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Ratio control: Theoretical basis and practical implementation

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ABSTRACT

Despite its widespread use in the process industry, a theoretical basis for ratio control has been lacking. It is sometimes described as a special case of feedforward control, but this interpretation is misleading. Feedforward control requires an explicit process model, whereas ratio control does not. Instead, ratio control relies on physical insight: For systems that satisfy the scaling assumption, maintaining constant ratios between extensive variables when there are throughput changes, results in constant intensive variables. Moreover, the ratio setpoint can be generated by an outer feedback loop, again without requiring a process model. The paper further discusses practical aspects of ratio control implementation, including more advanced schemes, such as dual ratio control for handling actuator saturation and cross-limiting control.

1. Introduction

Ratio control is likely the oldest form of control, with roots in cooking recipes from thousands of years ago. A classic example is making porridge where the instruction "mix 1 cup of grain with 2 cups of water" can be scaled up easily. If you want three times as much porridge then you just multiply the amounts of grain and water by the same factor $\lambda = 3$. This is the essence of the scaling assumption, discussed in Section 2.

Industrially, ratio control is applied extensively. A common example is mixing reagents in a given ratio to a chemical reactor. One of the earliest automatic devices applying this principle is the carburetor (ca. 1890), which mixes air and fuel in a desired ratio for combustion engines.

Fig. 1 shows a typical ratio-control strategy for maintaining constant product composition y for a continuous mixing process. Note that no model is needed to design and implement this control strategy, except for the insight that a constant ratio between extensive variables (in this case, the flow ratio $R = F_2/F_1$) will contribute to keeping the property variable y constant. In the figure, y is the composition, but in general y could be any intensive variable, including temperature, viscosity, taste or color. In Fig. 1, the ratio setpoint $R = (F_2/F_1)_s$ is obtained by "feedback trim" using an outer feedback controller (CC in this case). Here, the integral control action changes R until the measured property y equals its setpoint y_s at steady state.

The objective of the specific process in Fig. 1 could be to mix concentrate (1) (with composition x_1 [kg/kg]) with water (2) (with $x_2 = 0$) to obtain a diluted product (F) with a given concentrate composition y [kg/kg]. The concentrate flowrate F_1 [kg/s] is a disturbance and may have large variations. The simplest control system would be to implement a feedback composition controller (CC) that directly manipulates the dilution water F_2 to keep the measured concentration y at a given setpoint. However, the composition measurement may be unreliable and have a large delay (say, 10 min), and this motivates the use of ratio control, where $R = F_2/F_1$ is kept constant on a faster time scale to compensate for fast variations in F_1. The integral action in the outer feedback controller (CC) removes the effect of uncertainty at steady state by updating the ratio setpoint $(F_2/F_1)_s$ when, for example, there are disturbances in the inlet concentrations (x_1 and x_2 in Fig. 1) or errors and drift in the flow measurements (which affect the measured ratio F_2/F_1).

Dynamic simulations of the mixing process in Fig. 1 are shown in Fig. 2. Inflow F_2 is the manipulated variable and the responses for the product composition y (with setpoint $y_s = 0.2$) are shown with the following control strategies:

1. No control: $F_2 =$ constant.
2. Ratio control only: $F_2 = R^* \cdot F_1$ with constant R^* (nominal value).
3. Ratio with feedback control: $F_2 = R \cdot F_1$ where R is obtained by feedback control[1] (proposed, see Fig. 1).
4. Feedback control only: F_2 is obtained by feedback control.
5. Conventional linear (additive) combination of feedforward and feedback control: $F_2 = R^* \cdot F_1 + F_{2,c}$ where R^* is constant and $F_{2,c}$ is obtained by feedback control.

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[1] When combined with feedback control, ratio control is sometimes referred to as multiplicative feedforward control because the measured disturbance ($d = F_1$ in this case) is multiplied with the output from the feedback controller (R in this case). Conventional (linear) feedforward control is then referred to as additive feedforward control.

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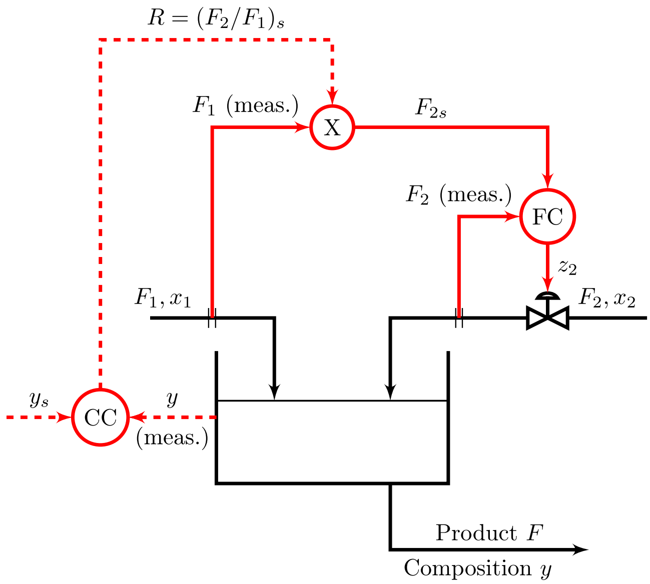

Fig. 1. Recommended ratio-control strategy for mixing process The outer feedback controller (CC) adjust the ratio $R = (F_2/F_1)_s$ to keep the product composition y at its setpoint y_s. The signals denoted (meas.) need to be measured or estimated.
Comment: To satisfy the steady-state mass balance $F = F_1 + F_2$ [kg/s], the product outflow F should be set by overflow or by a level controller (not shown on the flowsheet).

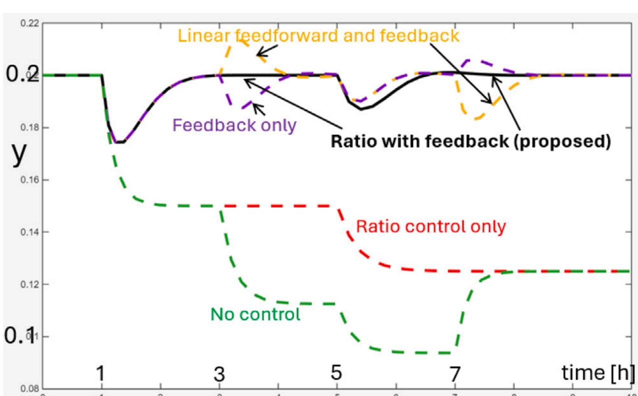

Fig. 2. Dynamic simulation for mixing process in Fig. 1 with various control strategies. The product composition y is shown for flowrate disturbances in F_1 at $t = 3$ and $t = 7$ [hours], and composition disturbances in x_1 at $t = 1$ and $t = 5$ [hours].

The proposed combination of ratio and feedback control (black solid line) gives the best response. The ratio part $F_2 = R \cdot F_1$ completely eliminates the throughput disturbances in F_1 at times $t = 3$ and $t = 7$ [hours], while the feedback part updates R to handle the feed composition disturbances in x_1 at $t = 1$ and $t = 3$ [hours] (with some transient offset). On the other hand, in this case, the conventional linear feedforward and feedback control (orange line) does not give any improvement compared to feedback only (purple line). The reason is that the feedforward part overreacts to the flow disturbance, because the feedforward gain R is not updated when there are disturbances in x_1. Further details about the case study are given in Appendix A (Fig. 16).

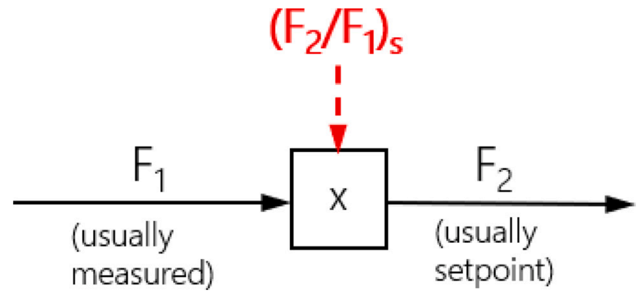

Fig. 3. Multiplication element to compute $F_2 = (F_2/F_1)_s \cdot F_1$ where F_1 is the basis and $(F_2/F_1)_s$ is the desired ratio.

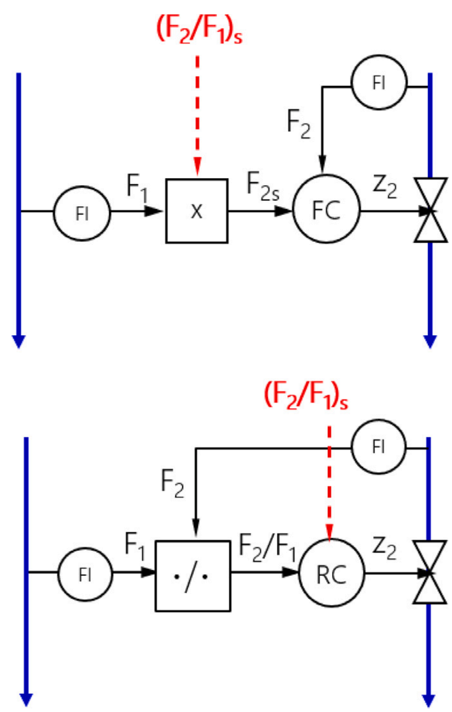

Fig. 4. Two alternatives schemes for implementing ratio control [1–3].
(a) Ratio control with multiplication element and flow controller (FC) (used in this paper). (b) Ratio control with division element and ratio controller (RC) (*not* recommended).

In terms of making food, the inner ratio control ($F_2 = R \cdot F_1$) corresponds to following the recipe given in the cook book (which provides a nominal value for R), while the outer feedback (CC) corresponds to correcting R by tasting the product.

The ratio control scheme in Fig. 1 uses a multiplication element,[2] as shown more clearly in Fig. 3. With the flow disturbance F_1 as the basis, the multiplication element computes the flowrate $F_2 = R \cdot F_1$ with $R = (F_2/F_1)_s$. The computed value F_2 is typically sent as a setpoint F_{2s} to a flow controller (FC) which manipulates the valve position z_2, as shown in Fig. 1 and Fig. 4a.

An alternative (but not recommended) implementation of ratio control, is to use a division element as shown in Fig. 4b. Here, we

[2] The "multiplication element" x in Fig. 3, was in earlier literature given some rather non-descriptive names, such as "ratio relay" (e.g., Buckley [1], page 157) and "ratio station" (e.g., Shinskey [2], page 161), presumably because performing multiplication was not straightforward with old analog (usually pneumatic) control equipment. Unfortunately, the non-descriptive term "ratio station" is still used in some publications and textbooks.

compute the ratio $R = F_2/F_1$ (measured) using a division element and send this to a ratio controller (RC) with setpoint $R_s = (F_2/F_1)_s$. Thus, we have a division instead of a multiplication, and a ratio controller (RC) instead of a flow controller (FC). However, this scheme is not recommended, primarily to avoid complications related to avoiding division by zero (e.g., Love [4], page 182) and also to avoid the related problem of introducing nonlinearity into the inner RC control loop [2], see Section 3.3.2.

Liptak [5] writes that "ratio systems maintain a relationship between two variables to provide regulation of a third variable". However, it should not be any variables. It is shown in Section 2 that the first two should be extensive variables (typically flowrates, F_1, F_2) whereas the third must be an intensive property variable (y).

Ratio control is often viewed as a special case of feedforward and decoupling control, because the "basis" in ratio control may be viewed as a disturbance d (as in feedforward control) or as another process input u_i (as in decoupling). For example, Shinskey [6] (page 173) writes that "ratio control systems are feedforward systems" and Liptak ([5,7] writes that "ratio control actually portray the most elementary form of feedforward control". However, this is misleading. First, ratio control does not require that we measure the disturbance. For example, in Fig. 1, where the disturbance is the inflow F_1, one could measure the outflow F and set the ratio F_2/F (rather than F_2/F_1). Second, feedforward control is model-based whereas ratio control is data-based: Conventional feedforward control requires explicit process models for how the disturbance d and the input u affect the output y (e.g., $\Delta y = G\Delta u + G_d\Delta d$ for the linear case). The feedforward controller then inverts the input model to compute the input u based on the measured d (e.g. $\Delta u = -G^{-1}G_d\Delta d$). On the other hand, ratio control it based on data (measurements of u and d) and the physical insight that a constant ratio u/d will keep the variable y constant (and no explicit model is needed for this). We do not even need a model to set the ratio setpoint, because this, as shown in Fig. 1, may be set by an outer feedback loop.

There are thousands of papers with applications of ratio control, but the literature on the theory and fundamentals of ratio control is scarce. The textbooks of Young [8] and Hengstenberg et al. [9] (p. 1351) show ratio control systems similar to the proposed one in Fig. 4a. Buckley [1] (Fig. 17.7) shows an example of a cascade system (similar to Fig. 1) where a composition controller updates the desired ratio of the two feeds to a reactor. Shinskey [2] considers the two schemes in Fig. 4 and to avoid nonlinearity in the control loop (involving FC or RC), he recommends the scheme in Fig. 4a with an FC and a multiplication element. A good and simple treatment of ratio control is Riggs [10]. He uses the multiplication block for implementing ratio control and also shows how to include feedback correction, similar to Fig. 1, for a neutralization process. Riggs [10] is the only one who, to my knowledge, links ratio control to the scaling property. More recently, Hägglund [11,12] has suggested flexible and robust ways of implementing ratio control.

The main theoretical assumption behind the use of ratio control is that the system satisfies the scaling property, which is discussed in Section 2. In Section 3, we discuss implementation of ratio control. In Section 4, we discuss more complex implementations, including the use of "dual" ratio control for the case when the manipulated variable u may saturate, and cross-limiting control for ratio control of combustion processes.

2. The scaling assumption and the theoretical basis of ratio control

Despite its long history, it seems that no one has provided a theoretical basis for ratio control. Therefore, its use is usually based on intuition. However, intuition and physical insight have limitations, and because of the lack of theoretical basis for ratio control, it is sometimes used incorrectly. The aim of this Section is to rectify this situation.

2.1. The scaling assumption

Ratio control is based on the scaling assumption. To state the scaling assumption, we need to understand the difference between intensive and extensive variables (e.g., [13]:

- *Intensive variables* are properties that do not depend on the size of the system. Common examples are composition, density, viscosity, pressure and temperature. Note that a ratio is an intensive variable.
- *Extensive variables* scale with the size of the system. Examples include flowrate, heatrate, volume, mass, energy and area.

For a steady-state process the scaling assumption may be formulated as follows:

For a process that satisfies the **scaling assumption***, we have that scaling (changing) <u>all</u> independent extensive variables (X) by the <u>same</u> factor $\lambda > 0$, with all independent intensive variables (x) constant, scales (changes) all dependent extensive variables (Y) by the same factor λ and keeps all the dependent intensive variables (y) constant.*

Here, for a continuous process, the factor λ represents a change in the throughput. To state mathematically the *scaling assumption*, we divide the independent variables (disturbances and inputs in control) into two classes:

- Independent intensive variables: $x \in \mathbb{R}^m$ (typically, feed compositions).
- Independent extensive variables: $X \in \mathbb{R}^n$ (typically, feed flow rates)

The dependent variables, which also are either intensive (y) or extensive (Y), are functions of the independent variables, and (generally) the steady-state model of the system may be written as

$$y \text{ intensive}: \quad y = g(x, X) \tag{1a}$$

$$Y \text{ extensive}: \quad Y = F(x, X) \tag{1b}$$

Generally, y and Y are vectors and g and F are vector-valued functions. For the system to satisfy the scaling assumptions, the nonlinear functions g and F must be homogeneous in the extensive variables of order $h = 0$ and $h = 1$, respectively. That is, if all the independent extensive variables of the system (X) are multiplied by a constant λ, then the dependent intensive variables (y) are unchanged (since $\lambda^h = 1$ for $h = 0$) whereas the dependent extensive variables (Y) are multiplied by the constant λ (since $\lambda^h = \lambda$ for $h = 1$). (e.g., Callen [14] and Appendix C in Modell and Reid [13]). Mathematically, for the scaling assumption to hold, both the following relationships must hold (*scaling assumption*):

$$y \text{ intensive}: \quad g(x, \lambda X) = g(x, X) \tag{2a}$$

$$Y \text{ extensive}: \quad F(x, \lambda X) = \lambda F(x, X) \tag{2b}$$

The scaling assumptions (2) holds generally for thermodynamic systems in equilibrium [14]. Individual steady-state mass and energy balances generally satisfy (2b) [15].

2.2. Implications of the scaling assumption for control

The terms "independent" and "dependent" variables have the meaning of "inputs/disturbances" and "states/outputs" from a control point of view. The objective of control is to keep selected dependent variables at given (constant) setpoints. It then follows that for control, (2a) is the key property, as it says that y (intensive variable) is constant if all extensive variables X are scaled by the same factor λ. However, (2b) is also important since it emphasizes that all the extensive variables X and Y need to increase by the same factor λ. If only one extensive variable fails to do this, for example, because it is kept constant (X_j = constant for some j), then the scaling assumption does not hold. This also implies that there can at most be one extensive disturbance variable X_d.

Note that the scaling assumptions (2), assume that the independent intensive variables x (for example, feed composition and feed temperature) are constant. Although this may not be satisfied, it is usually not a serious limitation for practical use of ratio control if they change relatively slowly, because such disturbances can be handled by an outer feedback controller which adjusts the ratio setpoint(s) (e.g., CC in Fig. 1); see also the Discussion section.

To understand better the control implications of the scaling assumption, assume that we operate the system at a nominal point where the n independent extensive variables have the values X^0. Then from (1)

$$y^0 = g(x, X^0) \tag{3a}$$

$$Y^0 = F(x, X^0) \tag{3b}$$

Next, we operate the system at a point where the n independent extensive variables have been changed by the same factor λ. For illustration, consider a case with $n = 3$ and we have:

$$X_1 = \lambda X_1^0 \tag{4a}$$

$$X_2 = \lambda X_2^0 \tag{4b}$$

$$X_3 = \lambda X_3^0 \tag{4c}$$

The resulting values of the dependent variables y and Y are given by (1) and if the system satisfies the scaling assumption (2), we must have:

$$y \text{ intensive}: \quad y = y^0 \tag{5a}$$

$$Y \text{ extensive}: \quad Y = \lambda Y^0 \tag{5b}$$

It is sufficient to specify $n-1$ ratios to keep y constant ($y = y^0$). To prove this, note that since λ is free to choose, we may eliminate λ so that the n equations in (4), may be reduced to $n-1$ equations (=2 equations in our case), which may be written as

$$\frac{X_1}{X_1^0} = \frac{X_2}{X_2^0} = \frac{X_3}{X_3^0}$$

or re-arranged as

$$\frac{X_2}{X_1} = \frac{X_2^0}{X_1^0} \quad \text{and} \quad \frac{X_3}{X_1} = \frac{X_3^0}{X_1^0} \tag{6}$$

Note here that ratios of extensive variables are intensive variables.

More generally, for a system satisfying the scaling assumption, specifying any $n-1$ uncorrelated intensive variables (y) will determine uniquely all extensive variables (except for the scaling factor $\lambda > 0$ which is free to choose). To prove this, assume that we control $n-1$ intensive variables chosen from the vector y (in our case, $n-1 = 2$), that is, they are kept constant at their nominal values:

$$y_1 = y_1^0$$

$$y_2 = y_2^0$$

In addition, to specify the system, we assume that one extensive variable X_d (throughput, basis) is specified; say X_d is X_1 which may be the feedrate to the system. Since $X_1 = \lambda X_1^0$ this indirectly set the value of the scaling factor λ. We assume (as before) that the intensive independent variables (x) are fixed. Using (1a), we then get $n-1$ equations in $n-1$ unknowns

$$g_1(x, [X_1, X_2, X_3]) = y_1^0 \tag{7a}$$

$$g_2(x, [X_1, X_2, X_3]) = y_1^2 \tag{7b}$$

With X_1 given, the $n-1 = 2$ unknowns are X_2 and X_3. We assume the steady-state equations admit a locally unique solution for the extensive variables, except for the trivial scaling degree of freedom λ. This means that with X_1 given, (7) yields unique values for X_2 and X_3. Now, since the system satisfies the scaling assumption, and we have satisfied $y = y^0$ in (7), it follows from the uniqueness of the solution that the solution for X_2 and X_3 must satisfy (4). QED.

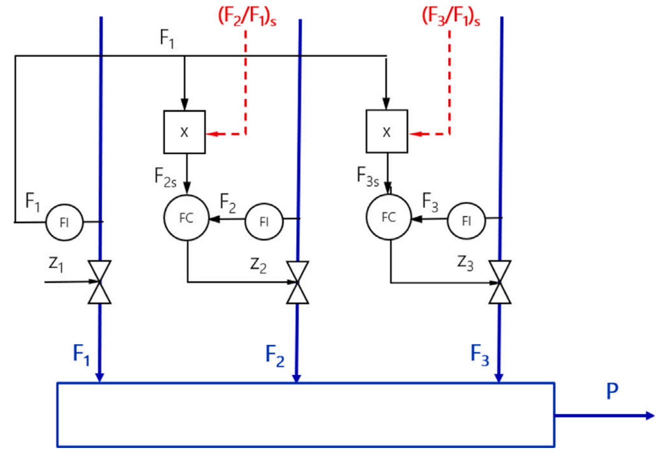

Fig. 5. Ratio control for process with $n = 3$ independent extensive variables (flows) where F_1 is a disturbance, F_2 and F_3 are manipulated, and the objective is to keep constant properties y for the product P. Here, we need to set two ratios, which in this case are selected as F_2/F_1 and F_3/F_1.

2.3. Rules for use of ratio control

From the above discussion and derivations, we arrive at the following rules for when to use ratio control:

- **Rule R1.** The selected controlled variable(s) is implicitly assumed to be an intensive variable (y), for example, composition, density, viscosity, taste or temperature.
- **Rule R2.** The system must satisfy the scaling assumption (2).
- **Rule R3.** Since all extensive variables must be scaled by the same factor λ, there can only be *one* independent extensive variable disturbance, X_d. The variable X_d is sometimes called the "basis", "wild variable", "master variable", "throughput disturbance" or "throughput manipulator" (TPM).
- **Rule R4.** If the system has n independent extensive variables X where one of them is free to choose (the throughput disturbance), then from (2) we need to manipulate the remaining $n-1$ variables (in X) to keep $n-1$ ratios (or more generally, $n-1$ dependent intensive variables y) constant. For a change (disturbance) in the throughput X_d (basis, wild flow) this will (at steady state) result in keeping *all* dependent intensive variables y constant, including the selected controlled variable(s) mentioned in Rule R1.

Rule R3 is particularly important and frequently violated in practice, typically, by fixing an independent extensive variable which should have been manipulated in ratio to the basis.

To illustrate Rule R4, consider the process in Fig. 5 which has $n = 3$ independent extensive variables (flowrates F_1, F_2, F_3). If the process satisfies the scaling assumption (say, it is a mixing process or a distillation column), then manipulating $n-1 = 2$ of these variables (F_1, F_2 or F_3) to keep $n-1 = 2$ ratios (or more generally, $n-1 = 2$ dependent intensive variables) constant, will keep *all* dependent intensive product variables constant at steady state. Fig. 5 shows one particular control structure, where F_1 (measured) is assumed to be the throughput disturbance X_d ("basis") and F_2 and F_3 are manipulated to keep the ratios F_2/F_1 and F_3/F_1 constant.

An application of the double ratio control scheme in Fig. 5 is a distillation column (Fig. 6) where the control objective is to keep constant product compositions (in D and B). The throughput disturbance is the feedrate ($X_d = F_1 = F$), and the manipulated variables for composition control are the liquid reflux ($F_2 = L$) and vapor boilup ($F_3 = V$). In Fig. 6, this is achieved by fixing the ratios $F_2/F_1 = L/F$ and $F_3/F_1 = V/F$. Assuming that the distillation column satisfies

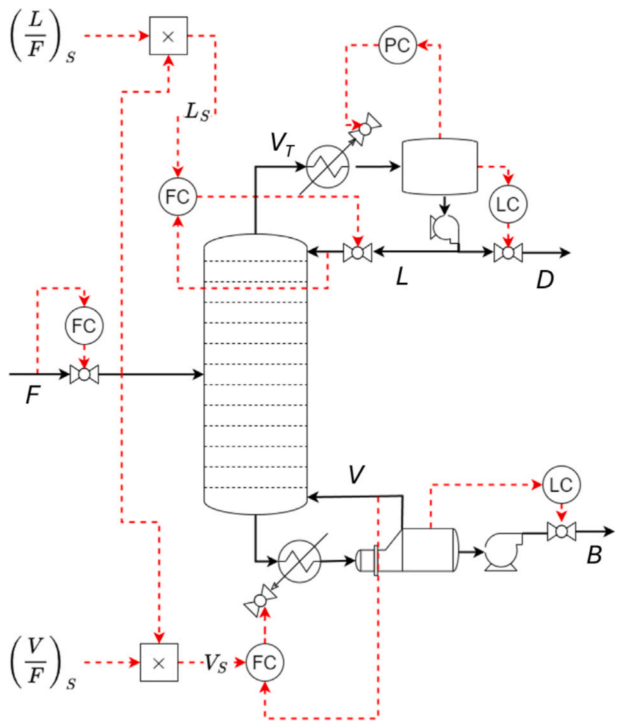

Fig. 6. Ratio control of distillation column with fixed ratios L/F and V/F which (ideally) results in constant product compositions y_D and y_B (in streams D and B) for disturbances in the feedrate F (assuming constant feed composition, constant pressure, vapor–liquid equilibrium and constant stage efficiencies).
In the discussion section, it is argued that the scheme is not recommended if V saturates (e.g., the heat supply at 100% valve opening) because then only one ratio (L/F) is kept constant, and this turns out to be worse than keeping L constant.

the scaling assumption,[3] all the intensive variables (including product compositions) will remain constant with the structure in Fig. 6, in spite of disturbances in the feedrate F. In addition, to handle disturbances in feed composition (x_F) and other sources of uncertainty, an outer temperature controller (TC) should be added to manipulate either $(L/F)_s$ (with temperature measured in the top section of the column) or $(V/F)_s$ (with temperature measured in the botton section). *Comment:* There are many other possible control structures for distillation. Fig. 6 uses the common LV-configuration where D and B are used for level control, V_T (cooling) is used for pressure control, and L and V remain as degrees of freedom for composition control. Even with the LV-configuration, there are many alternative ratio control schemes, for example, using the ratios L/D and V/B [16]. Note that these ratios do not involve F, so it is not necessary to measure the extensive variable disturbance X_d (F in this case) to apply ratio control; see also Rule R6.

Ratio control is very flexible and we may state three additional rules:

- **Rule R5.** When setting a ratio $R = X_2/X_1$, it is not required that the extensive variables X_1 and X_2 have the same units. For example, X_1 could be a flowrate F_1 (in units m^3/h) and X_2 a heat input Q (in units J/s)
- **Rule R6.** It is not required to measure the extensive disturbance variable X_d (basis, "wild flow", e.g., F_1 in Figs. 1 and 5) to apply ratio control, but in this case we need to use a ratio to another measured extensive variable X_1.
- **Rule R7.** It is also not required that any slave controllers for X_2 (e.g., FCs for F_2 and F_3 in Fig. 5) directly manipulate their own valves (z_2 and z_3), although we want the response from valve position to measured flow to be fast with a large gain to make the slave loop fast and non-interactive.

To illustrate Rule R6, consider Fig. 5 where F_1 is the measured throughput disturbance and we fixed the ratios F_2/F_1 and F_3/F_1. We would get the same steady state (and constant product composition y) if we obtained the setpoints for F_2 and F_3 by fixing instead the ratios F_2/P and F_3/F_2, where $P = F_1 + F_2 + F_3$ is the measured product outflow. As just mentioned, the same applies to a distillation column with feedrate F as the disturbance, where one may fix the ratios L/D and V/B rather that L/F and V/F, and still achieve constant product compositions (y).

To illustrate Rule R7, assume in Fig. 5 that we cannot measure F_3, but we can measure the outflow P which is affected by z_3. In this case, we get the same result at steady state if the flow controller (FC) that manipulates z_3 sets the ratio $(P/F_1)_s$, instead of the ratio $(F_3/F_1)_s$. Furthermore, in distillation, the boilup $F_3 = V$ is rarely manipulated directly by its own valve. Rather, the valve is located on the other side of the heat exchanger as illustrated in Fig. 6.

Finally, note from Rule R4 that we may replace a ratio by feedback control of a measured dependent intensive variable (y), which may not be the selected controlled variable (y_{CV}). One reason we may want to do this, is because "pure" ratio schemes may be sensitive to errors in flow measurements. If y correlates well with the controlled variable y_{CV}, then feedback control of y will also correct (partly) for intensive variable (x) disturbances,

2.4. *When does the scaling assumption hold?*

The scaling assumption holds for all thermodynamic equilibrium systems. From this, the scaling assumption (and thus the use of ratio control) applies to many process units, including

- Mixers
- Equilibrium reactors
- Equilibrium distillation with a constant number of theoretical stages

More generally, for the scaling assumption (2a) to hold, we must assume that the *process efficiencies* remain constant when there are throughput changes.

> *We define a process efficiency as the ratio between two extensive variables.*

Note that a flow split fraction fits this definition. Normally, the two extensive variables are defined such that the efficiency is in the range 0 to 1, although this is not required from the above general definition.

Consider the foundation for process system engineering models, which are the mass, component, and energy balances. In compact form, they become at steady-state for each unit and for the overall process:

$$0 = F_{\text{in}} - F_{\text{out}} \qquad [\text{kg/s}] \tag{8a}$$

$$0 = c_{A,\text{in}} F_{\text{in}} - c_{A,out} F_{\text{out}} + G_A \qquad [\text{mol A/s}] \tag{8b}$$

$$0 = h_{\text{in}} F_{\text{in}} - h_{\text{out}} F_{\text{out}} + Q + W \qquad [\text{J/s}] \tag{8c}$$

Here, c_A [mol A/kg] is the concentration of component A, and h [J/kg] is the specific enthalpy. c_A and h are intensive variables.

The actual model equations may be more complicated with multiple inflows and splits resulting in multiple outflows. Nevertheless, it is clear that the mass balance (8a) satisfies the scaling assumption, that is, if

[3] Distillation columns with constant pressure and a fixed number of theoretical stages with vapor–liquid equilibrium on each stage, satisfy the scaling assumption.

$F_{in} = \lambda F^0_{in}$ then $F_{out} = \lambda F^0_{out}$, assuming that any flow split fractions remain constant. Here, λ denotes the throughput scaling factor.

Without reaction ($G_A = 0$), the individual component balances (8b) also satisfy the scaling assumption, if the concentrations (intensive variables) are constant, and the following holds for separator splits:

- Separation processes with constant component splits or constant component recoveries or constant product compositions or thermodynamic equilibrium satisfy the scaling assumption.

With reaction, the component balance (8b) satisfies the scaling assumption provided the reaction term G_A scales with the throughput, that is, if the amount of A formed by reaction (which may be negative if A is consumed) can be written as $G_A = \lambda G^0_A$. This holds for equilibrium reactors (as already stated), and also for reactors with constant conversion: It then follows:

- Reactors with constant reaction conversion satisfy the scaling assumption.

There are many ways to define conversion, for example, as the ratio of "amount of A reacted" to the "amount of A fed", which fits the definition of an efficiency. Other related assumptions are constant yield, selectivity and recovery.

The energy balance (8c) satisfies the scaling assumption provided the supplied heat Q and work W both scale with the throughput, that is, if $Q = \lambda Q^0$ and $W = \lambda W^0$.

For a heat exchanger, the independent extensive variables are the flowrates of hot and cold fluid (F_h and F_c). *For heat exchangers without phase change*, the heat transfer from the hot to the cold side can be written in the form (e.g., [17])

$$Q = \epsilon \cdot Fc_P \cdot (T_{h,in} - T_{c,in}) \tag{9}$$

where ϵ is the heat exchanger efficiency and we define $Fc_P = \min\{F_c c_{Pc}, F_h c_{Ph}\}$. We have that $\epsilon \to 1$ for an infinitely large heat exchanger (with $A = \infty$). From (9) it follows that the heat transfer Q scales with throughput (and satisfies the scaling assumption) if the heat exchanger efficiency ϵ is constant. Here, ϵ is a function of the heat capacity ratio $C = (F_h c_{Ph})/(F_c c_{Pc})$ and the number of transfer units,

$$\mathrm{NTU} = UA/(Fc_P)$$

Thus, the use of raio control for heat exchangers without phase change assumes that NTU is constant, independent of the throughput F. This does not immediately seem likely since the area A is constant during operation. Nevertheless, ratio control is suggested by Smith [18] (page 204) and simulations show that it works quite well in terms of keeping constant temperatures. The reason is that the heat transfer coefficient U increases with throughput because of better heat transfer on both the hot and cold sides. The Dittus-Boelter correlation says that the individual heat transfer typically increases with flowrate to the power 0.8 (i.e., $U \sim F^{0.8}$).

- In summary, although heat exchangers without phase change do not truly satisfy the scaling assumption (which would require $U \sim F$ to get a constant ϵ), it may be close enough for many practical applications.

Next, consider *heat exchangers with phase change* (boiling or condensation):

- For heat exchangers with phase change, the scaling assumption often applies because the heat transfer Q is directly proportional to the amount of fluid changing phase (F_h or F_c).

If the heat exchanger does not satisfy the scaling assumption, then it may still be possible that overall process satisfies the scaling assumption if Q is manipulated to keep a constant temperature using feedback control:

- The heat transfer Q scales with throughput (and satisfies the scaling assumption) if the temperature after the heat exchange is assumed constant. Here, temperature may controlled by manipulating a bypass or one of the flows (F_c or F_h) - not necessarily in constant ratio.

For the shaft work W, the following applies for a variable speed machine:

- The shaft work from turbomachinery (pump, compressor or turbine) satisfies the scaling assumption if the supplied work W is set in proportion to the throughput (ratio control) and the thermodynamic efficiency η for the machine is assumed constant.

The value of the efficiency η does not affect the energy balance (8c), because the lost work to friction will be supplied as heat Q. However, it is necessary that the efficiency η be constant so that the pressure change (intensive variable) is independent of throughput. The efficiency η may often be assumed constant around the nominal point of operation. If the turbomachinery does not have variable speed, then the supplied work W cannot be freely manipulated, and the scaling assumption does not apply and the pressure change across the turbomachinery depends on throughput.

In most cases, a process engineer can determine from physical insight whether the scaling assumption is satisfied or not: Will all dependent intensive variables y remain constant when all n independent extensive variables are scaled by the same factor λ, or not? If in doubt, one can always check if the system satisfies (2a) and (2b).

2.5. When does the scaling assumption not hold?

There are many process units where the scaling assumption does not hold and therefore ratio control should not be used, or at least used with care.

First, the scaling assumption does not hold if the efficiency, split fraction or conversion depends on the size of the unit (e.g., as given by its volume or area), because size usually cannot be manipulated during operation. This includes, for example, reactors where kinetics are important, and separators where the separation efficiency depends on throughput. For example, for a CSTR reactor where the reaction kinetics determine the conversion, we have $G_A = r_A V$ where r_A [mol A/m^3,s] is the reaction rate and V [m^3] is the reactor volume. To satify the scaling assumption, G_A should scale with the throughput. Here, r_A depends on intensive variables (concentration, temperature and pressure) and should be constant when there are throughput changes. Thus, to satisfy the scalinmg assumption, the reactor volume V (extensive variable) should increase with throughput. Howeever, to optimize operation, V is usually kept constant at its maximum value (to maximize conversion G_A), so the scaling assumption does not hold.

Second, from Rule R3, ratio control requires that *all* extensive variables be scaled by the same factor. For example, if the process has a compressor where the duty W cannot be manipulated (because it has fixed speed) then the scaling assumption does not hold, unless the compressor is the only extensive variable (throughput) disturbance. Rule R3 generally implies that we need to be careful when applying ratio control to processes with many independent extensive variables, such as distillation columns. The reason is that it may happen that one of independent extensive variables is constant, for example, because it reaches saturation, and then the scaling assumption does not apply (see Discussion section).

3. Implementation of ratio control

The recommended implementation of ratio control for F_2/F_1, with a multiplication element and a flow controller (FC), was shown in the introduction, see Figs. 1, 3 and 4a.

The implementation with a division element and a ratio controller (RC) in Fig. 4b is *not* recommended. Flower and Parr [19] writes that it is "an intuitive, but incorrect, method of ratio control" where "the loop gain varies with throughput" (see (14)). Love [4] writes that it is "commonly used throughout industry, although in two different accounts, the indirect method [using the multiplication element] is superior". The "two different accounts" are the potential of zero division and the nonlinearity (see (14)).

3.1. Flow controller

A flow controller (FC) is an important part of most ratio control strategies. In Figs. 1 and 4a, the ratio setpoint $R = (F_2/F_1)_s$ can be implemented accurately by measuring F_1 and F_2 and using a flow controller for F_2 with setpoint

$$F_{2s} = (F_2/F_1)_s \cdot F_1 \tag{10}$$

The flow controller for F_2 usually uses the corresponding valve position z_2 as the manipulated variable (MV). The response from z_2 (MV) to F_2 (CV) is usually very fast and often the dynamic process model is assumed to be static, $\Delta F_2 = k_v \Delta z_2$. In such cases, the best controller is a pure I-controller with integral gain $K_I = \frac{1}{k_v \tau_c}$, where τ_c is the desired closed-loop time constant [20]. A typical value for τ_c for a flow controller is between 5 s and 15 s. It is also possible to use a PI-controller, where the integral time τ_I is typically equal to the process time constant τ for the valve, and the controller gain K_c may be selected such that the integral gain $K_I = K_c/\tau_I$ is the same as for the pure I-controller, which gives $K_c = \frac{1}{k_v}\frac{\tau}{\tau_c}$.

It is possible to implement ratio control without a feedback flow controller (FC), for example, using feedforward control from F_{2s} to the valve position z_2. However, this is usually not sufficiently accurate because it involves inverting the valve equation $F_2 = f(z_2)$. The function $f(z_2)$ is nonlinear and uncertain and also depends on the pressure drop over the valve which may vary. Of course, this uncertainty also affects the FC for F_2, because the valve gain $k_v = \partial f/\partial z_2$ will vary. This will change the effective closed-loop time constant, but fortunately it has no effect in steady state because of the integral action in the FC for F_2.

In summary, the main role of the flow controller is to ensure that the manipulated extensive variable F_2 truly scales with the basis variable F_1 at steady state, as required by the scaling assumption

Flower and Parr [19] refer to the flow controller for F_2 as the slave loop and to F_1 as the master flow. If there is also a flow controller for the master flow F_1, then we may use the setpoint $F_{1,s}$ instead of F_1 as the basis for the multiplication element when computing the setpoint for F_2 in (10) ([1], page 158). This (using F_{1_s} as a replacement for F_1) may be advantageous to speed up the response for F_2, but the disadvantage is that logic must be added for the case when the F_1-controller is not active or if F_1 does not reach its setpoint, for example, because of valve saturation for z_1 [12]. It is also possible to use a "blend" of the two variables as the basis, for example, $\gamma F_{1,s} + (1-\gamma)F_1$ where γ is an adjustable parameter between 0 and 1 [11]. Instead of using the setpoint $F_{1,s}$ as a replacement for F_1, a better solution may be to tune the flow controller for F_2 to be fast (with a small value of τ_c) or to add some "feedforward action" from $F_{1,s}$ to z_2.

3.2. Outer feedback controller

We mentioned already in the introduction (Fig. 1) the benefits of adding an outer feedback controller to update the setpoint R for the ratio control. We have the following rule:

- **Rule R8.** An outer feedback controller with integral action can manipulate the ratio setpoint R to give perfect control of the intensive controlled variable ($y = y_s$) at steady state, in spite of uncertainty. In addition, this updated setpoint R for the ratio control, maintains the perfect steady-state disturbance rejection property for throughput disturbances, also this in spite of uncertainty.

Here, the "uncertainty" includes intensive variable disturbances, model changes and measurements errors in the extensive variables (usually flow) measurements, for example, measurement errors for F_1 and F_2 when implementing $F_2 = RF_1$.

To illustrate Rule R8, consider the dynamic simulations in Fig. 2 (black line) with further details in Fig. 16. We see that the perfect disturbance rejection of flowrate disturbances for ratio control ($F_2 = RF_1$) is maintained at $t = 3$ and $t = 7$ [h], because R has been by updated from 1 (at $t = 0$h) to 0.5 (at $t = 3$h) and further to $R = 0.25$ (at $t = 7$). This is not the case with conventional "additive" feedforward and feedback control, where R is fixed at 1.

In terms of tuning of the outer feedback controller (CC in Fig. 1), there should in general be a time scale separation of at least 4 (preferably larger) between the faster flow controller (FC) and the slower outer controller (CC) [21]. This is necessary to avoid that the tuning of the flow controller (FC) will affect the performance and tuning of the outer loop (CC). The closed-loop time constant of a flow controller (FC) can typically be 5 s to 15 s, which means that the closed-loop time constant for the outer controller (CC) should at least be 20 s to 1 min.

3.3. Nonlinearity

We here show that the ratio implementation with the multiplication element in Figs. 1 and 4a, results in a linear response with a constant process gain, independent of throughput changes, both for the inner flow controller (FC) as well as for the outer property controller that sets the ratio setpoint (CC in Fig. 1).

3.3.1. Outer loop nonlinearity (for controller CC in Fig. 1)

One advantage with ratio control is that it makes the response from the ratio setpoint $R = (F_2/F_1)_s$ to y (for the outer controller; CC in Fig. 1) independent of the flow disturbance (F_1). In some sense this is obvious, because the scaling assumption, which is the basis for ratio control, says that y will be constant when the ratio R is constant, independent of the values of the extensive variables.

Nevertheless, to understand it better, consider the simple mixing process in Fig. 1 where F_1 is the disturbance and F_2 is the manipulated variable (the true MV is usually the valve position z_2, but we assume that we have a flow controller for F_2 as shown in Fig. 1). Assume that y is the fraction of a given component in the product and that x_1 and x_2 are the feed fractions. Then, from the component material balance, y is the average of the feed fractions,

$$y = \frac{x_1 F_1 + x_2 F_2}{F_1 + F_2} \tag{11}$$

Next, linearize this model for two cases:

1. *Without ratio control.* In this case, the MV for the outer loop (CC) is F_2 (rather than $R = F_2/F_1$). The process as seen from the outer loop is then from F_2 to y, and the linearized model becomes $\Delta y = K \Delta F_2$ with gain
$$K = \frac{F_1^*(x_2^* - x_1^*)}{(F_1^* + F_2^*)^2} \tag{12}$$
where superscript * denotes the value at the point of linearization. Note that the gain K depends on the disturbance F_1 and becomes infinite as the throughput $F_1 + F_2$ approaches zero. This nonlinearity applies when we use conventional feedback and feedforward control.
2. *With ratio control.* Here, the process as seen from the outer loop is from $R = F_2/F_1$ to y, and the linearized model becomes $\Delta y = K_R \Delta R$ with gain
$$K_R = \frac{x_2^* - x_1^*}{(1 + R^*)^2} \tag{13}$$
With a constant ratio $R^* = F_2^*/F_1^*$, the gain K_R is independent of the throughput disturbance F_1 (as expected).

In summary, the use of ratio control makes the process gain for the outer loop (CC in Fig. 1) independent of the throughput (e.g., F_1). This makes it possible to use linear controllers over a wider range of throughputs.

3.3.2. Inner loop nonlinearity (for controllers FC and RC in Fig. 4))

We here compare the two ratio control schemes in Fig. 4 using the analysis of Shinskey [2] (page 160) and show that the implementation with a multiplication element is better than with a division element.

For the scheme with the multiplication element in Fig. 4a, we have linearity for the inner flow controller FC (provided the valve is linear). This follows since for a linear valve, the response for the FC, from the manipulated variable z_2 (valve position) to the controlled variable F_2, is

$$\Delta F_2 = k_v \Delta z_2$$

where k_v is a constant.

On the other hand, for the scheme with the division element in Fig. 4b, we have a strong nonlinearity for the inner ratio controller RC. This follows since the response for the RC, from the manipulated variable z_2 (valve position) to the controlled variable $R = F_2/F_1$, is (again assuming a linear valve)

$$\Delta R = \frac{k_v}{F_1}\Delta z_2 = \frac{1}{F_1}\Delta F_2 \tag{14}$$

The gain becomes infinite if F_1 goes to zero. It does not help to control the inverse ratio, $R' = F_1/F_2$, because here we get $\Delta R' = -\frac{F_1}{F_2^2}\Delta F_2$ and the gain becomes zero if F_1 goes to zero and infinite if F_2 goes to zero.

3.4. Dynamic compensation

The scaling assumption, which is the basis of ratio control, applies to the steady-state behavior. To achieve better dynamic behavior for the multiplication scheme in Fig. 4a, one may introduce dynamic compensation as shown in Fig. 7. As with conventional feedforward control, we may use a lead–lag element with delay on the form

$$\text{Dynamic element} = \frac{Ts+1}{(\tau_1 s+1)(\tau_2 s+1)}e^{-\theta s} \tag{15}$$

where any of the parameters $(T, \tau_1, \tau_2, \theta)$ may be zero. An example for first-order order with delay models for u and d is given in the discussion section; see (26). Note that the steady-state gain is 1. Thus, the dynamic compensation does not alter the steady-state correctness of ratio control, which follows solely from the scaling assumption.

The dynamic element is usually applied to the measured throughput signal (F_1 in Fig. 7), to avoid that the dynamic element becomes a part of the outer control loop which adjusts the ratio setpoint $R = (F_2/F_1)_s$.

Dynamic compensation may be viewed as introducing a model-based feedforward on top of the model-free steady-state ratio structure. For example, for the distillation column Fig. 6, we may add a dynamic element on the measurement of F for the ratios L/F and V/F. We may use the time constant τ_1 to lag the measurement of F, because it takes some time for a change in F to effect the column ends (which is where we want to control the composition y and where L and V enter). In other cases, if the flow control loop (FC) for V is slow, then we may try to speed up the response for V by choosing $T > \tau_1$ in the dynamic element. In this case, T could be the closed-loop time constant of the FC-loop.

Luyben [22,23] compares linear *dynamic* feedforward control (which he calls "additive" feedforward control) with *static* ratio control (which he calls "multiplicative" feedforward control), and concludes that the "additive" structure is often better. However, this is not a fair comparison, because dynamic compensation can easily be added also to ratio control, as shown in Fig. 7. This would make ratio control the preferred choice (assuming that the scaling assumption holds), because ratio control gives the correct feedforward action at steady state.

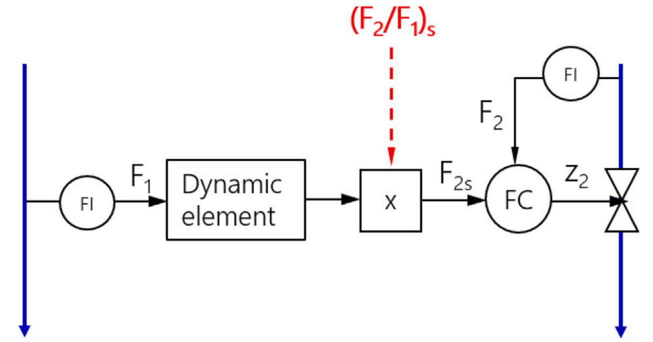

Fig. 7. Ratio control with dynamic compensation. The dynamic element (e.g., (15)) has no effect at steady state.

4. More complex ratio control implementations

4.1. Dual ratio control

Conventional ratio control, as shown Fig. 1, fails if the manipulated variable (MV) used to control the ratio (z_2 in Fig. 1) saturates (e.g., at a fully open valve position) or if the MV is temporarily set by another controller (override). If keeping the desired ratio has high priority, then the solution is to use *"dual" ratio control, where we let a second manipulated variable (MV) (e.g., z_1) take over the task of ratio control when the original MV (e.g., z_2) cannot be manipulated.* This is an example of MV-MV switching for which there generally are three alternatives [21]:

1. Split-range control (SRC) (one controller with two MVs)
2. Split-parallel control (SPC) (two controllers, one for each MV, with setpoint separation)
3. Valve position control (VPC) (the second MV, e.g., z_1, is used, when necessary, to avoid saturation for the original MV, e.g. z_2)

The last alternative (VPC) has the advantage that the original MV (z_2) is always used to control the ratio. However, this means that this MV (z_2) is not allowed to fully saturate (at 100%), which may result in an economic loss. Furthermore, the outer VPC loop (e.g., using z_1 to control z_2 at 80% position) may be too slow to avoid saturation in z_1, which may lead to temporary loss of ratio control.

If tight ratio control is required, then the second alternative (SPC) is not desirable as it requires a quite large setpoint separation to work well [24]. This has also been confirmed by simulations (not included) for the dual ratio control case.

It therefore seems that the first alternative (SRC) is usually the best for dual ratio control, and this is in line with industrial practice (K. Forsman, personal communication, 2025).

An example of dual ratio control using split-range control is shown in Fig. 8. The process has two feeds (F_1 and F_2) and the main control objective is to maintain a desired ratio $(F_2/F_1)_s$, and a secondary control objective is to keep F_1 at a given setpoint F_{1s}, but the secondary objective is given up if we reach saturation (for F_1 or F_2). Nominally, without saturation for F_1 or F_2, we use valve position z_2 to control the ratio F_2/F_1, and valve position z_1 to control the flowrate F_1. If F_2 reaches saturation (i.e., $z_2 = 100\%$), then the split range block makes z_1 take over the ratio control. However, this means that we need to switch the controlled variable (CV) for z_1 from flow control (F_{1s}) to ratio control ($(F_2/F_1)_s$). This CV-CV switching normally requires a MIN-selector [21] as shown in Fig. 8, but we show below that for split-range control (SRC), a multiplication element may be used instead.

With the implementation in Fig. 8, the measured value of F_1 is multiplied with the ratio setpoint $(F_2/F_1)_s$ to compute the setpoint F_{2s}. This is sent to the split-range flow controller FC-SR which computes the internal variable u and sends it to the logic SR-block. The SR-block

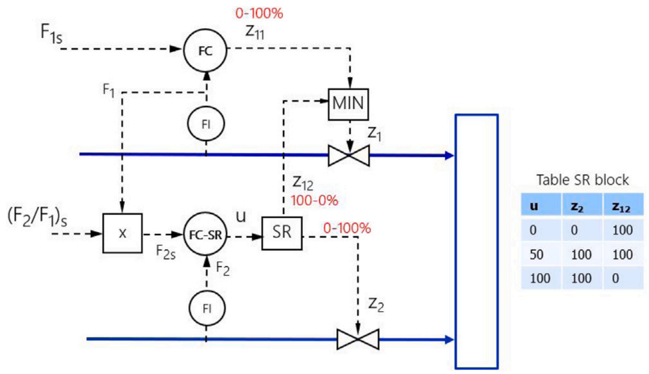

Fig. 8. "Dual" ratio control using split-range control (SRC) to handle saturation (100%) for the nominal MV (z_2).

manipulates the original MV (z_2) when u is less than 50% and the second MV (z_1) when u is above 50%. The split value (here, 50%) is a tuning parameter that can be selected to modify the effective gains for the two control loops. Fig. 8 includes a table showing how the SR-block maps the input signal u to the valve positions z_2 and z_{12}, assuming that the split value is 50% and that both valve positions are in the range 0%–100%.

The proposed strategy works well as shown by dynamic simulations in Fig. 9 (for controller tunings, see Appendix). There are flowrate setpoint changes at times 1, 2 and 3 [min] and ratio setpoint changes at times 5, 6 and 8 [min]. Until $t = 6$ [min], the ratio F_2/F_1 is controlled using F_2 (more exactly, using valve position z_2) and the flowrate of F_1 is controlled using F_1 (more exactly, using z_1). The increase in ratio setpoint from 1 to 1.4 at $t = 6$, results in F_2 reaching its maximum value (F2MAX=1 [kg/s], corresponding to $z_2 = 100\%$) and the split range block switches to using F_1 for ratio control. To reach the new ratio setpoint of 1.4, F_1 is reduced to 1[kg/s]/1.4=0.71 [kg/s] which is below its setpoint (F1SP=0.8 kg/s), that is, we have to give up control of F_1. This results in an "unavoidable offset" for F_1 because ratio control has priority and there exists no feasible steady state solution that satisfies $F_2/F_1 = 1.4$, $F_2 \leq 1$ kg/s and $F_1 = 0.8$ kg/s.

Importantly, the SRC scheme in Fig. 8 is able to maintain ratio control under all conditions, as desired.

However, the dynamic response for reaching the new ratio setpoints at $t = 6$ and $t = 8$ is a bit delayed. For example, at $t = 6$, the switch to using z_1 for ratio control does not occur immediately when we reach saturation ($z_2 = 100\%$) and z_{12} in Fig. 8 drops below 100%: The reason for this "limbo effect" is that z_{11} (the output from the flow controller for F_1) is initially at 80% (in this case), and it takes some time before the integral action in the split-range flow controller (FC-SR) increases u from 50% to 60%, so that z_{12} drops to 80% where switching with the MIN-selector occurs.

The "limbo" effect is discussed in Appendix E in Zotică et al. [25] who propose to speed up the switching by using bias updates. However, a much simpler solution is proposed by the modified SRC scheme in Fig. 10. Here, the signal z_{12} from the split-range block (which must be in the range 1 to 0) is multiplied by the valve position z_{11} desired by the flow controller and the switch occurs immediately when z_{12} drops below 1. The MIN-selector may be omitted because $z_{12} \leq 1$. Initially, when z_{12} drops below 1, there will be some "fighting" between controllers FC and FC-SR, because both controllers are connected to z_1, but the fighting will stop quite soon due to the anti-windup action in controller FC which manipulates z_{11}. The split-range controller FC-SR does not have anti windup. The proposed "multiplication trick" in Fig. 10 results in much better ratio control at times 6 and 8 as shown by the simulation in Fig. 11. In conclusion, the modified split-range scheme in Fig. 10 is recommended for dual ratio control applications.

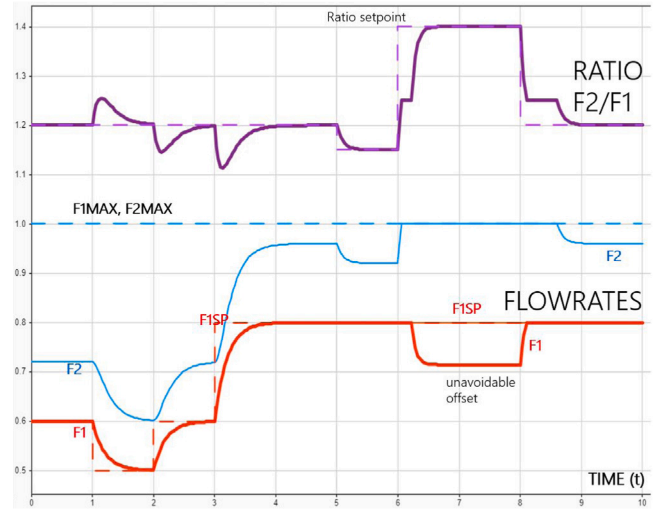

Fig. 9. Simulation of dual ratio SRC scheme in Fig. 8. The limbo effect, caused by a delay in transition between z_1 and z_2 involving the MIN-selector, is seen for the ratio at times 6 and 8.

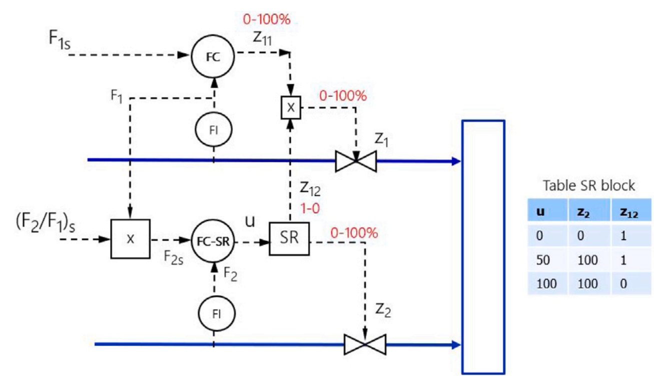

Fig. 10. Modified (proposed) split-range control (SRC) scheme for "dual" ratio control where the MIN-selector in Fig. 8 is replaced by multiplication (by a signal in the range 1 to 0) to avoid the limbo effect.

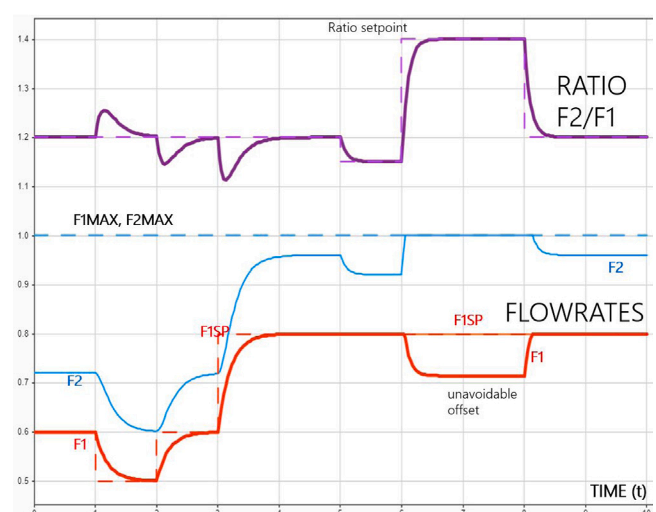

Fig. 11. Simulation of modified (proposed) dual ratio SRC scheme in Fig. 10 which avoids limbo effect during transition between using F_1 and F_2 for ratio control.

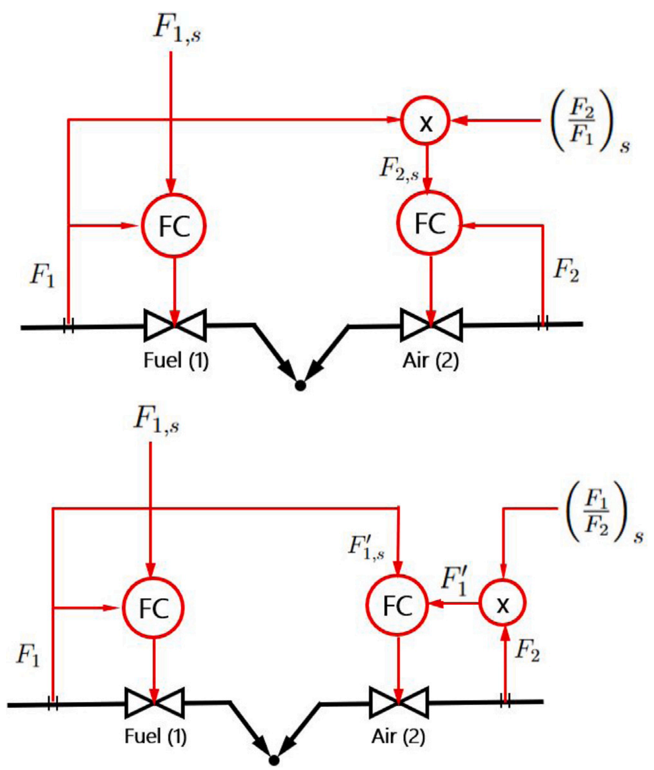

Fig. 12. Conventional ratio control implementations for combustion. The two schemes are equivalent for practical purposes. The second scheme with the inverse ratio is shown as an intermediate step to the cross-limiting scheme in Fig. 13.

An alternative to the dual ratio schemes in Figs. 8 and 10 is the "tracking ratio station" scheme of Hägglund [12]. However, this scheme is rather complicated to implement, and, more seriously, it results in undesirable frequent switching.

4.2. Cross-limiting ratio control

Another common, but fairly complex, application of ratio control is cross-limiting control in combustion power plants [26,27]. Here, the objective is to mix air (2) and fuel (1) in a given ratio, but during dynamic transients, when there will be deviations from the given ratio, one should make sure that there is always an excess of air, that is, we should always have $F_1/F_2 < (F_1/F_2)_s$.

A conventional ratio control scheme for combustion processes is shown in Fig. 12. The setpoint for the ratio, $(F_1/F_2)_s$, could be set by a feedback controller (not shown) which controls, for example, the remaining oxygen after combustion. The setpoint for the fuel, $F_{1,s}$ could be set by a feedback controller (not shown) which controls, for example, the power or the steam pressure. The conventional scheme in Fig. 12 is not able to maintain excess air under all dynamic transients, because normally (and always in the linear case) if $F_1/F_2 < (F_1/F_2)_s$ during one transient then $F_1/F_2 > (F_1/F_2)_s$ for the opposite transient (e.g., see the ratio for changes down and up in the flow setpoint in Fig. 9 at times 1 and 2).

Interestingly, the scheme in Fig. 13 with crossing min- and max-selectors achieves $F_1/F_2 < (F_1/F_2)_s$ during almost all transients. The scheme is widely used in industry and is described in many industrial books (e.g., [26,27]).

A dynamic simulation is shown in Fig. 14 for an increase in flow setpoint ($F_{1,s}$) at times 1 an 3, decrease at time 5, and finally a change down and up in the ratio setpoint at times 7 and 9. How does it work?

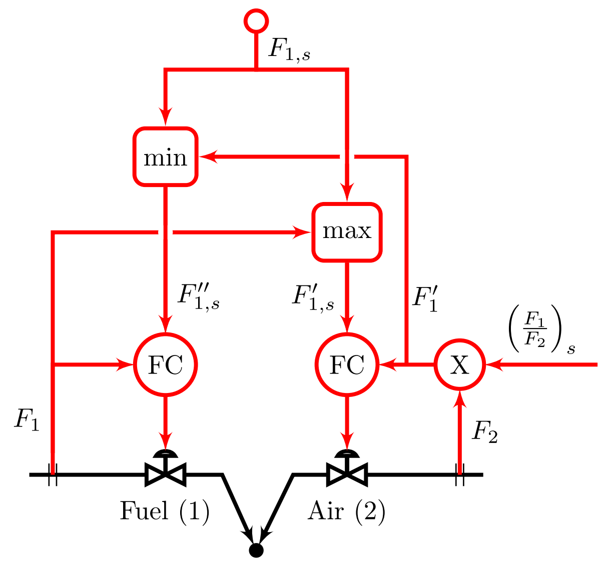

Fig. 13. Cross-limiting ratio control for combustion where air (2) should always be in excess of fuel (1) [27,28].

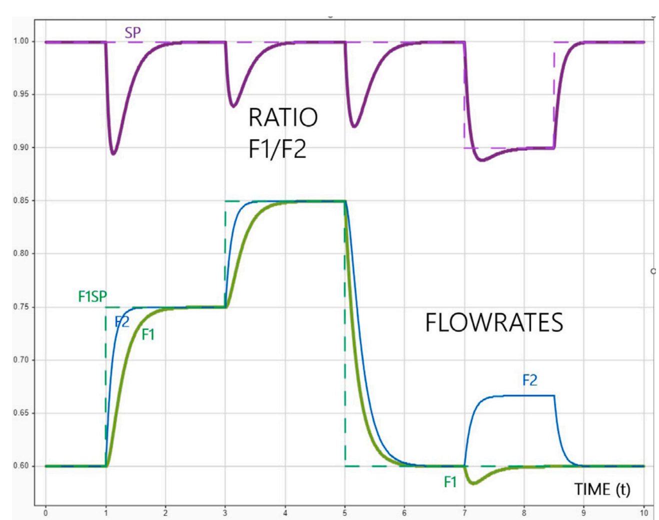

Fig. 14. Simulation of the cross-limiting ratio control scheme in Fig. 13.

Let us first consider the change in fuel rate (F_1), which is the most important, and achieves $F_1/F_2 < (F_1/F_2)_s$ during all transients. When we increase the fuel setpoint ($F_{1,s}$), the air flow (F_2) will increase first, while the MIN-selector holds back the fuel increase. On the other hand, when we decrease the fuel setpoint ($F_{1,s}$), the fuel flow (F_1) decreases first while the MAX-selector holds back the air flow (so it remains high for a longer time). In summary, we always have excess air during the dynamic transients in fuel rate.

However, when we suddenly decrease the ratio setpoint $(F_1/F_2)_s$ (at time 7), the ratio F_1/F_2 is above its setpoint for a short time. This situation is unavoidable, since it is not possible to react immediately to a setpoint change using feedback control. In any case, large and sudden setpoint changes in the fuel ratio are not expected, so this is unlikely to be a problem in practice.

5. Discussion

5.1. Previous work on the scaling assumption

In the literature, there appears to be no clear statement of the scaling assumption (2) and its relationship to ratio control. The one who comes closest is Riggs [10] who writes the following in the first paragraph in his section on ratio control: "Many processes scale directly with the feedrate to the process, e.g. distillation columns and wastewater neutralization. For distillation columns, all the liquid and vapor flow rates within the column are directly proportional to the column feed rate if the product purities are maintained and the tray efficiency is constant".

The idea of scaling all flowrates (and more generally all extensive variables) with respect to a "basis" is extensively used for mass and energy balance calculations in process engineering. However, its theoretical basis, namely the scaling assumption, is not mentioned in most textbooks. One exception is Reklaitis [15] who proves the homogeneity (2b) (with order $h = 1$) of the material balance equations (page 40) and shows that this leads to the scaling assumption: "If any set of flows F_i satisfies the balance equations and if α is any number, the flow rates αF_i also satisfy the balance equations". He notes that the same applies to the energy flows in the energy balance (page 460). However, this is not enough to guarantee that we satisfy (2a) (with order $h = 0$), that is, that we get the same steady-state solution with constant intensive variables (y).

Riggs [10] and Skogestad [29] both state that constant process efficiencies are required for the scaling assumption and (2a) to hold. Skogestad [29] writes (page 66): "An initial basis for a selected stream is often chosen, for example 1 kg or 100 mol/s. If necessary, we can later rescale (up or down) all the streams to the desired quantity. Mass, energy, volumes, etc. (all extensive variables) will scale with the same factor provided that the efficiencies of the units remain constant". This sounds reasonable, except that no clear definition of a "process efficiency" is given.

The scaling assumption holds for systems in thermodynamic equilibrium. In thermodynamics, (2b) (with $h = 1$) is used to derive Euler's theorem and from this we may derive the fundamental equation of thermodynamics, Legendre transformations, and the Gibbs–Duhem equation [13]. Skogestad [30] states the scaling assumption and stresses the difference between extensive and intensive variables. Following Modell and Reid [13], he derives Euler's theorem in thermodynamics and uses this to derive consistency relationships (Eq. (27) in Skogestad [30]) for linear steady-state models for systems that satisfy the scaling assumption, and shows that many published models do not satisfy this and therefore are incorrect. The counterpart of these consistency equations in thermodynamics is the integrated Gibbs–Duhem equation.

5.2. Limitation on the use of ratio control for distillation

Rule R4 says that for an application with $n = 3$ independent extensive variables at steady state, we need to keep $n - 1 = 2$ ratios (or other intensive variables) constant. For distillation (Fig. 6) with 3 independent extensive variables (F, L, V) this means that we need to keep two ratios constant (and not only one), for example L/F and V/F as in Fig. 6. However, from Rule R3, we can only have a single extensive disturbance, which in Fig. 6 is assumed to be the feedrate F. This means that ratio control should *not* be applied to a distillation column with a fixed heat input V and fixed (given) feedrate F, because then we in reality have two extensive disturbances (V and F). Nevertheless, reflux-to-feed (L/F) ratio control is commonly recommended (e.g., [8] (p. 321), [23]), without stating that also V must change in proportion to F. In fact, with constant heat input (constant V), a constant reflux ratio L/F /Fig. 6) gives the wrong (opposite) response in reflux L to a change in feedrate F, that is, it is worse than simply keeping reflux L constant (!). This is easy to explain: In distillation, the light feed components should go to the top product (D) and the heavy feed components to the bottom product (B). An increase in the feedrate F brings in more light components and thus we need to increase D. From a steady-state material balance at the top of the column, we have $D = V_T - L$ (see Fig. 6). Here, with a liquid feed, $V_T \approx V$, so with a constant V we want to decrease L to increase D. This is not achieved with L constant, but it is even worse to keep the ratio L/F constant, as this increases L when F increases, which is the opposite (!) of what we want. The resulting poor composition response is confirmed by dynamic simulations [31].

In summary, for distillation, we need to be careful about applying ratio control because we can only have one extensive variable disturbance X_d (Rule R3) (typically the feedrate) and since $n = 3$, we need to set two ratios, or one ratio and another intensive variable (e.g., temperature), or two other intensive variables (Rule R4). In particular, reflux-to-feed (L/F) ratio control is not a good choice for cases where the feedrate (liquid) is a disturbance and the heat input (V) may saturate.

Since it is less likely that the reflux L saturates, a better choice may be to use ratio control in the bottom (using V/F or V/B) and let the reflux L control composition or a sensitive temperature. This will maintain the desired material balance split (D/F) even if V saturates.

5.3. Generalized ratio control using transformed inputs

One limitation with ratio control is that it relies on an outer feedback loop to updatew R when there are disturbances in the independent intensive variables (x). If such disturbances are important and the outer feedback loop is too slow (for example, because of a time delay for the measurement of y), we may want to make use of model-based feedforward control. For the static case, a simple and powerful nonlinear feedforward approach is to use ideal transformed inputs [21,32], which provides a model-based generalization of ratio control.

Consider the steady-state model (the subscript 0 is used to emphasize that we use a static model)

$$y = f_0(u, d) \tag{16}$$

where y is the controlled variable (CV), u is the manipulated variable (MV) and d is the disturbance variable (including both extensive and intensive disturbances). All variables are vectors in the general case. The ideal transformed input v_0 (controller output) is selected as the right-hand-side of the steady-state model,

$$v_0 = f_0(u, d) \tag{17}$$

For implementation, one needs to invert the model by solving (17) with respect to u for given values of v_0 and d. Assuming invertibility, we can formally write the solution as [32]

$$u = f_0^{-1}(v_0, d) \tag{18}$$

At steady state, the resulting transformed system then trivially becomes

$$y = v_0 \tag{19}$$

That is, we have $y = I v_0$, so we have perfect feedforward control, decoupling and linearization at steady state. It looks like magic, but it works in practice. To achieve perfect control, we must assume that all disturbances d are measured (or at least estimated), but if this is not the case then one may use a simpler variant of f_0 as the transformed input v_0, where we fix the value of unmeasured disturbances to get partial feedforward or decoupling. To correct for model error and unmeasured disturbances, the value (setpoint) for v_0 may be adjusted by an outer controller C (usually a decentralized PID controller).

As an example, consider the simple mixing process in Fig. 15 where $u = F_2$ is the manipulated variable (the true MV is usually the valve position z_2, but we assume there is a flow controller for F_2 as shown in

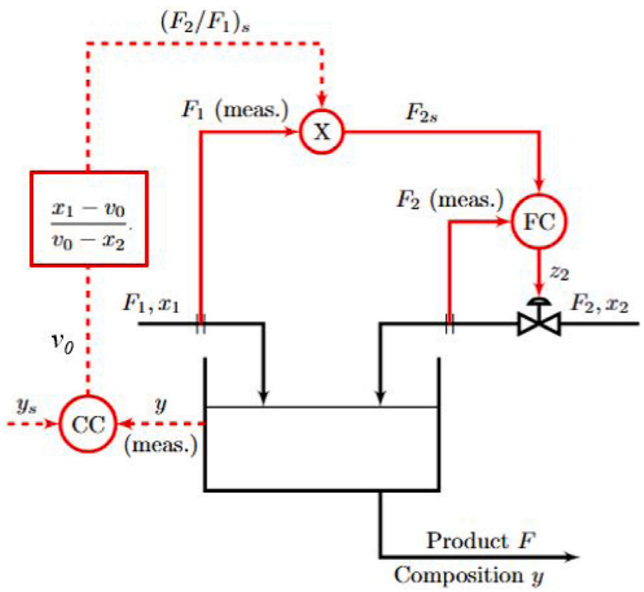

Fig. 15. Improved ratio control strategy for mixing process using transformed input v_0. The feed compositions x_1 and x_2 that enter the computation block, need to be measured or estimated.

Fig. 15) and y (product composition) is the controlled variable. The disturbances (d) are F_1 and the feed compositions x_1 and x_2 (mass fractions). From the steady-state component material balance, we have that y is the weighted average of the feed mass fractions (recall (11))

$$y = f_0(u, d) = \frac{x_1 F_1 + x_2 F_2}{F_1 + F_2} \tag{20}$$

Here, $u = F_2$, and the transformed input is $v_0 = \frac{x_1 F_1 + x_2 F_2}{F_1 + F_2}$. Inverting this expression, we find how the process input $u = F_2$ depends on v_0 (which is the controller output):

$$u = F_2 = f_0^{-1}(v_0, d) = \frac{x_1 - v_0}{v_0 - x_2} F_1 \tag{21}$$

This may realized[4] using the improved ratio control scheme in Fig. 15 where, compared to Fig. 1, we have added a computation block which uses (21) to compute the setpoint for $R = F_2/F_1$. In theory, this gives perfect steady state feedback control for all disturbances, and will result in constant composition y at steady state, even without a feedback controller (CC) to update v_0. However, this assumes that the model is correct and that we measure or estimate the disturbances accurately. This is not realistic, so in practice we should include a feedback controller (CC) to adjust v_0 on a slow time scale.

5.4. Normalized ratio

In the paper, we have considered the ratio

$$R = F_2/F_1$$

(or sometimes the inverse $R' = F_1/F_2$) which is simple to implement using a multiplication element. A somewhat more complex alternative is the normalized ratio [32,33]

$$R_N = \frac{F_2}{F_1 + F_2} \tag{22}$$

Note that

$$R = \frac{R_N}{1 - R_N} \tag{23}$$

The normalized ratio R_N is easy to implement using this expression, for example, the outer controller (e.g., CC) computes R_N and then (23) is used to compute R which is sent to the multiplication element, $F_2 = R \cdot F_1$.

From (23), the two ratios are equivalent in the sense that fixing one keeps the other constant. However, the normalized ratio has some properties that may make it better for implementation [32]. First, R_N is always in the range 0 to 1, whereas R may vary between 0 and ∞. Second, for many applications, the ratio R_N is a special case of the ideal static transformed input (v_0) and provides additional linearization (e.g. see (20)) To understand this better, consider again the mixing process in Fig. 1. The steady-state model Eq. (11) becomes [33]

$$y = x_1(1 - R_N) + x_2 R_N$$

which is linear in R_N, so the linearized model as seen from the outer controller (CC in Fig. 1) becomes $\Delta y = K_N \Delta R_N$ with [33]

$$K_N = x_2 - x_1 \tag{24}$$

Notice that the gain K_N for the normalized ratio R_N in (24) is independent of both the ratio (R or R_N) and the throughput F_1, whereas the gain K_R for the ratio R in (13) depends on the ratio R.

However, the process gain K_N will still vary if there are changes in $x_2 - x_1$, and in particular this may be a problem if the quality difference $x_2 - x_1$ is small, because then a relatively small change in either x_1 or x_2 will cause a proportionately large change in the difference. This problem is noted by King [33] who considers a very similar mixing example to Fig. 1, but with temperature rather than composition. King [33] proposes to add some feedforward action to compensate for changes in the temperature difference $T_2 - T_1$. Probably, the best way to add steady-state feedforward action is to use the transformed input $v_0 = f_0(u, d)$ in (20) as the outer controller output, which results in the implementation in Fig. 15. Here, the steady-state model $\Delta y = K_0 \Delta v_0$ has a constant process gain $K_0 = 1$.

5.5. Linear analysis of ratio control

Ratio control is a feedforward approach (usually working on a fast time scale). It relies on using feedback control (usually working on a slower time scale) to adjust the ratio setpoint R for handling model uncertainty and unmeasured disturbances. To better understand this, we perform a linear analysis.

Without control, the effect of small changes in the input u and disturbances d on the output y is given by (from the superposition principle for linear systems):

$$\Delta y = G(s)\Delta u + G_d(s)\Delta d$$

where Δ denotes deviation variables (e.g., $\Delta d = d - d^*$ where $*$ denotes the nominal steady-state at which the system is linearized) and s is the Laplace variable (which is frequently omitted in the following). Consider now the use of conventional linear feedforward and feedback control, where the two contributions from measurements of d and y are added together: $\Delta u = \Delta u_d + \Delta u_y$ where $\Delta u_d = C_d(s)\Delta d$ (linear feedforward) and $\Delta u = -C(s)\Delta y$ (linear negative feedback). The resulting closed-loop response becomes (upon eliminating Δu) :

$$\Delta y = S S_d G_d \Delta d$$

[4] There is a singularity at $v_0 = x_2$ in the inverse transformation (21). This is because the product composition y is physically bounded to be within the values x_1 and x_2 of the two feeds. If one encounters one of the bounds $v_0 = x_1$ or $v_0 = x_2$ during dynamic transients, then this will not lead to instability, but the setpoint $(F_2/F_1)_s$ should be set to its lower or upper bound to avoid negative values (this logic should be added to the computation block in Fig. 15). On a longer time scale, the feedback controller (CC in Fig. 15) will recover the system by updating v_0 to drive the system to the desired steady state where $y = y_s$, also when the measurements x_1 and x_2 are incorrect.

Here the feedback sensitivity is $S = (I + GC)^{-1}$ (for a scalar system, the identity matrix I becomes 1) and the feedforward sensitivity is $S_d = I + GC_dG_d^{-1}$. The frequency response is obtained by setting $s = j\omega$. It then follows that the effect of the disturbance d on the output y is reduced at frequencies ω [rad/s] where the product of the two sensitivities is less than 1, that is, when $|S(j\omega)| \cdot |S_d(j\omega)| < 1$. In general, with integral action in the controller, the feedback sensitivity $|S(j\omega)|$ is small at low frequencies, up to about the bandwidth frequency ω_B, where it may have a peak and then approach 1. Thus, the reason for adding feedforward control is to make $|SS_d|$ small at high frequencies (in particular, at $\omega > \omega_B$) where the disturbance effect $|G_d(j\omega)|$ (without control) is larger than can be accepted.

The ideal feedforward controller (which makes $S_d = 0$) is $C_{id} = -G^{-1}G_d$. This C_{id} may not be realizable, for example, if G has higher-order dynamics or G has more delay than G_d. However, more importantly, feedforward control assumes that the disturbance d is measured and relies on having accurate process models (for G and G_d).

Now, consider the relationship to ratio control, which may be viewed as a special case of feedforward control where d is the extensive variable disturbance (basis, "wild flow") and C_d is static (although dynamics may be added as discussed below). For simplicity, assume that the corresponding process models from u and d to y are first-order with delay:

$$G(s) = K\frac{e^{-\theta s}}{\tau s + 1}, \quad G_d(s) = K_d\frac{e^{-\theta_d s}}{\tau_d s + 1}$$

The ideal feedforward controller becomes

$$C_{id}(s) = -G^{-1}G_d = -\frac{K_d}{K}\frac{\tau_d s + 1}{\tau s + 1}e^{-(\theta_d - \theta)s}$$

At steady state, this gives $\Delta u = C_{id}(0)\Delta d = -\frac{K_d}{K}\Delta d$. We have shown in this paper that for processes that satisfy the scaling assumption, ratio control gives perfect disturbance rejection at steady state. This means that adjusting u to keep a constant ratio $R = u/d$, will keep the intensive variable y constant at steady state, in spite of disturbances in d. In summary, with ratio control we have $u = Rd$ (or $\Delta u = R^*\Delta d$ in linearized form), and we just showed that with ideal steady-state feedforward control, we have $\Delta u = C_{id}(0)\Delta d = -\frac{K_d}{K}\Delta d$. Since both schemes give perfect control ($\Delta y = 0$), it then follows that, *in general for ratio control* (where u and d are extensive variables) we must have that

$$R^* = C_{id}(0) = -\frac{K_d}{K} \tag{25}$$

This is an important result. Indeed, (25) is satisfied for the mixing example (Fig. 1) where we have $u = F_2$ and $d = F_1$. From (12) we derive

$$K_d = \frac{F_2^*(x_2^* - x_1^*)}{(F_1^* + F_2^*)^2}, \quad K = -\frac{F_1^*(x_2^* - x_1^*)}{(F_1^* + F_2^*)^2}$$

and it follows that $C_{id}(0) = -K_d/K = F_2^*/F_1^* = R^*$ (as expected).

Even more importantly, if we combine ratio control with an outer feedback, then we do not even need to know the value of R, because R will be updated automatically by the feedback to maintain perfect disturbance rejection for throughput disturbances at steady state, in spite of disturbances, model changes and errors for the flow measurements used for ratio control (Rule R8). In other words, with ratio control, the outer feedback loop will update R to match the ideal feedforward control gains in (25).

To achieve good dynamic disturbance rejection with static ratio control, we are implicitly assuming that the dynamics for the disturbance d and input u (which are both extensive variables) are similar. Fortunately, this is often satisfied in practice, for example, when both are feed flows to the process, like for the mixing process. In other cases, when the dynamics are not the same, we may include a dynamic element (see Fig. 7) which for the case with first-order with delay models becomes

$$\text{Dynamic element} = \frac{\tau_d s + 1}{\tau s + 1}e^{-(\theta_d - \theta)s} \tag{26}$$

Here we must assume $\theta_d \geq \theta$ for the dynamic element to be realizable (with no prediction).

5.6. Assumption of constant intensive variables (x)

The scaling assumption, which is the basis for ratio control, assumes that the independent intensive variables x (for example, feed composition and feed temperature) are constant. This may at first seem to be a serious limitation. However, from the superposition principle for linear systems, small changes (disturbances) in x will not affect the benefits of ratio control, which is to reject throughput disturbances. Longer-term large static changes in x are handled by the outer feedback loop, which in addition, by updating the ratio R, maintains perfect steady-state disturbance rejection for the throughput disturbance (Rule R8). This property of Rule R8 also follows from (25) and is a very important benefit of ratio control compared to conventional model-based feedforward control (both linear and nonlinear) where R is kept constant.

5.7. MPC and ratio control

Unlike conventional feedforward control, ratio control does not need an explicit model of how the controlled property variable y depends on the extensive variables (X). This makes ratio control more powerful and simpler to apply than many people think. However, it also implies that conventional model-based control approaches, like model predictive control (MPC), are not ideally suited for implementing ratio control.

First, MPC may be a good solution if maintaining a given setpoint for R is a primary control objective. Then R may be defined as a controlled variable (CV) for MPC. One application is MPC for air-to-fuel ratio (AFR) control in engines [34,35]. In this case, we may use a conventional industrial linear MPC implementation (with MV constraints). MPC is also a good alternative to achieve "dual" ratio control (as in Figs. 8 and 10). However, using R as a CV, requires a ratio element to compute the measured ratio (e.g., $R = X_2/X_1 = F_2/F_1$). This is similar to the ratio controller (RC) implementation in Fig. 4b and it shares some of its problems, including the need for logic to avoid division by zero, being undefined at low loads (approaching "zero divided by zero") and having nonlinearity from the MV (e.g., F_2) to the CV (e.g., $R = F_2/F_1$; see (14)). Nevertheless, these issues may be handled in practice. For example, the nonlinearity issue may be handled with gain scheduling or using nonlinear MPC.

Next, consider the mixing process in Fig. 1, where the main measured disturbance is the flowrate $d = F_1$ and the primary objective is to use the input $u = F_2$ (MV) to control the composition y (CV). Assuming that there is some delay associated with the measurement of y, we know intuitively that it may be an advantage to use ratio control, where $u/d = F_2/F_1$ is kept constant on a fast time scale. How are we going to make MPC do this? As discussed next, it is not so obvious.

The seemingly obvious solution is to supply MPC with a model, such as (11), which tells MPC that y will be constant when the ratio $u/d = F_2/F_1$ is constant. This should work well if we also tell MPC that there is some delay associated with y, so that it knows that feedforward action from $d = F_1$ will improve control of y. However, note that the use of ratio control gives nonlinear feedforward action, so this require a nonlinear MPC implementation (which is not commonly used in industry today). Also, what should we do if we do not have a good model for how y depends on u and d, for example, if y is a more complex property variable, such as viscosity or color? In this case, we may use the indirect approach of supplying MPC with an artificial model which tells MPC that y will be constant if we keep the ratio u/d constant.

Alternatively, for linear MPC, we may use a cascade implementation where MPC is combined with another controller (e.g., PID control), and the ratio is defined as either a CV or MV for MPC. This avoids some of the modeling issues and allows for using a linear MPC implementation. The simplest, as was discussed initially, is to let MPC be a slave controller and define the ratio as a controlled variable (CV) for MPC. The ratio setpoint is set by a separate slower master controller which controls y (like CC in Fig. 1). A possible disadvantage with this implementation is that ratio control may be slow because MPC is often operating with relatively infrequent updates, for example, every minute. To make the response faster, an alternative cascade implementation is to use MPC as the master controller. Here, the ratio setpoint, like $R_s = (F_2/F_1)_s$, is defined as a degree of freedom (MV) for MPC, and the ratio setpoint is implemented by the faster regulatory layer, below MPC, for example using the simple implementation in Fig. 4a or a "dual" ratio implementation (e.g., Fig. 10).

In summary, ratio control may be handled with MPC using various architectures, but there is no standard approach and implementation may require some effort and analysis.

6. Conclusion

Ratio control is very simple to use and it gives nonlinear feedforward action without needing an explicit process model. In spite of its widespread use, the literature gives no guidelines for when and how to use ratio control. To fill this gap, in this paper, it is argued that the theoretical basis for ratio control is the scaling assumption which says that we get the same steady-state solution if we increase all extensive variables (flows and heat rates) by the same factor compared to a basis. The scaling assumption is formulated mathematically in (2). From this we derived eight rules (R1-R8) for the use of ratio control.

- **Rule R1.** The selected controlled variable(s) is implicitly assumed to be an intensive variable (y), for example, composition, density, viscosity, taste or temperature.
- **Rule R2.** The system must satisfy the scaling assumption (2).
- **Rule R3.** Since all extensive variables must be scaled by the same factor λ, there can only be *one* independent extensive variable disturbance, X_d. The variable X_d is sometimes called the "basis", "wild variable", "master variable", "throughput disturbance" or "throughput manipulator" (TPM).
- **Rule R4.** If the system has n independent extensive variables X where one of them is free to choose (X_d), then from (2) we need to manipulate the remaining $n-1$ variables (in X) to keep $n-1$ ratios (or more generally, $n-1$ dependent intensive variables y) constant. For a change (disturbance) in X_d this will (at steady state) result in keeping *all* dependent intensive variables y constant, including the selected controlled variable(s) mentioned in Rule R1.
- **Rule R5.** When setting a ratio $R = X_2/X_1$, it is not required that the extensive variables X_1 and X_2 have the same units. For example, X_1 could be a flowrate F_1 (in units m^3/h) and X_2 a heat input Q (in units J/s)
- **Rule R6.** It is not required to measure the extensive disturbance variable X_d to apply ratio control, but in this case we need to use a ratio to another measured extensive variable X_1.
- **Rule R7.** It is also not required that any slave controllers for X_2 (e.g., FCs for F_2 and F_3 in Fig. 5) directly manipulate their own valves (z_2 and z_3), although we want the response from valve position to measured flow to be fast with a large gain to make the slave loop fast and non-interactive.
- **Rule R8.** An outer feedback controller with integral action can manipulate the ratio setpoint R to give perfect control of the intensive controlled variable ($y = y_s$) at steady state, in spite of uncertainty. In addition, this updated setpoint R for the ratio control, maintains the perfect steady-state disturbance rejection property for throughput disturbances.

In most cases, a process engineer can determine from physical insight whether the scaling assumption (Rule R2) is satisfied. Furthermore, it is not critical for the use ratio control that the scaling assumption is fully satisfied, as an outer feedback loop (Rule R8) can update the ratio setpoint on a slow time scale.

The paper has also discussed the practical implementation of ratio control using a multiplication element. Fig. 1 shows a typical cascade implementation where an outer loop controls the intensive variable y and sets the ratio setpoint (Rule R8). Note that no explicit process model is needed to implement this solution. More advanced implementations are dual ratio control for the case with saturation (Fig. 11) and cross-limiting control to keep one component (typically oxygen) in excess during dynamic transients (Fig. 13). Ratio control can be generalized to include model information using the idea of transformed inputs (Fig. 15).

In the writing of this paper, the main objective has been to provide a theoretical basis and guidelines for when and how to use ratio control. The main new results are the scaling assumption as the theoretical basis of ratio control and rules R1-R8 for when to use ratio control. In terms of implementation, the multiplication trick for dual ratio control is new, and it may be applied more generally to split-range control to avoid the "limbo" effect during switching.

Declaration of competing interest

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Appendix A

A.1. Simulation of mixing process (Figs. 1 and 2)

The simulations are for a mixing tank (Fig. 1) with dynamic model

$$m\frac{dy}{dt} = F_1(x_1 - y) + F_2(x_2 - y) \tag{27}$$

In the simulations, $m = 200$ kg $= 0.2$ t [ton], $x_2 = 0$ [kg/kg] and the desired product composition is $y_s = 0.2$ [kg/kg]. Nominally (at $t = 0$), we have $x_1^* = 0.4$ [kg/kg], $F_1^* = F_2^* = 0.5$ t/h and $R^* = F_1^*/F_2^* = 1$. From (11) and (27) this gives $y^* = 0.2$ (as desired) at steady state. There are four disturbances: At $t = 1$ h, x_1 is reduced from 0.4 to 0.3, and at $t = 5$ h it is further reduced to 0.25. In addition, F_1 is reduced by 40% (from 0.5 to 0.3 t/h) at $t = 3$ h and returns to its nominal value of 0.5 t/h at $t = 7$ h.

The flow controller (FC) for implementing the desired change in F_2 is assumed to be perfect, that is, with fast dynamics and no measurement error for F_2 (which anyway would be corrected for by the outer PI feedback loop).

The feedback PI controller (CC) is tuned at the nominal operating point using the SIMC rules [20]

$$K_c = \frac{1}{K}\frac{\tau}{\tau_c + \theta} \tag{28a}$$

$$\tau_I = \min\{\tau, 4(\tau_c + \theta)\} \tag{28b}$$

with the closed-loop time constant selected as τ_c= 0.4 h. The time constant for the process is equal to the residence time $\tau = m/F$ which is nominally 0.2 h, and the time delay θ is assumed to be zero, resulting in an integral time $\tau_I = 0.2$ h. For conventional feedback controller (with F_2 as the MV), the process gain in (12) becomes $K = \frac{F_1^*(x_2^* - x_1^*)}{(F_1^* + F_2^*)^2} = \frac{0.5\cdot(0-0.4)}{1^2} = -0.2$ and the SIMC controller gain is $K_c = -2.5$. For the ratio

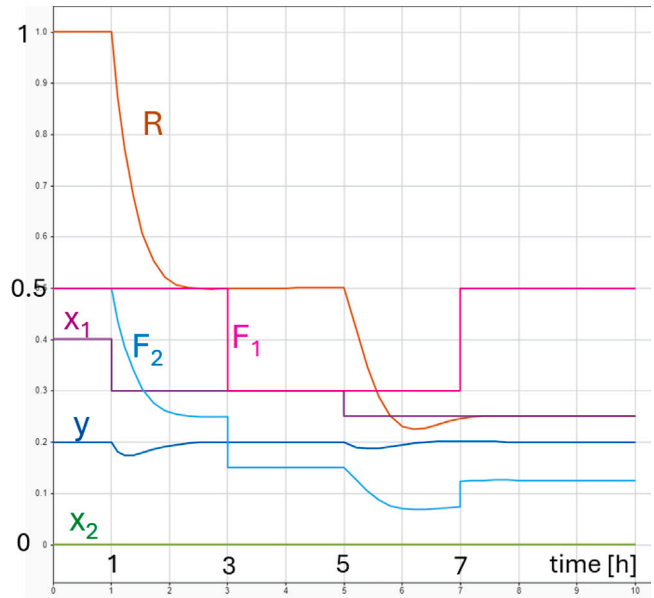

Fig. 16. Details of dynamic simulation for mixing process in Fig. 1 with the proposed ratio plus feedback control strategy.

feedback control (with R as the MV), the process gain in (13) becomes $K_R = \frac{x_2^*-x_1^*}{(1+R^*)^2} = \frac{(0-0.4)}{2^2} = -0.1$ and the SIMC controller gain is $K_c = -5$.

The response in product composition y are simulated for five control schemes in Fig. 2. Fig. 16 shows more details (including R and F_2) for the proposed ratio with feedback control strategy (black line for y in Fig. 2)).

Ratio control without feedback (red line in Fig. 2) gives perfect disturbance rejection for the flow disturbance in F_1 at $t = 3$ h and $t = 7$ h, but it cannot handle the unmeasured composition disturbance in x_1 at $t = 1$ h and $t = 5$ h. Also note that without feedback control, linear feedforward control and ratio control (red line) are identical, since both give $F_2 = R \cdot F_1$ where R is a constant (in our case, $R = 1$).

The only way to handle the unmeasured disturbances in x_1 is by feedback. The purple line in Fig. 2 shows the response with feedback only ($K_c = -2.5$). It has transient deviations for the four disturbances, but the system returns fairly quickly to steady state (with a closed-loop time constant $\tau_c = 0.4$ h).

The proposed combination of ratio and feedback control (black solid line in Fig. 2) (with $K_c = -5$) gives the best response as it eliminates the throughput disturbance in F_1 at $t = 3$ h and $t = 7$ h. Note from Fig. 16 that to correct for the composition disturbance in x_1, the feedback controller decreases the steady-state value of $R = F_2/F_1$ from 1 to 0.5 for $t > 1$ h, and further to 0.25 for $t > 5$ h.

Interestingly, in this case the conventional combination of linear feedforward and feedback control (orange line in Fig. 2) does not give any improvement compared to feedback only (purple line) in terms of handling the disturbance in F_1 (at $t = 3$ and $t = 7$). The reason is that the linear feedforward controller overreacts by changing F_2 too much when the ratio R is less than 1: It gives $\Delta F_2 = R^* \cdot \Delta F_1$ with $R^* = 1$ (nominal value), rather than the correct $\Delta F_2 = R \cdot \Delta F_1$, where R should vary depending on compositions (x_1, x_2, y).

Comment: At $t = 5$ h, the response in Fig. 2 with feedback control only (purple line) is seemingly a little better (faster) than with the proposed ratio control strategy (black line). Actually, this "improvement" is not desired. It is an unwanted effect because of nonlinearity which increases the process gain K when the disturbance F_1 is small. This nonlinearity can be seen from (12) which can be rewritten as

$$K = \frac{x_2^* - x_1^*}{F_1^*(1+R^*)^2}$$

which, except for the extra term F_1^*, is the same as the expression for K_R in (13).

A.2. Simulation of dual ratio control

In the simulations of dual ratio control in Figs. 9 and 11, the process model is assumed to be static. The process model is simply

$$F_1 = k_1 z_1, \quad F_2 = k_2 z_2$$

with $k_1 = k_2 = 0.01$ [(kg/s)/%] . Thus, the valves (process) are linear and static with constant gain. Both valves (z_1, z_2) are constrained to be between 0% and 100%. In the simulations, only the constraint $z_2 =$ 100% (corresponding to F2MAX=1 kg/s) is encountered, Note here that constraints for the other input ($z_1 = 100\%$) will not affect ratio control.

All dynamics come from the two flow controllers (FC and FC-SR) which are the same for both the nominal and modified control schemes. FC and FC-SR are both pure I-controllers that give a loop transfer function $5/s$ (corresponding to a closed-loop time constant $\tau_c = 0.2$ [min]). The FC for F_1 has anti-windup with tracking coefficient K_t (using the PID controller in Matlab Simulink). To be clear, the PI-controller is [21]

$$u(t) = u_0 + K_c e(t) + \int_{t_0}^{t} (K_I e(t') + K_t e_T(t'))dt' \tag{29}$$

with $e(t) = y - y_s$ and $e_T = \tilde{u} - u$. Controller FC for $y = F_1$ has $u = z_{11}$, $\tilde{u} = z_1$ and $K_c = 0, K_I = 500$ and $K_t = 500$. Controller FC-SR for $y = F_2$ has u as the input to the static SR-block, $K_c = 0, K_I = 500$ and $K_t = 0$, that is, this controller does not have anti windup because it is always active.

A.3. Simulation of cross-limiting control

In the simulations of cross-limiting control in Fig. 14, the valves are assumed to be linear ($\Delta F = k_v \Delta z$) and the only dynamics come from the two flow controllers. These are pure I-controllers and give a loop transfer function $5/s$ for F_1 (corresponding to a closed-loop time constant $\tau_{c1} = 0.2$ [min]) and $10/s$ for F_2 (corresponding to a closed-loop time constant $\tau_{c2} = 0.1$ [min]). Note that the cross-limiting solution works well in all cases, independent of the tuning of the flow controllers. No constraints are encountered in these simulations, and in general one has to stay away from input (valve) constraints for cross-limiting control to work as expected.

Appendix B. Supplementary data

The MATLAB/Simulink files for the dynamic simulations are available at the home page of Sigurd Skogestad.

Data availability

No data was used for the research described in the article.

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