures). Because pressure drops of HENs are relatively small, neither variation of the total pressure drop or the pressure drop distribution between the exchangers have appreciable effect on the dynamics.

2.5.3 Actuator and sensor dynamics

The dominating time constants for control valves to manipulate bypass flows and thermocouples for temperature measurements are often between 2 and 10 seconds. From comparison with time constants for energy holdup of single heat exchangers given in the introduction, it is clear that actuator and sensor dynamics may be important. The dynamic HEN model should therefore include actuator and sensor dynamics to correctly assess controllability.

2.5.4 Experiences with SIMULINK

The model was implemented in SIMULINK, a program for simulating dynamic systems with a graphic interphase (see SIMULINK User's Guide for details). A SIMULINK representation of a network with 4 streams and 4 exchangers is shown in Fig. 2.21. Our main reason for choosing SIMULINK was its close integration with MATLAB, which we already used for control design and analysis. Both MATLAB data and MATLAB programs may be used in the simulation, and output-data are available in the MATLAB workspace for further analysis after the simulation. SIMULINK provides a

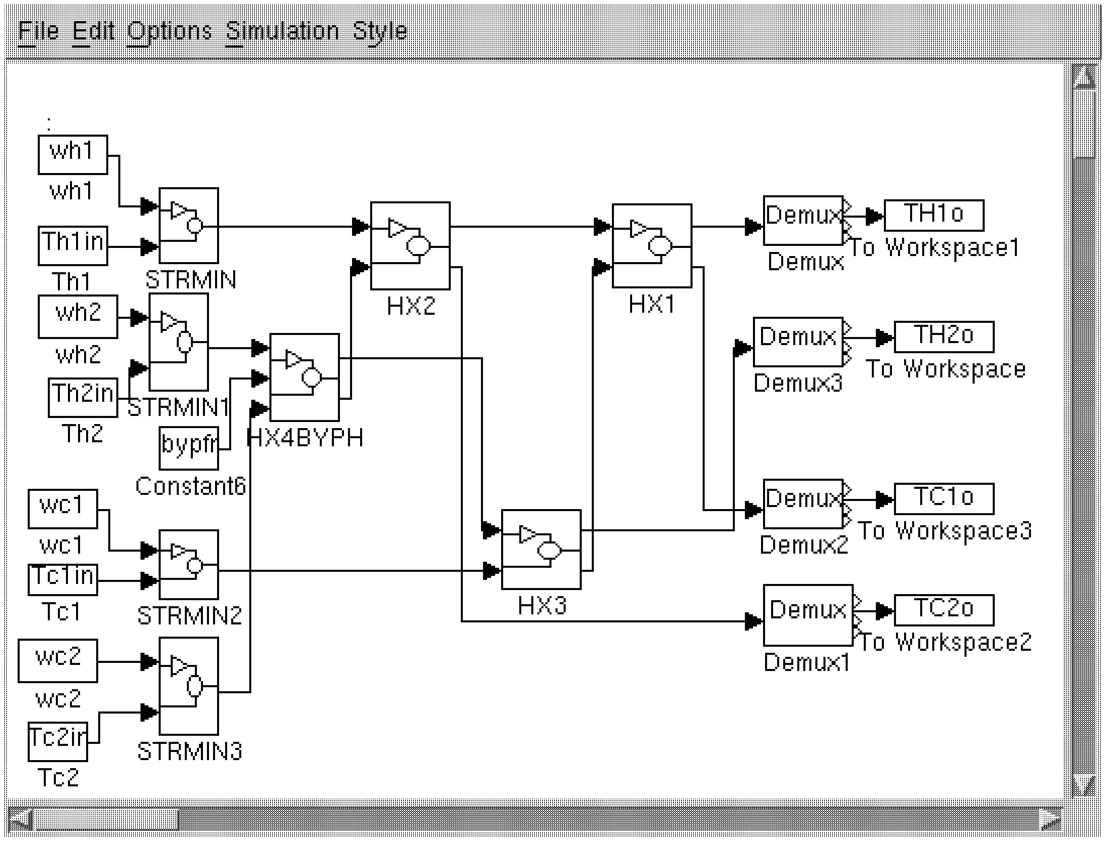

Figure 2.21: SIMULINK representation of heat exchanger network.

graphical interphase which enables the user to quickly and correctly set up the process flowsheet from a library of standard and user-developed moduls or *blocks*. This facility was especially helpful for heat exchanger network applications where there are only four different process units (splitter, mixer, pipe and heat exchanger). During simulation, the integration can be followed graphically as it proceeds through scope blocks, and parameters may be corrected without having to restart. We found these facilities helpful, and in general we have good experiences with SIMULINK. However, dynamic simulation with SIMULINK involves some disadvantages and problems, too. Some of the problems are common to sequential modular dynamic simulators; algebraic loops cannot be handled, and different boundary conditions requires different dynamic modules. With compressible fluids, the integration routines had problems with slow convergence and even numerical instability. These problems are however mainly due to the stiffness of the problem. The flow (mass) dynamics are typically several orders of magnitude faster than the temperature (energy) dynamics. A few other problems are probably due to the fact that SIMULINK was not developed with typical process engineering applications in mind. The difficulties with reusing old steady-state data for similar problems and the graphical restrictions making it impossible to draw typical countercurrent process-units were irritating and not user-friendly.

Nomenclature

A - Heat exchanger area, $[m^2]$

a - Parameter in Eq. 2.31, $[-]$
b - Parameter in Eq. 2.31, $[-]$
C - Parameter in Eq. 2.12, $[-]$
c - Spec. heat capacity, $[J/kgK]$
D - Diameter, $[m]$
F - Molar flowrate, $[kmole/s]$
f - friction factor, $[-]$
h - Heat transfer coefficient, $[W/m^2K]$
i - Index (of tube side), $[-]$
j - Index (of shell/wall side), $[-]$
K - Steady-state gain in Eq. 2.16
k - Conductance, $[W/mK]$
L - Flow trajectory array, $[-]$
M - Molar (weight), $[kg/kmole]$
N - Number (of cells), $[-]$
n - moles, $[kmole]$
P - Thermal effectiveness, $[-]$
P - Pressure, $[N/m^2]$
p - Prandl number exponent, $[-]$
Q - Conducted heat, $[J/s]$
q - Volumetric flowrate, $[m^3/s]$
R - Gas constant, $8314.3[J/kmoleK]$
R - Heat capacity rate ratio, $[-]$
r - Reynold number exponent, $[-]$
T - Temperature, $[K]$
t - time, $[s]$
U - Overall heat transfer coefficient, $[W/m^2K]$
U - Internal energy, $[J]$
V - Volume, $[m^3]$
v - linear velocity, $[m/s]$
w - Heat capacity flowrate, $[kW/K]$

greek
β - Volume fraction, $[-]$
ΔT - Temperature difference, $[K]$
ϵ - Roughness (of tube), $[m]$
μ - viscosity, $[kgm/s]$
ρ - density, $[kg/m^3]$
τ - time constant, $[s]$

superscripts
c - cold side/fluid
g - gas
h - hot side/fluid

l - liquid
s - shell side/fluid
t - tube side/fluid
w - wall between fluids
0 - nominal (reference)

subscripts
B - baffles
F - convection
f - flow
hx - for total exchanger
i - inlet
k - heat transfer
lm - logarithmic mean (hybrid)
o - outlet
m - main body (of exchanger)
Nu - Nusselt
P - tube passes
Pr - Prandl
p - at constant pressure
Re - Reynold
TU - transfer units
v - at constant volume
w - (molar) weight

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Appendix 1: Derivation of Steady-State Gains

The steady-state energy balance for heat exchangers may be expressed by three equations:

$$Q = w^h(T_i^h - T_o^h) = w^c(T_o^c - T_i^c) = UA\Delta T_{hx} \tag{2.17}$$

where superscript h means hot side, c cold; subscript o outlet, i inlet, Q heat duty and $w = FM_w c_p$ heat capacity flowrate and $U = h^h * h^c/(h^h + h^c)$ is the overall heat transfer coefficient. The overall temperature driving force of the heat exchanger, ΔT_{hx}, depends on the flow configuration. Assuming countercurrent flow yields (Kern, 1950):

$$\Delta T_{hx} = \Delta T_{lm} = \frac{(T_i^h - T_o^c) - (T_o^h - T_i^c)}{\ln \frac{T_i^h - T_o^c}{T_o^h - T_i^c}} \tag{2.18}$$

Temperature disturbances

We now define the heat capacity (flow) ratio:

$$R^c = \frac{w^c}{w^h} \qquad R^h = \frac{w^h}{w^c} = \frac{1}{R^c} \tag{2.19}$$

and solve Eq. 2.17 for T_o^h:

$$T_o^h = T_i^h - R^c(T_o^c - T_i^c) \tag{2.20}$$

By substituting Eq. 2.18 and 2.20 into the energy balance, one may express the cold outlet temperature in terms of the two inlet temperatures. The steps are:

$$w^c(T_o^c - T_i^c) = UA\frac{(T_i^h - T_o^c) - ((T_i^h - R^c(T_o^c - T_i^c)) - T_i^c)}{\ln \frac{T_i^h - T_o^c}{T_i^h - R^c(T_o^c - T_i^c) - T_i^c}} \tag{2.21}$$

$$(T_o^c - T_i^c)\ln\frac{T_i^h - T_o^c}{T_i^h - R^c(T_o^c - T_i^c) - T_i^c} = -\frac{UA}{w^c}(1 - R^c)(T_o^c - T_i^c) \tag{2.22}$$

We now eliminate $(T_o^c - T_i^c)$ from boths sides and get rid of the logarithm:

$$\frac{T_i^h - T_o^c}{T_i^h - R^c(T_o^c - T_i^c) - T_i^c} = \exp(-\frac{UA}{w^c}(1 - R^c)) \tag{2.23}$$

For simplicity we introduce the number of transfer units:

$$N_{TU}^c = (UA)/w^c \tag{2.24}$$

and get

$$T_o^c = \frac{1 - \exp(-N_{TU}^c(1 - R^c))}{1 - R^c \exp(-N_{TU}^c(1 - R^c))} T_i^h + \frac{(1 - R^c) \exp(-N_{TU}^c(1 - R^c))}{1 - R^c \exp(-N_{TU}^c(1 - R^c))} T_i^c \tag{2.25}$$

For further simplification we define $a = \exp(-N_{TU}^c(1 - R^c))$ and express Eq. 2.25 and 2.20 on matrix form:

$$\begin{bmatrix} T_o^h \\ T_o^c \end{bmatrix} = \begin{bmatrix} \frac{1-R^c}{1-R^c a} & \frac{R^c(1-a)}{1-R^c a} \\ \frac{1-a}{1-R^c a} & \frac{a(1-R^c)}{1-R^c a} \end{bmatrix} \begin{bmatrix} T_i^h \\ T_i^c \end{bmatrix} \tag{2.26}$$

The effect of an inlet temperature disturbance on the outlet temperature on the opposite side may be called the thermal effectiveness:

$$P^c = \frac{\Delta T_o^c}{\Delta T_i^h} \tag{2.27}$$

From Eq. 2.26 we see that P^c is a function of the two dimensionless parameters R^c and N_{TU}^c only.

$$P^c = \frac{1 - a}{1 - R^c a} = \frac{1 - \exp(-N_{TU}^c(1 - R^c))}{1 - R^c \exp(-N_{TU}^c(1 - R^c))} \tag{2.28}$$

Substituting P^c into Eq. 2.26 yields:

$$\begin{bmatrix} T_o^h \\ T_o^c \end{bmatrix} = \begin{bmatrix} 1 - R^c P^c & R^c P^c \\ P^c & 1 - P^c \end{bmatrix} \begin{bmatrix} T_i^h \\ T_i^c \end{bmatrix} \tag{2.29}$$

Flowrate disturbances

The steady-state transfer function from flowrates to output temperatures of a single heat exchanger may be expressed as:

$$\begin{bmatrix} T_o^h \\ T_o^c \end{bmatrix} = \begin{bmatrix} \frac{\partial T_o^h}{\partial w^h} & \frac{\partial T_o^h}{\partial w^c} \\ \frac{\partial T_o^c}{\partial w^h} & \frac{\partial T_o^c}{\partial w^c} \end{bmatrix} \begin{bmatrix} w^h \\ w^c \end{bmatrix} = (T_i^h - T_i^c) \begin{bmatrix} -\frac{\partial P^h}{\partial w^h} & -\frac{\partial P^h}{\partial w^c} \\ \frac{\partial R^h P^h}{\partial w^h} & \frac{\partial R^h P^h}{\partial w^c} \end{bmatrix} \begin{bmatrix} w^h \\ w^c \end{bmatrix} \tag{2.30}$$

The derivative of the thermal effectiveness (P^h) with respect to heat capacity flowrate (w^h) is like P^h itself independent of temperature, i.e., it is a function of number of transfer units (N_{TU}^h), heat capacity flowrate ratio (R^h) and flow configuration only. For countercurrent heat exchangers:

$$\frac{\partial P^h}{\partial w^h} = \frac{1}{w^h} \frac{(R^h(a - 1) + N_{TU}^h(b^h(1 - R^h)))a}{(1 - R^h a)^2}; \quad R^h \neq 1 \tag{2.31}$$

where $a = \exp(-N_{TU}^h(1 - R^h))$ and $b^h = R^h \frac{mh^c}{h^h + h^c} - \frac{mh^c}{h^h + h^c} + 1$

$$\frac{\partial P^h}{\partial w^h} = \frac{1}{w^h} \frac{N_{TU}^h(N_{TU}^h + 2(1 - \frac{mh^c}{h^h + h^c}))}{2(N_{TU}^h + 1)^2}; \quad R^h = 1 \tag{2.32}$$

$$\frac{\partial P^h}{\partial w^c} = \frac{1}{w^c}\frac{R^h(a-1) + N_{TU}^c(b^c(1-R^h)))a}{(1-R^h a)^2}; \quad R^h \neq 1 \tag{2.33}$$

where $b^c = R^h \frac{mh^h}{h^h+h^c} - \frac{mh^h}{h^h+h^c} + 1$

$$\frac{\partial P^h}{\partial w^c} = \frac{1}{w^c}\frac{N_{TU}^h(N_{TU}^h + 2(1-\frac{mh^h}{h^h+h^c}))}{2(N_{TU}^h+1)^2}; \quad R^h = 1 \tag{2.34}$$

Bypass manipulations

First we consider a single bypass with direct effect. With the standard simplifying assumptions one may derive the gain from input to output analytically. An energy balance over the hot stream mixer downstream match 1 yields:

$$T_o^h = (1-u^h)\hat{T}_o^h + u^h T_i^h \tag{2.35}$$

To get the analytic gain, we must differentiate this expression with respect to the bypass fraction:

$$\frac{\partial T_o^h}{\partial u^h} = (-1)\hat{T}_o^h + (1-u^h)\frac{\partial \hat{T}_o^h}{\partial u^h} + T_i^h \tag{2.36}$$

where $\hat{T}$ is the temperature of stream from the heat exchanger to the mixer.

$$\hat{T}_o^h = (1-P^h)T_i^h + P_1^h T_i^c \tag{2.37}$$

$$N_{TU}^h = \frac{UA}{w^h} \quad U = \frac{h^h h^c}{h^h + h^c} \quad h^h = h^{h0}(\frac{w^h}{w^{h0}})^m \quad R^h = \frac{w^h}{w^c} \tag{2.38}$$

$$\frac{\partial \hat{T}_o^h}{\partial u^h} = \frac{\partial T_{o,hx}^h}{\partial w^h}\frac{\partial w^h}{\partial u^h} = \frac{\partial \hat{T}_o^h}{\partial w^h}(-w^h) = \frac{\partial \hat{T}_o^h}{\partial P_1^h}\frac{\partial P^h}{\partial w^h}(-w^h) = -(T_i^h - T_i^c)\frac{\partial P^h}{\partial w^h}(-w^h) \tag{2.39}$$

$$\frac{\partial T_o^h}{\partial u^h} = (P^h + (1-u^h)\frac{\partial P^h}{\partial w^h}w^h)(T_i^h - T_i^c) \tag{2.40}$$

Note that the temperature term is the inlet temperature difference.

Appendix 2: Dynamic Model on State-Space Form

Balance equations

Mole balance

$$\dot{n}(i) = F(i-1) - F(i) \tag{2.41}$$

For incompressible fluids:

$$\dot{n}(i) = 0 \tag{2.42}$$

For ideal gases:

$$\dot{n}(i) = \frac{d(P(i)V(i)/RT(i))}{dt} = \frac{V(i)}{R}\frac{d(P(i)/T(i))}{dt} \tag{2.43}$$

$$\frac{V(i)}{R}(\frac{1}{T(i)}\frac{dP(i)}{dt} - \frac{P(i)}{T(i)^2}\frac{dT(i)}{dt}) = F(i-1) - F(i) \tag{2.44}$$

Energy balance

$$\dot{U}(i) = H(i-1) - H_i \pm Q_i \tag{2.45}$$

For constant specific heat capacities and reference temperature $T^0 = 0K$:

$$\dot{U}(i) = c_v V(i) \frac{d(\rho(i)T(i))}{dt} = c_p M_w (F(i-1)T(i-1) - F(i)T(i)) \pm Q(i) \tag{2.46}$$

For incompressible fluids where $c_v \approx c_p$:

$$c_p \rho(i) V(i) \frac{dT(i)}{dt} = c_p M_w F(T(i-1) - T(i)) \pm Q(i) \tag{2.47}$$

For ideal gases where $\rho_= P(i) M_w / RT(i)$ and $c_v = c_p - R/M_w$ the energy balance yields:

$$\frac{(c_p - R/M_w)V(i)}{R} \frac{dP(i)}{dt} = c_p(F(i-1)T(i-1) - F(i)T(i)) \pm Q(i) \tag{2.48}$$

The expression for the time derivative of the pressure ($\dot{P}(i)$) may be substituted into the mole balance:

$$\frac{V(i)}{R}\left(\frac{1}{T(i)} \frac{(c_p(F(i-1)T(i-1) - F(i)T(i)) \pm Q(i))R}{(c_p - R/M_w)V(i)} - \frac{P(i)}{T(i)^2}\frac{dT(i)}{dt}\right) = F(i-1) - F(i) \tag{2.49}$$

Gas model

The resulting equations for the fluid temperatures on state-space form become:

$$\frac{dT(i)}{dt} = ((c_p(F(i-1)T(i-1) - F(i)T(i)) - \frac{h(i) * A(i)}{M_w}\Delta T(i))/(c_v) - (F(i-1) - F(i))T(i))\frac{RT(i)}{P(i)V(i)} \tag{2.50}$$

for the wall temperatures:

$$\frac{dT^w(i)}{dt} = (h^h(i)\Delta T^{wh}(i) - h^c(i)\Delta T^{wc}(i))A(i)/(\rho_w c_p^w V(i)) \tag{2.51}$$

and finally the fluid pressures:

$$\frac{dP(i)}{dt} = (c_p(F(i-1)T(i-1) - F(i)T(i)) - \frac{h(i)A(i)}{M_w}\Delta T(i))\frac{R}{V(i)(c_v)} \tag{2.52}$$

Liquid model

For liquids, we assume that the density is constant, and only temperatures are state variables. The resulting equations for the fluid temperatures on state-space form become:

$$\frac{dT(i)}{dt} = (T(i-1) - T(i) - \frac{hA(i)}{FM_w c_p}\Delta T(i))\frac{FM_w}{\rho V(i)} \tag{2.53}$$

The corresponding equations for the wall tanks are

$$\frac{dT^w(i)}{dt} = (h^h \Delta T^{wh}(i) - h^c \Delta T^{wc}(i))\frac{A(i)}{\rho^w c_p^w V^w(i)} \tag{2.54}$$

When the wall capacitance is neglected, the fluid temperatures may be calculated from:

$$\frac{dT(i)}{dt} = (T(i-1) - T(i) - \frac{UA(i)}{FM_w c_p}\Delta T(i))\frac{FM_w}{\rho V(i)} \tag{2.55}$$

Area and volume

The most important geometrical parameters are the heat transfer area A_{hx} and volume for each fluid V_{hx}^t and V_{hx}^s. For existing units, the geometrical parameters are known, or may be computed from design drawings. During design, the volume may be computed from area and volume fraction. i.e. for the shell side:

$$V_{hx}^s = \frac{A_{hx}\beta^s}{A_{spec}} \tag{2.56}$$

where the specific area per volume A_{spec} and volume fraction β^s are functions of heat exchanger type, tube diameter, tube thickness, pitch type and pitch length. For simple shell and tube heat exchanger types without fins, the specific area and volume fractions are approximately constant, and we will use the values of $A_{spec} = 80, \beta^t = 0.4, \beta^s = 0.4$ and $\beta^w = 0.2$. We assume that the area and volume of all compartments are the same, i.e. for the shell side:

$$A^s(i) = A_{hx}/N^s \tag{2.57}$$

$$V^s(i) = V_{hx}\beta^s/N^s = \frac{A_{hx}}{200N^s} \tag{2.58}$$

We further assume that the film coefficient (h) always referes to the outer heat transfer area in order to make it unnecessary to consider the surface area difference between the inside and the outside of the tubes. Area for the tube and volume for the tube and wall compartments may then be computed analogously:

$$A^t(i) = A_{hx}/N^t \tag{2.59}$$

$$V^t(i) = V_{hx}\beta^t/N^t = \frac{A_{hx}}{200N^t} \tag{2.60}$$

$$V^w(i) = V_{hx}\beta^w/N^s = \frac{A_{hx}}{400N^s} \tag{2.61}$$

Note that the number of wall compartments is equal to the number of shell compartments.

Pressure drop/flowrate

The pressure drop is usually neglected in HEN calculations, and flowrates and inlet temperatures may then be considered as disturbances. However, the reason for the flowrate disturbances are pressure variations, and for compressible fluids flowrate must be computed from the pressure drop. The pressure drop may also be used to estimate film heat transfer coefficients as in detailed heat exchanger design calculations (Jegede, 1990). The pressure drop can be calculated as:

$$\Delta P_{hx} = \Delta P_i + \Delta P_m + \Delta P_o \tag{2.62}$$

i.e., the sum of the pressure drop of the inlet or entrance region, the main body of the heat exchanger and outlet or exit region. ΔP has the form of

$$\Delta P = C\rho\frac{v^2}{2} \tag{2.63}$$

For example, the pressure drop of the main body on the tube side may be computed as

$$\Delta P_m^t = f\frac{L}{D_i^t}\rho\frac{v^2}{2} \quad f = f(N_{Re}, \frac{\epsilon}{D^t}) \tag{2.64}$$

and there are analogous, although a bit more complicated, expressions for the main part of the shell side (i.e., Kern, 1950). The pressure drop for the entrance and exit region may often be considerable compared to the pressure drop of the main body, but is highly dependent on the design. We will invariably fit pressure drop coefficients to steady-state data (assumed, estimated or measured), and assume a typical distribution of the pressure drop between the inlet, main body and outlet of the exchanger.

$$C_i = C_o = C_m/3 \tag{2.65}$$

i.e., that the pressure drop of the inlet and outlet each account for approximately 20% of the total pressure drop. The pressure drop of the main body is distributed between the different compartments to give the following equations:

$$C(0) = C_i + C_m/2N^s = 0.20C + 0.60C/2N^s \tag{2.66}$$

$$C(i) = C_m/N^s = 0.60C/N^s; \quad i = 1, 2, .., N^s - 1 \tag{2.67}$$

$$C(N^s) = C_o + C_m/2N^s = 0.20C + 0.60C/2N^s \tag{2.68}$$

Liquids

Usually, liquids are assumed to be incompressible, so that the flow becomes constant throughout the exchanger. The molar flowrate may be computed from

$$F = \frac{A_f}{\sqrt{C_{hx}/2}M_w}\sqrt{\Delta P_{hx}\rho} \tag{2.69}$$

where A_f is the cross-sectional area and M_w the molecular weight

Gases

For gases, the density is computed from pressure, temperature and physical properties through a state equation. We assume the gas to be ideal and compute the density for compartment i on the tube side as c$\rho = PM_w/RT$. The pressure drop is assumed to be concentrated between the compartments, and we need the corresponding density to enter in the pressure drop equations. Pressure drop in valves is considered to depend on upstream and downstream pressures and upstream temperature. Since the pressure ratio (P(i)/P(i+1)) is normally close to one, we have simply used the aritmetric average of the pressures to compute the density.

$$\rho(i) = \frac{M_w}{R}\frac{P(i) + P(i+1)}{2T(i)} \tag{2.70}$$

Note that $\rho(i)$ by this convension is the density between compartment i and $i+1$. Analogously, $F(i)$ is the molar flow from compartment i and $i+1$, see Fig. 2.1. Substituting this equation in Eq. 2.69 yields for compartment i:

$$F(i) = A_f\sqrt{\frac{1}{RM_wC(i)}}\sqrt{\frac{P(i)^2 - P(i+1)^2}{2T(i)}} \tag{2.71}$$

With constant temperature, this is the well known simplified flowrate equation for gases. Note that the pressure drop across the tank must be positive, also dynamically.

Heat conduction/Film coefficients

We compute the heat conduction to or from a fluid (Q) with a standard expression of the following form:

$$Q = hA\Delta T \tag{2.72}$$

where h is the (total) transfer coefficient, A the heat transfer area and ΔT is the temperature driving force. The heat transfer coefficient is computed from the simple Dittus-Boelter equation:

$$N_{Nu} = CN_{Re}^{r}N_{Pr}^{p} \tag{2.73}$$

where N_{Nu} is Nusselts number hD/k; N_{Re} is Reynolds number $\rho Dv/\mu$ and N_{Pr} is Prandls number $c_p\mu/k$ and C is a constant. The exponential dependencies r and p are between 0 and 1 with typical values $0.55 < r < 0.8$ and $0.3 < p < 0.4$. When the exact exchanger type and configuration is unknown, we will normally use $r = 0.8$ as suggested by Jegede (1990) and $p = 0.33$. This yields the following expression for the film heat transfer coefficient:

$$h = CD^{r-1}\rho^{r}\mu^{p-r}c_p^{p}k^{1-p}v^{r} \tag{2.74}$$

For constant property fluids this may simplified to:

$$h = C'w^{r} \tag{2.75}$$

where w is the heat capacity flowrate. For ideal gases the Reynolds number can be expressed as:

$$N_{Re} = \frac{\rho D q}{\mu A_f} = \frac{\rho D F M_w}{\mu A_f \rho} = \frac{D F M_w}{\mu A_f} \tag{2.76}$$

Since the the Reynolds number is independent of density, Eq. 2.75 can be used for ideal gases as well. For our applications, the nominal film coefficients are specified rather than the constant in the Dittus-Boelter equation. The film coefficient may then be expressed as:

$$h = h^0 (\frac{w}{w^0})^r \tag{2.77}$$

where h^0 is the nominal or reference film coefficient and w^0 the flowrate this coefficient referes to. We assume that wall conductance and possible resistance due to fouling are included in the film heat transfer coefficients for the fluids.

Temperature driving force

We first assume that the wall between the fluids is neglected. The driving force of a lumped parameter heat exchanger model is then simply

$$\Delta T(i) = T^t(i) - T^s(i) \tag{2.78}$$

However, we want to number the compartments on both sides in the direction of flow. Eq. 2.78 is only correct for cocurrent flow. Obviously, Eq. 2.78 may easily be adjusted to fit a countercurrent flow configuration for a fixed number of cells. We want to be able to handle different flow configurations and number of cells, so we have introduced tube and shell side trajectory arrays L^t and L^s. The flow trajectory arrays are defined so that $L^t(i)$ gives the mixing-tank number on the shell side that exchange heat with tube side mixing tank i. $L^s(i)$ for the shell side is defined analogously. This convenient notation is based on an idea by Correa and Marchetti (1987), but modified so that both sides can be treated similarly. The temperature driving force for a lumped model of any heat exchanger type can then be expressed as:

$$\Delta T^t(i) = T^t(i) - T^s(L^t(i)) \qquad \Delta T^s(i) = T^s(i) - T^t(L^s(i)) \tag{2.79}$$

We have derived the flow trajectory arrays as function of number of cells for some usual flow configurations. These are given in appendix 3.

More mixing-tanks on the tube side than the shell side

In practice, the degree of mixing on each side will often be quite different. For shell and tube heat exchangers, the flow on the shell side will be considerably more backmixed than the flow in the smooth tubes. To take this difference into account, we have included the possibility to have two mixing-tanks on the tube side per mixing-tank on the shell side. The resulting flow trajectories are given in the appendix 3. The temperature driving force of the shell side must be computed as the mean of the temperatures of the corresponding mixing-tanks on the tube side. If the shell trajectory array L^s points to the final of the two tanks, the appropriate expression for the driving force is ($j = L^s(i)$):

$$\Delta T^s(i) = T^s(i) - (T^t(j) + T^t(j-1))/2 \tag{2.80}$$

Logarithmic mean as the driving force

Most data for heat exchangers and heat exchanger networks are given assuming a distributed parameter model. For 1-1 countercurrent or parallel heat exchangers, the overall temperature driving force then becomes the logarithmic mean temperature difference (LMTD or ΔT_{lm}):

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\log(\Delta T_1/\Delta T_2)} \tag{2.81}$$

where ΔT_1 and ΔT_2 are the temperature differences on each end of the exchanger. In order to be able to compare with other work, we have included the possibility to use LMTD in our lumped cell model. For countercurrent flow the expression for cell i on the tube side becomes

$$\Delta T_1 = T^t(i-1) - T^s(j) \qquad \Delta T_2 = T^t(i) - T^s(j-1) \tag{2.82}$$

and parallel flow

$$\Delta T_1 = T^t(i-1) - T^s(j-1) \qquad \Delta T_2 = T^t(i) - T^s(j) \tag{2.83}$$

where $j = L^t(i)$ in both cases.

Wall capacitance

Wall capacitance may be included by introducing a third mixing tank for the wall in the cell. We assume that the mixing tanks of the wall is numbered as the mixing tanks of the shell side fluid, and the temperature driving forces for the tube and shell sides may be expressed as (assuming equal number of cells on each side):

$$\Delta T^t(i) = T^t(i) - T^w(j)) \qquad \Delta T^s(i) = T^s(i) - T^w(i) \tag{2.84}$$

whereas the driving force of the wall becomes:

$$\Delta T^{wt}(i) = T^w(i) - T^t(j) \qquad \Delta T^{ws}(i) = T^w(i) - T^s(i) \tag{2.85}$$

where $j = L^t(i)$ in both cases as before. When the number of mixing tanks is not equal, the driving force from the wall to the tube side must be adjusted as explained in appendix 3.

Appendix 3: Flow Trajectory

In this appendix, the flow trajectories of some common flow configurations are given. The trajectories are derived by considering one tube pass at a time. The corresponding trajectories of other configurations may be derived by using this approach, too. Note that the mathematical function $rem(i, j)$ means the remainder of the integer division i/j, e.g., $rem(1, 2) = 0$ means that the integer i is even.

Equal number of mixing tanks on each side

For all these case the number of mixing tanks on the tube side is equal to the number of mixing tanks on the shell side or the wall; $N^s = N^t$. We also have that $L^t(L^s(i)) = i$ so that the shell side trajectory array is immediately known from the tube side trajectory or vice versa.

1-1 exchanger with countercurrent (C) flow

This is the default case.

$$L^t(i) = N^t - i + 1; \quad 1 \leq i \leq N^t \tag{2.86}$$

1-1 exchanger with parallel (P) flow

This case is straight forward:

$$L^t(i) = i; \quad 1 \leq i \leq N^t \tag{2.87}$$

1-2 exchanger with countercurrent-parallel (CP) flow

This case has 2 tube passes and one shell pass, and the first tube pass is countercurrent. The number of compartments must be even.

$$L^t(i) = -2i + N^t + 2; \quad 1 \leq i \leq N^t/2 \wedge rem(i, 2) = 1 \tag{2.88}$$

$$L^t(i) = -2i + N^t + 1; \quad 1 \leq i \leq N^t/2 \wedge rem(i, 2) = 0 \tag{2.89}$$

$$L^t(i) = 2i - N^t; \quad N^t/2 < i \leq N^t \wedge rem(i, 2) = 1 \tag{2.90}$$

$$L^t(i) = 2i - N^t - 1; \quad N^t/2 < i \leq N^t \wedge rem(i, 2) = 0 \tag{2.91}$$

1-2 exchanger with parallel-countercurrent (PC) flow

This case has 2 tube passes and one shell pass, and the first tube pass is parallel or cocurrent. The number of compartments must be even.

$$L^t(i) = 2i; \quad 1 \leq i \leq N^t/2 \wedge rem(i, 2) = 1 \tag{2.92}$$

$$L^t(i) = 2i - 1; \quad 1 \leq i \leq N^t/2 \wedge rem(i, 2) = 0 \tag{2.93}$$

$$L^t(i) = -2i + 2N^t + 2; \quad N^t/2 < i \leq N^t \wedge rem(i, 2) = 1 \tag{2.94}$$

$$L^t(i) = -2i + 2N^t + 1; \quad N^t/2 < i \leq N^t \wedge rem(i, 2) = 0 \tag{2.95}$$

Two tube compartments per shell compartment

One may argue that there will be less back-mixing on the tube side than the shell side. Therefore we include the flow trajectories for some common flow configurations with two mixing tanks on the tube side per mixing tank on the shell side, i.e., $N^t = 2N^s$ for all these cases. There are no restrictions on the number of cells. By definition the shell side trajectory referes to the final compartment on the tube side, so that:

$$L^t(L^s(i)) = i; \quad i = 2, 4, 6, ..N^t \tag{2.96}$$

Thus, the shell side trajectory array is known from the tube side trajectory for these cases, too.

1-1 exchanger with countercurrent (C) flow

The well-known countercurrent heat exchanger has the following trajectory.

$$L^t(i) = N^t/2 + 1 - (i+1)/2; \quad 1 \leq i \leq N^t \wedge rem(i, 2) = 1 \tag{2.97}$$

$$L^t(i) = N^t/2 + 1 - i/2; \quad 1 \leq i \leq N^t \wedge rem(i, 2) = 0 \tag{2.98}$$

1-1 exchanger with parallel (P) flow

This case is straight forward.

$$L^t(i) = (i+1)/2; \quad 1 \leq i \leq N^t \wedge rem(i, 2) = 1 \tag{2.99}$$

$$L^t(i) = i/2; \quad 1 \leq i \leq N^t \wedge rem(i, 2) = 0 \tag{2.100}$$

1-2 exchanger with countercurrent-parallel (CP) flow

This case has 2 tube passes and one shell pass, and the first tube pass is countercurrent. The number of shell side compartments must be even.

$$L^t(i) = -i + N^t/2 + 1; \quad 1 \leq i \leq N^t/2 \wedge rem(i, 4) \leq 1 \tag{2.101}$$

$$L^t(i) = -i + N^t/2 + 2; \quad 1 \leq i \leq N^t/2 \wedge rem(i, 4) = 2 \tag{2.102}$$

$$L^t(i) = -i + N^t/2; \quad 1 \leq i \leq N^t/2 \wedge rem(1, 4) = 3 \tag{2.103}$$

$$L^t(i) = i - N^t/2 - 1; \quad N^t/2 < i \leq N^t \wedge rem(i, 4) = 0 \tag{2.104}$$

$$L^t(i) = i - N^t/2 + 1; \quad N^t/2 < i \leq N^t \wedge rem(i, 4) = 1 \tag{2.105}$$

$$L^t(i) = i - N^t/2; \quad N^t/2 < i \leq N^t \wedge rem(i, 4) \geq 2 \tag{2.106}$$

1-2 exchanger with parallel-countercurrent (PC) flow

This case has 2 tube passes and one shell pass, and the first tube pass is parallel or cocurrent. The number of shell side compartments must be even.

$$L^t(i) = i - 1; \quad 1 \leq i \leq N^t/2 \wedge rem(i, 4) = 0 \tag{2.107}$$

$$L^t(i) = i + 1; \quad 1 \leq i \leq N^t/2 \wedge rem(i, 4) = 1 \tag{2.108}$$

$$L^t(i) = i; \quad 1 \leq i \leq N^t/2 \wedge rem(1, 4) \geq 2 \tag{2.109}$$

$$L^t(i) = -i + N^t + 1; \quad N^t/2 < i \leq N^t \wedge rem(i, 4) \leq 1 \tag{2.110}$$

$$L^t(i) = -i + N^t + 2; \quad N^t/2 < i \leq N^t \wedge rem(i, 4) = 2 \tag{2.111}$$

$$L^t(i) = -i + N^t; \quad N^t/2 < i \leq N^t \wedge rem(i, 4) = 3 \tag{2.112}$$

Chapter 3

Control of Heat Exchanger Networks - I: Dynamic Behaviour and Control Limitations

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Abstract

The dynamic behaviour of heat exchanger networks with emphasis on phenomena that may cause control problems is studied. Specifically, it is explained how structural singularities, right-half-plane zeros, time delays, input constraints and interactions may arise. Structural singularity is due to insufficient number of downstream paths from the inputs to the outputs. Right-half-plane zero in heat exchanger networks are caused by competing effects from two parallel downstream paths from an input to an output, and may occur in networks with split streams or networks where the heat load on inner matches are used as manipulated inputs. Considerable time delays mainly occur for networks where the temperature effect must traverse a heat exchanger from inlet temperature to outlet temperature on the same side. Input constraints and interactions may become important for networks where both outlet temperatures of one heat exchanger are controlled outputs. These insights may be used both during network design and control design to improve the controllability of the resulting system.

3.1 Introduction

The objective of this paper is to identify and explain important dynamic characteristics of HENs, in particular behaviour or phenomena that may limit the controllability of the plant. The term controllability in this context has the meaning *inherent control characteristics* or *achievable control performance* irrespective of the controller. Furthermore, a simple decentralized controller is preferred since it will make the plant easier to operate. Thus, we will also discuss dynamic characteristics that may make it necessary to use an advanced controller.

Propagation of disturbances and input manipulations and the following control limitations will be discussed:

- structural singularities
- right half plane zeros
- time delays
- input constraints
- interactions
- nonlinearities

Inputs, i.e., disturbances and manipulated variables, in HENs only affect the outputs if there is a "downstream path" (Linnhoff and Kotjabasakis, 1986) between the input and the output. Absence of a downstream path yields structural singularity. For disturbances structural singularity is desirable (e.g., Georgiou and Floudas, 1990), whereas for manipulated inputs it must be avoided as the plant becomes uncontrollable. In this paper we consider structural singularities, explain their origins and divide them into categories.

A right half plane (RHP) transmission zero represents a fundamental limitation of the achievable control performance (Rosenbrock, 1970; Morari, 1983). Plants with RHP zeros may have inverse responses and fast and efficient control is impossible. Thus, identification of possible RHP-zeros are very important when comparing alternative designs. The main contribution of this paper is to explain how RHP-zeros and inverse responses may occur in HENs.

Time delays represent another fundamental limitation of the achievable control performance (Ziegler and Nichols, 1943; Rosenbrock, 1970). For HENs the time delay is due to mass and energy holdup in the heat exchangers and mass holdup in the connecting pipes. Holt and Morari (1985) show that controllability of some HEN can be improved by *increasing* the time delay between the exchangers. This may at first seem counterintuitive, but some thought reveals that it is generally an advantage to increase delay on off-diagonal elements in multivariable systems as it reduces the interaction.

Adequate disturbance rejection is important both for flexibility and controllability. Disturbances should have small effects and the manipulated inputs large and fast effects on the outputs, otherwise problems with input constraints will occur. Input constraints are difficult to address with a phenomenon approach because they strongly depend on scaling.

The relative gain array, RGA, (Bristol, 1966, 1978) and some of the related controllability measures (Hovd and Skogestad, 1993) are used for evaluating interactions between control loops. We explain that interactions will always be present in HENs, but that the interactions may be small or only one-way. When both outputs of one heat exchanger are controlled outputs, there will usually be strong interaction between the control loops. However, control problems due to large RGA elements (Skogestad and Morari, 1987) will usually not occur.

Heat exchangers have been considered to be extremely nonlinear (Shinskey, 1979) and control of single heat exchangers are sometimes used in articles on nonlinear control (e.g., Alsop and Edgar, 1989; Khambanonda *et al.*, 1991). Both thermal effectiveness and heat transfer coefficients depend on the flowrate, and this is the main cause for the nonlinearity. However, results from bypass control of single heat exchangers in Mathisen *et al.* (1993)* indicate that the nonlinearity is usually of secondary importance, and we will not consider any further in this paper.

To study the dynamic behaviour of HENs, we use the dynamic multicell model described in Mathisen *et al.* (1993).

3.2 Propagation of disturbances and manipulations

3.2.1 Steady-state behaviour of heat exchangers

The steady-state energy balance for heat exchangers is

$$w^h(T_i^h - T_o^h) = w^c(T_o^c - T_i^c) = UA\Delta T_{hx} \tag{3.1}$$

where T is temperature, w heat capacity flowrate, superscript h means hot side, c cold side; subscript i inlet, o outlet; U overall heat transfer coefficient and A is the exchanger area. The overall temperature driving force of the heat exchanger, ΔT_{hx}, depends on the flow configuration.

We now introduce the thermal effectiveness for the hot and the cold side:

$$P^h = (T_i^h - T_o^h)/(T_i^h - T_i^c) \tag{3.2}$$

$$P^c = (T_o^c - T_i^c)/(T_i^h - T_i^c) \tag{3.3}$$

and it is clear that thermal effectiveness is bounded between zero and unity.

Then the steady-state equation for heat exchangers may be expressed as (Appendix 1 of Ch. 2; Kern, 1950):

$$\begin{bmatrix} T_o^h \\ T_o^c \end{bmatrix} = \begin{bmatrix} 1 - P^h & P^h \\ P^c & 1 - P^c \end{bmatrix} \begin{bmatrix} T_i^h \\ T_i^c \end{bmatrix} \tag{3.4}$$

*corresponds to chapter two of this thesis

Importantly, P^h is a function of heat capacity flowrate ratio $R^h = w^h/w^c$, the number of transfer units $N_{TU}^h = UA/w^h$, and flow configuration only. Moreover, there is a simple relation between the thermal effectiveness of the cold and the hot side: $P^c = R^h P^h$.

Countercurrent flow configuration yields the best thermal effectiveness, and it may be expressed as:

$$P^h = \frac{1 - \exp(-N_{tu}^h(1 - R^h))}{1 - R^h \exp(-N_{tu}^h(1 - R^h))} \tag{3.5}$$

Expressions for other flow configurations are given by Heggs (1985) and Martin (1990).

3.2.2 Transfer functions

For control, relations between inputs and outputs may be conveniently expressed through deviation variables and transfer functions. With a single heat exchanger, the outputs are the outlet temperatures. We denote the transfer function from inlet temperatures G_T, from heat capacity flowrates as G_w, and from manipulated inputs (usually bypass fractions) as G.

3.2.3 Steady-state effects of temperature variations

Since the thermal effectiveness is independent of temperature, Eq. 3.4 shows that temperature variations propagate *linearly* in heat exchangers. Usually inlet temperatures are disturbances rather than manipulators, but temperatures in HENs are often manipulated indirectly by changing the heat load on an upstream exchanger.

Single heat exchangers

The steady-state transfer function from inlet to outlet temperatures of a single heat exchangers (G_T at zero frequency) is from Eq. 3.4 simply:

$$G_T(0) = \begin{bmatrix} 1 - P^h & P^h \\ P^c & 1 - P^c \end{bmatrix} \tag{3.6}$$

In Fig. 3.5 it is illustrated how the four extreme combinations of thermal effectiveness affect propagation of temperature variations. Note that an inlet temperature variation may have a large effect on both, one or none of the outlet temperatures depending on heat exchanger parameters.

Multivariable singularity

The transfer matrix G_T looses rank if:

$$det(G_T) = 0 \tag{3.7}$$

At steady state this is fulfilled if and only if

$$P^h = 1 - P^c \Leftrightarrow P^h = \frac{1}{1 + R^h} \tag{3.8}$$

In section 3.4 it is shown that manipulating an inner match in HENs with match heat load loops may change both the hot and the cold inlet temperature of another match. The effects will have opposite signs so that the conditions for singularity in Eq. 3.9 or 3.10 may be fulfilled.

Propagation of temperature variations in HENs

Fact 1: A positive (negative) temperature change in a HEN has a positive or zero (negative or zero) effect on all other temperatures.

Proof: The hot and cold thermal effectiveness in the steady-state equation for single heat exchangers are physically bounded by zero and unity. The boundaries are independent of heat exchanger type and fluid properties, and from Eq. 3.4 it is then clear that a temperature disturbance cannot change sign (or even increase in size) when traversing a heat exchanger. In HENs there may be feedback loops, but this feedback will always be positive from the same reasons (see Fact 2).

Fact 2: Temperature disturbances are naturally dampened in HENs.

Proof: Let g_T^k denote the transfer function between an inlet temperature and an outlet temperature of exchanger k. From Eq, 3.6 we see that g_T^k will be given by the thermal effectiveness and be bounded between zero and unity.

Case I: The downstream path is not part of a loop. A loop exists if there is a natural feedback in the HEN, i.e., a stream temperature variation affect itself (a network with a loop is shown in Fig. 1.4). With no loops we have a simple series interconnection and the overall transfer function becomes:

$$g_T^{tot} = \prod_{k=1}^{N_{hx}} g_T^k \tag{3.11}$$

It then follows from $0 < g_T^k < 1$ that

$$0 < g_T^{tot} < 1 \tag{3.12}$$

Case II: The downstream path is part of a loop. Let g_{direct} denote the direct transfer function and g_{loop} the loop transfer function. Since the loop gain is positive, the "closed-loop" transfer function is given by:

$$g_T^{tot} = \frac{g_{direct}}{1 - g_{loop}} \tag{3.13}$$

where g_{direct} and g_{loop} are given by expressions similar to Eq. 3.11 and thus are bounded between zero and unity. Note that loops in HEN increase the gain since $1/(1-g_{loop}) > 1$, but still $g_T^{tot} < 1$ as shown next. Let k denote the match where we "enter" the loop, see Fig. 3.6. Then g_{direct} contains the term g_T^k and g_{loop} the term $1 - g_T^k$ (only true at steady-state), that is we may write:

$$g_T^{tot} = \frac{\hat{g}_{direct} g_T^k}{1 - \hat{g}_{loop}(1 - g_T^k)} \tag{3.14}$$

Figure 3.6: Illustrati

Substituting $h = 1 - g_T^k$ yie

Figure 3.7: Flowrat

$$\frac{\partial P^h}{\partial w^h} = $$

where $\alpha = \exp(-N^h_{TU}(1 - F$

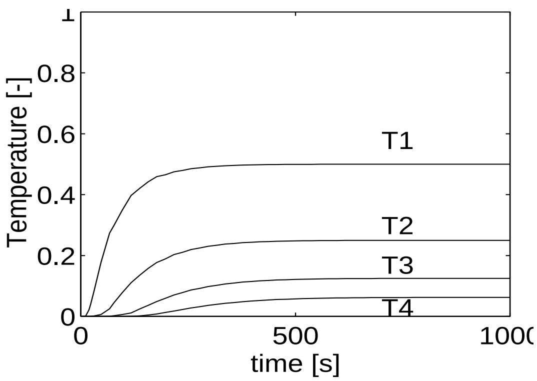


Figure 3.8: a) Temperature disturbance

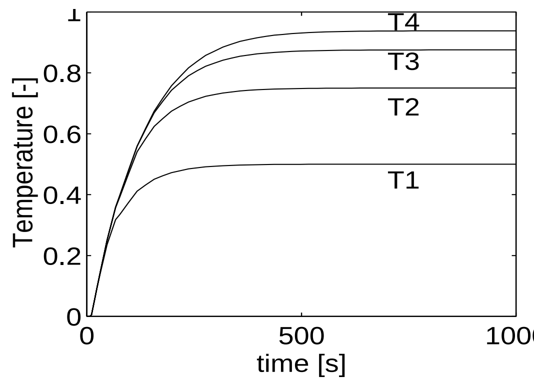


Figure 3.9: b) Flowrate disturbance

Figure 3.10: Step responses for 4 identical heat exchangers in series

In cases where all single heat exchanger transfer functions are identical Eq. 3.23 may be simplified using the result for geometric series:

$$g_w^{tot} = g_w(\,1 + g_T + (g_T)^2 + ... + (g_T)^N\,) = g_w \frac{1-(g_T)^{N+1}}{1-g_T} \approx \frac{g_w}{1-g_T} \quad N \geq 2 \quad (3.25)$$

since $g_T < 1$. Note that the approximation is not limited to steady-state.

This shows that flowrate disturbances are fundamentally different from temperature disturbances and that they may be difficult to reject. This is illustrated in Fig. 3.10 where the temperature and flowrate disturbance propagation over identical heat exchangers in series are compared. The disturbances have been scaled so that the steady-state effect over the first exchanger is 0.5 for both disturbances. The effect of the temperature disturbance decreases towards zero whereas the effect of the flowrate disturbance increases towards unity with increasing number of heat exchangers in series.

Fact 3: A hot (cold) stream flowrate increase has a positive or zero (negative or zero) effect on all temperatures.

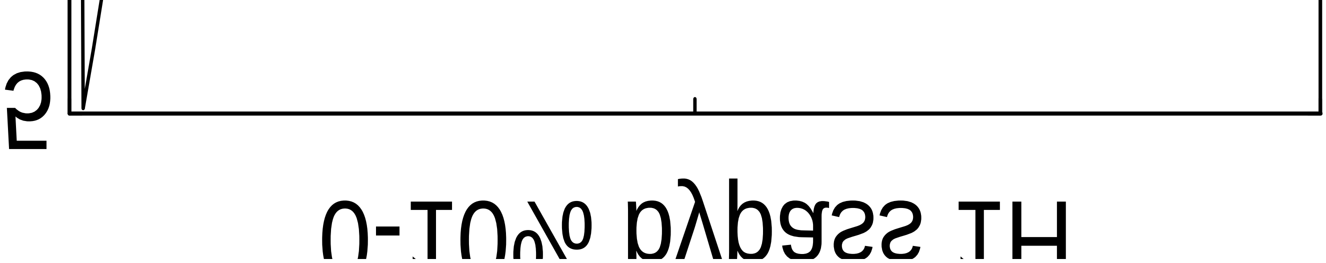
Proof: Case 0: A single heat exchanger. In this case Fact 3 holds since $(\partial P^h/\partial w^h)$ and $(\partial P^c/\partial w^c)$ are negative and $(\partial P^h/\partial w^c)$ and $(\partial P^c/\partial w^h)$ are positive, respectively. Note that the heat load on a heat exchanger increases with both hot and cold flowrate.
Case I: A single match on the disturbed stream. In this case the disturbance will propagate as a pure temperature disturbance from the cold side throughout the HEN.
Case II: Several matches on the disturbed stream. All outlet temperatures of the matches on the disturbed stream will increase from the primary flowrate effect. The downstream matches on the disturbed stream will get a secondary temperature effect, but the secondary effect will strengthen the primary effect. Further downstream the disturbance propagates as a temperature disturbance.

Figure 3.11: a) N

Figure 3.14:

effect on the final temperat

state of may be slow becaus


5
0-10% bypass TH

Fact 6: Bypass manipulations of matches may depending on problem parameters either increase or decrease a temperature (output) if and only if there are parallel (independent) downstream paths from the two sides of the manipulated match to the temperature.

Proof: Case I: One downstream path from the bypassed match to output. The effect of the bypass manipulation will then propagate as one temperature disturbance, and the direction of the effect is independent of problem parameters from Fact 1.
Case II: Dependent downstream paths. Two downstream paths exist there is both a primary effect from the hot (cold) side and a secondary effect from the cold (hot) side of the manipulated match to the output. The downstream paths are dependent if the secondary effect from the cold (hot) side affect the output through the hot (cold) side of the manipulated match. The two effects are opposing from Fact 5, but analogously to the proof of Fact 2 it may be shown that the primary effect always dominates.
Case III: Parallel (independent) downstream paths. Parallel and independent downstream paths exist if neither of the downstream paths traverses the manipulated match, and may only occur if an inner match is manipulated. From Fact 5 the effects are opposing, and both effects may be expressed through individual heat exchanger effects, i.e., by Eq. 3.11. With downstream paths to the same temperature either one may dominate depending on problem parameters.

3.3 Structural singularities

Singularities arise when an input or a combination of inputs have no effect on the output(s), and identification of possible singularities is very important for controllability assessment. Singularites that depend on plant structure only are structural, whereas parameter singularities occur if the plant is singular for some specific parameter combinations only. Structural singularities for HENs are discussed by Georgiou and Floudas (1990), who suggested to design HENs that are structurally singular from the disturbances. This may be good idea for plants with one or a few dominating disturbances, but is impossible in the general case where all inlet temperatures and flowrates should be regarded as disturbances. Work on parametric singularities in HENs, and their impact on control, seems to be missing in the literature.

Mathematically, a plant $G(s)$ is functional controllable if the rank of $G(s)$ is equal to the number of outputs, and is not if $G(s)$ somehow is singular (Rosenbrock, 1970). Since the requirement is fundamental and often easy to check, it can be recommended as the first test of a proposed control structure. In the following we describe four cases of structural singularities in HENs.

3.3.1 No downstream paths from inputs

An obvious way to get a structural singular system is when none of the inputs affect one of the outputs, i.e., a row in the plant transfer function only has zero elements. For HENs this occurs if there is no downstream path from any of the inputs (i.e. matches if bypasses are used) to one of the outputs. In Fig. 3.22a none of the inputs affect output y_1, which yields a structural singular system. The plant transfer function for

This means that there is an important connection between steady-state and dynamics of HENs, and that network and control structures that may give RHP zeros are easy to identify. For HENs we divide the cases of parametric singularity into five categories; 1) Downstream mixers (i.e., downstream a heat exchanger), 2) Upstream mixers, 3) Splits, 4) Inner matches and 5) Combining matches. The first four categories are monovariable zeros, whereas the final category is a multivariable zero. In the following we identify and describe the competing effects and show how the response may or may not be inverse depending on problem parameters. Fig. 3.34 summarizes the main results, but we will present numerical examples and show how they may occur in HENs in the following sections. Note that these control problems may occur in designs derived with modern synthesis techniques.

3.4.1 Downstream mixers

In section 3.2.6 we explained that remixing a bypass flow with the main stream through the exchanger (Fig. 3.34a) give competing effects, but that the faster flow effect always dominate yielding overshoot-type responses (Fig. 3.34b). This means that only left-half-plane (LHP) zeros may occur.

In other cases mixing may yield a right-half-plane (RHP) zero or parametric singularity. Mixing a stream downstream a heat exchanger with an independent stream as in Fig. 3.34c may yield an inverse response. When mixing two hot (cold) streams, such mixers may give an inverse response if the manipulated stream through the exchanger is the colder (hotter) stream. A numerical example is shown in Fig. 3.34d. The negative effect of increasing the flowrate of the colder of the two streams to the mixer may be counteracted by the slower, positive effect of increasing the temperature of this stream. The two effects are the same as when remixing a bypass flow, but in this case the slower effect may dominate because they are independent.

A stream split with downstream mixing may yield two RHP-zeros. This is discussed in section 3.4.3.

3.4.2 Upstream mixers

Mixers upstream heat exchangers may also yield an inverse response, see Fig. 3.34e. When mixing two hot (cold) streams, such upstream mixers may give inverse response if the manipulated stream is the colder (hotter) stream. Increasing the manipulated stream increases the flowrate to the heat exchanger, a positive effect, but decreases the inlet temperature to the heat exchanger, a negative effect. Either one may dominate depending on problem parameters. Note that there are opposing effects to both outlet temperatures, but that the problem parameters that give parametric singularity are different. Furthermore, two RHP-zeros are possible to the opposite side, see the numerical example in Fig. 3.34f. At high frequency the flowrate effect dominates, but the temperature effect may dominate at intermediate frequency due to the countercurrent flow in the heat exchanger even when the flowrate effect dominates at steady-state.

3.4.3 Splits (parallel mass flows)

Split fractions may be used as manipulated inputs instead of bypass fractions. For example, in Fig. 3.34g a hot stream is split.

Manipulating split fractions gives competing effects; the heat loads on the exchangers on one of the branches increase whereas the heat loads on the exchangers on the other branch decrease. This may give an inverse response, see Fig. 3.34h. The sum of the heat load changes may be positive or negative depending on operating point, and at some intermediate split fraction the steady-state gain will be zero. Such parallel heat exchange may be considered as a special case of downstream mixing, but with a simultaneous change in both the mixed flows. With different residence times, the sign of the gain may change twice. Initially, the temperature of the remixed stream may increase or decrease depending on the operating point. At some intermediate time, the faster of the two branches will dominate, and the sign may have changed. Finally, the slower of the two branches may have reversed the sign again at steady-state. The presence of the inverse response requires very long time constants in the corresponding control loops, and such split fractions should only be used for supervisory control (energy optimization), not for regulatory control (rejection of dynamic disturbances). Note that the split fraction may safely be used to control one of the streams that are not split (in this case one of the cold streams) as this gives no competing effects.

3.4.4 Combination of series and parallel heat exchange

Designs with a combination of series and parallel heat exchange have been suggested both to conventional HEN problems and flexibility problems. An example is shown in Fig. 3.38a. The main motivation for such designs is to reduce the number of units. However, the possibility for RHP-zeros make these designs less desirable. Manipulating the flow that combines the two branches (u_1 in Fig. 3.38a) gives competing effects due to an upstream mixer. Increasing this flow, increases the flow through match 2 which increases the heat load. However, increasing u_1 will also decrease the inlet temperature to match 2, which decreases the heat load. Either the positive flowrate effect or the negative temperature effect may dominate, and an inverse response is possible, see Fig. 3.38b.

3.4.5 Inner match (parallel energy flows)

Inner matches are matches with downstream matches on both sides. If an inner match is bypassed, and the inner match is part of a loop, there may be two parallel downstream paths to a stream temperature. As explained in section 3.2.6 the effects will be opposing, and parametric singularity and a RHP-zero may occur. In Fig. 3.41a, input u_1 (hot bypass on match 4) affects output y_1 (outlet temperature of hot stream 1) through matches 3 and 1 (positive gain) and through matches 2 and 1 (negative gain). These downstream paths are independent as neither traverse the manipulated match (match 4). A typical step response is shown in Fig. 3.41b. For this example all heat capacity flowrates are equal.

3.4.6 Multivariable RHP-zero: Combining match

In section 3.2.3 we explained that using the inlet temperatures to control the outlet temperatures of a heat exchanger may yield a multivariable singularity and RHP-zero. Here we would like to point out that this may occur even for simple HENs where bypasses are used as manipulated inputs. If two upstream matches are used to control the two outputs on a double output match as in Fig. 3.42 this may occur. Since all four transfer functions in a 2×2 system or subsystem traverses match 1 from an inlet to an outlet temperature and combines the input/output transfer functions, we denote it a *combining match*. All input combinations that gives a combining match may have a multivariable singularity, the criterion for singularity also holds true if there are additional matches between the combining match and one of the outputs or between one of the inputs and the combining match. An important rule for bypass placement may be derived: Never use a pair of bypasses that affect the outputs through a combining match.

3.5 Time delays

Time delays in control loops of HENs are due to

1. Mass holdup (fluid capacitance) and energy holdup (wall capacitance) in heat exchangers

2. Mass holdup (fluid capacitance) in connecting pipes and bypasses

3. Actuator and measurement dynamics

We will explain how the time delay vary with plant structure through the example in Fig. 3.19a. For all the examples we approximate actuator and measurement dynamics with a delay of 10 seconds.

3.5.1 Direct effect bypasses

For incompressible fluids, the initial effect from increasing the bypass fraction is immediate except for possible response delay from actuator and measurement. A typical response is shown in Fig. 3.19b.

3.5.2 Bypass on upstream match on the controlled stream

When heat exchanger dynamics are approximated with a lumped model, a high model order is recommended (e.g., Mathisen *et al.*, 1993)* . In order to get the effect of an apparent time delay for temperature changes. A typical example would be to bypass match 2 to control output y_1 in Fig. 3.19a. The step response is shown in Fig. 3.19c.

When upstream matches are bypassed and used as manipulators, the pipe holdup between the matches affect the delay in the response. For comparison, a time response

*corresponds to chapter two of this thesis

with a pipe holdup between matches 2 and 1 equal to the match holdup is included in Fig. 3.19c (dashed line).

3.5.3 Bypass on upstream match on opposite side

Most heat exchangers are countercurrent, and this may make it desirable to use upstream matches on the opposite side of the controlled stream, e.g., match 3 in Fig. 3.19a. The step response from a cold bypass around match 3 is shown in Fig. 3.19d. A comparison between Fig. 3.19c and d indicate that the apparent delay from bypass 3C is smaller than from bypass 2H as expected because the heat exchangers are countercurrent. Note that additional pipe holdup between matches 3 and 1 may alter this preference order.

3.6 Input constraints

Input constraint problems largely depend on problem parameters, e.g., on the size of disturbances (larger inlet temperature and flowrate variations). However, in section 3.4 we discussed cases where mainly the heat exchanger network and control configuration gave input constraint problems due to competing effects. From the discussion in sections 3.2 and 3.5 it is also clear that long downstream paths (several heat exchangers) between manipulated inputs and controlled outputs will give problems with input constraints. Here we will discuss how multi-bypasses may yield input constraint problems for some special multivariable problems.

3.6.1 Operating range with multi-bypass

For some types of HENs it may be desirable to use a multi-bypass. One important example is HENs that include a *double output match*. Double output matches are matches where both outlet temperatures are controlled outputs, and it may be recommended to install a multi-bypass as shown in Fig. 3.45a to get a fast response to both outputs.

The operating range may seem unchanged after installing the multi-bypass since the match duties are the same both with zero and unity bypass fractions. However, this a multivariable problem, and the operating range is in fact much smaller with the bypasses in Fig. 3.45a compared two single bypasses, see Fig. 3.45b. Note the following: 1) The control range with single bypasses are limited by straight lines. Heat exchangers are linear in temperature, and the effect of bypass manipulations propagates linearly downstream the remixer. 2) Closing single bypass 2H has a positive effect on the heat heat load on match 1. The hot outlet temperature of match 1 is the inlet temperature to match 1, and increasing this temperature increases the temperature driving forces of match 1 increases. 3) Closing the multi-bypass decreases both match duties. The effect on match 2 is equal to the single bypass 2H. The effect on match 1 will be the combined effect of a flowrate decrease and a temperature decrease. Since both effects are negative, the overall effect is a large decrease in the heat load on match 1 and this gives a line that curves downwards on the plot.

The control range when using a multi-bypass may be reduced to $20\% - 40\%$ of the control range with two single bypasses, and this may give severe problems with input constraints.

3.7 Interactions

In this section we consider the effect of interactions as given by the elements of the plant transfer function G. A decentralized control system with simple PI or PID controllers is desirable. Interactions between the control loops may seriously affect the control performance, or even make independently tuned control loops unstable. The interactions may be one-way or two-way, and two-way interactions will usually be worst. In the following we will explain the structural differences between networks with one- and two-way interactions, and networks without interactions.

3.7.1 Plants without interaction

HENs without loop interaction are plants where only final utility exchangers are manipulated inputs. Only for such plants, the plant transfer function becomes diagonal: $G = diag(g_{ii})$. Manipulating the heat load on a process exchanger will always affect at least one hot and one cold network outlet temperature, and we assume that all network outlet temperatures are controlled outputs.

3.7.2 Plants with one-way interaction

A HEN include a loop if there is a downstream path from one of matches via at least one other match and back to the match one started from (i.e. natural feedback loops). Matches part of loops yield two-way interactions, so to get plants with only one-way interaction such matches must not be manipulated. Many HENs with minimum number of *process exchangers* yield plants with only one-way interaction. An example is shown in Fig. 3.46. The plant transfer function is in this case tridiagonal:

$$G = \begin{bmatrix} g_{11} & g_{12} \\ 0 & g_{22} \end{bmatrix} \tag{3.31}$$

Pairing of the control loops is of course straightforward as RGA becomes equal to the identity matrix (when numbering the inputs as in the figure).

3.7.3 Plants with two-way interaction

Network with loops

For HENs with heat load loops consisting of only process heat exchangers, there may exist an inner match with two parallel downstream paths to a controlled output, see section 3.4.5. But even without such manipulators, HENs with process heat load loops give systems with interaction. A simple example where the heat load loop consists of two matches between the same two streams is shown in Fig. 3.47.

Double output matches

When both outputs of one match are controlled outputs, using single direct effect bypasses yield a singular system, see section 3.3.4. Using upstream matches on both sides yields a combining match with a possible multivariable RHP zero, see section 3.4.6. Therefore, one needs one bypass around the double output match, and one bypass around an upstream match. The resulting system is a 2×2 control problem with two-way interaction between the control loops. Since the structure of the problem is symmetric assume that there is an upstream match on the hot side only, see Fig. 3.50a. In Fig. 3.50a hot, single bypasses are used. The steady-state gains may be expressed as

$$G(0) = \begin{bmatrix} g_{11}(0) & g_{12}(0) \\ -R_1^h g_{11}(0) & \frac{R_1^h P_1^h}{1-P_1^h} g_{12}(0) \end{bmatrix} \tag{3.32}$$

The ratio $g_{21}(0)/g_{11}(0)$ is equal to $-R_1^h$ because input u_1 manipulates the heat load of the match with the two outputs. The ratio $g_{22}(0)/g_{12}(0)$ is equal to $P^c/(1-P^h)$ from Eq. 3.6 because input u_2 affect both outputs through the hot inlet temperature to the double output match.

With the plant transfer function on this form, it is straightforward to compute the relative gain matrix (RGA):

$$\Lambda(0) = \begin{bmatrix} P_1^h & 1-P_1^h \\ 1-P_1^h & P_1^h \end{bmatrix} \tag{3.33}$$

The fact that $\lambda_{11}(0) = P_1^h$ shows that the preferred pairings are independent of temperatures and thus independent of the upstream match. Because thermal effectiveness P is bounded by $0 < P < 1$, so is $\lambda_{ij}(0)$. The thermal effectiveness often is about 0.5 and varies with operating point, so simple decentralized controllers may behave poorly. As large RGA-elements cannot occur, the plant will not be particularly sensitive to model uncertainty, and decoupling controllers may be applied (Skogestad and Morari, 1987). As an alternative one may use a multi-bypass around matches 2 and 1. This makes it possible to get direct effect bypasses in both control loops which is desirable. However, due to the restrictions on the operating range discussed in section 3.6.1, the multi-bypass should be installed in addition too, not instead of the single bypass on the upstream match. Also note that the use of multi-bypasses so that two inputs manipulate the same heat load yields RGA elements that exceeds unity at steady-state.

Double output match and output on upstream match (3×3)

An interesting special case of HENs with double output matches occurs if an outlet on the upstream match also is a controlled output. An example is given in Fig. 3.50b.

The steady-state gain matrix may be expressed as:

$$G(0) = \begin{bmatrix} g_{11}(0) & g_{12}(0) & g_{13}(0) \\ -R_1^h g_{11}(0) & \frac{R_1^h P_1^h}{1-P_1^h} g_{12}(0) & \frac{R_1^h P_1^h}{1-P_1^h} g_{13}(0) \\ 0 & -\frac{R_2^h}{1-P_1^h} g_{12}(0) & \frac{1-P_2^h}{P_2^h(1-P_1^h)} g_{13}(0) \end{bmatrix} \tag{3.34}$$

where the ratios $g_{32}(0)/g_{12}(0)$, $g_{23}(0)/g_{13}(0)$ and $g_{33}(0)/g_{13}(0)$ are derived from Eq. 3.6 similarly as for the 2×2 system.

Eq. 3.34 yields the following RGA at steady-state:

$$\Lambda(0) = \begin{bmatrix} P_1^h & (1-P_1^h)(1-P_2^c) & (1-P_1^h)P_2^c \\ 1-P_1^h & P_1^h(1-P_2^c) & P_1^h P_2^c \\ 0 & P_2^c & 1-P_2^c \end{bmatrix} \tag{3.35}$$

Note that the RGA only depends on the thermal effectiveness of the two matches with controlled outputs, and that all RGA elements are between zero and unity.

3.8 Conclusions

Important dynamic charactericstics may be described through the following facts:

1. A positive (negative) temperature change in a HEN has a positive or zero (negative or zero) effect on all other temperatures.

2. Temperature disturbances are naturally dampened in HENs.

3. A hot (cold) stream flowrate increase has a positive or zero (negative or zero) effect on all temperatures.

4. Bypass manipulations propagate as a temperature increase from the hot side and a temperature decrease from the cold side of the bypassed match.

5. Bypass manipulations of matches may increase or decrease a temperature (output) depending on problem parameters if and only if there are parallel (independent) downstream paths from the two sides of the manipulated match to the temperature.

6. All heat exchanger networks are open-loop stable.

Understanding about the dynamic behaviour is used to identify control limitations and explain how they may occur in HENs. Of particular importance is inverse responses due to RHP zeros. Possible RHP zeros in HENs may be divided into five categories:

1. Downstream mixers. Mixing two independent streams downstream a heat exchanger may give competing flowrate and temperature effects, and there may be one monovariable RHP zero.

2. Upstream mixers. Mixing to independent streams upstream a heat exchanger may also give compteting flowrate and temperature effects, and there may be two monovariable RHP zeros.

3. Splits. Stream splits give parallel mass flows, and remixing the streams give parallel downstream paths that may give two monovariable RHP zeros.

4. Inner matches. Manipulating inner matches may give parallel energy flows, and a heat load loop consisting of only process exchangers may give independent, parallel downstream paths that may give a monovariable RHP zero.

5. Combining matches. For multivariable 2×2 problems, a multivariable RHP zero may occur if all four input-output effects traverse the same match.

The results on control limitations may be used to divide plants (HEN with control configuration) into different categories according to their expected control characteristics. The classification is based on network and control structure only, so any conclusions must be used with caution.

1. Plants with no interaction

2. Plants with single, direct effect bypasses and one-way interaction only

3. Plants with direct effect bypasses

4. Plants with fast, but not direct effect bypasses.

5. Plants with appreciable apparent time delay.

6. Plants with right half plane zeros.

7. Plants that are structural singular at steady-state

8. Plants that are structural singular (also dynamically)

Plants with no interaction are obtained by manipulating final heaters and coolers on all streams. This usually gives excess units and high capital cost. Furthermore, a certain heat load must be maintained for all the utility exchangers for all operating points. This will give excess utility requirements and high energy cost. Still, these plants are preferred from a control point of view. Thus, there is definitely a trade-off between controllability, energy and capital in HENs.

One way to resolve the trade-off could be to allow one-way interactions only. Plants with one-way interaction will usually have close to minimum number of units with many final utility exchangers.

However, two-way interactions between control loops manipulating matches are often a minor control problem. Good performance may be obtained with simple inverse-based controllers. Thus, plants of category three and four may sometimes be overall optimal, too.

Plants with considerable time delay or important RHP-zeros should be avoided.

Nomenclature

For the dynamic model

A - Heat exchanger area, $[m^2]$
a - Parameter in Eq. 3.17
b - Parameter in Eq. 3.17

h - Heat transfer coefficient, $[W/m^2K]$
N - Number, $[-]$
P - Thermal effectiveness, $[-]$
R - Heat capacity rate ratio, $[-]$
T - Temperature, $[K]$
t - Time, $[s]$
U - Overall heat transfer coeff., $[W/m^2K]$
w - Heat capacity flowrate, $[kW/K]$

greek
ΔT - Temperature difference, $[K]$
τ - Time constant, $[s]$

For control
$G(s)$ - Process transfer function matrix
$G_T(s)$ - Temperature disturbance transfer function matrix
$G_w(s)$ - (Heat capacity) flowrate disturbance transfer function matrix
$G_d(s)$ - Augmented disturbance transfer function matrix ($[G_T\, G_w]$)
$g_{ij}(s)$ - ij'th element of $G(s)$
$\hat{g}(s)$ - part of g
$g_{dik}(s)$ - ik'th element of $G_d(s)$
$u(s)$ - Vector of manipulated inputs
$y(s)$ - Vector of outputs

greek
$\Lambda(s)$ - Relative gain matrix
$\lambda_{ij}(s)$ - ij'th element of $\Lambda(s)$

superscripts
c - cold side/fluid
h - hot side/fluid
k - exchanger k
tot - total

subscripts
$direct$ - direct
hx - exchanger
i - inlet or index for outputs
j - index for manipulted inputs
k - index for disturbances or stream segments
$loop$ - loop
o - outlet
TU - transfer units

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Chapter 4

Control of Heat Exchanger Networks - II: Control Configuration Design

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Abstract

Heat exchanger networks are usually controlled with decentralized controllers, and the control configuration may be very important for the control performance. Usually single, direct effect bypasses are preferred for control, but the network structure or exchanger properties may make it necessary or desirable to bypass upstream exchangers or use multi-bypasses over several exchangers. Moreover, energy considerations may favour bypasses on upstream exchangers or to manipulate split fractions. Thus, there is a both a trade-off between alternative single bypasses, and a trade-off between alternative types of manipulators. From an energy point of view split fractions are preferred to single bypasses which are preferred to multi-bypasses, whereas the order may be reversed when considering control.

4.1 Introduction

In this paper we consider control configuration design for heat exchanger networks (HEN), i.e. we select control objectives, controlled outputs and manipulated inputs and determine the best pairing. Nisenfeld (1973) first addressed this problem. He used the steady-state RGA to evaluate interactions and decide on control configuration of a HEN.

Marselle *et al.* (1982) and Calandranis and Stephanopoulos (1988) recommend to manipulate the heat load on the final exchanger. This heuristic often yields good results, but it has some important limitations and short-comings:

1. Justification. The rule is presented without examples and with little argumentation.

2. Type of manipulator. Usually there are several ways to manipulate an exchanger, e.g., hot and cold single bypasses, multi-bypasses and splitters.

3. Bandwidth. Manipulating the final exchanger is best at high frequencies, but manipulating an upstream match may be better for long term control, e.g, if the heat load on the final exchanger is small.

4. Utility consumption. Besides the control objectives concerning the target temperatures, there are important objectives concerning energy, and manipulating an upstream exchanger may decrease the utility consumption.

5. Double output matches. When both outlet temperatures of one match are controlled outputs, manipulating an upstream match is required.

6. Pairing. The pairing problem for HENs is straightforward when single, direct effect bypasses are used exclusively, in other cases the appropriate pairings may depend on the bandwidth.

This paper gives a more complete and general treatment of the control configuration design problem for HENs where these issues are addressed. A stepwise procedure that screens the large number of input combinations and performs a systematic comparison of the remaining alternatives through controllability measures is presented. To compute controllability measures a dynamic model is needed. The dynamic model is obtained from lumped, multicell models of the individual heat exchangers, see Mathisen *et al.*, (1993)* for details.

4.2 Problem description

Control configuration design consists of the following tasks:

1. Selection of control objective(s)

*corresponds to chapter two of this thesis

consumption. Thus, the secondary control objective is also important, and must be taken into consideration. Minimization of the utility consumption may also be part of the regulatory control level, or taken into consideration at the higher, supervisory control level.

In conclusion, the control objectives of HENs concerns either temperatures or energy. We want a control configuration that fulfills both criteria, and must decide whether they are compatible.

4.2.2 Selection of controlled outputs

The controlled variables or outputs in HENs may be network outlet temperatures, intermediate temperatures or heat loads. The controlled outputs should be kept:

1. at their setpoints or targets,
2. within a given range,
3. below an upper limit or above a lower limit *or*
4. as close to a limit as possible (i.e. at maximum or minimum values)

The first type of controlled outputs is needed to fulfill the primary control objective and these outputs are often referred to as hard targets. The fourth type is included to be able to fulfill the secondary objective concerning energy. Type two and three describe the additional constraints in the system, and these outputs are often referred to as soft targets.

4.2.3 Selection of measurements

In most cases selection of measurements is straightforward in HENs because the controlled outputs are usually stream temperatures. Temperature measurements are fast, inexpensive, reliable and easy to maintain. In some cases it may be desirable to control the heat load, e.g. when the heat exchanger is a reboiler or a condenser. In that case two temperature measurements and one flow measurement on the opposite side of the exchanger (without phase-shift) are often used to estimate the heat load.

4.2.4 Selection of actuators

Manipulated inputs or actuators in HENs may be

1. Utility flowrates
2. Bypass fractions
3. Split fractions
4. Process stream flowrates
5. Exchanger area (e.g. flooded condenser)

4.3 Complexity of the bypass selection problem

4.3.1 Bypasses for flexibility and controllability

In practice it may be necessary to place bypasses for three reasons:

1. *Feasibility.* The heat exchangers must have sufficient area to maintain the specifications for all possible operating points, i.e. the HEN must be able reject static disturbances. In a specific operating point this area may be too large and may be effectively reduced by the use of bypass streams.

2. *Minimization of utility consumption.* Even if a HEN has the heat exchanger areas and bypasses necessary to maintain feasible operation for all steady-state operating points, it may be possible to minimize the utility consumption by including additional bypasses. Two possible definitions of a flexible HEN is a feasible HEN with a specified maximum utility consumption or a feasible HEN with specified maximum total annual cost.

3. *Controllability.* In a specific operating point one needs degrees of freedom (bypasses) to get satisfactory control behavior in the presence of dynamic disturbances. The optimal location for the bypass is generally different depending on whether it is for flexibility or controllability. An operable HEN may be defined as a flexible and controllable HEN with a simple (e.g., decentralized) control system.

4.3.2 Possible bypass locations

"Bypass" is usually thought of as a pipeline around an arbitrary side of a single process heat exchanger. The steady-state nonlinear effect or operating range from bypassing the hot side of an exchanger is equivalent to bypassing the cold side, it is simply determined by the heat load. However, the dynamic effect is different. Consequently, one needs to differ between a hot and a cold bypass when addressing control. One should also consider multi-bypasses over more than one exchanger because this may become beneficial if the installation cost of actuators and/or controllability is taken into account. All possible bypass locations around the process exchangers of a simple HEN is shown in Fig. 4.3. The bypasses are identified with a number for the match and a letter H or C for hot or cold side. For example, the hot stream bypass on match 1 is denoted $1H$ and $21H$ denotes a multi-bypass around matches 2 and 1 (in that order).

4.3.3 Number of bypasses

The possible number of bypasses may be derived from an degree of freedom analysis. We consider a general HEN with

- N_{hx} process heat exchangers
- N_y controlled outputs ("hard" targets)

4.3.4 Combinatorial nature

Assume that N_{byp} single bypasses are to be selected, and that there are N_{hx} process exchangers in the network. Since we must differentiate between the hot and the cold side, there are two different bypass selections per match, so the number of bypass combinations is:

$$\frac{2N_{hx}!}{N_{byp}!(2N_{hx}-N_{byp})!} \tag{4.3}$$

However, using two single bypasses on one match yields structural singularity at steady-state (Mathisen *et al.*, 1994a)* . If one assumes that there cannot be more than one bypass per match the number of alternative sets is:

$$2^{N_{byp}}\frac{N_{hx}!}{N_{byp}!(N_{hx}-N_{byp})!} \tag{4.4}$$

Also this yields a fast-growing combinatorial problem. In practice, one may also use multi-bypasses and the number of bypasses may be any number of bypasses between the target and well above the number of process exchangers (see Eq. 4.1). Expressions computing the number of alternative bypass combinations in the general case together with a numerical example can be found in Appendix 1. Even for small examples with only 4 streams and 6 units, the number of alternative sets may be above 2000. In addition, if we use decentralized control, there are $N_{byp}!$ different possible pairings for each of the configurations in Eq. 4.4.

The rapid growth of this combinatorial problem makes it difficult to apply techniques which involve searching over all alternatives. Therefore it is desirable to develop simplified methods and to obtain insights and establish simple heuristics.

4.4 Controllability measures

Controllability measures are used to evaluate the inherent control properties of the process without having to do a controller design. A disadvantage with most measures for analyzing controllability is that they have to be recomputed for each control configuration.

We will use the measures listed below to evaluate controllability or dynamic resilience of HEN. Further justification for their use is given by Hovd and Skogestad (1993).

Scaling. Many controllability measures depend on scaling, and we always assume that the process transfer function matrix $G(s)$ and the disturbance transfer function matrix G_d is scaled so that allowed magnitude of the manipulators ($u's$), disturbances ($d's$) and controlled outputs ($y's$) should vary between 0 and 1 at all frequencies. The scaling should based on process knowledge. in this paper the following scalings are used:

- Disturbances in the supply temperatures (ΔT_s) : $\pm 10^0 C$

*corresponds to chapter three of this thesis

- Disturbances in the heat capacity flowrates (Δw) : $\pm 20\%$
- Manipulated inputs (from nominal bypass fractions) (Δu) : $\pm 10\%$
- Controlled outputs (target temperatures) (ΔT_t) : $\pm 3^0 C$

Singularities and time delays. When evaluating if a set of N_{byp} bypasses may be an appropriate configuration to control the N_{byp} target temperatures we first require that $G(s)$ is structural controllable.

A right half plan (RHP) transmission zero of the plant transfer function limits the achievable bandwidth regardless of the controller used (Rosenbrock, 1970). When decentralized control is used, one should also avoid RHP zeros in the elements in order to maintain stability of the individual loops. For HENs, plants that may have a RHP zero may also become parametric singular, and we disregard such bypass combinations.

Time delays represent another fundamental limitation of the achievable bandwidth, and plants with large time delays are discouraged.

Input constraints. A rough indicator for a *good* configuration is that, for each output y_i, there is one $|g_{ij}| > 1, \omega < \omega_B$ (with the variables scaled as indicated above). Otherwise we will propably get problems with input constraints when we want to make a change in y_i of magnitude 1. In addition, to get a simple controller design, it is desirable that the other elements $|g_{ij}|$ in row i of $G(s)$ is approximately zero. This does not take into account the magnitude of the disturbances or multivariable effects, and a better indication is easily derived from the requirement of *perfect disturbance rejection.*

$$y(s) = G(s)u(s) + G_d(s)d(s) \tag{4.5}$$

For square systems:

$$y(j\omega) = 0 \Rightarrow u(j\omega) = G^{-1}(j\omega)G_d(j\omega) \tag{4.6}$$

One should avoid configurations with elements in $|G^{-1}G_d|$ larger than 1. Specifically if $||G^{-1}G_d||_\infty$ (the largest row sum) in the frequency range important for control, then the nominal bypass fractions (overdesign) must be increased. If that is impossible due to driving force constraints on the exchangers the set of bypasses should be discarded.

Interactions (use of RGA). The relative gain array (RGA) is defined as (Bristol, 1966; 1978):

$$\Lambda(s) = G \times G^{-T} \tag{4.7}$$

where $\times$ means element by element multiplication (Schur product). The RGA is used as a measure of interactions in a general sense, and bypasses that minimize interactions are preferred.

In particular, one should avoid cases with large RGA-values at frequencies close to the closed-loop bandwidth because such plants are fundamentally difficult to control (irrespective of the controller)

Pairing (use of RGA). We want to control the HEN with decentralized control loops and use the relative gain array (RGA) as function of frequency to the decide the best pairing, i.e. what bypasses should be used to control what target temperatures. We like to pair so that the RGA-value is close to one around the the expected bandwidth of the system. To ensure stability of individual loops and remaining subsystem when one loop fails, pairing on negative steady-state values should be avoided.

Disturbance rejection (use of G_d and CLDG). The frequency-dependent open-loop disturbance gain matrix (G_d) may be used to check whether feedack control is required. Since G_d is scaled according to expected disturbances and allowed output deviations, feedback control is required if $|G_d(s)|$. It is usually sufficient to check G_d at steady-state.

For decentralized control some other measures are even more useful to evaluate disturbance rejection. We assume from now on that the manipulators are numbered after the pairing is decided so that u_1 is used to control y_1 etc. Then the controller matrix C is diagonal with elements c_i.

The offset of the targets of the closed loop system is:

$$e(s) = y(s) - r(s) = -S(s)r(s) + S(s)G_d(s)d(s) \tag{4.8}$$

where S(s) is the sensitivity function $(I + GC)^{-1}$, $r(s)$ is the vector of setpoints and $d(s)$ the disturbances.

At low frequency the offsets may be approximated by

$$y(s) - r(s) \approx -S_{diag}(s)G_{diag}G^{-1}r(s) + S_{diag}(s)G_{diag}G^{-1}G_d(s)d(s) \tag{4.9}$$

where G_{diag} consists of the diagonal elements (g_{ii}) of G and S_{diag} is defined as $(I + G_{diag}C)^{-1}$, i.e. has elements $1/(1 + g_{ii}c_i)$ (Hovd and Skogestad, 1993). The closed-loop disturbance gain (CLDG) is defined as:

$$\Delta = G_{diag}G^{-1}G_d \tag{4.10}$$

The elements are denoted δ_{ik} and represents the closed-loop disturbance gain from disturbance k to output i when we do the design for each individual loop.

Since G_d and G are scaled the magnitude $|\delta_{ik}|$ at a given frequency directly gives the necessary loop gain $|g_{ii}c_i|$ at this frequency needed to reject this disturbance. The frequency where $|\delta_{ik}(j\omega)|$ crosses 1 gives the minimum bandwidth requirement for this disturbance. It should be less than the bandwidth that can be achieved in practice, which will be limited by time delays, RHP zeros etc.

Setpoint tracking (use of PRGA). In a similar manner the performance relative gain array (PRGA) defined as $\Gamma = G_{diag}G^{-1}$ can be used to evaluate set-point tracking of the system. However, in process control disturbance rejection is often the major concern, and since PRGA (Γ) will generally be small when CLDG ($\Delta = \Gamma G_d$) is small, evaluation of setpoint tracking can normally be omitted.

4.5 Single bypasses for monovariable problems

In this and the next section we use single bypasses as manipulators, multi-bypasses around several matches and splitters will be considered in sections 4.7 and 4.8.

There are three ways of changing the outlet temperature of a heat exchanger:

1. by changing the flowrate through the exchanger, e.g., by use of a bypass

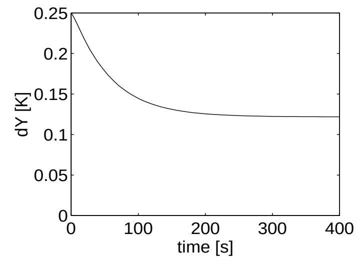

Figure 4.7: a) Bypass 1H

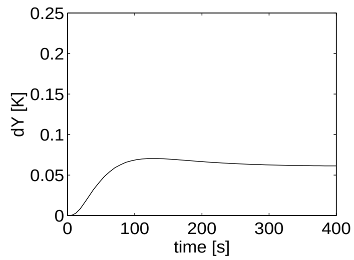

Figure 4.8: b) Bypass 2H

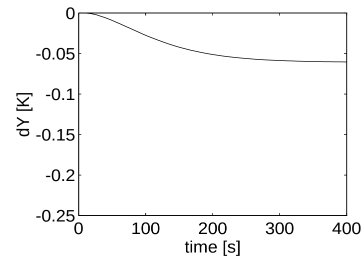

Figure 4.9: c) Bypass on match 3H

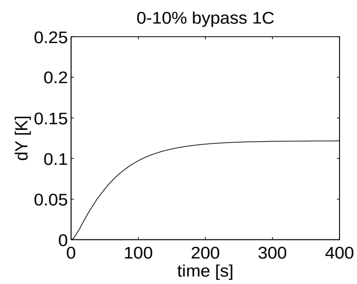

Figure 4.10: d) Bypass 1C

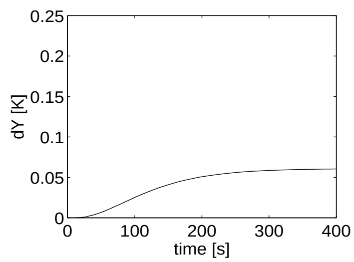

Figure 4.11: e) Bypass 2C

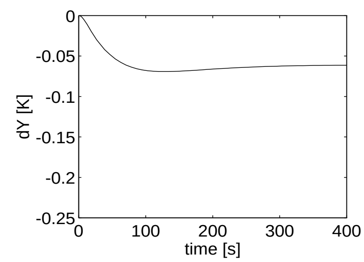

Figure 4.12: f) Bypass 3C

Figure 4.13: Example 1. Step response from various bypasses to output y_1.

The difference between the linear gains and the operating range ("average gain') illustrates the nonlinearity. Since all heat exchangers have the same set of parameters, the gain differences between the alternative bypass locations must be due to structural differences. We note the following: 1) The gains from the hot single bypasses are equal to the gains from the cold bypasses around the same matches because the heat capacity flowrates are equal ($R = 1$). 2) The gains from the upstream match on the hot side (match 2) are equal to, but have the opposite signs as the gains from the upstream match on the cold side (match 3). The negative gain from the bypasses around match 3 may be used to increase the feasibility range and may be important from flexibility considerations. 3) The gains from match immediately upstream the controlled output (match 1) are twice as large as (the absolute value of) the gains from the upstream matches (matches 2 and 3) This is because the gain from $2H$ and $3C$ is reduced (with 50%) by the thermal effectiveness of match 1. 4) The gains from the upstream matches are equal to the gains from the final match times the thermal effectiveness. 5) The magnitude of the linear gains are smaller than the magnitude of the operating range.

In Fig. 4.13 the responses from all six bypasses are compared, whereas the phase of the frequency response is shown in Fig. 4.16. The following points about the dynamics should be noted: 1) The response from bypass $1H$ is immediate because this bypass has a direct effect on the output (y_1), and the asymptotic phase shift from bypasses $1H$ is zero 2) The asymptotic phase shift from bypass $1C$ is $-\pi$. The additional phase shift compared to bypass $1H$ is due to the wall ($-\frac{\pi}{2}$) and the hot side mixing tank of match 1 ($-\frac{\pi}{2}$). 3) The asymptotic phase shift from bypass $3C$ is $-3\pi/2$. The additional phase

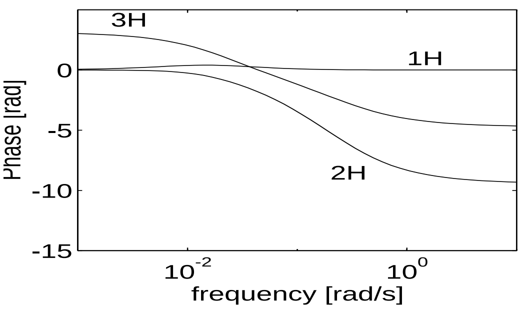

Figure 4.14:

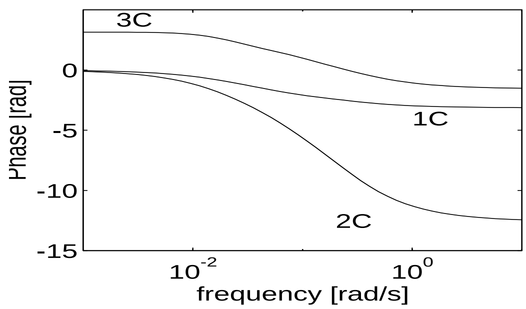

Figure 4.15:

Figure 4.16: Example 1. Phase shift from various bypasses to output y_1.

shift $(-\frac{\pi}{2})$ compared to bypass $1C$ is due to the heating of the cold side mixing tank. 4) The asymptotic phase shift from bypass $3H$ is larger than $3C$. This is due to the heat transfer from the hot to the cold side of match 3. 5) The asymptotic phase shift from bypass $2H$ is $-N\pi/2$ where N is the number of mixing tanks on the hot side of match 1 ($N = 6$ in example). This gives a "dead-time-response", and is typical for heat exchanger responses from the inlet to the outlet temperature on the same side. 6) The asymptotic phase shift from bypass $2C$ is larger than $2H$. This is due to the heat transfer from the cold to the hot side of match 2. 7) The response from bypass $3C$ is faster than bypass $2H$ because the heat exchanger is countercurrent. ($2H$ is more slowed down due to mass holdup on the hot side than $3C$ is slowed down due to wall capacitance. With a parallel exchanger the order is usually reversed, then bypass $3C$ would be slowed down due to both mass holdup and energy holdup in the metal parts.)

The results are summarized in Fig. 4.6 where the speeds of response from the alternative bypasses are indicated.

From the steady-state gains and the phase of the frequency response, one may "rank" the bypasses as follows in terms of controllability:

1. The bypass with the direct effect ($1H$)

2. The bypass around the final match on the opposite side ($1C$)

3. The bypasses around the upstream match on the opposite side ($3C$ or, slightly worse, $3H$)

4. The bypasses around the upstream match on the same side as the controlled output ($2H$ or, slightly worse, $2C$)

Bypasses even further away from the controlled output will be worse, and should be discouraged.

General expression. One crucial question is how sensitive this priority order is to problem parameters. To answer this question we consider the following general gain expression for a single bypass such as bypass 1H in example 1 (Mathisen *et al.* (1994a):

$$g_{11} = (P_1^h + \frac{\partial P_1^h}{\partial w_1^h})(T_1^h i - T_1^c i) \tag{4.11}$$

where $(T_1^h i - T_1^c i)$ is the inlet temperature difference of the match 1 (the bypassed match) and the derivative of the thermal effectiveness, e.g. $\partial P_1^h / \partial w_1^h$, is with respect to dimensionless heat capacity flowrate.

When comparing bypasses on alternative a large operating range which largely corresponds to large linear gains are preferred to avoid problems with input constraints. From Eq. **??** it is clear that bypasses on matches with high thermal effectiveness and large inlet temperature difference are preferred.

Bypasses on the same match have the same operating range, but the linear gain will be larger for the smaller stream. Since one may design the bypass valves such that the maximum allowed bypass is a fraction of the total flow, a bypass on the smaller stream may yield less problems with input constraints. Moreover, installing a bypass pipe and a control valve on the smaller stream is usually least expensive option, too.

Bypasses on match 2, which is upstream on the same side as the output, is favored by low effectiveness of the final match ($P_1^h \approx 0$), whereas bypasses on match 3, which is upstream on the opposite side as the output, is favored by a high effectiveness of the final match ($P_1^c \approx 1$). A high thermal effectiveness of match 1 favours both bypasses on match 1 and match 3, and gain from bypasses on match 3 will therefore seldom exceed the gain from bypass $1H$ at steady-state. Since bypasses on match 2 is favoured by a low thermal effectiveness of match 1, the gain from these bypasses may more often exceed the gain from bypass $1H$ than bypasses on match 3. However, at higher frequency, bypass 3C may be acceptable due its moderate asymptotic phase shift whereas bypasses on match 2 should not be used.

Conclusions

The main conclusion from this section is that the direct effect bypasses ($1H$) are best in most cases, at high frequency they are always best. A heuristic for bypass placement for control may be suggested:

Bypass placement rule 1 (main bypass placement rule): Prefer bypasses with direct effects on the outputs.

Corollary. For cases with two bypasses with the same downstream paths to the outputs, prefer the one with the shorter downstream path.

In cases where the controlled stream has a larger heat capacity flowrate than the opposite stream, bypassing the opposite side may be recommended. The following heuristic is suggested:

Bypass placement rule 2 (heat capacity flowrate ratio): Prefer bypasses on the heat exchanger side with the smaller heat capacity flowrate.

Bypasses on upstream matches (or other matches for that matter) is preferred by large thermal effectiveness and large inlet temperature differences. This may be stated in two further rules for bypass placement.

Bypass placement rule 3 (thermal effectiveness): Prefer bypasses on heat exchangers with large thermal effectiveness.

Figure 4.18: a) N

Figure 4.21: a) Proposed

w_1, $y_2 - w_2$ and $y_3 - w_3$)

Step 2: Parametric singularity

Second, we consider the structure of the remaining 40 plants, and apply bypass placement rule 7. This eliminates bypass combinations with possible parametric singularities and RHP-zeros in the elements due to competing effects. Match 5 is *an inner match with downstream paths from both the hot and the cold side* to both outputs, and manipulating the heat load on such matches gives competing effects that may give a severe RHP-zero, low gains or even parametric singularity (Mathisen *et al.*, 1994a). Eliminating all bypass combinations including match 5 reduces the number of bypass combinations from 40 to 24.

Then we note that match 1 affects output y_1 from the hot side and output y_2 from the cold side. This makes match 1 *a combining match*. If upstream matches on each side are used as inputs, the resulting plant may have a multivariable RHP-zero. Thus, the 4 bypass combinations using matches 2 and 4 should be disregarded. The structure of the remaining 20 bypasses combinations are structural controllable and cannot become parametric singular.

Step 3: Bypass side

The linear gains will generally be larger when bypassing the smaller of the two heat exchanger streams, e.g., the gain from bypass $2H$ (g_{13}) is larger than the gain from $2C$ (g_{14}). Since match 2 affect both outputs from the hot side, there will be additional delay from bypass $2C$ compared to $2H$ due to the wall capacitance. Thus, from the corollary to rule 1 and rule 2 we may conclude that bypass $2H$ is preferred to $2C$, and with similar argumentation that $4C$ is preferred to $4H$.

By disregarding all bypass combinations including $2C$ and $4H$, only 12 alternatives remain for closer analysis (i.e., $1H2H$, $1H3H$, $1H3C$, $1H4C$, $1C2H$, $1C3H$, $1C3C$, $1C4C$, $2H3H$, $2H3C$, $3H4C$ and $3C4C$).

Step 4: Input constraints

After the previous steps, six alternative inputs remain for consideration, and we define the input vector as:

$$u = \begin{bmatrix} 1H & 1C & 2H & 3H & 3C & 4C \end{bmatrix}$$

The steady-state gain then becomes:

$$G^{all}(0) = \begin{bmatrix} 0.50 & 0.28 & 0.21 & 0.12 & 0.18 & -0.41 \\ -0.62 & -0.35 & 1.05 & 0 & 0 & -0.35 \end{bmatrix}$$

The magnitude of the inputs (bypasses) needed for perfect control is given by elements of $G^{-1}G_d$. Here G corresponds to the 2 appropriate columns of G^{all}. Unfortunately $G^{-1}G_d$ has to be recomputed for each of the 12 control configurations. With decentralized control, there are five alternative bypasses for control of y_1 and four alternative bypasses for control of y_2. The required manipulations for perfect disturbance rejection are shown in Fig. 4.27a and 4.27b, respectively. Note the following: 1) The required manipulation increases at high frequency except for the loop involving a bypass with a

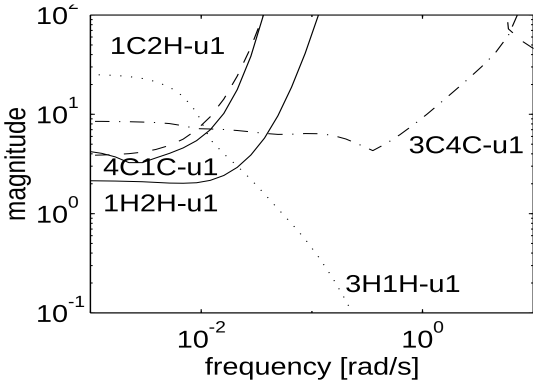

Figure 4.25: a) Magnitude of input u_1

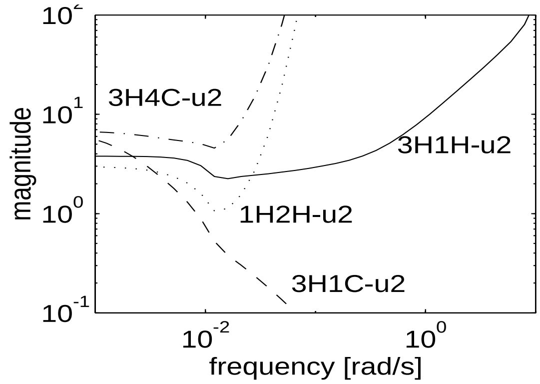

Figure 4.26: b) Magnitude of input u_2

Figure 4.27: Example 5. Required manipulations for perfect control (row sums of $G^{-1}G_d$).

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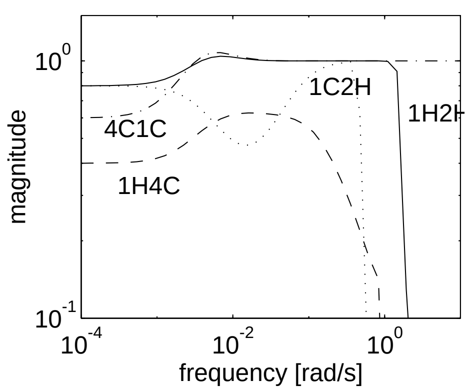

Figure 4.28: Example 5. RGA,1,1 for four control configurations

direct effect, namely $3H \leftrightarrow y_1$ and $1C \leftrightarrow y_2$. At high frequency $3H1C$ is the preferred control configuration. However, at low frequency $3H$ is the worst input to control output y_1 and may yield problems with input constraints. 2) Using bypass $1H$ to control $y1$ seems to be best on low frequency. Since $1C$ cannot be used together with $1H$, the preferred control configuration based on a linear analysis of input constraints is $1H2H$.

Interactions and preferred pairings

The 1,1-element of the RGA (λ_{11}) is 1.0 at all frequencies for all combinations with the first bypass on match 3. This can be seen directly from the network structure, since there is no downstream path from match 3 to output y_2. Thus, the best pairing for decentralized control is obvious for 8 of the 12 remaining bypass combinations. The 1,1-element of the RGA for the 4 other combinations is shown in Fig. 4.28. From the RGA-plot, configuration $1H2H$ seems to be the best, and $1H4C$ worst because this configuration show strong interaction both at steady-state and around the bandwidth.

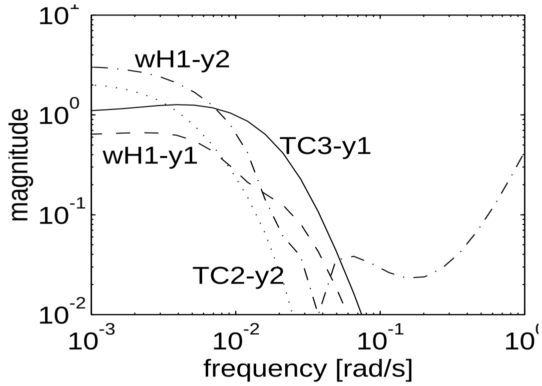

Figure 4.29: a) Control configuration $1H2H$

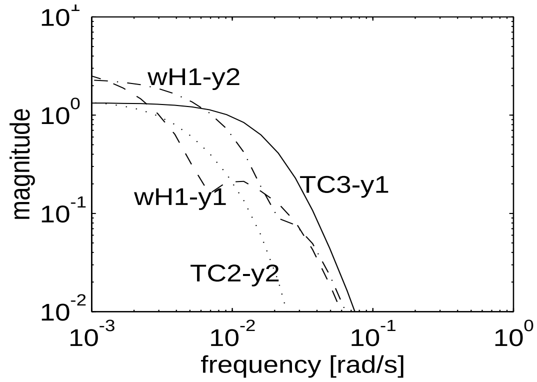

Figure 4.30: b) Control configuration $3H1C$

Figure 4.31: Example 5. Selected (worst case) elements of the CLDG.

Disturbance rejection with decentralized control

To discriminate between the two configurations which was best based on the required manipulation for perfect control $1H2H$ (at low frequency) and $3H1C$ (at high frequency), the closed loop disturbance gain (CLDG) introduced by Hovd and Skogestad (1993) may be helpful. The CLDG is defined as $G_{diag}G^{-1}G_d$. The worst temperature disturbances to reject was found to be T_s^{C3} on output y_1 and T_s^{C2} on y_2 for both configurations. The worst flowrate disturbance is w^{H1} on both outputs for both configurations.

The most important information from the CLDG-plot (Fig. 4.31a and b) is the frequency were the curves crosses 1.0. For all cases and both loops the necessary bandwidth is ≈ 0.02 rad/sec, see Fig. 4.31. For case $3H1C$ with direct effects from both inputs to the corresponding outputs, the speed of the response will be about 0.05 to 0.5 rad/sec, i.e. fast enough.

Conclusions from example 5

Feedback control is necessary. Since two single bypasses are to be placed on the hot or the cold side of the five matches in the network, there are 45 alternative bypass combinations.

The fast method for selection of bypass combination is to only apply the main rule 1 for bypass placement, i.e., select bypasses $3H$ and $1C$ directly. The detailed method is to apply all the five rules to eliminate the majority of the altenatives, (i.e., 33 combinations) and perform a controllability assessment through controllability measures of the remaining 12 combinations. The controllability measures may then be used to select pairings and identify interactions, input constraint problems and/or high bandwidth requirements. For this example, the analysis indicate that $1H2H$ is the best control configuration. It is preferred to other configurations using matches 1 and 2 due to smaller interactions. Configuration $1H2H$ is preferred bypass combinations using the final matches 1 and 3 mainly from input constraints at low frequency, at higher frequencies $3H1C$ is best as expected. Thus, this conclusion depends on the bandwidth.

4.7 Multi-bypasses

4.7.1 Operating range with monovariable problems

We now reconsider example 1, and assume that multi-bypasses may be present, too. For this network two double bypasses may be installed, $21H$ and $31C$, see Fig. 4.3. The multi-bypasses affect the controlled output by the same mechanisms as the corresponding single bypasses. Firstly, multi-bypass $21H$ affect the output both by changing the hot inlet temperature to match 1 as the single bypasses around match 2 and by changing the heat capacity flowrate of the final exchanger as the single bypasses around match 1. Secondly, multi-bypass $31C$ affect the output both by changing the cold inlet temperature to match 1 as the single bypasses around match 3 and by changing the heat capacity flowrate of the final exchanger as the single bypasses around match 1. For our numerical example, the linear steady-state gains from the multi-bypasses are:

	$21H$	$31C$
dy_1/du	0.29375	0.18125

whereas the operating ranges are:

	$21H$	$31C$
$y_1(u=0) - y_1(u=1)$	0.50	0.25

Note the following: 1) The gain from the multi-bypass $21H$ is larger than the sum of the gains from the corresponding single bypasses. 2) Multi-bypass $31C$ combine single bypasses with opposite effects. Still, the linear gain from the multi-bypass is larger than the dominating single bypass (the operating ranges are equal).

For this example the multi-bypass around matches 2 and 1 ($21H$) has larger gains than anyone of the single bypasses on matches 2 or 1. In general this fact holds at steady-state only, but since multi-bypass $21H$ has a direct effect on the output, the gain from the multi-bypass will be larger also at higher frequencies.

Fact: Multi-bypasses always have larger gains than any one of the corresponding single bypasses.

The gain for the effect of a single bypass on an upstream match on the opposite side (e.g., $3H$ or $3C$) may become larger than a multi-bypass with a direct effect (e.g., $21H$, but this is not probable since the gain from match 3 seldom will exceed match 1, see section 4.5. Thus, we need to compare the multi-bypass on the opposite side (e.g. $31C$) with the multi-bypass with the direct effect at steady-state. Since match 2 is unaffected by bypass $31C$ the operating range of $31C$ cannot exceed the operating range of $21H$. The linear gain from $31C$ may exceed $21H$ at conditions where match 3 is favoured and match 2 is disfavoured.

Figure 4.33: a) N

- Single hot (cold) bypass-controlled output where the final match has a downstream path to a hot (cold) utility-controlled output: 1) (prefer) Single bypass on final match 2) (avoid) Multi-bypass with direct effect 3) (avoid) Single bypass on upstream match.

- Single hot (cold) bypass-controlled output where an upstream match has a downstream path to a hot (cold) utility-controlled output: 1) (prefer) Multi-bypass or single bypass with direct effect reset with single bypass around upstream match 2) (accept) Single bypass around upstream match (depends on performance specifications, disturbances and delay)

- Two controlled outputs on same match: 1) (prefer) Multi-bypass and single bypass with direct effects where multi-bypass is reset with single bypass around upstream match 2) (accept) Single bypasses around final and upstream matches (depends on performance specifications, disturbances and delay)

4.8 Splitters

4.8.1 Final splits

A final split exists if the remixed stream temperature is a controlled output. An example is shown in Fig. 4.38a. In this case there are four ways to control the stream temperature of a remixed stream; 1) with the split fraction, 2) with a bypass on a match upstream the split (if present) 3) with a bypass on one of the matches on the stream bransches, 4) with a total bypass. The first and second options are bad for control see Mathisen *et al.* (1994a). The third and fourth options may be poor from an energy point of view (for pinch problems). In conclusion, no option fulfills both objectives.

4.8.2 Effect on utility consumption

Fact: Manipulating the split fraction of split streams is preferred to increasing the bypass fraction of any one of the matches on the stream branches.

Proof: Assume there are one match on each branch of the split stream. Manipulating the split fraction decreases the heat load on the match on one of the branches. A corresponding reduction may be achieved by increasing a bypass fraction around the match. However, manipulating the split fraction will increase the heat load on the match on the opposite branch, whereas the single bypass manipulation does not affect the other match. The extension to split streams with several matches on each branch is straightforward.

To illustrate the effect on utility consumption, we reconsider the network in Fig. 4.38a, but assume that there is a final cooler on the split stream $H1$. All the stream output temperatures may then be controlled by manipulating utility flowrates. The split fraction will affect the total heat transfer of the two matches, and the split fraction should be manipulated to minimize the utility consumption. Manipulating split fractions to

E.

3. Prefer bypasses on heat exchangers with large thermal effectiveness.

4. Prefer bypasses on heat exchangers with the large inlet temperature difference.

5. Prefer bypasses with downstream paths to utility-controlled outputs of the same type as the bypass-controlled output.

6. Disregard bypass combinations that yields structural (functional) singularity.

7. Disregard bypass combinations if the resulting structure yields competing effects.

Rule 1 is the main rule, but it may sometimes be in conflict with the other rules. If the main rule is in conflict with rule 2, 3 or 4, the preferred manipulation may depend on the bandwidth. If the main rule is in conflict with rule 5, a control/energy trade-off between alternative single bypasses exists. Moreover, it may sometimes be recommended to use multi-bypasses due to increased operating range or splitters due to decreased energy requirements. There is another control/energy trade-off between alternative actuator types: for control multi-bypasses may be preferred to single bypasses which are preferred to splitters whereas the order is usually reversed when considering energy.

4.9.2 Stepwise procedure for bypass selection

We have looked at the problem of selecting bypasses and appropriate pairings for decentralized control and evaluation of controllability or dynamic resilience of HEN. We suggest the following stepwise approach (as all matrices are assumed to be scaled, "large" means greater than unity):

1. G: Discard bypass combinations where G is structural singular (rule 6)

2. RHP-zeros: Discard combinations where G may become parametric singular (rule 7)

3. Time delay: Discard combinations with excessive time delay.

4. $G^{-1}G_d$: Discard combinations if large (both at steady-state and dynamically).

5. Interactions RGA: Prefer combinations with smaller interactions (RGA approximately equal to the identity matrix, both at steady-state and dynamically)

6. Decentralized control CLDG: Prefer combinations with smaller closed-loop disturbance gains and smaller bandwidth requirements.

4.9.3 Pairing

The main rule for bypass placement is to use single bypasses with direct effects to control all bypass-controlled outputs, and the preferred pairings are obvious in such cases. Adhering to the main rule for bypass placement becomes impossible with double output matches, and a 2×2 pairing problem results. It is shown that the preferred pairing at steady-state based on the relative gain array is given by the thermal effectiveness for such HENs. The result has also been extended to 3×3 problems and may be very helpful for control configuration designers because the thermal effectiveness of a match may be computed from inlet and outlet temperatures *or* heat capacity flowrate ratio, number of heat transfer units and flow configuration.

Nomenclature

$C(s)$ - Diagonal controller transfer function matrix
$c_i(s)$ - Controller element for output i
$d(s)$ - Vector of disturbances.
$e(s) = y(s) - r(s)$ - Vector of output errors
$G^{all}(s)$ - Augmented process transfer function matrix with all possible inputs
$G(s)$ - Process transfer function matrix
$G_d(s)$ - Disturbance transfer function matrix
$g_{ij}(s)$ - ij'th element of $G(s)$
$g_{dik}(s)$ - ik'th element of $G_d(s)$
n - Number (of units)
N_{byp} - Number of bypasses
N_c - Number of cold process streams
N_h - Number of hot process streams
N_{hx} - Number of process exchangers
N_{ux} - Number of heaters and coolers
P - Thermal effectiveness, $[-]$
R - Heat capacity rate ratio, $[-]$
$r(s)$ - Reference signal (setpoint) for outputs
$S(s)$ - Sensitivity function $S = (I + GC)^{-1}$
$u(s)$ - Vector of manipulated inputs.
$u_{y=0}(s)$ - Vector of manipulated inputs necessary for perfect control.
$y(s)$ - vector of outputs

greek
$\Delta(s)$ - Closed loop disturbance gain matrix
$\delta_{ik}(s)$ - ij'th element of $\Delta(s)$
$\Gamma(s)$ - Performance relative gain matrix
$\gamma_{ij}(s)$ - ij'th element of $\Gamma(s)$
$\Lambda(s)$ - Relative gain matrix
$\lambda_{ij}(s)$ - ij'th element of $\Lambda(s)$
Θ - Dimensionless temperature difference
ω - Frequency

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Appendix 1: Combinatorics of the bypass selection problem

The number of bypasses N_{byp} may vary between zero (all hard targets are controlled by utility exchangers) and the number of process heat exchanger N_{hx}. The number of alternative sets of bypass selections with exclusively single bypasses (Eq. 4.4) must then be replaced by the following sum:

$$\sum_{k=0}^{Nhx} 2^k \frac{N_{hx}!}{k!(N_{hx}-k)!} \tag{4.15}$$

where the summation (with index k) is over the number of bypasses.

If bypasses over two exchangers in series is allowed, the structure of the network must be known to decide the number of alternative sets of bypass selections. Suppose N_i is the number of different i exchangers in series in the HE, i.e., N_2 is the number of different pairs of exchangers in series etc. One is to take the utility exchangers into consideration. If N_{byp} bypasses are to be placed and single and double bypasses are considered, the number of alternative sets of bypass selections is:

$$\sum_{j_2=0}^{max(0,min(N_{byp},N_2))} 2^{N_{byp}-j_2} \frac{N_{hx}!}{(N_{byp}-j_2)!(N_{hx}-N_{byp}+j_2)!} \frac{N_2!}{j_2!(N_2-j_2)!} \tag{4.16}$$

where the summation (with index j_2) is over the number of double bypasses. Suppose that the total number of units (process exchangers and utility exchangers) are n and that all possible multi-bypasses are to be considered. One needs $n-1$ summations,

2. The number of bypasses between zero and the number of process exchangers ($0 \leq N_{byp} \leq N_{hx}$) and only single bypasses: 81 alternatives (Eq. 4.15)

3. Two bypasses ($N_{byp} = 2$) and single or double bypasses: 87 alternatives (Eq. 4.16).

4. Two bypasses ($N_{byp} = 2$) and single or double or *triple* bypasses: 116 alternatives (Eq. 4.17).

5. The number of bypasses between zero and the number of process exchangers ($0 \leq N_{byp} \leq N_{hx}$) and single or double bypasses: 1159 alternatives (Eq. 4.18). The number of bypasses between zero and the number of process exchangers ($0 \leq N_{byp} \leq N_{hx}$) and single or double or triple bypasses: 2073 alternatives (Eq. 4.19).

An approximate value of the maximum number of alternative double bypasses (N_2) can be calculated from the number of streams and exchangers, i.e. without knowing the exact network structure:

$$N_2 \approx 2N_{hx} + N_{ux} - (N_h + N_c + N_{spl}) \tag{4.20}$$

where N_ux is the number of utility exchangers, N_h hot streams, N_c cold streams and N_{spl} stream splits. Eq. 4.20 may be generalized to triple bypasses and higher order bypasses as shown in Eq. 4.21 and 4.22:

$$N_3 \approx 2N_{hx} + N_u - 2(N_h + N_c + N_{spl}) \tag{4.21}$$

$$N_i \approx 2N_{hx} + N_u - (i-1)(N_h + N_c + N_{spl}) \tag{4.22}$$

For the example from Townsend and Morari, $N_{ux} = 2$, $N_h = 2$, $N_c = 2$ and $N_{spl} = 0$ which gives $N_2 = 6$ and $N_3 = 2$. Consequently, the approximate formula is an exact formula in this case.

Appendix 2: Data for the Examples

The major part of the steady-state data for the examples have been presented in the main part of the paper. Here we include the full parameter set for the first example to indicate how the dynamic parameters are obtained.

Example 1

The stream data for example 1 is shown in Table 4.1. The basic heat exchanger data is given in Table 4.2. The resulting nominal steady-state parameters and time constants for heat exchangers are given in Table 4.3.

Stream	$T_s[-]$	$T_t[-]$	$w_0[kW/^oC]$	$h_0[W/m^2,^oC]$
$H1$	1	0.5	20	400
$H2$	0.5	0.25	20	400
$C1$	0	0.5	20	400
$C2$	0.5	0.75	20	400

Table 4.1: Stream data for example 1

Match	$U_0[W/m^2,^oC]$	$m^h = m^c[-]$	$A_{hx}[m^2]$	$V^h = V^c = 4.5V^w$
1,2 and 3	200	0.8	1000	45/8
Match	$\rho^h = \rho^c[kg/m^3]$	$c_p^h = c_p^c[J/kg,K]$	$\rho^w[kg/m^3]$	$c_p^w[J/kg,K]$
1,2 and 3	800	2000	7000	800

Table 4.2: Basic heat exchanger data for example 1

Match	$R^h = R^c[-]$	$N_{TU}^h = N_{TU}^c[-]$	$P^h = P^c[-]$	$\tau_F^h = \tau_F^c[s]$	$\tau_k^w[s]$
1,2 and 3	1	1	0.5	45	8.75

Table 4.3: Nominal steady-state parameters and time constants for example 1

Chapter 5

Optimal Operation of Heat Exchanger Networks

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Abstract

Research on heat exchanger networks focuses on design whereas operational and control issues have largely been neglected. In this paper a simple, parameter-independent method to ensure steady-state optimal operation is presented. It is shown that for several important classes of heat exchanger networks the sign of the transfer matrix elements between manipulated inputs and controlled outputs may be determined from structure only. This "sign matrix" may be utilized to select the heat exchangers to be bypassed in order to minimize utility consumption. The applicability of the proposed procedure is explained through several examples including networks with loops, splits and multi-bypasses over several exchangers. Manipulation of stream splits are shown to be preferred to bypass manipulations, and single bypasses are preferred to multi-bypasses. For networks where the procedure cannot guarantee optimality, conventional on-line optimization must be used. However, the sign matrix may still be used to formulate heuristic rules for bypass selection that give practically useful, optimal or near-optimal solutions.

5.1 Introduction

In this paper we discuss steady-state operation of existing heat exchanger networks (HENs) or proposed HEN designs. Thus, we assume that both the HEN structure and heat exchanger areas are fixed.

The controlled variables or outputs in HENs are usually the stream outlet temperatures. The control objective is to keep these outputs at their setpoints or targets. At steady-state HENs have one degree of freedom per exchanger, the heat load. During operation, the heat load on the utility exchangers (i.e., heater and coolers) are manipulated indirectly by adjusting the utility flowrate. The heat load on matches (i.e., process heat exchangers) are usually manipulated by adjusting a bypass flow on the hot or the cold side, but liquid level in flooded condensers or a recycle flow may also be used. Since we only discuss stationary effects, these alternative methods are equivalent, and *by bypassing a match we mean in this paper to reduce the heat load in a general sense*. We will also discuss multi-bypasses (i.e., bypasses over several matches in series) and split fractions, which will change the heat load on multiple matches.

Utility exchangers are assumed to be the final exchangers if present. Target temperatures downstream of utility exchangers are controlled by manipulating the utility flowrate, and we call them utility-controlled outputs. The remaining target temperatures must be controlled by manipulating match heat loads through bypass and split fractions, and we denote them bypass-controlled outputs. The operation objective is to fulfill all target temperatures with minimum utility consumption, and this may be achieved by minimizing the hot temperatures upstream the coolers or by maximizing the cold temperatures upstream the heaters.

Problem definition

The optimal steady-state operation problem or network optimization problem (Marselle *et al.*, 1982) may be formulated as (Mathisen *et al.*, 1992) :

$$\min_{u} \quad (T_{t-1}^{Hj} - r_t^{Hj})w^{Hj} \quad \text{(minimize hot utility)} \tag{5.1}$$

$$\begin{array}{lcll}
\text{subject to:} & & & \\
T_t^{Hi} - r_t^{Hi} & = & 0 & \text{(hot and} \\
T_t^{Cj} - r_t^{Cj} & = & 0 & \text{cold target temperatures)} \\
r_t^{\hat{H}i} - T_{t-1}^{\hat{H}i} & \leq & 0 & \text{(positive or zero heat load coolers} \\
T_{t-1}^{\hat{C}j} - r_t^{\hat{C}j} & \leq & 0 & \text{and heaters)} \\
-u & \leq & 0 & \text{(bypass and split fractions above 0} \\
u - 1 & \leq & 0 & \text{and below 1)}
\end{array}$$

where w means heat capacity flowrate, T_t stream temperatures (controlled outputs), r_t the reference values for the controlled outputs (setpoints) and u split or bypass fractions (manipulated inputs). Supercript Hi (Cj) denotes the set of hot (cold) streams, and $\hat{H}j$ ($\hat{C}j$) the subset of the hot (cold) streams that are utility-controlled, and subscript

$(t-1)$ means the stream upstream the final utility exchangers. Energy balances for the exchangers and mixers, and mass balances for the splitters and the mixers yield additional equality constraints. Note that only hot utility is included in the cost function as the cold utility will be given from an energy balance.

Assumptions: It is assumed that a single hot and a single cold utility is used (e.g., steam and cooling water) and that the problem has a pinch (i.e., both hot and cold utility is needed). We also assume that the target temperatures to be equality constraints.

In this paper, we present a procedure that may be used to solve the optimal operation problem by hand, i.e, it may be used to

- replace numerical optimization (e.g., Model Predictive Control, MPC)
- check and understand the solution from a numerical optimization
- decide where to install bypasses (i.e. control configuration design)
- understand interactions between design and operation/control

Note that we only consider steady-state optimal operation and do not take dynamic considerations like speed of response into account. One may need additional bypass flows to allow for feedback control.

As mentioned, Marselle *et al.*, (1982) first defined and discussed the optimal operation problem for HENs. They state that ”the direction of change of the decision variables (i.e., manipulated inputs) to yield an improvement of the objective is not apparent and has to be established through on-line experiments, a lengthy procedure ...”. The main contribution of our paper is to show that this is possible for many different types of HENs. Our approach is to determine the sign of the gain from the alternative manipulated inputs to all the outputs. We then use this ”sign matrix” to determine what matches that should be bypassed prior to the implementation.

We have already introduced matches, utility exchangers and bypass-controlled and utility-controlled outputs. A downstream path between an input and an output in HENs exists if the input affects the output so that the gain is structurally not zero. Two parallel downstream paths between a match and an output exist if there are downstream paths from both the hot and the cold outlet temperatures of the match to the output. Inner matches are matches with downstream matches on both the hot and the cold side.

5.2 Theory

5.2.1 Facts about input propagation

The procedure for minimizing utility consumption while maintaining all target temperatures is based on four facts on how disturbances and manipulations propagate in HENs:

F1: A positive (negative) temperature change has a positive (negative) effect on all downstream temperatures.

F2: Temperature disturbances are naturally dampened.

F3: A flowrate increase of hot (cold) streams has a positive (negative) effect on all downstream temperatures.

F4: Bypass manipulations propagate as a temperature increase from the hot side and a temperature decrease from the cold side of the bypassed match.

The facts are derived from single heat exchanger equations, see (Mathisen, 1994) for details.

5.2.2 The sign matrix

The sign matrix matrix defined below is used as part of the procedure:

Definition: Let u denote all match heat load manipulations (bypasses), y all controlled outputs, and g_{ij} the transfer function between input j (bypass split fractions) and output i (temperatures, both utility-controlled or bypass-controlled), i.e., $y(s) = G(s)u(s)$. Furthermore, let Q denote the "process heat transfer, and define sign(Q) such that:

$$[\text{sign}(Q)]_{ij} = \begin{cases} - & \text{if } g_{ij} > 0 \,\forall G \text{ and output } i \text{ is a hot stream} \\ + & \text{if } g_{ij} > 0 \,\forall G \text{ and output } i \text{ is a cold stream} \\ + & \text{if } g_{ij} < 0 \,\forall G \text{ and output } i \text{ is a hot stream} \\ - & \text{if } g_{ij} < 0 \,\forall G \text{ and output } i \text{ is a cold stream} \\ 0 & \text{if } g_{ij} = 0 \,\forall G \\ \pm & \text{otherwise} \end{cases} \tag{5.2}$$

Note that the sign is defined oppositely for hot and cold outputs since the desired or positive effect of increasing process-process heat transfer (and reducing utility consumption) is achieved when hot temperatures decrease and cold temperatures increase. Deviations (errors, e) in the controlled outputs are defined similarly:

$$e_i = y_i - r_i \quad \text{if output } i \text{ is a hot stream}$$

$$e_i = r_i - y_i \quad \text{if output } i \text{ is a cold stream}$$

which means the deviation error is negative if the target is exceeded.

Depending on the kind of downstream path from the considered input (manipulated match) to the considered output (stream temperature), the sign matrix may be constructed from the following rules:

- From hot side of match to hot temperature: $-$
- From hot side of match cold temperature $+$

- From cold side of match to hot output $+$
- From cold side of match to cold output $-$
- No downstream path: 0
- From both the hot and the cold side $\pm$

The idea is to use the sign matrix is as follows : To keep the bypass-controlled temperatures constant (i.e., deviation $e = 0$) one must manipulate some bypass (input). We prefer inputs which at the same time reduce the utility consumption, and this information is given by the sign matrix.

Relocating the utility exchangers

Because the sign matrix is based on information about the match structure only, it is not influenced by the existence of final utility exchangers. This means that we use the same sign matrix to optimize any set of bypass-controlled outputs. Still, the location of the utility exchangers influence both the procedure and the optimal set of bypasses.

5.3 Procedure

The optimal operation problem was defined in Eq. 9.1. We will here consider the following closely related problem: *Given a deviation (e) in a bypass-controlled output, which bypass (heat load) should be adjusted and in which direction in order to reset this output to its setpoint (reference value r) while at the same time minimizing the utility consumption?* The objective of this paper is to answer this problem based on structural information only, that is, without knowledge of temperatures, flowrates, heat heat loads etc. With an answer to this problem, one can relatively easily design a "rule-based" algorithm to optimize the operation, for example, using a simple decentralized feedback control loops with some additional logic. This could then replace a detailed numerical optimization based on solving Eq. 9.1, which would require a detailed steady-state model of the network (effect of all $u's$ on all $y's$).

The suggested sequential procedure for how to find the optimal operating point by hand is presented with only a few brief comments. Additional comments and explanations are given in the examples. All selections are based on the information about the HEN structure only.

Step 0: Initialize inputs that have a negative or zero effect on all hot (cold) utility-controlled outputs ($-$ or 0 in the corresponding elements of the sign matrix) to zero (i.e., minimize). Let index h denote hot streams that are utility-controlled and index c cold streams that are utility-controlled. Then this may be expressed as

$$\text{Set } u_j = 0 \text{ if} \quad q_{hj} \leq 0; \quad \forall h \quad or \quad q_{cj} \leq 0; \quad \forall c \tag{5.3}$$

Repeat the remaining steps for each bypass-controlled output i:

Step 1: Find the set of possible manipulated inputs for each bypass-controlled output, that is all inputs j where:

$$q_{ij} \neq 0 \tag{5.4}$$

In words, find the elements with $+$, $-$ or $\pm$ in the corresponding row (row i) of the sign matrix.

Step 2: Prefer the manipulations that has the most desirable side-effect on the utility consumption. Compare both hot and cold utility consumption. Assume that undetermined ($\pm$) effects may be both positive and negative. *Prefer in the following order:*

a) Manipulations that have positive effect on hot (cold) utility consumption. Mathematically

$$\begin{aligned} q_{ij}q_{hj}e_i \geq 0; \quad & \forall h \quad \text{(and at least one is positive)} \\ q_{ij}q_{cj}e_i \geq 0; \quad & \forall c \quad \text{(and at least one is positive)} \end{aligned} \tag{5.5}$$

Note that saturated inputs must be disregarded, for example, if the requirement to get $e_i = 0$ is to *decrease* a bypass that is already zero.

b) Manipulations that have a zero or mixed effect on hot (cold) utility consumption, i.e., all cases not included in a or c.

c) Manipulations that have negative effect on hot (cold) utility consumption. Mathematically

$$\begin{aligned} q_{ij}q_{hj}e_i \leq 0; \quad & \forall h \quad \text{(at least one negative)} \\ q_{ij}q_{cj}e_i \leq 0; \quad & \forall c \quad \text{(at least one negative)} \end{aligned} \tag{5.6}$$

Step 3: a) (Heuristic) Prefer the manipulated input closer to the bypass-controlled output (among inputs in the same group along the same downstream path). b) Disregard inputs with undetermined ($\pm$) effects on the bypass-controlled output.

Note the following: 1) Selecting the manipulated input closer to the output along the same downstream path (Step 3a), which is usually preferred from dynamic considerations, will usually have positive or no effect on utility consumption in such cases. 2) Both the effect on hot and cold utility consumption may be be used to discriminate between alternative inputs in Step 2. 3) The procedure as presented disregard multivariable effects, e.g., there must be enough degrees of freedom which means that the same manipulations cannot be used for multiple outputs. These points will be further explained through the examples.

5.3.1 Example 1 - HEN with minimum number of units

As an introductory example consider a HEN structure with minimum number of units, see Fig. 5.1. There are three matches in the network and the input vector may be defined as:

$$u^T = \left[\text{ bypass on match 1 } \quad \text{ bypass on match 3 } \quad \text{ bypass on match 4 } \right]$$

whereas the output vector i

Figure 5.1: Exa

	Primary u	Secondary u
T_t^{H1} exceeded $e_1 < 0$	$\Delta u_2 < 0$	$\Delta u_1 > 0$
T_t^{H1} not reached $e_1 > 0$	$\Delta u_1 < 0$	$\Delta u_2 > 0$
T_t^{C2} exceeded $e_4 < 0$	$\Delta u_2 < 0$	$\Delta u_3 > 0$
T_t^{C2} not reached $e_4 > 0$	$\Delta u_3 < 0$	$\Delta u_2 > 0$

Table 5.1: Example 1. Priority order for the inputs (bypasses) for each of the bypass-controlled outputs.

The results are summarized in Table 5.3.1. Decreasing input u_2 is preferred for both outputs when they are exceeded. Only after this input saturates to zero other inputs should be used, but from the initialization this is likely to be the case. When output y_1 is not reached, input u_1 should be decreased. Only after this input saturates to zero, input u_2 may be used. When output y_4 is not reached, input u_3 should be decreased. Only after this input saturates to zero, input u_2 may be used.

If neither outputs are reached with zero bypass fractions, increasing input u_2 is preferred for both outputs. The optimal operating may then be found by increasing u_2 until either bypass-controlled output is met. If output y_1 is met first input u_1 may be used to control this output whereas input u_2 is further increased until output y_4 is met.

Match 1 is the final unit on a hot stream and immediately upstream a cooler, whereas match 4 is the final unit on a hot stream and immediately upstream a heater.

Additional examples are given in Section 5.5.

5.4 Further facts for minimizing energy

As illustrated in the above example the following facts apply about final matches and matches immediately upstream utility exchangers:

F5: Bypassing matches immediately upstream a cooler (heater), increases the cold (hot) utility consumption.

Proof: Assume the manipulated match is immediately upstream a cooler. From F3 increasing the bypass fraction will decrease the heat load on the manipulated match. To maintain the target temperature downstream the cooler, this heat load reduction must be compensated with a corresponding increase in the heat load on the cooler. If there is a downstream path from the cold side of the manipulated match to another cooler the heat load on this cooler will decrease (from F4). From F2 the hot side (negative) effect will always dominate because the downstream path from the cold side must traverse at least one additional match to reach a cooler. Analogous reasoning may be made for matches immediately upstream heaters.

F6: Bypassing final matches on hot (cold) streams reduces or has no effect on cold (hot) utility consumption and increases or has no effect on hot utility consump-

tion.
Proof: Follows directly from F4 or the rules for setting up the sign matrix.

Since we always initialize (minimize) inputs (bypasses) with a known negative effect the process heat transfer, the saturated inputs are disregarded in the following examples. Furthermore, the reasoning are based on physical arguments rather than mathematics.

Additional utility exchangers

Adding utility exchangers will not change the sign matrix, but may affect the optimal operating point. To illustrate this point, assume that a final cooler on stream $H1$ is added to example 1.

- Step 0: Although input u_2 now have a mixed effect on the coolers, it has a negative effect on the only heater so all inputs may still be initialized to zero.
- Step 1: Bypasses u_2 and u_3 affect the bypass-controlled output y_4.
- Step 2: When output y_2 is exceeded input u_2 fulfills criteria a), but this input is saturated so input u_3 must be used. When output y_2 is not reached input u_3 fulfills criteria a), but this output is saturated so u_2 must be used.

Finally, assume that all streams are utility-controlled. In this case, there is no way to determine the bypass fraction around the inner match (match 3). Adjusting the bypass around this match may increase or decrease the utility consumption (+ and − to hot outputs y_1 and y_2, and − and + to cold outputs y_3 and y_4). This point about inner matches is stated in F7.

F7: Bypassing inner matches may increase or decrease the utility consumption.

Multi-bypasses

In industrial HENs, multi-bypasses or bypasses over several matches in series are sometimes used to increase the speed of response or increase the control range. To illustrate the effect of multi-bypasses, we reconsider example 1 and assume that there are multi-bypasses $31C$, i.e., bypass around matches 1 and 3 on the cold side, and $34H$, i.e., bypass around matches 3 and 4 on the hot side. The sign matrix may then be extended with another two columns:

$$u^T = \begin{bmatrix} \text{match 1} & \text{match 3} & \text{match 4} & \text{multi-bypass 31C} & \text{multi-bypass 34H} \end{bmatrix}$$

Multi-bypasses decrease the heat load of both the bypassed matches, and this may be used to derive the columns in the sign matrix for the multi-bypasses:

$$sign(Q) = \begin{bmatrix} - & + & 0 & - & + \\ 0 & - & - & - & - \\ - & - & 0 & - & - \\ 0 & + & - & + & - \end{bmatrix} \tag{5.8}$$

The bypass controlled outpu

Figure

- Step 1: All bypasses affect output y_4.
- Step 2: Two different cases may be identified. 1) If the bypass-controlled output is exceeded inputs u_2, u_3 or u_4 may be used. Bypassing match 4 (u_4) is preferred to inputs u_2 and u_3 because it has a positive effect on stream $C1$ and the hot utility consumption. 2) If the output is not reached bypass u_1 must be used.

Note that increasing these bypasses (u_1 and u_4) are preferred until they saturate. The corresponding match loads have then dropped to zero, which changes the match structure and the sign matrix. The procedure may then be repeated to identify that u_2 should be increased when u_4 saturates whereas there are no other alternatives when u_1 saturates.

Additional outputs (internal temperatures)

In the previous examples, the controlled outputs are the network outlet temperatures of the process streams. Sometimes it is required to also control internal temperatures, e.g. due to material constraints on the heat exchangers. To illustrate that the procedure may handle internal constraints, assume that the hot outlet temperature of match 1 $T_t^{1h} = y_5$ must be controlled, too. The sign matrix is extended with a fifth row:

$$sign(Q) = \begin{bmatrix} - & - & + & - \\ - & + & - & - \\ - & - & - & + \\ + & - & - & - \\ - & + & + & - \end{bmatrix}$$

Now, assume that output y_1 is controlled with u_4, and that all other bypass fractions are zero. If output y_5 is exceeded, bypass match 1. If output y_5 is not reached, a bypass on match 2 or 3 may be used. Bypassing match 3 is preferred because match 2 affect the output via match 3. The following rules holds:

F9: The bypass closer to the output is preferred among two bypasses that affect the bypass along the same downstream path.
Proof: Fact F9 is a consequence of fact F2 since heat load changes are dampended like temperature changes.

Note that the hot utility consumption increases for both cases. There is a penalty for adding constraints.

5.5.2 Example 3 - HEN with parallel downstream paths

Although the network in Fig. 5.2 has more than minimum number of matches and include a loop, there is no match with two parallel downstream paths to the same output. Such parallel downstream paths may exist when the number of matches is one above minimum. To illustrate this point, we use the HEN in Fig. 5.3. This

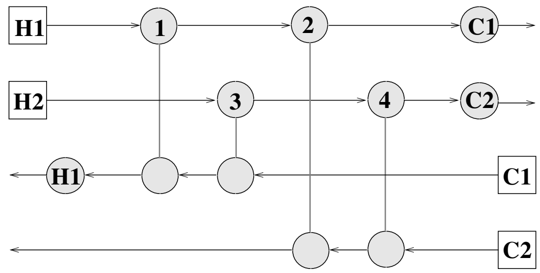

Figure 5.3: Example 3 - HEN with a match (match 3) with parallel downstream paths

network include a match (match 3), which has parallel downstream paths to some of the outputs. With same input and output vector as in the previous example, the sign matrix is:

$$sign(Q) = \begin{bmatrix} - & - & \pm & + \\ 0 & 0 & - & - \\ - & 0 & - & 0 \\ + & - & \pm & - \end{bmatrix}$$

The $\pm$ elements in column 3 indicate that there are downstream paths from both the hot and the cold side of match 3 to outputs T_t^{H1} and T_t^{C2} which make the sign matrix dependent on problem parameters.

Let us apply the proposed procedure to this network.

- Step 0: Set all bypass fractions to zero.
- Step 1: All inputs affect the bypass-controlled output.
- Step 2: Two different cases: 1) If the bypass-controlled output y_4 is exceeded, match 2 or match 4 may be bypassed. Manipulating match 3 may also give the desired effect, increasing input u_3 has a negative effect on hot utility consumption whereas inputs u_2 and u_4 have no effect on the heater. 2) If the output is not reached, bypass u_1 may be increased. Manipulating match 3 may also give the desired effect, and the sign matrix cannot be used to discriminate between the two possible inputs.
- Step 3: If output y_4 is exceeded, prefer bypass u_2 to bypass u_4 since it is closer to the output (from a). If output y_4 is not reached, prefer bypass u_1 to bypass u_3 since u_3 u_3 has an undetermined effect on the output (from b).

Note that bypassing match 2 or match 4 is equivalent from an energy point view since neither match influence the heater on stream $C1$.

5.5.3 Example 4 - HEN with a split stream

Usually streams that are split in HENs are remixed before the network outlet. When controlling such mixed streams, there are two (parallel) downstream paths from the splitter to the controlled output, see Mathisen *et al*, 1992. Therefore, the proposed

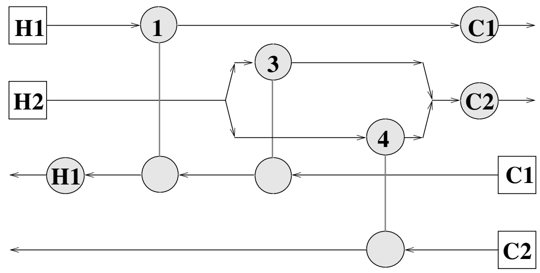

Figure 5.4: Example 4 - HEN with a split stream

procedure may not be applied to split-designs in general. Certain special situations may still be handled, see Fig. 5.4 where stream $H2$ is split to enable parallel heat exchange of matches 3 and 4. The input vector is defined as

$$u^T = \begin{bmatrix} \text{match 1} & \text{match 3} & \text{match 4} & \text{split H2} \end{bmatrix}$$

The sign matrix for this split structure is:

$$sign(Q) = \begin{bmatrix} - & + & 0 & - \\ 0 & - & + & \pm \\ - & - & 0 & + \\ 0 & 0 & - & - \end{bmatrix}$$

where the split fraction is defined as the fraction of the flowrate to the upper of the two branches. The $\pm$ entry in $[\text{sign}(Q)]_{24}$ indicates that increasing the split fraction may have a positive or a negative effect on the heat load of the cooler depending on the problem parameters. We try to apply the procedure to this network:

- Step 0: Set all bypass fractions to zero, and the split fraction to 0.5

- Step 1: Two different cases: 1) If output y_4 is exceeded, bypass u_4 or the split fraction may be increased. 2) If output y_4 is not reached, the split fraction may be decreased. If output y_4 is exceeded, increasing the split fraction is preferred because it has a positive effect the heater on stream $C1$.

In both cases, manipulating the split fraction is the preferred choice and the following fact holds:

F10: Manipulating the split fraction is preferred (from an energy point of view) to increasing the bypass fractions on the stream branches.

The fact holds in general, but note that the procedure may not be used to determine the optimal split fraction for all possible sets of utility-controlled streams, it is required that one of the streams exchanging heat with the split stream is bypass-controlled.

	Primary u	Secondary u
T_t^3	Bypass on B	Bypass on A
T_t^5	Bypass on C	Bypass on D or X
T_t^7	Bypass on E	Bypass on D
T_t^8	Bypass on G	Bypass on X
T_t^{10}	Bypass on F	Bypass on E or D

Table 5.2: Aromatic plant design from Linnhoff *et al.*, 1982. Priority order for the alternative manipulated inputs (single bypasses) for each of the bypass-controlled outputs.

to select the matches to be bypassed as there will be fewer alternatives. 3) The sign matrix include no $\pm$ elements. This means that none of the matches has downstream paths from both the hot and the cold side to one of the controlled outputs.

We apply the suggested procedure on this HEN structure.

- Step 0: Set all inputs to zero
- Step 1: We first consider bypass-controlled outputs $T_t^5 = y_5$. Two different cases: 1) If output y_5 is exceeded, match C should be bypassed. 2) If output y_5 is not reached, match D or X should be bypassed.
- Step 2: Bypass match D or match X

Similarly, one may find the possible single bypasses for the other 4 bypass-controlled outputs, and the priority order for all the outputs that ensures minimum utility consumption are shown in Table 5.2. The first column gives the preferred manipulated inputs during normal operation, i.e., when all bypass-controlled outputs would be exceeded if all bypass fractions were set to zero. The second column gives the secondary or alternative manipulated inputs when the primary input saturates to zero. The following information may be extracted from the table: 1) Bypasses on matches A and B are only manipulated to control output y_2, and y_2 cannot be controlled with any other bypasses. The reason is simply that the design consists of two subnetworks. This is easy to determine by inspection or mathematical manipulation of the sign matrix, too. 2) Bypasses on the six other matches are manipulated to control the four other bypass-controlled outputs. 3) During normal operation a bypass around the final match is used as manipulated input. This means that the preferred bypass combination for energy coincides with the best bypass combination for control. This very desirable feature frequently occurs for structures with minimum number of matches. 4) If one of the bypasses around the final matches saturates (to zero bypass fraction), an upstream match must be bypassed. This results in a energy penalty. The new bypass must of course introduce an additional degree of freedom, i.e., it cannot already be used to control one of the other outputs. 5) Selecting between alternative bypasses when the bypass-controlled output is not reached, (e.g., between a bypass on D and E for output y_1), has no effect on the utility consumption.

5.6 Discussion

5.6.1 Implementation

The optimal operating points may be implemented with a decentralized control system with varying configuration. Control logics must then be used to select the set of inputs for the different operating conditions. Alternatively, a decentralized control system with constant configuration may be used. To be able to maintain the same control configuration for all operating points, additional manipulated inputs must be used for constraint handling. That is, if any of the *regulatory* manipulated inputs approach saturation, an additional *supervisory* manipulated input is put into action to avoid reaching the limit. The (steady-state) utility consumption for such a decentralized control system with constant configuration will be larger than the utility consumption with varying configutration. The dynamic characteristics will however usually be superior with a constant control configuration, because the inputs often has a direct effect on the output. Thus, there is an interesting trade-off between dynamic and steady-state properties.

5.6.2 Inner matches and splits

The proposed procedure may be applied to many practically important HEN structures. The main limitations are due to inner matches and splits. For HENs with inner matches and splits, the procedure may not be applied to all combinations of utility-controlled streams. The requirement is that one of parallel downstream paths only affect hot or cold utility consumption. For the split example in Fig. 5.4, which may be handled with the procedure, the lower split branch only affect cold utility consumption.

The results indicate that no-split designs without inner matches should be preferred during synthesis of HENs because such designs are simple to operate optimally. Moreover, designs without inner matches and splits will the optimal bypasses for energy coincides with the optimal bypasses for control for the normal case where all bypass-controlled outputs are exceeded with zero bypass fractions.

5.6.3 Assumptions

We here discuss the three main assumptions mentioned in the introduction.

Single utilities

The case of multiple utilities cannot be handled with the proposed procedure as is. One may be able to apply the procedure to a multiple utility problem if one gave different priority to the different utility levels, and optimized one level at a time. Consider, for example the common problem of multiple hot utility levels (i.e. two steam pressure levels) and only one cold utility type (i.e. cooling water). One may then state the optimization problem as:

1. Minimize cold utility. The total heat load of the hot utilities would then be determined by the problem parameters.

2. Minimize hottest (most expensive) hot utility.

In this way, the procedure may be applied to some practical important multiple utility level problems.

Equality constraints

In some cases the performance specifications on the controlled temperatures should be formulated as inequality constraints rather than equality constraints. Typically this occur for hot streams where environmental regulations set an upper limit on the temperature. The procedure may still be used by simply disregarding inactive inequality constraints.

Pinch problem

In order to get a meaningful optimization problem the problem must be a pinch problem since utility consumption is determined from problem parameters for threshold problems. Interestingly, threshold problems become meaningful optimization problems if some target temperature constraints are removed (e.g. inactive inequalities constraints instead of equality constraints). Consider Fig. 5.1 and assume that the target temperature constraint on stream $H1$ is removed. This gives a threshold problem because there is no point in wasting cold utility. However, a meaningful optimization problem consisting of minimizing hot utility consumption still exists. This optimization problem is equivalent to the original problem.

5.7 Conclusions

A procedure for finding the optimal operating point in terms of the input combination that minimizes the utility consumption which is based on structural information only is proposed. A sign matrix for how alternative manipulations (bypass and split fractions) affect the outputs (stream temperatures) is constructed from the network structure. The idea is to manipulate the outputs with the most positive (or least negative) effect on the temperatures upstream the heaters and coolers. The procedure may be used for all networks without inner matches or split fractions. For networks with inner matches and splits the procedure may only be used in special cases.

Nomenclature

A	Heat exchanger area	$[m^2]$
e	Error	$[W/m^2K]$
G	Process transfer matrix	$[-]$
Q	"process heat transfer" (Eq. 5.2)	
r	reference (setpoint)	$[K]$
T	Temperature	$[K]$
u	Manipulated input	$[-]$
y	Controlled output (temperature)	$[K]$
w	Heat capacity flowrate	$[W/K]$

Superscripts

c	cold side/fluid of heat exchanger
Ci	set of cold streams
Cj	set of utility-controlled cold streams
h	hot side/fluid
Hi	set of hot streams
Hj	set of utility-controlled hot streams

Subscripts

c	index for cold utility-controlled outputs
i	index for outputs
j	index for inputs
h	index for hot utility-controlled outputs
t	target temperature
$t-1$	temperature upstream utility-controlled output

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Chapter 6

Control Considerations in Heat Exchanger Network Design

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Abstract

Control and operational aspects may influence design of heat exchanger networks. Inappropriate heat exchanger network design or control configuration design may give fundamental control limitations in terms of right-half-plane zeros or time delays. Control configuration design, which for heat exchanger networks mainly consists of placing bypasses, has been considered in a previous paper. Here it is discussed how network structure, heat exchanger area and the volume in the connecting pipes and the heat exchangers, affect control and operation. The conclusions on network structure are given in terms of a set of heuristics that may be taken into account during heat exchanger network synthesis.

6.1 Introduction

The importance of the interactions between process design or process synthesis and process operation and control was first addressed as early as in 1943. Ziegler and Nichols (1943) point out that the controller and the process form a unit and should by considered simultaneously. They introduced the term *controllability* as a link between these two areas. Recently, the interest in this topic has increased. This is reflected in articles from academia and industry and complete conference sessions or workshops organized by chemical engineering and control institutions like AIChE (American Institute of Chemical Engineers) and IFAC (International Federation of Automatic Control). Perkins (1989), Morari (1992) and Wolff *et al.* (1992) review present techniques and methods to address the interactions between process design and process control.

Synthesis of heat exchanger networks (HENs) is generally recognized to be the most mature field within process synthesis (e.g., Gundersen and Næss, 1988). The HEN synthesis problem is well-defined, and this problem is one of the few particular problems where synthesis methods that consider operation and control has been suggested.

Kotjabasakis and Linnhoff (1986) suggest a structural or heuristic approach. They point out that the downstream path from large disturbances to critical targets should be increased or broken to reduce the effect of the disturbances. This work provides valuable insight on disturbance propagation in HENs, but does not address dynamics and control issues. Moreover, the suggested heuristics seems difficult to apply to real problems where all inlet streams vary to some degree and all or most target temperatures are controlled outputs.

Georgiou and Floudas (1990) use structural matrices and point out that networks that are structural singular from disturbances to outputs are preferred. The main idea seems to be similar to breaking a downstream path suggested by Kotjabasakis and Linnhoff (1986), and the same limitations apply.

Huang and Fan (1992) use an knowledge-based approach and classify both expected inlet stream and allowed outlet stream variations in three groups according to their relative size. This is an interesting approach, but the results largely depend on the how the variables are classified, and this classification seems arbitrary. Moreover, it is not differed between temperature and flowrate disturbances, and these disturbances propagate differently through HENs.

Papalexandri and Pistikopoulos propose to take various operational considerations into account in a mathematical programming formulation (e.g., Papalexandri and Pistikopoulos, 1992; 1994). One problem seems to be that the resulting mixed-integer nonlinear (MINLP) problems become very complex.

We have chosen to include insight about dynamics, control and operation of HENs as heuristic constraints because it is very difficult to quantify operability. These simple heuristics may be included in any synthesis procedure.

Control problems in HENs may arise from a number of reasons (Mathisen *et al.*, 1994a)* :

1. Parallel downstream paths. If there are two parallel downstream paths from

*corresponds to chapter three of this thesis input to output, the two paths will always have competing effects, and may yield a right-half-plane zero or even parametric singularity at steady-state.

2. Large number of matches between manipulated input and controlled output. The input will then have small and slow effect on the output, which gives problems with input constraints both at steady-state and dynamically.

3. Competing flow and temperature effects due to stream mixing. If two independent streams are mixed, increasing the flow of one of the streams may yield a right-half-plane zero or even parametric singularity at steady-state.

4. Interactions (manipulated inputs affect multiple controlled outputs). Interactions will always exist in HENs, and may deteriorate both steady-state and dynamic behaviour. However, interactions may also reduce the utility consumption, the underlying assumption when installing a HEN is that the increased interaction is outweighted by the decreased utility consumption. Still, among designs with similar cost, designs with smaller interactions are preferred.

5. Small temperature changes over the manipulated matches. The bypasses (manipulated inputs) will then have small effects on the outputs. Thus, bypassing streams with phase shift and small matches is discouraged.

6. Large pipe residence times between input and output. This will give a slow response and problems with input constraints dynamically.

7. Small temperature driving forces. The disturbances will then have large effects on the outputs, which often give problems with input constraints at steady-state. HEN problems with parallel hot and cold composite curves often result in such control problems. They may also occur for bad designs with excessive non-vertical heat transfer.

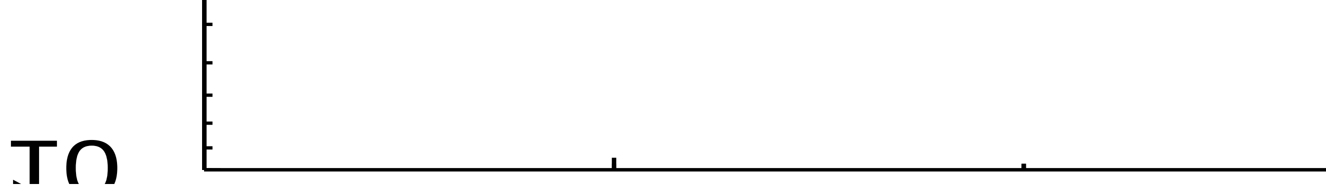
Encouragingly, an appropriate control configuration will usually reduce these control problems. In previous papers we have discussed control configuration design (i.e., bypass placement) for rejection of dynamic disturbances (Mathisen and Skogestad, 1994)* and steady-state optimal operation in terms of minimization of utility consumption (Mathisen *et al.*, 1994b)† In this paper we assume that an optimal, or at least reasonable, control configuration has been selected and discuss how process modifications may improve the controllability. We mainly consider network structure, but some comments to heat exchanger area or heat load distribution and network volume are also given (section 6.5).

6.2 Series matching sequence

Most HENs without splits are straightforward to control provided an appropriate control configuration has been selected. Single, direct effect bypasses as manipulated

*corresponds to chapter four of this thesis

†corresponds to chapter five of this thesis



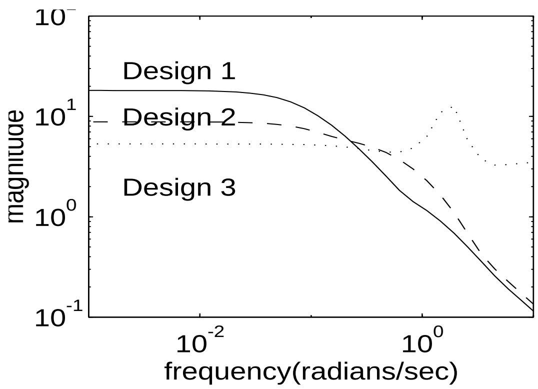


Figure 6.12: a) The necessary input for perfect control, $||G^{-1}G_d||_\infty$

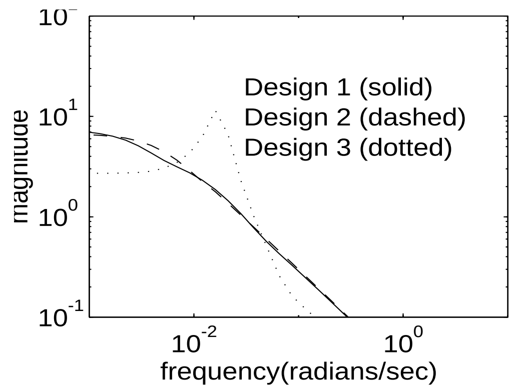


Figure 6.13: b) Closed-loop disturbance gains, $||G_{diag}G^{-1}Gd||)_\infty$

Figure 6.14: Design 1 (non-resilient) and designs 2 and 3 (resilient) of Problem 4 in Townsend and Morari (1984).

like inlet temperatures, heat capacity flowrates, expected disturbance variations and allowed target temperature deviations all affect the controllability of a given HEN structure. However, a final match should not be the first match on streams with large and fast disturbances. An example is given in Fig. 6.17 where the bypass-controlled output y_1 is on stream $H1$ and the disturbance on stream $C1$. With the series matching sequence in Fig. 6.17a disturbance d_1 has a fast effect on output y_1, especially since heat exchangers are countercurrent. With the opposite series matching sequence shown in Fig. 6.17b output y_1 is protected by match 2, and the effect of the disturbance is both slowed down and reduced. The steady-state effect of the disturbance will be particularly well dampened if the thermal effectiveness of the final match is high.

Kotjabaskis and Linnhoff (1986) discuss how changing the series matching sequence may break a downstream path from a disturbance to a controlled output. Georgiou and Floudas (1990) use structural matrices to achieve the same result, i.e., a network that is singular from disturbance to controlled output. These approaches are helpful with one or a few dominating disturbances, but cannot be used to consider industrial cases with steady-state and dynamic disturbances and performance requirements on all or most streams.

6.3 Series or parallel heat exchange

Selection between series and parallel heat exchange represents one of the difficult tradeoffs in HEN synthesis. Stream splitting may reduce area requirements and heat exchanger cost, but will usually increase the piping cost. During HEN synthesis piping costs are usually neglected, and often no-split and split designs are derived in parallel and compared in terms of cost. The no-split design is preferred unless the split design is considerably cheaper. It also seems generally accepted that no-split designs are preferred from a control point of view (e.g., Yee and Grossmann, 1990), but the argumentation is not based on an analysis of controllability.

- Piping cost. As mentioned, piping cost will usually increase with parallel heat exchange, and piping cost is usually neglected during design of HENs.

- Number of control valves. In order to exploit the potential benefits of parallel heat exchange, it will often be required to install an additional control loop. This yields a more complex control system that will be more expensive to purchase and maintain.

- Controller type. In order to exploit the potential benefits of parallel heat exchange, it will often be required to perform a numerical optimization of the split fraction. With simple, no-split design optimal operation may be ensured without such an optimization (Mathisen *et al.*, 1994b).

- Flow-dependent heat transfer coefficients. Heat transfer coefficients depend on flowrate, but this is usually disregarded during design of HENs even for flexibility problems. The flow dependency favour series designs to parallel designs. In a design with two exchangers in parallel, both exchangers must be designed for maximum flow (i.e. maximum allowed pressure drop), and this will correspond to different operating points. Thus, both exchangers cannot be operated optimally (with maximum heat transfer coefficient) at the same operating point (Mathisen and Skogestad, 1992)* .

- Minimization of fouling. Reduced flowrate through an exchanger, even for a short period of time, may give a considerable degradation of exchanger performance for the rest of the campaign. Because exchanger flowrates tend to vary more with parallel exchangers, fouling may be an argument for HEN structures without split streams.

However, it should be noted that parallel heat exchange may in some cases improve disturbance rejection properites. Large flowrate disturbances on the bypass-controlled stream may be difficult to reject with series heat exchange. The reason is that the steady-state effect of flowrate disturbances may increase through heat exchangers in series, see Mathisen *et al.*, (1994a).

6.3.3 Combination of series and parallel heat exchange

Wood *et al.*, (1985) suggest to use a combination of series and parallel heat exchange in order to reduce the number of units. A simple example is shown in Fig. 3.38. A similar combination of bypasses and splits have been suggested for synthesis of HENs for a set of operating points, e.g., Floudas and Grossmann (1987). This arrangement give opposing effects and may give a RHP-zero or even parametric singularity (Mathisen *et al.*, 1994a), and is discouraged. Note that such designs frequently are the result of requiring the exchanger minimum approach temperature (EMAT) to be equal to the heat recovery approach temperature (HRAT). By allowing EMAT to be lower than HRAT, designs without a combination of series and parallel heat exchange with lower

*result also presented in chapter nine of this thesis

Figure 6.27: I

6.5 Further design issues affecting operation

6.5.1 Adding or moving utility exchangers

Manipulating the utility flowrate through heaters or coolers to control the downstream process temperature can be recommended because the effect is often both fast and large, and there is no side-effects. Changing the utility flowrate, changes the heat transfer throughout the utility exchanger, and the effect on the controlled output is only slowed down by the wall capacity. Moreover, the delay due to wall capacitance may be removed by installing a bypass on the process side, and use cascade control.

The steady-state gain depends on the heat load of the utility exchanger. For existing designs, the load are often large for many (if not all) of the steady-state operating points. If utility exchangers are added to existing designs to improve controllability, the heat load will usually be quite small to avoid excessive utility consumption. Therefore, adding utility exchangers to improve controllability is seldom a good idea. An important exception may be designs with double output matches. The design from Linnhoff *et al.*, (1982) in Fig. 6.24 is such an example. A design modification may be to add a cooler on stream C3, see Fig. 6.24b.

However, adding utility exchangers may improve the feasible operating range considerably unless the utility levels (i.e., steam pressure or cooling water temperature) are restrictive.

6.5.2 Area of final matches

First, we consider HENs where area is added to final matches that are used for regulatory control. Adding area on these matches makes it possible to increase the nominal bypass fraction at the steady-state operating point, and improve the dynamic disturbance rejection properties. Larger static disturbances may also be rejected, i.e., the operating range will increase.

6.5.3 Area of matches upstream utility exchangers

Usually matches with downstream utility exchangers on one stream have downstream matches on the other stream, and they are usually not used for regulatory control. Adding area to such matches will always reduce the utility consumption. The heat load reduction on the downstream utility exchanger will always dominate over possible heat load increases on other utility exchangers of the same type (heaters or cooler), see Mathisen *et al.* (1994b). Such matches upstream utility exchangers may be final matches on the opposite stream. In such cases adding area may have a positive effect on both disturbance rejection properties and utility consumption. Occasionally matches may have downstream utility exchangers on both streams. Increasing the heat load on such matches by adding area yield a corresponding reduction in the utility consumption.

6.5.4 Area of inner matches

Inner matches are matches with downstream matches on both streams, and they are usually not used for regulatory control. Increasing the heat load by adding area to such matches will decrease the heat load on both the downstream matches. At least one of these will often be used for regulatory control, and the operating range will decrease. The overall effect on utility consumption may be positive or negative, see Mathisen *et al.*, (1994b).

6.5.5 Volume of the connecting pipes

The best control strategy for disturbance rejection is usually to control all outputs with direct effect bypasses (Mathisen and Skogestad, 1994). Applying direct effect bypasses will usually eliminate problems with large pipe residence times. For illustration, see Fig. 6.32a, and assume that the direct effect bypass (i.e., u_2) is used to control output y_1. We have assumed that the hot stream has considerable pipe volumes wherever possible (i.e., P1 through P9). None of the pipe volumes P1 through P5 affect controllability because they are upstream the manipulated input. Moreover, the pipe volumes P6 through P8 have very little effect on the controllability since changing the bypass flow has an immediate (with incompressible fluids) or very fast (with compressible fluids) effect on the temperature downstream the mixer. The only pipe connection with an appreciable effect on control performance is P9 between the mixer after the bypass and to the controlled output (y_1). In cases when the pipe residence time in the pipe from the final, bypassed match to the controlled output is a control problem, repiping the bypass as shown in Fig. 6.32b or simply moving the temperature measurement as shown in Fig. 6.32c should be considered.

Alternatively, the upstream bypass around match 1 (i.e., u_1) may be used to control output y_1. In that case the pipe connections P5 through P9 are completely downstream the mixer, and may all affect controllability. Control problems may occur for plants where the two heat exchangers are physically far apart, for example if match 1 is in another plant section. The residence time in pipe connections P1 and P5 may then be considerable and limit the controllability, and it may be considered to move the bypass as shown in Fig. 6.32d).

6.5.6 Volume of heat exchangers

Heat exchanger volume mainly depend on heat exchanger area; for shell and tube exchangers commonly used in the chemical process industries the specific area per volume is approximately $100m^2/m^3$. However, space, weight and heat transfer coefficient considerations may make it preferable to apply shell and tube exchangers with fins or compact heat exchangers like plate or spiral exchangers. Such exchangers may have a specific area per volume that may be an order of magnitude larger than ordinary shell and tube exchanger. Due to smaller fluid residence times, they may be preferred from a control point of view. However, above we argued that the pipes immediately before and after the bypassed match, i.e., P7 and P8 when input u_2 is used to control output y_1 in

1. Avoid designs with final splits.
2. Avoid designs with double output matches.
3. Avoid designs with a combination of series and parallel heat exchange.
4. Prefer designs without splits.
5. Prefer designs with few process exchangers.
6. Prefer designs without inner matches.

Note that heuristic 4 includes heuristics 1 and 3, but we include all heuristics since violations of the first three heuristics are worse than violations of the last three. Also note that designs without inner matches must have few process exchangers so that heuristic 5 and 6 largely are overlapping. Still, we include both heuristics because this is convenient when integrating these heuristics in existing synthesis procedures like the pinch design method.

Nomenclature

$d(s)$ - Vector of disturbances.
$G(s)$ - Plant transfer function matrix
$g_{ij}(s)$ - ij'th element of $G(s)$
$G_d(s)$ - Disturbance transfer function matrix
$g_{dik}(s)$ - ik'th element of $G_d(s)$
$u(s)$ - Vector of manipulated inputs.
$u_{y=0}(s)$ - Vector of manipulated inputs necessary for perfect control.
$y(s)$ - vector of outputs

greek
$\Delta(s)$ - Closed loop disturbance gain matrix
$\delta_{ik}(s)$ - ij'th element of $\Delta(s)$
$\Lambda(s)$ - Relative gain matrix
$\lambda_{ij}(s)$ - ij'th element of $\Lambda(s)$

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Chapter 7

Operability Considerations during Heat Exchanger Network Synthesis with the Pinch Design Method

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Abstract

Controllability, flexibility and controller complexity are important issues for the design of heat exchanger networks. In a previous paper heuristics about operable heat exchanger networks have been derived. Here it is shown how operability may be taken into account during interactive synthesis methods like the pinch design method. The operability heuristics are used to select between topological alternatives both in the synthesis and the evolutionary stage. The resulting designs generally have minimum or near minimum cost, good controllability and simple network structures and control configurations that are easy to operate.

7.1 Introduction

Heat exchanger networks (HENs) must be controllable to reject disturbances at a steady-state operating point. Controllability is a property of the plant, and may only be improved by changing the control configuration (i.e., bypass placements) or the process. In a previous paper we have derived heuristics that may be used to ensure that HENs are operable. (Mathisen *et al.*, 1994). In this paper we discuss how the gained insight about operability may be taken into account during synthesis of heat exchanger networks with sequential, interactive techniques like the pinch design method. When designing HENs one is faced with a number of design options both during the synthesis and the evolutionary stages. Heuristics are used to select between the different options, but topology traps may still be a serious problem. Furthermore, operability considerations are partly or completely neglected. We suggest operability heuristics that may be used as a supplement to the existing guidelines. These heuristic may help the designer both during the synthesis stage and the evolutionary stage. During synthesis recommendations regarding the number of utility exchangers and their placement are essential. During evolution recommendations regarding the heat load loop to be broken and the exchanger to be removed may be derived from the operability heuristics. The operability heuristics are illustrated with some simple examples, and it is shown that they typically give HENs with similar total annualized cost and better operability than the designs derived with the pinch design method.

7.2 The pinch design method

The operability heuristics suggested in this paper may be applied to any sequential synthesis procedure, but we consider the pinch design method (Linnhoff *et al.*, 1982) since it is the most popular technique. It is assumed that the reader is familiar with the pinch design method, but we will give a short description of the main steps and heuristics and give some comments regarding control and operation. Like most synthesis techniques, the pinch design method may be divided into three main steps:

1. Targeting
2. Synthesis
3. Evolution

7.2.1 Targeting

Targets for energy, units, area and cost are derived prior to the synthesis stage. By optimizing the trade-off between operating and investment cost through so-called Supertargeting (Ahmad *et al.*, 1990), a reasonable value for the minimum temperature difference ΔT_{min} is obtained. The hot and cold pinch temperatures are also determined. The targets for utility consumption may be computed from the rule that no energy should be transferred across the pinch by using the problem table algorithm (PTA, Linnhoff *et al.* 1982). Designs achieving the utility targets are called maximum energy recovery or minimum energy requirement (MER) designs. Alternatively, the utility consumption may be determined graphically from the composite curves. The composite curve is a temperature-enthalpy (TQ) diagram where all the hot streams are added into the hot composite curve and and all the cold streams into the cold composite curve.

The target for units is then computed from the number of streams (including utilities) above and below pinch. Since some process streams will exist on both sides of the pinch, the unit target for MER designs will be higher than the overall unit target ($U_{min,MER} > U_{min}$). An area target may be computed based on the assumption that all heat transfer is vertical, i.e., that the heat capacity flow ratios of all matches are equal to the ratio between the hot and the cold composite curves in the corresponding enthalpy interval. It should be clear that this is an optimistic assumption. The cost target is then computed from the unit target and the area target assuming the area to be equally distributed among the units. Since exchanger costs are estimated with an economy of scale cost law, this assumption is conservative. The cost target is often quite accurate for pinch problems because the optimistic assumption about vertical heat transfer is approximately cancelled by the conservative assumption of equal area distribution (Ahmad and Linnhoff, 1990).

Note that the targets are usually derived assuming that the HEN is to be operated at the nominal operating point at all times. Both long-term and short-term variations, that will be present in all plants, are neglected. The variations will make it more difficult to exploit the installed match area, and favour a larger HRAT which increases the targets for utility consumption. There is no general relation between the number of units and disturbance rejection properties. However, a HEN with fewer units is usually easier to operate. The target for area is based on vertical heat transfer. From a control point of view this is good because the ability to reject disturbances (i.e., resiliency) increases with better utilization of the driving forces (Townsend and Morari, 1984).

7.2.2 Synthesis

In the synthesis stage of the pinch design method the problem is divided at the pinch and the part above the pinch is designed independently from the part below the pinch. The energy target is reached by applying hot utility only above the pinch and cold utility only below the pinch. The units are placed in sequence. The unit target is achieved by maximizing the heat loads of the units. This tick-off heuristic ensures that the heat load of each unit matches the remaining heat load of at least one of the process streams. In order to ensure that the exchanger approach temperature is greater or equal to ΔT_{min} on both ends, the cold heat capacity flowrate should be greater than the hot heat capacity flowrate above pinch and vice versa below pinch. This tends to give excessive stream splitting at the pinch, so it has been suggested to specify an exchanger minimum approach temperature (EMAT) that is smaller than the heat recovery approach temperature (HRAT) instead of one common ΔT_{min} as in the original pinch design method.

The user is faced with a number of design options in the synthesis stage. Later refinements of the original pinch design method like driving force plots (DFP, Linnhoff and Vredeveld, 1984) and remaining problem analysis (RPA, Ahmad and Linnhoff, 1990) may be used to find MER designs with total area close to the target. However, MER design with minimum area may not have minimum cost because the total cost also depends on how the area is distributed among the exchangers. Furthermore, it may be impossible to derive the optimal design from the best MER design in the following evolutionary stage due to topology traps (Gundersen and Grossmann, 1988; Trivedi *et al.* 1989; Sagli *et al.*, 1990). Here we will illustrate how operability considerations may be used to select between different design options. Note that the heuristics presented here are a supplement to, not a replacement of, the existing procedure based on the driving force plot and the remaining problem analysis. They will not eliminate the problems with topology traps, but make it more probable that the final design is easy to control and operate.

7.2.3 Evolution

The MER designs derived with the pinch design method tend to be expensive and complicated with many splits and heat load loops. Therefore, one tries to simplify the design by heat load loop breaking in a third evolutionary stage. The loop breaking imply that either area and/or utility consumption must increase, i.e., the restriction on either EMAT and/or HRAT must be relaxed. It may be difficult to select the heat load loop to break, and heat load loops may be broken by removing either one of two units. In the original pinch design method, the heuristic is to select the heat load loop with the smallest unit, and to remove this unit. These two heuristics works well in most cases, but may also give excessive energy penalties. A number of alternative heuristics have been proposed, the most recent by Trivedi *et al.*, (1990). These authors point out that the pinch and the fixed inlet temperatures limit the energy relaxation. Some matches cannot increase their heat load without a corresponding increase in the utility consumption, whereas other matches cannot be removed without making another match infeasible.

Note that breaking heat load loops simplifies the HEN and this will usually make it easier to control and operate. Breaking heat load loops will usually make the heat exchange less vertical, reduce the average temperature driving force for the matches and may make the design less resilient to disturbances. However, breaking of heat load loops usually also involve relaxation of energy, which tend to make the design more resilient. Furthermore, breaking heat load loops may result in designs that violates the operability heuristics suggested in this paper. We will explain how the operability heuristics may be a supplement to the existing heuristics for breaking heat load loops from Linnhoff *et al.* (1982) and the ones from Trivedi *et al.*, (1990).

7.3 Operability heuristics

Operability considerations are difficult to quantify mathematically. Therefore, we have proposed a set of heuristics that describe HENs with good operability (Mathisen and

Skogestad, 1994)* . The heuristics are only based on structure in order to make them general.

O1. Avoid designs with final splits. A final split exists if the split streams are not recombined before entering the final unit, i.e., when a remixed split stream is a controlled output.

O2. Avoid designs with double output matches. Double output matches are process exchangers where both the hot and the cold outlet temperatures are controlled outputs.

O3. Avoid designs with a combination of series and parallel heat exchange. In the original pinch design method and most other pinch-based methods one may not derive such designs so this heuristic is automatically fulfilled.

O4. Prefer designs without splits.

O5. Prefer designs with few process exchangers.

O6. Prefer designs without inner matches. Inner matches are process exchangers with downstream process exchangers on both the hot and the cold side.

Note that heuristic O4 includes heuristic O1. A violation of heuristic O1 is more severe than a violation of heuristic O4.

These heuristics may be used directly for analysis of HENs, i.e. for comparing the expected operability of alternative designs. When applying the pinch design method on HEN problems, it may be helpful to derive more specific design (synthesis/evolution) heuristics.

7.3.1 Synthesis stage

The pinch heuristics are:

- Low energy requirement. Apply hot utility only above pinch, and cold utility only below pinch.

- Few units. Maximize the heat load on the matches (tick-off rule). This heuristic ensures (in most cases) that the unit target (based on decomposition at pinch) is achieved.

- Low area requirement. In order to achieve the area target, the heat capacity flowrate ratio of the matches should be equal to the heat capacity flowrate ratio of the composite curves in the corresponding enthalpy interval (assuming equal film coefficients for the streams). This requires an excessive or "maximum" number of matches as well as a considerable number of stream splits. In practice, one tries to minimize the area target while achieving the unit target.

*corresponds to chapter six of this thesis

Often MER designs which include many splits and heat load loops and violate several of the operability heuristics may easily be relaxed into operable designs in the evolutionary stage. Thus, the operability heuristics are softer in the synthesis stage than during analysis. In the synthesis stage one should rather violate the operability heuristics than the pinch heuristics about utility aconsumption and number of units. However, the requirement on total area, which always will be somewhat relaxed to achieve the unit target, may be further relaxed. This will be illustrated in example 1.

The operability heuristics for the synthesis stage are:

No final splits

Pinch-based methods decompose the problem at the pinch, and stream splitting at the pinch is often required to be able to adhere to the pinch heuristics on heat capacity flowrate ratios for the pinch matches. Stream splitting at the pinch may be accepted from heuristic O1 because this heuristic only address final splits. However, heuristic O1 may be applied when ticking-off streams. Streams that are split at the pinch should be remixed before the final unit:

S1. Tick-off streams using series heat exchange.

Above pinch the heuristic applies to all cold streams and to hot streams that only exist above pinch. Below pinch it applies to all hot streams and to cold streams that only exist below pinch.

No double output matches

In the synthesis stage, heuristic O2 only applies to matches where the hot stream only exists above pinch or matches where the cold stream only exists below pinch. Since the problem is decomposed at the pinch, ticking off both streams simultaneously is only possible for such matches.

S2. Tick-off at most one stream per match.

Note that we now consider the problem without decomposition at the pinch, i.e., the heuristic only applies to matches between streams that have their target temperatures in the same pinch region.

No splits

Stream splitting at the pinch is often unavoidable from an energy point of view and unfavourable from an area point of view. Sometimes series heat exchange may be chosen to parallel, and then this heuristic may be applied as is, i.e., S4=O4.

Few process exchangers

In the synthesis stage MER designs with minimum number of units ($U = U_{min,MER}$) are generated. Still, the number of process exchangers may vary. Ticking-off hot streams rather than cold streams above pinch, and cold streams rather than hot streams below pinch, will minimize the number of process exchangers while achieving the unit target.

S5. Tick-off hot streams first above pinch, and cold streams first below pinch.

No inner matches

In the synthesis stage, the other heuristics will often ensure that there are no inner matches in one pinch region. However, inner matches are quite common for MER designs, and they are considered in the evolutionary stage.

7.3.2 Evolutionary stage

In the evolutionary stage one needs heuristics to select: 1) What heat load loop to be broken; 2) How to break the heat load loop, i.e., what match in the heat load loop to remove.

The pinch heuristics for loop-breaking are (Linnhoff *et al.*, 1982):

- Break the heat load loop with smallest unit.
- Remove the smallest unit in the selected heat load loop.

Although it is quite easy to construct counterexamples (e.g., Trivedi *et al.*, 1990), removing the smallest unit yields a small energy penalty and reduces overall cost in most cases. Note that two heuristics are required because the smallest unit in the design may not be part of a heat load loop.

Trivedi *et al.* (1990) suggest several refinements to these heuristics for energy relaxation, e.g., that heat load loops with process exchangers should be broken before heat load loops with utility exchangers. This is largely compatible with the operability heuristics as selecting match heat load loops ensures that a match rather than a utility exchanger is removed, and often eliminates inner matches. Attempts have also been made to formulate heuristics to remove splits (e.g., Nishida *et al.*, 1977).

We will try to explain how the operability heuristics may be used to suggest all structural changes of the network such as removing or adding units or splits.

Final splits

Final splits may be dealt with by including a small utility exchanger at the end of the split stream, but basic pinch rules must not be violated.

E1a. Change designs with final splits to series heat exchange.

E1b. Place coolers on hot streams with final splits below pinch and heaters on cold streams with final splits above pinch.

Double output matches

The evolutionary heuristics about double output matches are straightforward to derive:

E2a. Do not remove a match that gives another match a second controlled output.

E2b. Avoid double output matches below pinch by adding coolers and double output matches above pinch by adding heaters.

After a design has been relaxed, matches may straddle the pinch, and either a heater or a cooler may be used.

No splits

Parallel heat exchange may be changed to series heat exchange in two ways:

E4a. Change designs with splits to series heat exchange.

E4b. Remove matches with parallel heat exchange.

Only towards the end of the evolution one should try to eliminate remaining splits because the splits add degrees of freedom that may be exploited to minimize the energy and/or area penalty when removing matches.

Few process exchangers

The obvious way to reduce the number of process exchangers in the evolutionary stage is to combine process exchangers that are adjacent on both streams. This is sometimes used actively in the synthesis stage of sequential design procedures e.g., Trivedi *et al.*, (1988); Wood *et al.*, (1991). Trivedi *et al.* (1990) argue that heat load loops involving only process exchangers should be broken before heat load loops involving utility exchangers. This is usually a reasonable heuristic because initially small utility exchangers may often increase in size in the evolutionary stage because energy requirements are relaxed. Trivedi *et al.* (1990) also experienced that further evolution of structures with few utility exchangers tends to be difficult, i.e. requires large energy penalties. Thus, removing utility exchangers may give a topology trap.

E5a (Trivedi). Break heat load loops involving only process exchangers before heat load loops involving utility exchangers.

E5b. Remove process exchangers rather than utility exchangers.

Note that this heuristic may be in conflict with the pinch heuristic about removing the smallest unit.

Inner matches

E6a. Break heat load loops involving inner matches.

E6b. Remove process exchangers that eliminates inner matches.

Inner matches may be eliminated by removing the inner match itself or the downstream match on the hot or the cold side.

Stream	$T_s[^oC]$	$T_t[^oC]$	$w[kW/^oC]$	$h[W/m^2,^oC]$
$H1$	150	60	20	100
$H2$	90	60	80	100
$C1$	20	125	25	100
$C2$	25	100	30	100

Annualized cost of process and utility exchangers ($[\$/year]$):

$$(C_1 + C_2 A_{hx}^m) F_{install} F_{payback} = 8600 + 670 A_{hx}^{0.83} (m^2) 3\frac{1}{3}$$

Cost of utilities ($[\$/year]$):

$$(C_{HU} Q_{HU} + C_{CU} Q_{CU}) F_{online} 8760 = (3e{-}5 Q_{HU} + 3e{-}6 Q_{CU}) 8600$$

Table 7.1: Stream and cost data for Example 1 from Linnhoff and Hindmarsh (1983)

7.3.3 Area optimization

After a unit has been deleted, the areas of the units in the remaining HEN structure should be optimized, preferably by relaxing constraints on both HRAT and EMAT. This requires a suitable optimization procedure. In the interactive pinch design method HRAT is usually relaxed whereas EMAT is kept by shifting heat through a heat load path. In some cases there may be several possible heat load paths, and it may be difficult to pick the best one. Trivedi *et al.*, (1990) points out that a path that decreases the heat load on pinch matches should be selected.

We would like to point out that in some cases it may be possible to select a path that increases the temperature driving forces, i.e., reduces the area requirement of all the traversed matches. Such paths follow cold streams countercurrently and hot streams cocurrently.

7.4 Examples

7.4.1 Example 1 from Linnhoff and Hindmarsh

The first example is a four stream problem from Linnhoff and Hindmarsh (1983), which has been extensively studied by Gundersen and coworkers (Gundersen and Grossmann, 1988; Sagli *et al.*, 1990; Gundersen *et al.*, 1991). The problem data are given in Table 7.1.

Synthesis stage

From supertargeting HRAT was selected to 20K, and the pinch temperature is $90/70^0C$ caused by stream H2. Minimum hot and cold utility requirements are $1075kW$ and $400kW$, respectively. Sagli *et al.* (1990) derived MER designs with EMAT=20K.

Stream	$T_s[^oC]$	$T_t[^oC]$	$w[kW/^oC]$
$H1$	250	70	3.5
$H2$	170	70	6
$C1$	60	160	5
$C2$	110	260	4

Table 7.3: Stream data for example 2 from Trivedi *et al.*, 1990)

if the requirement on EMAT or HRAT is relaxed unless the composite curves are almost parallel in the pinch region. For these reasons we feel that heuristic O3, in its given simple and general form, is justified.

Conclusions from example 1

The general conclusion is that the operability heuristics works well for this example. The heuristics help the designer both in the synthesis and the evolutionary stage to resolve difficult design options, and a near-minimum cost design that fulfills all operability heuristics is derived. Some specific conclusions:

- There are three design options in the synthesis stage, and thus eight possible MER designs. By applying the operability heuristics two of the three options may be resolved. The resulting "operable" MER designs are the two most expensive MER designs.

- By applying the heuristics during the evolutionary stage either one of the two operable MER designs gives the least expensive six unit design. This could indicate that applying the operability heuristics reduces the problems with topology traps. The best six unit design violates none of the operability heuristics and may be accepted as the final design.

- A five unit design that is slightly less expensive than the best six unit design exists. This five unit design includes a double output match and should be rejected.

7.4.2 Example 2 from Trivedi et al.

We now consider a simple four stream example studied by Trivedi *et al.* (1990). An interesting feature of this problem is that one of the streams that do not cause the pinch ($C1$) only exists at one side of the pinch. The stream data are given in Table 7.3. No film coefficients or cost data are given.

Synthesis stage

Trivedi *et al.* (1990) use $HRAT(= EMAT) = 10^0C$ which gives a pinch point at $170/160^0C$ generated by stream $H2$. Above pinch there is only one hot and one cold

Stream	$T_s[^oC]$	$T_t[^oC]$	$w[kW/^oC]$	$h[W/m^2,^oC]$
$H1$	160	110	7.032	1600
$H2$	249	138	8.44	1600
$H3$	227	106	11.816	1600
$H4$	271	146	5.6	1600
$C1$	96	160	9.144	1600
$C2$	116	217	7.296	1600
$C3$	140	250	18	1600

Annualized cost of process and utility exchangers ([\$/$year$]):

$$(C_1 + C_2 * A_{hx}^m)F_{install}F_{payback} = 0 + 670A_{hx}^{0.6})3\frac{1}{3}$$

Cost of utilities ([\$/$year$]):

$$(C_{HU}Q_{HU} + C_{CU}Q_{CU})F_{online}8760 = 0.08Q_{HU} + 0.02Q_{CU}$$

Table 7.4: Stream and cost data for Example 3 from Ciric and Floudas (1991)

Conclusions from example 2

- With cold (hot) streams only below (above) pinch the heuristics about avoiding final splits and double output matches may be used during the synthesis stage.

- Both MER designs may be evolved into a simple design which fulfills all operability heuristics, but the preferred MER design yield a design where the operability heuristics in a way are more than fulfilled because the design include only two process exchangers and is decoupled. Such designs are very easy both to control and to operate optimally, i.e., use a bypass on match 3 to control T_t^{C1} and never bypass match 2 as this match only affects utility controlled outputs (see Mathisen *et al.*, 1994). Note that this design may be arrived at by using the existing synthesis and evolutionary heuristics, too. The operability heuristics only help finding it.

7.4.3 Example 3 from Ciric and Floudas

In order to illustrate the procedure for more realistic problems, consider the following industrial problem presented by Ciric and Floudas, (1991). We use the problem data from Trivedi et al., (1990), which refer to Floudas and coworkers, but unfortunately there is a small discrepancy. The data are shown in Table 7.4.

match 2 is upstream match 5a with a large heat load and small temperature driving forces, there is an area incentive for removing match 2. The overall cost effect should be evaluated. Removing match 5b cannot be recommended from heuristic E2a since it would leave match 4 with two target temperatures, so that heuristic O2 is violated.

Finally, one may consider removing splits and utility exchangers. Removing the splitter on stream C3 is expected to yield a substantial energy penalty because the branch matches are large. Removing the small cooler on stream C4 is more tempting, but this would make EMAT equal to 6^0C for match 12. Since match 12 has the smallest temperature difference on the cold side, it is determined as the difference between an inlet temperature and a target temperature. Thus, removing cooler C4 cannot be recommended from flexibility reasons.

Conclusions from example 3

- The operability heuristics may be applied to industrial problems.
- Parallel heat exchange on the stream C3 with a large heat capacity flowrate and heat load is recommended.
- The possibility of generating a double output match restricts the evolutionary options.

7.5 Conclusions

It is shown how operability considerations may be taken into account during interactive synthesis with the pinch design method. From operability heuristics for analysis of HENs, more specific heuristics for the synthesis and evolutionary stages have been derived. The most important ones are:

S1. Tick-off streams using series heat exchange.

S2. Tick-off at most one stream per match.

S5. Tick-off hot streams first above pinch, and cold streams first below pinch.

E2a. Do not remove a match that gives another match a second controlled output.

E4b. Remove matches with parallel heat exchange.

E5b. Remove process exchangers rather than utility exchangers.

E6b. Remove process exchangers that eliminates inner matches.

The heuristics may be applied as a supplement to existing pinch heuristics to select between topological alternatives. The procedure is illustrated through several examples from the literature, and the resulting designs generally have near minimum cost, good controllability and simple network structures and control configurations that are easy to operate.

Nomenclature

$A_h x$ - Heat exchanger area $[m^2]$
N_{hx} - No. of process heat exchangers in HEN
U_{min} - Minimum number of units
T_t - target temperature
T_s - supply temperature
$u(s)$ - Vector of manipulated inputs.
$y(s)$ - vector of outputs
w - heat capacity flowrate $[W/K]$
MER - maximum energy recovery
$HRAT$ - heat recovery approach temperature
HLD - heat load distribution
$EMAT$ - exchanger minimum appraoch temperature
HEN - heat exchanger network

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Chapter 8

Operability Considerations in Heat Exchanger Network Synthesis with Mathematical Programming

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Abstract

One of the two main approaches for solving heat exchanger network synthesis problems is based on mathematical programming. In theory, mathematical programming formulations enable simultaneous optimization of network structure, exchanger areas and utility consumption, but in practice the minimum cost design is difficult to identify due to local optima and the combinatorial nature of the problem. Another problem, addressed here, is that the minimum cost solution may be suboptimal because of control or operating considerations. It is shown how novel heuristics for operability may be taken into account by imposing restrictions on the binary variables. Some mathematical programming models for heat exchanger network synthesis have the advantage that all the constraints are linear. To maintain this feature, the operability heuristics are formulated as linear inequalities. Designs with near minimum cost that fulfill all or all but one of the operability heuristics are identified for the tested examples.

8.1 Introduction

The heat exchanger network (HEN) synthesis problem consists of a set of process streams that are to be cooled or heated from supply to target temperatures and a set of hot and cold utilities. The task is to design the optimal heat exchanger network, and optimal generally means minimum total annualized cost. We use examples from the literature which include the following assumptions:

- Constant film coefficients, heat capacity flowrates, supply and target temperatures
- Ideal mixing
- Countercurrent heat exchangers
- Negligible pressure drops
- Capital cost of HEN includes only heat exchanger costs
- Economy of scale cost laws for the heat exchangers
- Operating cost includes only utility costs
- Single hot and cold utilities

Note that supply and target temperatures and heat capacity flowrates are fixed; the standard HEN synthesis problem is defined for a single steady-state operating point. Even with these simplifying assumptions, the HEN synthesis problem is difficult because it includes a large number of discrete and continuous variables. Two approaches for the solution of the synthesis problem are commonly used; thermodynamic approaches using pinch technology and algorithmic approaches using mathematical programming. We will consider algorithmic methods using mathematical programming in this paper.

The main advantage of mathematical programming is the ability to automate the synthesis task and simultaneously optimize heat recovery level, HEN structure and exchanger areas. Furthermore, forbidden, restricted or required matches may easily be included in the problem formulation. However, solving synthesis problems with matematical programming is a difficult task:

1. Algorithms. There are no conventional mathematical algorithms for solving mixed-integer nonlinear programming (MINLP) problems. Grossmann (1990) reviews recent developments and applications to process synthesis and engineering. Non-convexities, e.g. due to economy of scale cost laws for the heat exchangers, may make existing algorithms cut into and eliminate part of the feasible region.

2. Local optima. Even with fixed binary variables the non-linearities, e.g. due to using the logarithmic mean as the temperature driving force in the heat exchangers, may give local optima.

3. Combinatorial nature. MINLP problem size grows exponentially with the number of binary (integer) variables (worst case).

4. Operability. In addition to quantitative design objectives regarding investment and utility cost, there are qualitative design objectives related to the operability. Operability of HENs may be determined from controllability, controller complexity and flexibility.

We will mainly consider how operability may be taken into account. Floudas and Grossmann (1987a,1987b) formulate the steady-state flexibility problem for a set of discrete operating points, i.e., the multiperiod problem, and for continuously varying operating points, i.e., the parameter range problem. A main problem of their approach is that the heat recovery level is prespecified (Mathisen *et al.*, 1992).* More recently, Papalexandri and Pistikopoulos (1992, 1994) have addressed the operability problem with mathermatical programming. They formulate a very detailed MINLP-model where various aspects of operability are quantified. This yields a very complex model, and the solutions to the simple examples presented are not convincing.

The objective of this paper is to explain how some simple structural criteria for HENs with good operability may be included when solving HEN synthesis problems with mathematical programming. First we present the operability heuristics. Second, we formulate the heuristics as linear inequality constraints to the optimization problem. A key point is the use of binary variables for identifying the final match on each stream. The operability constraints are included in the stagewise model for simultaneous optimization of HEN synthesis problems introduced by Yee *et al.*, (1990). Finally, we illustrate that the suggested formulation works, i.e., that the resulting HEN designs fulfill the specified operability requirements on two problems from the literature.

8.2 Operability heuristics

Operability considerations are difficult to quantify mathematically. Therefore, we have proposed a set of heuristics that describe HENs with good operability (Mathisen and Skogestad, 1994)† . The heuristics are only based on structure in order to make them general. The heuristics are (in approximate order of importance):

O1. Avoid designs with final splits. A final split exists when a remixed split stream is a controlled output. An example is given in Fig. 8.5a.

O2. Avoid designs with double output matches. Double output matches are process heat exchangers where both the hot and the cold outlet temperatures are controlled outputs. An example is given in Fig. 8.5b.

O3. Avoid designs with a combination of series and parallel heat exchange. Such designs are suggested by Wood *et al.*, (1985) in order to reduce the number of units while maintaining the utility target. An example is given in Fig. 8.5c.

*results also presented in chapter nine of this thesis

†corresponds to chapter six of this thesis

2. Operability; no double output matches (02), no splits (O4) and no inner matches (O6).

In the optimizations we generate a number of solutions by disallowing the solutions previously found, and sometimes we also include constraints on the utility consumption and the number of units:

1. Disallow specific solutions (yields a solution order).
2. Bounds on utility consumption (HRAT).
3. Bounds on the number of units.

8.3 Synthesis with mathematical programming

The best *simultaneous* approach seems to be the stagewise model by Yee and coworkers (Yee *et al.*, 1990; Yee and Grossmann, 1990). We use an Outer approximation (Duran and Grossmann, 1986; Viswanathan and Grossmann, 1990a) implementation of Yee *et al.*'s model in GAMS (Brooke *et al.*, 1992) with Dicopt++ (Viswanathan and Grossmann, 1990b), where SCICONIC (Scicon ltd., 1986) is used to solve the MILP problem and MINOS (Murtagh and Saunders, 1985) is used to solve the NLP problem.

8.3.1 Yee's stagewise superstructure

Yee *et al.*'s model is thoroughly and clearly presented in Yee *et al.*, (1990) and Yee and Grossmann (1990). Later we will describe how the operability heuristics are implemented in this MINLP model. Therefore, we will briefly explain Yee *et al.*'s main idea which is the stagewise superstructure. The indices i, j, k are used to denote hot streams, cold streams and stages or temperature locations, respectively. The sets HP, CP and ST include the N^h hot streams, the N^c cold streams and the N^k stages, respectively.

The idea of dividing a HEN synthesis problem into temperature stages or intervals is old (e.g., Grossmann and Sargent, 1978), the novelity of Yee *et al.*'s approach is that the temperatures after the stages are treated as independent variables during the optimization. For example, the temperature of hot stream i at stage k, $T^h(i,k)$, is free to vary. The only requirement is that the temperature decreases monotonically downstream a hot stream and increases monotonically downstream a cold stream. This may be expressed as:

$$T^h(i,k) \geq T^h(i,k+1), \quad i \in HP \quad k \in ST \tag{8.1}$$

$$T^c(j,k) \geq T^c(j,k+1) \quad j \in CP \quad k \in ST \tag{8.2}$$

since it is assumed that the cold streams flow countercurrently to the hot streams as in the grid diagram of HENs. In stage k hot stream i may be matched with zero, one or several of the N^c cold streams. Thus, the hot streams must be split into N^c branches in each stage in the superstructure. Similarly, the cold streams are split into N^h branches in each stage. The resulting superstructure for a problem with two hot

represented in different ways if the number of stages is larger than required, i.e., each design may have duplicate solutions.

8.4 Formulation of logical constraints

8.4.1 Logic statements

The operability heuristics may be expressed through the logic statements below. For clarification separate statements are given for the hot and the cold streams:

A1 : Hot stream i is split and has parallel heat exchange in stage k

A2 : Cold stream j is split and has parallel heat exchange in stage k

B1 : Hot stream i has a final cooler

B2 : Cold stream j has a final heater

C1 : Hot stream i has no process exchangers downstream stage k

C2 : Cold stream j has no process exchangers downstream stage k

8.4.2 Binary variable for final matches

In Yee *et al.*'s MINLP formulations for HEN synthesis problems there is a binary variable $Z(i,j,k)$ for the occurrence of process exchamgers between hot stream i and cold stream j in stage k. Hot stream i is split if $\sum_{j \in CP} Z(i,j,k) > 1$. Similarly, cold stream j is split if $\sum_{i \in HP} Z(i,j,k) > 1$. Furthermore, the binary variables $Z^c_{util}(i)$ and $Z^h_{util}(j)$ denote the existence of a final cooler on hot stream i and a final heater on cold stream j, respectively. We introduce new sets of binary variables, $Z^h_{fin}(i,k)$ for the hot streams and $Z^c_{fin}(j,k)$ for the cold streams, to store information about final process exchangers. Specifically, the variable for the hot streams $Z^h_{fin}(i,k)$ is set to unity if stage k is the last stage hot stream i has a match, and for the all remaining stages $k+1$, $k+2$,..,N_k, i.e.:

$$\sum_{k1=k+1}^{N^k} \sum_{j \in CP} Z(i,j,k1) = 0 \Rightarrow Z^h_{fin}(i,k) = 1; \quad i \in HP,\ k \in ST \tag{8.3}$$

$$\sum_{k1=k+1}^{N^k} \sum_{j \in CP} Z(i,j,k1) \geq 1 \Rightarrow Z^h_{fin}(i,k) = 0; \quad i \in HP,\ k \in ST \tag{8.4}$$

The variable for the cold streams $Z^c_{fin}(j,k)$ is set to unity if stage k is the last stage cold stream j has a match, and for the all remaining stages $k-1$, $k-2$,..,1.

$$\sum_{k2=1}^{k-1} \sum_{i \in HP} Z(i,j,k2) = 0 \Rightarrow Z^c_{fin}(j,k) = 1; \quad j \in CP,\ k \in ST \tag{8.5}$$

$$\sum_{k2=1}^{k-1} \sum_{i \in HP} Z(i,j,k2) \geq 1 \Rightarrow Z^c_{fin}(j,k) = 0; \quad j \in CP,\ k \in ST \tag{8.6}$$

The value of the binary variable $Z^h_{fin}(i,k)$ can be determined by the following two equations:

$$N_k(1 - Z^h_{fin}(i,k)) - \sum_{k1=k+1}^{N^k} \sum_{j \in CP} Z(i,j,k1) \geq 0; \quad i \in HP,\ k \in ST \tag{8.7}$$

$$Z^h_{fin}(i,k) + \sum_{k1=k+1}^{N^k} \sum_{j \in CP} Z(i,j,k1) - 1 \geq 0; \quad i \in HP,\ k \in ST \tag{8.8}$$

The value of the binary value $Z^c_{fin}(j,k)$ is determined analogously:

$$N_k(1 - Z^c_{fin}(j,k)) - \sum_{k1=1}^{k-1} \sum_{i \in HP} Z(i,j,k1) \geq 0; \quad j \in CP,\ k \in ST \tag{8.9}$$

$$Z^c_{fin}(j,k) + \sum_{k1=1}^{k-1} \sum_{i \in HP} Z(i,j,k1) - 1 \geq 0; \quad j \in CP,\ k \in ST \tag{8.10}$$

8.4.3 Equations for the heuristic constraints

O1: No final splits

Heuristic O1 about final splits on hot and cold streams may be expressed through the following logic statements:

$$\neg(A1 \wedge \neg B1 \wedge C1) \Leftrightarrow \neg A1 \vee B1 \vee \neg C1 \tag{8.11}$$

$$\neg(A2 \wedge \neg B2 \wedge C1) \Leftrightarrow \neg A2 \vee B2 \vee \neg C2 \tag{8.12}$$

where $\neg$ is logical negation, $\wedge$ is logical conjunction and $\vee$ is logical disjunction.

The binary variable for final process exchangers makes it possible to express this heuristic with the following linear inequalities:

$$\sum_{j \in CP} Z(i,j,k) - 1 - N^c(1 - Z^h_{fin}(i,k) - 2N^c Z^c_{util}(i)) \leq 0; \quad i \in HP, \quad k \in ST \tag{8.13}$$

$$\sum_{i \in HP} Z(i,j,k) - 1 - N^h(1 - Z^c_{fin}(j,k) - 2N^h Z^h_{util}(j)) \leq 0; \quad j \in CP, \quad k \in ST \tag{8.14}$$

O2: No double output matches

Heuristic O2 about disallowing a match to be the final unit on both the hot and the cold stream may also be expressed through the logic statements:

$$\neg((\neg B1 \wedge C1) \wedge (\neg B2 \wedge C2)) \Leftrightarrow B1 \vee \neg C1 \vee B2 \vee \neg C2 \tag{8.15}$$

The heuristic may therefore be represented with the following equation:

$$Z(i,j,k) - (1 - Z^h_{fin}(i,k)) - Z^c_{util}(i) - (1 - Z^c_{fin}(j,k)) - Z^h_{util}(j) \leq 0; \quad i \in HP,\ j \in CP,\ k \in ST \tag{8.16}$$

O3: No combination of series and parallel heat exchange

We will use Yee *et al.*'s stagewise model to illustrate the effect of the operability constraints. Combinations of series and parallel heat exchange as decribed in heuristic three is then not included in the superstructure of allowed alternatives. Thus, heuristic three will automatically be fulfilled.

O4: No splits

Yee *et al.* (1990) point out that it is straightforward to disallow parallel heat exchange in the stagewise model. One may include the following restrictions to the binary variable for the occurence of process exchangers:

$$\sum_{i \in HP} Z(i,j,k) \leq 1; \quad j \in CP, \quad k \in ST \tag{8.17}$$

$$\sum_{j \in CP} Z(i,j,k) \leq 1; \quad i \in HP, \quad k \in ST \tag{8.18}$$

Since heuristic O4 includes heuristic O1, the constraints in Eq. 8.13 and 8.14 may be removed when constraints Eq. 8.17 and 8.18 are active.

O5: Few process exchangers

The fifth heuristic states that solutions with fewer process exchangers are preferred, e.g., if two solutions with the same number of units have similar cost, the solution with fewer process exchangers (or, equivalently, more utility exchangers) is preferred. One way to implement this heuristic is to restrict the number of process exchangers to some function of the number of streams and/or the number of units. In our experience, the number of streams ($N^h + N^c$) is a good upper bound for the number of process exchangers both for cost-optimal and controllable near-optimal designs. Such a restriction is very simple to include:

$$\sum_{i \in HP} \sum_{j \in CP} \sum_{k \in ST} Z(i,j,k) \leq N^h + N^c \tag{8.19}$$

O6: No inner matches

Heuristic O6 about disallowing inner matches is fulfilled if all the process exchangers are the final process exchanger on either the hot or the cold stream. Note that we in this case only consider *process exchangers*, there may be additional utility exchangers. This requirement is conveniently expressed through logical statements, i.e.:

$$\neg(\neg C1 \wedge \neg C2) \Leftrightarrow C1 \vee C2 \tag{8.20}$$

Since we have binary variables $Z^h_{fin}(i,k)$ and $Z^c_{fin}(j,k)$ to identify final matches, this may be implemented with the following equation:

$$Z(i,j,k) - Z^h_{fin}(i,k) - Z^c_{fin}(j,k) \leq 0; \quad i \in HP, \quad j \in CP, \quad k \in ST \tag{8.21}$$

Stream	$T_t[^oC]$	$T_t[^oC]$	$w[kW/^oC]$	$h[W/m^2,^oC]$
$H1$	150	60	20	100
$H2$	90	60	80	100
$C1$	20	125	25	100
$C2$	25	100	30	100
ST	180	180		100
CW	10	15		100

Cost of matches and utility exchangers:

$$C_{hx} = 8600 + 670A_{hx}^{0.83} \tag{8.22}$$

Cost of utilities:

$$ST : C_{HU} = 0.03\$/kWh \qquad CW : C_{CU} = 0.003\$/kWh \tag{8.23}$$

Operating hours per year: 8600 Payback time: 3 Installation Factor: 3

Table 8.1: Stream and cost data for Example 1 from Linnhoff and Hindmarsh (1983)

8.5 Examples

8.5.1 Example 1 from Linnhoff and Hindmarsh

The first example is a four stream problem from Linnhoff and Hindmarsh (1983). Later it has been extensively studied by Gundersen and coworkers (Gundersen and Grossmann, 1988; Sagli *et al.*, 1990; Gundersen *et al.*, 1991). The problem data are given in Table 8.1. To simplify the synthesis problem in order to limit the well-known problems with local optima, it was first decided to apply the cost constraints by limiting both the heat recovery level and the number of units. With HRAT fixed at 20^0C, there are 4 feasible heat load distributions (HLD) with five units, and ten feasible designs as each HLD has two or three feasible structures. For two of the ten designs three stages in Yee *et al.*'s model are needed, and with three stages 34 different combinations of the binary variables yield a feasible solution. We generated all the possible combinations with automatic integer cuts, and ideally, the cost should be monotonically increasing.

Explanation of tables. All the tables summarizing the results from the synthesis exercises are similar with one row for each different design. Several solutions were generated for each case by disallowing previous solutions, and column 1 gives the order in which the designs were identified. Note designs that do not utilize all stages may have duplicate (identical) solutions and may be identified repeatedly. Column 2 gives the heat load distribution of the design as defined by Gundersen and coworkers, i.e., number of units and order in terms of cost. Column 3 gives the structure of the design since each heat load distribution may have several feasible sequences. The first five unit design is for example denoted 2S34C2H1, where the first three digits (2, 3 and 4)

Order	$HLD^\dagger$	Structure	A^*_{tot}	$Cap.cost^*$	TAC^*	$Min.cost^\ddagger$	min. $TAC^\ddagger$
1-3-13-16-33	5.1	2S34C2H1	3164.8	720.3	1008.0	716.0	1003.7
2-12-17	5.1	2P34C2H1	3212.0	724.7	1012.3		
4-14-18	5.2	2P34C1H1	3229.1	734.0	1021.7	732.0	1019.7
5-6-15-27-32	5.2	2S34C1H1	3219.5	738.1	1025.8		
8-9-19	5.3	1P34C1H2	3614.7	801.3	1089.0	801.1	1088.8
7	5.3	S134C1H2	3858.8	842.6	1130.3		
10-11-20	5.4	1P34C2H2	4065.4	854.7	1142.4	853.2	1140.9
21-22-24-25-26	5.4	1S43C2H2	4086.8	860.0	1147.7		
23-29-30-31-34	5.3	1S43C1H2	4005.1	865.7	1153.4		
28	5.4	S134C2H2	4194.0	876.2	1163.8		

† As defined by Gundersen *et al.*, (1991), i.e., no. of units and order in terms of cost
∗ Based on the Chen approximation of LMTD
‡ Based on LMTD and split-optimized (Gundersen *et al.*, 1991)

Table 8.2: Five unit designs with fixed $HRAT = 20^0C$ (example 1)

gives the process exchanger numbers ordered from left to right in the grid diagram, the letter S indicate that matches 3 and 4 are in series and C2 and H1 indicate that there is a cooler on hot stream 2 and a heater on cold stream 1. By convention exchanger 1 matches streams $H1$ and $C1$, exchanger 2 $H1$ and $C2$, and so on. Columns 4, 5 and 6 give the total area, capital cost and total annualized cost of the designs. All values are a direct result of the model, i.e., the areas are based on Chen's approximation (Chen, 1987) of the logarithmic mean temperature difference (LMTD) and isothermal mixing of split streams. The two last columns give the capital cost and total annualized cost of the HEN structure when the exact value of LMTD is used and split fractions and heat load loops are (re-)optimized.

Encouragingly, the best design is found before the second etc., the only error being that the sixth best design was found before the fifth best. Discouragingly, the best design has five identical duplicates with three stages, and the fifth one is identified as the 33rd of the 34 feasible combinations of the binary variables, see column 1 of Table 8.2, which clearly illustrates the difficulties with local optima.

However, the main purpose of this paper is to illustrate how the operability considerations may be taken into account, and it turns out that a number of the feasible 5 unit designs should be rejected from a controllability point of view. For example, the best two designs with heat load distribution 5.1 violate heuristic O2 because match 2 is the final unit on both stream H1 and stream C2. We therefore included the operability constraints O1 and O2, and repeated the exercise. The results are shown in Table 8.3. The controllability heuristics disallow seven of the ten designs so that only three remains feasible. Two of the three designs have five duplicates and they were all found in the correct order, i.e., first the five duplicates of the best (controllable) design, then the second best design with no duplicates, and finally the five duplicates

Order	$HLD^{\dagger}$	Structure	A^{*}_{tot}	$Cap.cost^{*}$	TAC^{*}	$Min.cost^{\ddagger}$	min. $TAC^{\ddagger}$
169-186-194-201	6.1	2S43C2H1H2	3049.5	716.7	1004.3	716.0	1003.7
243	6.3	S21S43C2H1	2994.1	725.1	1012.8	720.4	1008.1
19-51-125	6.2	P12P34C1H1	2975.8	726.4	1014.0	719.6	1007.3
26-119	6.2	S21P34C1H1	2981.3	728.8	1016.4		
247	6.3	S21P34C2H1	3034.3	729.9	1017.6		
33	new	S2343C2H2	3164.31	730.1	1017.8		
76-179	6.6	1P34C1H1H2	3088.0	730.1	1017.8	728.6	1016.3
199-200-233-238	6.6	1S43C1H1H2	3100.3	735.4	1023.0		
32-244	6.3	2P134C2H1	3017.4	738.5	1026.2		
246	6.2	S21S43C1H1	3042.8	739.0	1026.6		

Table 8.4: Example 1: Six unit designs with fixed HRAT (example 1)

Order	$HLD^{\dagger}$	Structure	A^{*}_{tot}	$Cap.cost^{*}$	TAC^{*}	$Min.cost^{\ddagger}$	min. $TAC^{\ddagger}$
114	6.1	2S43C2H1H2	3049.5	716.7	1004.3	716.0	1003.7
58-146	6.1	1P34C2H1H2	3095.9	721.8	1009.5		
159	6.3	S21S43C2H1	2994.1	725.1	1012.8	720.4	1008.1
97-139-161	6.6	1S43C1H1H2	3100.3	735.4	1023.0		
8-70-165	6.3	2P134C2H1	3017.4	738.5	1026.2		
160	6.2	S21S43C1H1	3042.8	739.0	1026.6		

Table 8.5: Six unit designs with fixed HRAT fulfilling controllability requirements (example 1)

requirements may seem very restrictive, two of the least expensive designs fulfill them, the best operable six unit design is shown in Fig. 8.9b). The reason why the five units dummy solutions are being generated is partly because they are cheap (a dummy exchanger only costs $8.6k\$/yr$), and thus only partly due to the problems with local minima.

We then repeated the exercise with no restrictions on HRAT, and allowed the number of units to vary between 4 and 6. The first solution has the same structure as the best design with fixed HRAT and 5 units, but has a *lower* heat recovery. The third solution has the same structure as the best design with fixed HRAT and 6 units but has

Order	$HLD^{\dagger}$	Structure	A^{*}_{tot}	$Cap.cost^{*}$	TAC^{*}	$Min.cost^{\ddagger}$	min. $TAC^{\ddagger}$
38-53-61	6.1	2S43C2H1H2	3049.5	716.7	1004.3	716.0	1003.7
34-58-65	6.6	1S43C1H1H2	3100.3	735.4	1023.0		

Table 8.6: Six unit designs with fixed HRAT fulfilling operability requirements O2, O4 and O6 (example 1)

Order	$HLD^{\dagger}$	Structure	A^*_{tot}	$Cap.cost^*$	TAC^*	$Min.cost^{\ddagger}$	min. $TAC^{\ddagger}$
1	5.1	2P34C2H1	2923.4	680.7	1001.1		
3	6.1	1P34C2H1H2	3166.4	732.4	1009.2		
2	5 (new)	1P34H1H2	4178.6	846.9	1039.1		
4	4	1P34H2	4575.3	911.2	1085.4		

Table 8.7: Designs with free HRAT and four, five or six units (example 1)

Order	$HLD^{\dagger}$	Structure	A^*_{tot}	$Cap.cost^*$	TAC^*	$Min.cost^{\ddagger}$	min. TAC
9-10-36-39-40-41	6(5.1 dummy)	S234C2H1	3115.6	722.8	1007.9		
12	6.1	1P34C2H1H2	3166.4	732.4	1009.2		
7-16-25-26	6(5.1 dummy)	2P34C2H1	2923.4	689.4	1009.7		
44	6.6	1S43C1H1H2	3390.3	775.1	1012.7		
11	6.3	S21P34C2H1	3159.7	749.3	1016.3		
33	6.2	S12S34C1H1	3286.7	767.4	1020.8		
8-37	5.2	2S34C1H1	3115.6	722.8	1023.2		
18-24	6.3	2P134C2H1	2887.0	716.4	1023.4		
14-30	5(new)	14C2H1H2	2709.0	640.5	1044.5	639.7	1043.7

Table 8.8: Designs with free HRAT and 4, 5 or 6 units with controllability requirements O1 and O2 (example 1)

a slightly *higher* heat recovery. This is satisfactory, both for the supertargeting, and for Yee *et al.*'s model. However, the rest of the solutions are threshold designs, solution 2 has 5 units, and the rest of the solutions 4 units. The cost of the 4 unit threshold solutions are considerably higher than dozens of the pinch designs generated with fixed HRAT (compare Table 8.7 with the previous tables), i.e., several good solutions are lost.

We then included the controllability constraints and repeated the exercise. The results are summarized in Table 8.8. By including the controllability requirements (O1 and O2) several of the good designs generated with fixed HRAT are rediscovered. The controllability requirements seem to help the search, and reduce the probability of converging to an expensive threshold solution. Finally we included the operability requirements (O2, O4 and O6). The results are presented in Table 8.9. The best design ($1S43C2H1H2$) has the same structure as the best 6 unit design with fixed HRAT, but has a slightly higher heat recovery. It is only about 0.2% more expensive than the minimum cost design for this synthesis problem. Note that this design was not found without the operability requirements (O4 and O6). In addition to the solutions presented in the table, we get a number of threshold solutions as before, and also a number of even more expensive low heat recovery designs with a single process exchanger and three utility exchangers.

Order	$HLD^{\dagger}$	Structure	A^{*}_{tot}	$Cap.cost^{*}$	TAC^{*}	$Min.cost^{\ddagger}$	min. $TAC^{\ddagger}$
26	6.1	1S43C2H1H2	3166.7	734.5	1003.4		1002.4
2-3-4-5-7	6(5.1 dummy)	S234C2H1	3115.6	722.8	1007.9		
10	6.6	1S43C1H1H2	3390.3	775.1	1012.7		
1-6	5.2	2S34C1H1	3115.6	722.8	1023.2		
61	5(new)	14C2H1H2	2709.0	640.5	1044.5	639.7	1043.7

Table 8.9: Designs with free HRAT and 4, 5 or 6 units with operability requirements O2, O4 and O6 (example 1)

Conclusions on example 1

1. A large number of the possible designs are eliminated by the operability constraints.

2. Several near-minimum cost designs fulfilling all operability heuristics exist.

3. The problems with local optima decreases when including the operability constraints.

4. The search for the second best, third best etc. solution more often is trapped in local minima when HRAT is free than when HRAT is fixed. This confirms the results of Gundersen *et al.*, (1991).

8.5.2 Example 2 from Ahmad

The second example is a five stream problem from Ahmad (1985). This example illustrates that it may be expensive to include the operability constraints. All results from this example are based on the Chen approximation of LMTD and include no final optimization of split fractions or heat load loops. The problem data are given in Table 8.10.

Fixed heat recovery level

Previous authors (Gundersen and Grossmann, 1988; and Yee and Grossmann, 1990) use $HRAT = 10K$ for this example, and this heat recovery level fits well with our data for utility cost. With this heat recovery, the minimum number of units $U_{min,MER} = 8$. We first generated solutions with $HRAT = 10K$ and 8 units, with and without the operability heuristics. The results are presented in Table 8.11. Note that 1) The operability constraints cannot all be fulfilled with this fixed HRAT, no feasible solution without inner matches exists. 2) Disallowing parallel heat exchange in general (O4) increases the capital cost with 16% compared to the case where only final splits (O1) are disallowed. This shows that the operability heuristics may be expensive. 3) With operability constraints the best solution is found first whereas the cheapest design without operability constraints is the sixth solution. This could indicate that the operability heuristics help the algorithmic search.

Stream	$T_s[^oC]$	$T_t[^oC]$	$w[kW/^oC]$	$h[W/m^2,^oC]$
$H1$	159	77	228.5	400
$H2$	267	88	20.4	300
$H3$	343	90	53.2	250
$C1$	26	127	93.3	150
$C2$	118	265	196.1	500
ST	376	376		1000
CW	15	30		600

Cost of matches and utility exchangers:

$$C_{hx} = 8600 + 670 A_{hx}^{0.83} \tag{8.24}$$

Cost of utilities (not given by Ahmad, 1985):

$$ST : C_{HU} = 0.03\$/kWh \qquad CW : C_{CU} = 0.003\$/kWh \tag{8.25}$$

Operating hours per year: 8600 Payback time: 3 Installation Factor: 3

Table 8.10: Stream and cost data for Example 2 from Ahmad (1985)

Order	Units	HRAT	Heuristics	Structure	A_{tot}	$Cap.cost$	TAC
6	8	fixed	None	P46S2P135C1H2	7195.3	1538.7	4501.7
1	8	fixed	1,2	P46S21C1C2C3H2	7912.9	1588.5	4551.5
-	8	fixed	2,4,6	-	-	-	-
1	8	fixed	2,4	S6421C1C2C3H2	9673.7	1839.4	4802.4

Table 8.11: Cost of optimal 8 unit designs with $HRAT = 10K$ with and without operability constraints for example 2

For this example there is little incentive to increase the number of units with $HRAT = 10K$, whereas fewer units requires that the HRAT is relaxed. Thus, we next present designs with fewer units without restrictions on HRAT, see Table 8.12. Note that 1) Total area and capital cost increases with number of units whereas the total annual cost decreases due to lower utility costs. 2) With operability heuristics O1 and O2 and 8 units, the cost with free HRAT exceeds the cost with fixed HRAT. This shows that the algoritm has problems with local minima. 3) Including operability heuristics is expensive, for this example it may be recommended to allow violations of both heuristic O4 and O6.

8.6 Discussion

Additional examples. We have also tested the operability constraints on other examples.

Order	Units	HRAT	Heuristics	Structure	A_{tot}	$Cap.cost$	TAC
1	5	free	None	S6P35C1H2	2927.9	698.9	6597.0
1	5	free	1,2	S5C1C2C3H2	2282.1	569.9	8692.8
2	5	free	2,4,6	S5C1C2C3H2	2282.1	569.9	8692.8
1	6	free	None	S16C1C2C3H2	4255.4	933.7	5712.7
1	6	free	1,2	S16C1C2C3H2	4255.4	933.7	5712.7
5	6	free	2,4,6	S163C1C2C3H2	4153.4	930.8	5709.8
1	7	free	None	S1P46C1C2C3H2	4738.1	1046.6	5010.8
1	7	free	1,2	S1P46C1C2C3H2	4738.1	1046.6	5010.8
1	7	free	2,4,6	S164C1C2C3H2	4522.5	1010.0	5191.5
2	8	free	None	P46S2P15C1C2H2	7096.0	1505.1	4508.0
10	8	free	1,2	S62531C1C3H2	6642.0	1420.5	4919.9
20	8	free	2,4,6	S16a46bC1C2C3H2	4654.7	1078.0	5081.8

Table 8.12: Cost of optimal designs with free HRAT with and without heuristic constraints for example 2

Often the operability heuristics may be included with no or small effect on total annual cost, but it may in some cases be expensive to disallow splits in general (O4) or inner matches (O6).

Combinatorial growth. HEN synthesis problems are difficult combinatorial problems. An important side-effect of including the operability heuristics is that the feasible solution space is greatly reduced. Disallowing splits and/or splits or inner matches may reduce compution times although the number of constraints is increased.

Duplicate designs, solution order. The results indicate that GAMS often converges to neighbor solution instead of a identitical duplicate design after an integer cut. Yee *et al.* (1990) recommend to use only 2 stages for simple problems with 2 hot and 2 cold streams. We repeated the exercise with only 2 stages, and in this case the problems with incorrect solution order and duplicate designs are greatly reduced.

Isothermal mixing and remixing streams between stages. To compare cost of split and no-split designs, we included the above constraints in some of the optimization runs. Note that the two main limitations about Yee *et al.*'s model both affect the cost of series heat exchange compared to parallel heat exchange. The isothermal mixing requirement will favor series heat exchange to parallel heat exchange because the full benefit of parallel heat exchange through optimizing the split fractions will not be achieved. Consequently, this assumption will reduce the number of splits. However, the limitation that the streams must be recombined after each stage will tend to increase the number of splits because large streams will tend to be split in successive stages. Splitting in successive stages will very seldom be installed in real plants because of the cost involved with piping, installation and maintanance.

Few process exchangers. This heuristic is difficult to implement as a logical constraint because it is a soft heuristic. One possible way to favor utility exchangers would be to reduce the fixed cost term for utility exchangers. However, for comparison reason this approach is undesirable. Another option would be to restrict the number of matches to some function of the number of streams and/or the number of units. In our experience, the number of streams ($N^h + N^c$) is a good upper bound for the number of matches both for cost-optimal and controllable near-optimal designs. However, heuristic O6 about disallowing inner matches effectively limits the number of process exchangers, and heuristic O5 may be disregarded during design with mathematical programming.

Number of stages and inner matches. The complexity of the mathematical programming model greatly depends on the number of stages, and it is important not to use more stages than necessary. Still, the number of stages should be larger than maximum of the hot and the cold streams to be able to represent all solutions. We use $N^k = max(N^h, N^c) + 1$ in the examples. However, Yee *et al.* (1990) have experienced that $N^k = max(N^h, N^c)$ is sufficient in most cases. Interestingly, this will effectively disallow inner matches for simple problems with 2 hot and 2 cold streams as example 1, and the experience that two stages is enough for such problems indicate that disallowing inner matches is not very limiting.

Better techniques. Even though Yee *et al.*'s model only is nonlinear in the objective function, local optima is a problem. Better start values as discussed by Yee and Grossmann (1990) or better algorithms for the MILP and NLP subproblems may reduce, but not eliminate the problems. HEN synthesis problems are inherently nonconvex, and reformulations or linear approximations to avoid these nonconvexities should be considered. Quesada and Grossmann (1993) present an algorithm where global optimality may be guaranteed for HENs with fixed topology when linear area cost functions, arithmetic mean temperature for the driving force and isothermal mixing is used. Obviously, these assumptions may be very limiting and further advances are needed.

Penalty terms for violating operability heuristics. As mentioned in the introduction, the operability heuristics may be taken into account through penalty terms in the objective function. For example, violation of heuristic O4 may be taken into account by including a penalty or cost for additional piping. Papalexandri and Pistikopoulos (1992,1994) discuss MINLP models for HENs where cost of piping etc is included. An interesting approach would be to include penalties for violating operability heuristics together with penalty terms for non-vertical heat transfer as suggested by Gundersen and Grossmann (1988).

Single heat exchanger constraints. All heuristics discussed in this paper are related to the HEN structure only. This makes the heuristics simple and general, but operability also depends on exchanger and stream parameters. A possible extension could be to include constraints on the individual heat exchangers. For controllability it is advantageous to have large final units on streams because the heat loads of these units are manipulators in the regulatory control loops. The minimum size constraints may be given in terms of exchanger area, exchanger load, exchanger end temperature difference or stream temperature difference.

Maximum size constraints for the heat exchangers may also be included to avoid solutions with exchangers that are larger than the practical limit. The practical limit is usually decided by mechanical design (i.e., vibration), installation, plant-layout and pressure drop considerations. The practical limit for the total installed area in one heat exchanger unit is in the range $1000 - 3000m^2$. (Note that some of the exchangers have installed areas exceeding $1000m^2$ in most of the designs presented in this paper). Another limit may be the total length of the shell (i.e. 6 or 7 m). It may also be recommended to limit the thermal effectiveness, especially if shell and tube exchangers are to be implemented:

$$P^h = (T^h(i,k) - T^h(i,k+1))/(T^h(i,k) - T^c(j,k+1)) \leq P_{max} \tag{8.26}$$

$$P^c = (T^c(j,k) - T^c(j,k+1))/(T^h(i,k) - T^c(j.k+1)) \leq P_{max} \tag{8.27}$$

where we suggest a maximum thermal effectiveness $P_{max} = 0.8$ for countercurrent exchangers and $P_{max} = 0.5$ for exchangers with two tube passes per shell pass.

8.7 Conclusions

This paper discusses how novel heuristics for the structure of heat exchanger networks with good operability may be taken into account during heat exchanger network synthesis with mathematical programming. The heuristics are included in a stagewise model proposed by Yee *et al.* (1990), and the following observations have been made:

1. The operability heuristics may be expressed as logic statements and formulated as additional linear constraints. The operability constraints require that a new set of binary variables with information about final process exchangers on the different streams are included.

2. The most important heuristics about disallowing final splits (O1) and double output matches (O2) seem to be fulfilled without large effect on the cost.

3. The heuristics about disallowing all splits and inner matches may increase the cost considerably for some problems, but often near-minimum cost solutions fulfilling all operability heuristics may be identified.

4. The operability constraints reduce the feasible solution space, and the results indicates that the problems with local optima are reduced.

5. The problems with local optima increase without bounds on the heat recovery level (HRAT).

Nomenclature

A - Area $[m^2]$
C - Cost $[k\$/yr]$ or $[k\$/kWh]$
H - Film coefficient $[W/m^2, {}^0C]$
i - index for hot streams $[-]$
j - index for cold streams $[-]$
k - index for stages and temperature locations $[-]$
N - Nunmber

P - thermal effectiveness
T - Temperature
Z - Binary variable for occurence of process exchangers
Z_{fin} - Binary variable used to identify final process exchangers
Z_{util} - Binary variable for utility exchangers
w - heat capacity flowrate $[kW/^0C]$

Acronyms
HEN - heat exchanger network
HLD - heat load distribution
$HRAT$ - heat recovery approach temperature
$MINLP$ - mixed-integer nonlinear programming (MINLP)
$MILP$ - mixed-integer linear programming
NLP - nonlinear programming
TAC - total annual(ized) cost
ST - stages

Sets
HP - hot process streams
CP - cold process streams
ST - stages

superscripts
c - cold
CU - cold utility
h - hot
HU - hot utility k - stages

subscripts
hx - exchanger
max - maximum
MER - maximum energy recovery
min - minimum
s - supply
t - target
tot - total

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Chapter 9

Effect of Flexibility Requirements on Design of Heat Exchanger Networks

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Abstract

Due to varying steady-state operating conditions, it is important that heat exchanger networks are flexible. Surprisingly, we find for typical design examples from the literature, that the size of the matches are reduced when flexibility requirements are included, and that the simpler designs with less integration are favored. An incorrect problem formulation with a prespecified heat recovery level at each operating point yield the opposite result. Thus, it is especially important to simultaneously optimize approach temperatures, superstructure and area when addressing flexibility problems.

9.1 Introduction

The flexibility or static resiliency problem for heat exchanger networks (HEN) was first defined by Marselle *et al.* (1982). They define the problem in terms of the disturbance range or the union of the ranges of all supply temperatures and heat capacity flowrates. A network is flexible or resilient if it allows for maximum energy recovery (MER) corresponding to a minimum temperature difference ΔT_{min} or heat recovery approach temperature (HRAT) for this disturbance range. This definition is adapted from nominal HEN synthesis problems, where ΔT_{min} is a simple and good manner of taking the trade-off between energy cost and capital cost into account. For flexibility problems MER is inappropriate, and suggested improvements (e.g., Kotjabasakis and Linnhoff, 1986; Colberg *et al.* 1990) do not remove the fundamental problem. The optimal heat recovery level for one operating point greatly depends on the area requirements of the other points.

This paper may be divided in two parts. In part one, we discuss the effect on prespecifying the same heat recovery level for multi-period problems. i.e., flexibility problems that have a discrete set of operating points. The discussion is based on an example from Floudas and Grossmann (1987). This paper is still very much the state of the art for automatic generation of flexible HENs, although Pistikopoulos and coworkers (e.g., Papalexandri and Pistikopoulos, 1992; 1994) are in the process of changing this. In part one the HEN structure is the one suggested by Floudas and Grossmann (1987), and we discuss:

Optimal operation. For a HEN with given structure, exchanger areas and bypasses, and a given set of steady-state operating points (supply temperatures, heat capacity flowrates, heat transfer coefficients and target temperatures), find the set of manipulated inputs (bypass fractions and possible split fractions) that minimizes energy cost. The appropriate formulation is:

$$\min_{u} \quad (T_{t-1}^{Hj} - r_t^{Hj}) w^{Hj} \quad \text{(minimize hot utility)} \tag{9.1}$$

subject to:

$$\begin{array}{lcll}
T_t^{Hi} - r_t^{Hi} & = & 0 & \text{(hot and} \\
T_t^{Cj} - r_t^{Cj} & = & 0 & \text{cold target temperatures)} \\
r_t^{\hat{H}i} - T_{t-1}^{\hat{H}i} & \leq & 0 & \text{(positive or zero heat load coolers} \\
T_{t-1}^{\hat{C}j} - r_t^{\hat{C}j} & \leq & 0 & \text{and heaters)} \\
-u & \leq & 0 & \text{(bypass and split fractions above 0} \\
u - 1 & \leq & 0 & \text{and below 1)}
\end{array}$$

where w means heat capacity flowrate, T_t stream temperatures (controlled outputs), r_t the reference values for the controlled outputs (setpoints) and u split or bypass fractions (manipulated inputs). Supercript Hi (Cj) denotes the set of hot (cold) streams, and $\hat{H}j$ ($\hat{C}j$) the subset of the hot (cold) streams that are utility-controlled, and subscript $(t-1)$ means the stream upstream the final utility exchangers. Energy balances for the exchangers and mixers, and mass balances for the splitters and the mixers yield additional equality constraints. Note that only hot utility is included in the cost function as the cold utility will be given from an energy balance. We assume no limitation on the heat load on the utility exchangers, but one may alternatively assume given areas and maximum flowrates.

More simply, we compute the cost-effects of allowing different heat recovery levels (HRAT) for the different operating points with fixed heat exchanger areas.

Area optimization. For a HEN with given structure and a given set of steady-state operating points (supply temperatures, heat capacity flowrates, heat transfer coefficients and target temperatures), find the set of manipulated inputs (areas and possible split fractions) that minimizes exchanger and energy cost.

More simply, we optimize heat recovery levels of the different operating points together with the heat exchanger areas.

In both problems, we discuss the effect on the necessary number of bypasses.

In part two the complete multi-period problem is considered. i.e.,

Network synthesis. For a HEN problem with a given set of steady-state operating points (supply temperatures, heat capacity flowrates, heat transfer coefficients and target temperatures), find the set of manipulated inputs (structure, areas and possible split fractions) that minimizes exchanger and energy cost.

The discussion is based on a classical nominal HEN problem from Linnhoff and Hindmarsh (1983) where we have included flexibility requirements in terms of six additional operating points. With an appropriate problem formulation without constraints on approach temperatures (heat recovery approach temperature-HRAT or exchanger minimum approach temperature-EMAT), we illustrate that optimal integration decreases with increasing flexibility requirements, i.e., that total exchanger area and the number of matches decreases.

9.2 Optimal steady-state operation

Previous work with specified heat recovery

As mentioned, the conventional HEN flexibility problem has previously been solved assuming specified heat recovery levels for the various operating points. For example, Floudas and Grossmann (1987) use a uniform heat recovery approach temperature (HRAT) to set the utility requirement for all operating points. However, this constraint may make it impossible to utilize the installed area fully at all operating points. To illustrate this point consider example 2 from Floudas and Grossmann (1987) where the stream and cost data are shown in Table 9.1. For this flexibility problem, Floudas and Grossmann (1987) present the "optimal" design shown in Fig. 9.1 where the exchangers in parallel are bypassed in period 3 to get the specified heat recovery. The reason is simply that the area requirements for period 3 are lower than for period 1. The HRAT specification is equivalent to specifying the temperature of stream $C1$ into the parallel exchangers at 450^oC (10^oC lower than the target temperature of stream H1). With this temperature of the cold stream entering the parallel exchangers, there is more

Stream	$T_s[K]$ Per. 1	$T_s[K]$ Per. 2	$T_s[K]$ Per. 3	$T_t[K]$	$w[\frac{kW}{K}]$ Per. 1	$w[\frac{kW}{K}]$ Per. 2	$w[\frac{kW}{K}]$ Per. 3	$h[\frac{W}{m^2,K}]$
$H1$	640	620	620	460	9.9	9.9	8.1	200
$H2$	560	540	540	480	7.15	7.15	5.85	200
$H3$	540	520	520	480	3.3	3.3	2.7	200
$H4$	480	460	460	400	39.6	39.6	32.4	200
$H5$	460	440	440	310	7.7	7.7	6.3	200
$H6$	420	400	400	350	79.2	79.2	64.2	200
$C1$	300	300	300	650	29.7	29.7	24.3	200

Annualized cost of process heat exchangers and coolers:

$$(C_1 + C_2 A_{hx}^m) F_{install} F_{payback} = 4333 A_{hx}^{0.6}(m^2) 3\frac{1}{3}[\$/year]$$

Cost of heater (furnace):

$$(C_1 + C_2 Q_{HU}^m) F_{install} F_{payback} = 1.5246 Q_{HU}^{0.7}(W) 3\frac{1}{3}[\$/year]$$

Cost of utilities (C_{util}):

$$(C_{HU}Q_{HU}+C_{CU}Q_{CU})F_{online}8760 = (204.732{-}7Q_{HU}(W)+60.576e{-}7Q_{CU}(W))8400[\$/year]$$

Cooling water 300-330 K. Operation time equally distributed among the three operating points.

Table 9.1: Stream and cost data for Example from Floudas and Grossmann (1987)

area available than what is needed, so the parallel exchangers must be bypassed. By relaxing the HRAT specification, the temperature of stream $C1$ entering the parallel exchangers may be increased to 454^0C. The bypass around the parallel exchangers must then be closed in order to achieve the target temperatures of H1, H2 and H3. By closing the bypass, the utility cost of period 3 is reduced from \$77.85h^{-1} (Floudas and Grossmann, 1987) to \$74.72h^{-1} (new formulation). In fact, for this problem the bypass should be closed for all operating points, i.e. it should simply be removed.

Bypass placement for flexibility

From energy recovery considerations it will always be advantageous to manipulate split fractions instead of bypass fractions if possible. The split fractions in Fig. 9.1 may be manipulated to control two of the three hot outlet temperatures of the parallel exchangers. Thus, either the bypass on match 2 or match 3 may be removed, too. The optimal bypass placement for flexibility are shown in Fig. 9.4a.

The target or minimum number of bypasses may be computed as (Mathisen and

		Floudas & Grossmann (1987)	This work
Process HX area [m^2]	HX 1	94.68	55.12
	HX 2	72.75	43.33
	HX 3	35.58	20.95
	HX 4	385.88	234.65
	HX 6	142.34	116.15
Utility cost [$\$h^{-1}$]	Period 1	91.64	102.22
	Period 2	95.15	100.37
	Period 3	77.85	79.33
TAC [$k\$$]		1374	1217

Table 9.2: Area optimization with and without fixed HRAT

Stream	$T_s[^oC]$	$T_t[^oC]$	$w[kW/^oC]$	$h[W/m^2,^o C]$
$H1$	150	60	20	100
$H2$	90	60	80	100
$C1$	20	125	25	100
$C2$	25	100	30	100

Annualized cost of process exchangers (C_{hx}) and utility exchangers (C_{uhx}):

$$(C_1 + C_2 A_{hx}^m) F_{install} F_{payback} = 8600 + 670 A_{hx}^{0.83} (m^2) 3\frac{1}{3} [\$/year]$$

Cost of utilities (C_{util}):

$$(C_{HU} Q_{HU} + C_{CU} Q_{CU}) F_{online} 8760 = (3e{-}5 Q_{HU} + 3e{-}6 Q_{CU}) 8600 [\$/year]$$

Table 9.3: Stream and cost data from Linnhoff and Hindmarsh (1983)

only two bypasses which corresponds to the target number of bypasses, see Fig. 9.4b.

9.4 Network synthesis

In this part of the paper, we consider a simple four-stream example introduced by Linnhoff and Hindmarsh (1983), and later studied by Gundersen *et al.* (1991) among others. The nominal stream and cost data are given in Table 9.3. Gundersen *et al.* (1991) considered a number of alternative heat load distributions and network structures varying both the heat recovery level and the number of units. Nominal stream parameter values are given in Fig. 9.6, where the best nominal designs with four, five and six units are shown. We will use this example to address the structural relations between optimal or near-optimal flexible and nominal designs, and how flexibility

	1	2	3	4	5	6	7
Design	4P134	5S234	6S143	6P134	5S14	5S23	6S21S43
C_{hx}	856.5	535.6	595.9	591.5	434.4	400.6	629.1
C_{uhx}	511.7	148.3	139.8	142.9	207.1	222.2	122.2
C_{util}	174.2	314.2	266.8	273.0	402.1	441.2	258.4
TAC [$k\$$]	1081.8	998.1	1002.4	1007.4	1043.7	1064.0	1009.7

Table 9.4: Cost for nominal designs. Designs 1-3 adapted from Gundersen *et al.*, (1991)

Op. point	Nominal	1	2	3	4	5	6
Stream H1	150.	145.	155.	155.	140.	160.	160.
Stream H2	90.	85.	95.	95.	80.	100.	100.
Stream C1	20.	15.	25.	15.	15.	30.	15.
Stream C2	25.	20.	30.	20.	15.	35.	15.

Table 9.5: Supply temperature at the nominal and the six additional operating points

and $C2$ and so on. Process heat exchanger cost C_{hx}, utility exchanger cost C_{uhx} and operating cost C_{util} for the three designs are given in Table 9.4. In Table 9.4 we have also included four other network structures. Among the other possible designs with three process heat exchangers, we consider design $6P134$, which is a variation of design $6S143$ with exchangers 3 and 4 in parallel. We also consider two very simple designs with only two matches, $S14$ and $S23$. These designs may have been the result if one did not use pinch technology or mathematical programming. Finally, we have included one of the designs with 4 matches, i.e. $6S21S43$.

The threshold solution $4P134$ requires large matches (C_{hx} is large), and is the most expensive design in terms of total annualized cost (TAC). The simple designs with 2 matches (i.e. $5S14$ and $5S23$) are less expensive, but still not near-optimal because of higher cost of utility exchangers (C_{uhx} and larger utility consumption (C_{util}) than the best designs. The other four designs have a better trade-off between capital cost and utility consumption resulting in "near-optimal" TAC (within 1%).

9.4.2 Flexibility requirements

Now we include flexibility requirements to the problem studied by Gundersen *et al.*, (1991). The disturbance range is selected to $\pm 10K$ in inlet temperatures and $\pm 20\%$ in the flowrates. Then six additional operating points are specified. The first three correspond to maximum hot utility, maximum cold utility and maximum area requirement (Marselle *et al.*, 1982) with a disturbance range of $5K$ of the inlet temperatures and 10% of the flowrates. The other three operating points are similarly selected but with the full disturbance range (i.e., $10K$ and 20%) see Table 9.5 and 9.6.

Even though these temperature and flowrate disturbances are no larger than what

Op. point	Nominal	1	2	3	4	5	6
Stream H1	20.	18.	22.	22.	16.	24.	24.
Stream H2	80.	72.	88.	88.	64.	96.	96.
Stream C1	25.	27.5	22.5	27.5	30.	20.	30.
Stream C2	30.	33.	27.	33.	36.	24.	36.

Table 9.6: Heat capacity flowrates at the nominal and the six additional operating points.

	1 and 4	2	3	5	6	7
Design	7P134	7S234	7S143	6S14	6S23	8S21S43
C_{hx}	439.5	373.2	438.3	398.9	364.0	442.1
C_{uhx}	463.5	490.4	468.8	471.0	466.4	467.9
C_{util}	478.7	568.0	486.0	518.8	568.2	493.3
TAC	1381.7	1431.6	1393.0	1388.7	1422.6	1403.3

Table 9.7: Cost of flexible designs

might be expected in chemical process plants, none of the network structures given in 9.6 and in fact none of structures presented by Gundersen *et al.* (1991) can ensure feasible operation at all operating points even with infinite areas on all exchangers. However, feasible operation for the set of operating points may easily be obtained by increasing existing utility exchangers and installing new utility exchangers. For example, to the nominally best 5 and 6 unit designs ($5S234$ and $6S43$) one may add utility exchangers and get 7 units designs denoted $7S143$ and $7S234$. With these matches, utility exchangers must be placed on all streams to fulfill the flexibility requirements. Similarly, nominal design 6P134 may be made into the flexible design $7P134$.

To check whether these designs are optimal we have derived and compared most of the possible alternatives with 2, 3 or 4 matches. However, here we will include results only on those designs that can be derived by adding utility exchangers to the designs in Table 9.4. The results are presented in Table 9.4.2 where the numbers 1 to 7 refer to Table 9.4. With 2 matches, utility exchangers must be included on all streams to fulfill the flexibility requirements, and this gives designs $6S14$ and $6S23$. For the nominal design $6S21S43$ it also beneficial to place utility exchangers on all streams, and this gives design $8S21S43$.

The following comments to the result should be noted:

- The nominally *best* design $5S234$ yields the *worst* flexible design $7S234$, whereas the best flexible design $7P134$ has the process heat exchanger structure of the nominally expensive threshold solution $4P134$. This indicate that there is a topology trap when adding flexibility requirements to nominal designs.
- Since $S14$ and $S23$ give such good results, the more complicated structures have

% at nom. op. point	14.29%	50%	99%	100%
C_{hx}	438.3	456.3	461.0	595.9
C_{uhx}	468.8	465.1	464.7	139.8
C_{util}	486.0	433.7	374.9	266.8
TAC	1393.0	1355.0	1300.6	1002.4

Table 9.8: Effect of increasing the operation time at the nominal operating point for design $7S143$

similar area distributions to either design $S14$ or $6S23$ (i.e., the area of the additional matches of these designs are close to zero). This explains why the designs are similar in terms of TAC. Indeed, if one allows exchanger area to go to zero, there are only three different designs to consider; $7P134$, $6S14$ and $6S23$.

- **IMPORTANT!** The sizes and investment cost of the matches *decrease* compared to the nominal design in all cases. This is opposite of what has previously been believed (e.g., Marselle *et al.*, 1982). The reason is that area cannot be exploited as efficiently in the flexibility case because the matches will be bypassed for some of operating points. More generally, the reason is that the benefits of tight integration depends closely on matching the various parts of the process, and this is not possible when the operating points changes.

Effect of operation time at each operating point

We consider the effect of changing operating hours for the different cases by changing the operation time at the nominal operating point. The nominal operating point is used for 14.29% (as in flexibility cases above), 50%, 99% and 100% (i.e. operation at nominal operating point only) of the total operating hours. The remaining operating hours are equally distributed over the six other operating points.

The results for design $7S143$ are presented in Table 9.4.2, and it should be noted that the cost is only decreasing slowly with increasing operation at the nominal operating point. The flexibility requirements are very costly even as the operation time away from the nominal operating points approaches zero. The main reason is that much larger utility exchangers must be included to make all operating points feasible.

Similar results are obtained for other designs.

9.4.3 Control considerations

In order to determine an overall optimal solution, the simplifying assumptions in the problem formulation must be addressed. For HEN synthesis, both nominal problems and flexibility problems, two important assumptions is that cost of bypasses are neglected and control considerations disregarded.

Both control system investment and maintenace cost favor a simple control system, that is a decentralized control system with few controlled outputs. It is an additional

Design	7SP134	7SP134	7SP134	6S14	6S14
No. of bypasses	2	1	0	2	1
TAC	1381.7	1398.8	1438.5	1388.7	1403.6

Table 9.9: Cost effect of reducing the number of bypasses for flexibility

advantage if the same set of bypasses can cope with desirable and undesirable long-term changes (flexibility) and dynamic variations (controllability). When a simple control system cannot be implemented, it may often be advisable to either tolerate that the plant cannot fulfill all the flexibility or controllability requirements or redesign the entire HEN.

Number of bypasses for flexibility. The cost of bypasses of bypasses may be considerable. To check how the cost depends on the assumption that all process heat exchangers may be bypassed, we disallow the possibility of bypassing some of the matches. The required flexibility must then be fulfilled by adjusting the remaining bypasses and heat loads on the utility exchangers. The results for flexible designs $7P134$ and $6S14$ are shown in Table 9.9. For these designs two of the matches are bypassed in order to obtain the optimal operation in all operating points. Intuitively, one expects that the number of bypasses cannot be reduced without a large increase in TAC. However, for design $7P134$, one of the two bypasses (i.e. bypass on exchanger 4) is only utilized at one operating point, and the other bypass (i.e. bypass around exchanger 1) is only utilized at 2 operating points. For design $6S14$ the bypass around exchanger 4 is utilized for 2 operating points. Omitting the bypass around exchanger 4 only increases TAC with 1.2% (to 1398.8k\$). Also omitting the bypass around exchanger 1 increases the TAC with additional 2.8% (to 1438.5k\$). For design $6S14$ the bypass around exchanger 4 is utilized for 2 operating points, and omitting this bypass only increases TAC with 1.1% (to 1403.6k\$). The cost penalties are surprisingly low, so for this problem the results are not sensitive to the number of bypasses installed.

Bypasses for control. Because the designs include utility exchangers on all streams, one would immediately assume that controllability is not a problem. However, for some operating points one or several of the utility exchanger duties drop to zero. Bypasses around matches must then be used as manipulated variables. For design $S14$ the heat load on the coolers drop to zero for some operating points whereas the heat load on the heaters does not. Thus, it is easy to control this design, by placing the bypasses on the hot side of the matches they can be used to control the hot target temperatures with a direct effect for the operating points where the duty drops to zero. For design $6S23$ control will be a bit more complicated because both the coolers and the heater on stream $C2$ drop to zero for some (but not the same) operating point. Adhering to the main bypass placement rule for control becomes impossible, the bypass around exchanger 3 must be used to control stream $H2$ for one operating point and stream $C1$ for another operating point. For design $7P134$ both coolers and the heater on stream $C1$ drop to zero for some operating point. For control one would either have to use the same bypass to control two different outputs (at different operating points) or use the splitter. Neither solution adhere to the main bypass placement rule for control.

exchanger performance for the rest of the campaign. Therefore, fouling may be an important argument for HEN structures without splitters, and fouling may be important both for flexibility and control. The HEN must be flexible so that the target temperatures can be met throughout the campaign, i.e. the HEN must be able to cope with this undesirable long-term change. For minimization of fouling, additional intermediate temperatures must be controlled and some exchangers should not be bypassed. In some cases it may even be appropriate to install a recirculation stream around a fouling exchanger to keep the flowrate high during low plant load or use a recycle as an alternative to a bypass for reducing the heat load on the exchanger without reducing the flowrate through the exchanger.

Pressure drop. The pressure drop in HENs may be important for design as well as operation, especially for gas streams. The heat transfer coefficient generally increases with pressure drop, but the plant power requirements increases, too, so a trade-off exists. Due to pressure drop considerations, heat exchangers with multiple tube passes are often used. The temperature driving force decreases when the flows no longer are countercurrent. This increases area requirements and is unfavorable for flexibility. However, the apparent dead-time of the exchanger is decreased, so control is favored in cases where an exchanger not immediately upstream the output is used as a manipulator.

9.6 Conclusions

In part one of this paper we have shown that state-of-the art methods for automated synthesis of flexibility problems use an inappropriate problem formulation with a prespecified heat recovery level. The resulting designs are suboptimal in terms of heat recovery level and area distribution.

In part two of this paper the following important points about synthesis of flexible HENs have been illustrated with an example:

1. Flexibility designs may have less installed heat exchanger area than nominal designs,

2. Flexibility designs may have a simpler network structure than nominal designs.

3. Including flexibility requirements to nominal problems introduces another topology trap to HEN synthesis.

4. Control considerations favour simple designs with few process heat exchangers because few control loops are required and they are easy to control at all operating points.

Nomenclature

A	Heat exchanger area	$[m^2]$
C	Cost	$[\$/Wh]$
r	reference (setpoint)	$[K]$
T	Temperature	$[K]$
U	Overall heat transfer coefficient	$[W/m^2K]$
u	Manipulated input	$[-]$
y	Controlled output (temperature)	$[K]$
w	Heat capacity flowrate	$[W/K]$

Superscripts

c	cold side/fluid of heat exchanger
Ci	set of cold streams
Cj	set of utility-controlled cold streams
h	hot side/fluid
Hi	set of hot streams
Hj	set of utility-controlled hot streams

Subscripts

c	index for cold utility-controlled outputs
i	index for outputs
j	index for inputs
h	index for hot utility-controlled outputs
t	target temperature
$t-1$	temperature upstream utility-controlled output

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Chapter 10

Conclusions and Directions for Future Work

This thesis addresses different aspects of operability of heat exchanger networks. Simple and powerful methods exist for synthesis of minimum cost networks for a nominal operating point, but the optimal network should also be controllable and flexible and have a simple control system. The main contributions of this thesis is on dynamic behaviour, bypass placement and including operability considerations in design.

10.1 Dynamic behaviour of heat exchanger networks

Heat exchangers and all heat exchanger networks are stabile systems, and they are linear in temperature, but nonlinear in area and flowrates. Temperature variations in heat exchanger networks decrease when traversing heat exchangers since the thermal effectiveness is less than unity. Still temperature disturbances may require feedback control since inlet temperature variations may be much larger than the allowed deviations in the target temperatures. The effect of flowrate disturbances on temperature increases across matches on the disturbed stream, so they propagate fundamentally different than temperature variations. After propagating to other streams via a match, they propagate as temperature variations.

Inverse responses from right-half-plane (RHP) zeros may occur for HENs with parallel effects or parallel downstream paths. Such parallel effects may involve mass or energy flows and are quite common. Monovariable RHP-zeros from mass effects may occur with parallel heat exchange or a combination of parallel and series heat exchange. Monovariable RHP-zeros from energy effects may occur when manipulating inner matches, that is matches with downstream matches on both the hot and the cold side. The HEN may have exclusively series heat exchange, but must include one heat load loop involving only process exchangers. Multivariable (2×2) RHP-zeros may occur if both inputs affect both outputs via the same match. The simplest example is to control both outlet temperatures of a single heat exchanger by manipulating the heat load on upstream matches on both streams.

Two-way interactions in HENs are due to (cause-effect) loops or double output matches. Loops usually exist for HENs with heat load loops involving only matches. Double output matches are matches where both the hot and the cold outlet temperature are controlled outputs. Obviously, one of the outputs cannot be adjusted without affecting the other in such cases. Two-way interactions in HENs represent no fundamental control limitation as the directionality is week, but using simple, decentralized controllers with double output matches may be a problem.

10.2 Control configuration design for heat exchanger networks

The conventional heuristic for control configuration design is to manipulate the final exchanger. This heuristic is supported as it is shown that disturbances have a fast and often large effect on the outputs, and manipulating the final exchanger has the fastest and usually the largest effect on the output. Violating this heuristic tends to give problems with input constraints both dynamically (around the bandwidth) and at steady-state. However, there are a number of exceptions to the main rule for bypass placement:

- Double output matches. Structure makes it impossible to adhere to the main bypass placement rule when one match has two controlled outputs.
- Inner matches. With inner matches the energy consumption may be smaller when these upstream matches are bypassed instead.
- Operating range. Problem parameters may make the operating range larger when upstream matches are bypassed instead.
- Multi-bypasses. A multi-bypass usually increases the operating range compared to one single bypass
- Splitters. Manipulating split fractions decreases the utility consumption compared to manipulating bypasses.

It is shown that the preferred control configuration from an energy point of view may for many classes of HENs be determined from the HEN structure only. This may be utilized to maintain optimal operation without numerical on-line optimization.

A difficult pairing problem exists for HENs with double output matches. It is shown that the relative gain array is given by the thermal effectiveness of the double output match. This derivation more or less solves the pairing problem for HENs, and shows that the resulting control problem has no fundamental control limitation because all elements of the RGA is between zero and unity.

10.3 Operability considerations in heat exchanger network synthesis

Most techniques for HEN synthesis disregard flexibility and control issues. HENs where one single input controller for each target temperature fulfills all control objectives are preferred from operability considerations. HENs without double output matches, splits and inner matches fulfill this criteria and are easy to control and operate. Even though the operability heuristics eliminate the majority of the feasible solutions, they may often be included without large effect on cost. The reason is that most HEN synthesis problems have a large number of designs with near-minimum cost.

It is explained how the operability heuristics may be taken into consideration in HEN synthesis based on pinch technology and matematical programming. When applying pinch technology, the designer is faced with a number of difficult design options both when placing exchangers in the synthesis stage and when removing exchangers in the evolution stage. The effect of the alternative options on the total annualized cost is often very difficult to estimate, and an option fulfilling the operability heuristics should be selected.

When solving HEN synthesis problems with mathematical programming, a feature of some formulations is that all constraints are linear. It is shown that the operability heuristics may be included as linear constraints so that this favorable feature may be maintained. When applying the the resulting model on some literature examples, some of the results indicate that the problems with local minima are reduced after including the operability constraints.

Finally, it is explained that state-of-the-art methods for automated synthesis of flexibility problems use an inappropriate problem formulation with a prespecified heat recovery level. The resulting designs are suboptimal in terms of heat recovery level and area distribution.

By comparison of designs that are optimal for a nominal operating point with optimal flexible designs, it is shown that flexibility requirements introduce another topology trap in HEN synthesis. In contrast to previous results, it is shown that flexibility designs may have less installed heat exchanger area and a simpler network structure than nominal designs. Control considerations further favour simple designs with few process exchangers because they are easy to control at all operating points.

10.4 Future work

10.4.1 Dynamic models

As a continuation of the work on dynamic behaviour of HENs presented in chapters 2 and 3, it may be suggested to derive simple, heat exchanger models based on physical parameters. On-going research at the University of New South Wales (e.g., Ma *et al.*, 1992) may result in a such a model. With a simple, parameter-based model it may be easier to quantify possible control limitations at an early design stage. Problems with input constraints are particularly difficult to address from structural properties only.

10.4.2 Control configuration design

A possible extension of the work in chapters 4 and 5 would be to try to resolve the possible trade-off between the control objectives related to target temperatures and energy. In some case one has to select weights, and how this be done may be an interesting problem although general, parameter-independent results probably are hard to find.

The optimal operation procedure presented in chapter 5 may be implemented in industry. Such an automation should be possible, but still requires some work on how to rank *all* alternative inputs in different operting points so that the control logic may be programmed.

10.4.3 Integrated design and control

Further work is desired in order to confirm that the suggested operability heuristics are important. This work may be to apply the pinch design method and include the heuristics as explained in chapter 7 on more examples, or show how the heuristics may be used in other sequencial, thermodynamic-based methods, e.g. The Dual Temperature Design Method (Trivedi *et al.* 1989).

The implementation of the operability heuristics in the stagewise mathematical programming model by Yee *et al.*, (1990) is done in a simple manner. The operability heuristics greatly limits the feasible solutions space, and it should be possible to exploit this when setting up the superstructure, and/or when searching for the optimal design.

An interesting approach would be to implement the heuristics in a knowledge-based synthesis approach. A research group at the University of New South Wales working with an expert system for HEN synthesis (e.g., Suaysompol and Wood, 1991) has expressed interest in this thesis work.