Ratio control: Theoretical basis and practical implementation

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Abstract

Ratio control is the oldest control approach, dating back thousands of years (think of food recipes), but despite this, there exists no theoretical basis for its use. It is widely used in the process industry, in particular, for mixing processes and chemical rectors. It is sometimes viewed as a special case of feedforward control. However, feedforward control requires an explicit process model, but this is not needed for ratio control. Instead, ratio control is based on the physical insight that scaling all flows to keep constant flow ratios will result in constant product properties, and this scaling assumption is discussed in detail in the paper. Furthermore, the ratio setpoint may be set by an outer feedback loop, again without the need for a process model. The paper also discusses the practical implementation of ratio control, including dual ratio control for the case with saturation and cross-limiting control for keeping one component (typically oxygen) in excess during dynamic transients. Finally, it is shown that the multiplication trick proposed to avoid the limbo-effect for dual ratio control applies more generally to all split-range control solutions.

Keywords: control architecture, control structure design, feedforward control, PID control, advanced regulatory control,

1. Introduction

Ratio control is the oldest of all control methods, originating from cooking recipes thousands of years ago. Originally, the recipe for making porridge may have been: “Mix 1 cup of grain with 2 cups of water and let it boil for 10 minutes”. But then someone realized that if you want three times as much porridge then you just multiply all the amounts by the same factor $k = 3$. This is the scaling assumption, which is discussed in detail in Section 2. The

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generalized recipe then becomes: "Mix 1 part of grain with 2 parts of water and let it boil for 10 minutes". The amount in "1 part" (also known as the "basis") is adjustable, so this is a statement of ratio control.

Also, industrially, ratio control is extensively used. An example is reagents that are fed to a chemical reactor. The oldest automatic ratio device is probably the carburetor, invented by Karl Benz in 1888, for mixing of air and fuel in the correct ratio for combustion engines.

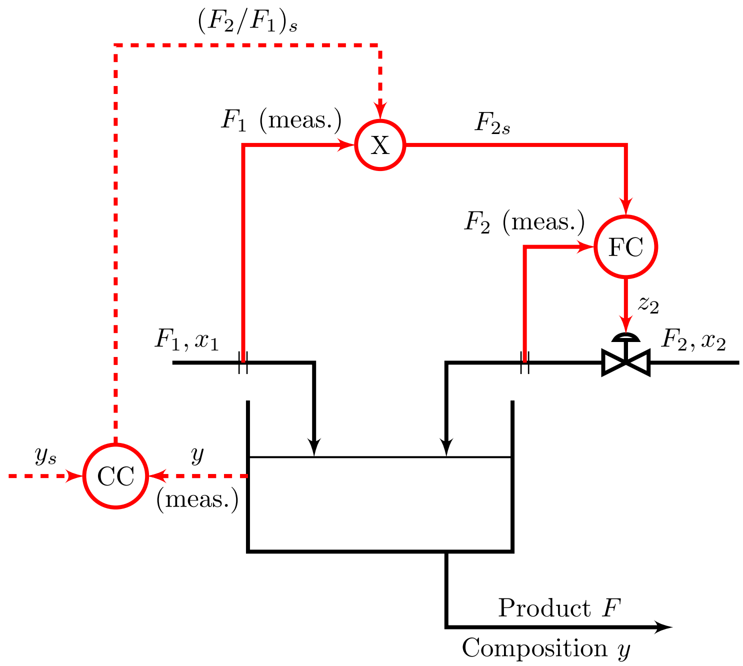

Figure 1: Recommended ratio control scheme for mixing process The ratio setpoint $(F_2/F_1)_s$ is set by an outer composition controller (CC). The signals denoted (meas.) need to be measured or estimated.

Figure 1 shows a typical ratio control scheme for a continuous mixing process. Note that no model is needed to design this control scheme, except for the insight that a constant flow ratio will keep the property variable y constant. In the figure, y is the composition, but in general y could be any intensive variable, including taste and color. In Figure 1, the ratio setpoint $R_s = (F_2/F_1)_s$ is obtained by "feedback trim" using an outer feedback controller (CC in this case) which drives the measured property y to its setpoint y_s.

The objective of the specific process in Figure 1 could be to mix concentrate (1) (with composition $x_1 = 1$ [kg/kg]) with water (2) (with $x_2 = 0$) to obtain a diluted product (F) with a given concentrate composition y [kg/kg]. The concentrate flowrate F_1 [kg/s] is a disturbance (at least as seen from the perspective of concentration control) and may have large variations. The simplest control system would be to implement a feedback composition controller (CC) which directly manipulates the dilution water F_2 to keep the measured concentration y at a given setpoint. However, the composition measurement may be unreliable and have a large delay (say, 5 minutes), and this motivates the use of ratio control where $R = F_2/F_1$ is kept constant on a fast time scale to compensate for variations in F_1. The combination of the outer feedback controller (CC) and ratio control (which makes use of an inner flow controller, FC) may be viewed as a special case of cascade control. The outer feedback controller (CC) will update the ratio setpoint when there are disturbances in the inlet concentration (x_1 in Figure 1) and will also correct for errors in the flow measurements for F_1 and F_2. Finally, note that there will be an inventory (level) controller that manipulates the product flow F (and makes $F = F_1 + F_2$ at steady state), but this is not shown in Figure 1.

In terms of making food, the inner ratio control corresponds to following the recipe given in the cook book, while the outer feedback (CC) corresponds to the corrections done by tasting the product.

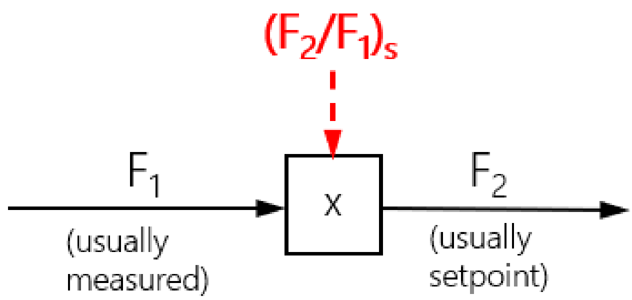

Figure 2: Multiplication element to compute $F_2 = (F_2/F_1)_s \cdot F_1$ where F_1 is the basis and $(F_2/F_1)_s$ is the desired ratio (setpoint).

The ratio control scheme in Figure 1 uses a multiplication element, as shown more clearly in Figure 2. With the flow disturbance F_1 as the basis, the multiplication element computes $F_2 = (F_2/F_1)_s \cdot F_1$. The computed value F_2 is typically sent as a setpoint F_{2s} to a flow controller (FC) which manipulates the valve position z_2, as shown in Figure 1 and Figure 3a.

An alternative (but not recommended) implementation of ratio control, is to use a division element as shown in Figure 3b. Here, we compute the ratio $R = F_2/F_1$ (measured) with a division element and send this to a ratio controller (RC). However, this scheme is not recommended, primarily to avoid division by zero (Love, 2007, page 182) and also to avoid the related problem of introducing nonlinearity into the inner RC control loop (Shinskey, 1967) as explained in more detail in Section 3.2.2.

Liptak (2006) writes that “ratio systems maintain a relationship between two variables to provide regulation of a third variable”. However, it should not be any variables. We show in Section 2 that the first two should be extensive variables (typically flowrates, F_1, F_2) whereas the third must be an intensive property variable (y).

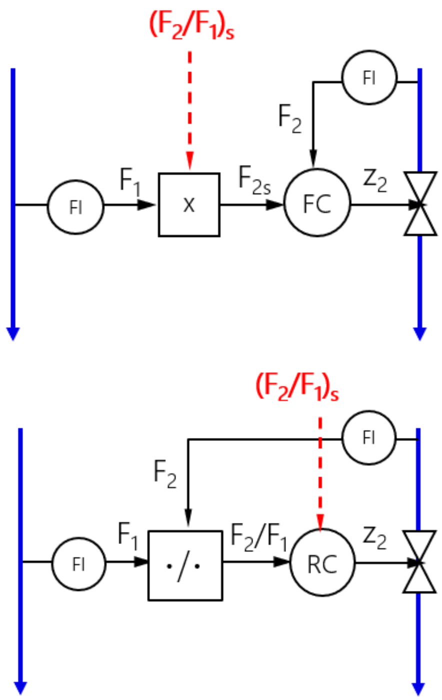

Figure 3: Two alternatives schemes for implementing ratio control (Buckley (1964), Figures 17.5 and 17.7) (Shinskey, 1967) (Luyben, 1973).
(a) Ratio control with multiplication element and flow controller (FC) (used in this paper).
(b) Ratio control with division element and ratio controller (RC) (not recommended).

More generally, we show in Section 2 that ratio control requires measuring (or estimating/inferring) as many extensive variables as there are independent extensive variables. These are typically flows and we may use flow meters (FI = flow indicator) as shown in the figures. Figure 4 shows ratio control for a

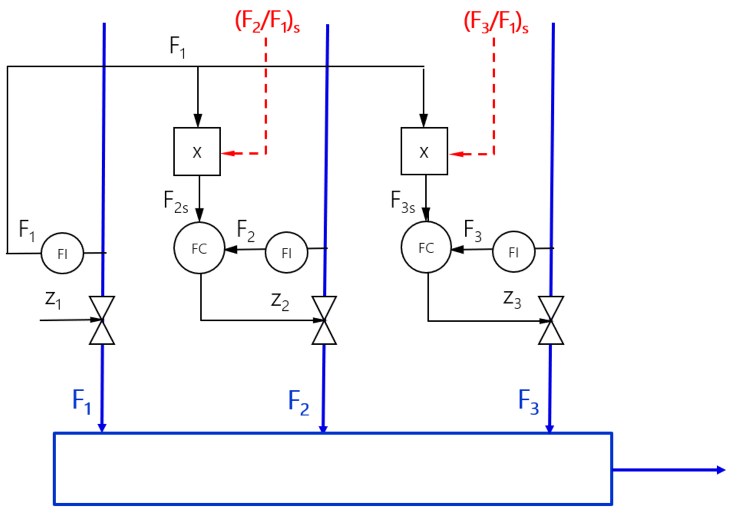

Figure 4: Ratio control for process with three independent extensive variables (flows) where F_1 is a disturbance and F_2 and F_3 are manipulated. Here, we need to set two ratios, which in this case are selected as F_2/F_1 and F_3/F_1.

case with three independent flows, where F_1 is a disturbance and F_2 and F_3 are manipulated. To apply ratio control for this case, we need to measure three flows and set two ratios. In Figure 4, the ratios are $R_2 = F_2/F_1$ and $R_3 = F_3/F_1$, that is, with F_1 as the basis.
Ratio control is very flexible:

1. It is not required that the ratio variables have the same units. For example, for the ratio $R = F_2/F_1$ we may have F_2 in kg/s and F_1 in m^3/h. It is also not required that the ratio variables are flows. For example, F_1 or F_2 could be the heat input Q or the compression power W (e.g., in units J/s).

2. It is not required that the flow controllers (e.g., FCs for F_2 and F_3 in Figure 4) directly manipulate their own valves (z_2 and z_3), although we want the response from valve position to measured flow to be fast with a large gain to make the FC-loop fast and non-interactive. For example, assume in Figure 4 that we cannot measure F_3, but we can measure the outflow $F_4 = F_1 + F_2 + F_3$ which is affected by z_3. In this case, we get the same result at steady state if a flow controller (FC) uses z_3 to set the ratio $(F_4/F_1)_s$ instead of $(F_3/F_1)_s$.

3. It is not required to measure the flow disturbance (F_1 in Figures 1 and 4) to apply ratio control, but we need a measured flowrate that depends on the flow disturbance. For example, in Figure 4 we would get the same result at steady steady if we obtained the setpoints for F_2 and F_3 by setting two other ratios, for example, $R_2 = F_2/F_4$ and $R_3 = F_3/F_2$ where F_4 is another measured extensive variable in the system that depends on F_1, e.g., the outflow $F_4 = F_1 + F_2 + F_3$.

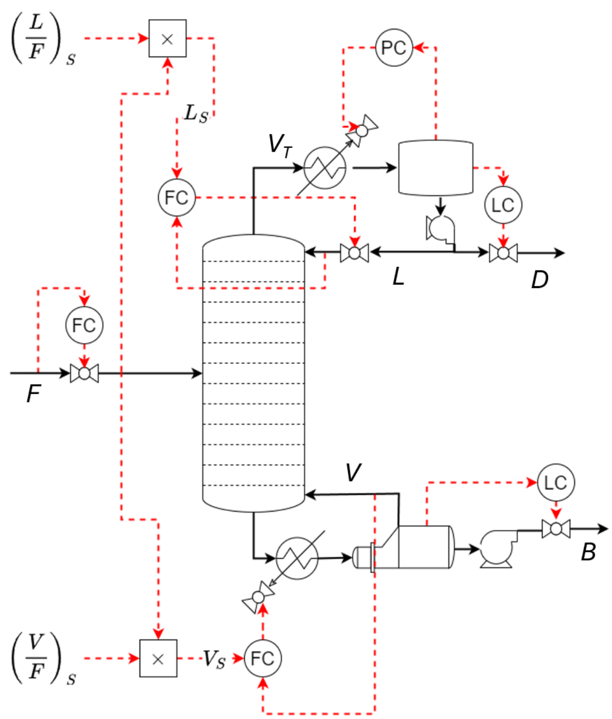

Figure 5: Ratio control of distillation column with fixed L/F and V/F. Another example of a control scheme that gives no steady-state change in the product compositions (assuming constant feed composition, constant pressure and constant stage efficiencies) is fixed L/D and x (using V to control x at a fixed setpoint where x is any composition or temperature in the column). Actually, all control schemes with two fixed intensive variables (flow ratios, compositions) that manipulate L and V give the same steady state for disturbances in the feedrate F (assuming vapor-liquid equilibrium with constant stage efficiencies).

An application of Figure 4 is a distillation column, see Figure 5, where the disturbance is the feedrate ($F_1 = F$), the manipulated variables are liquid reflux ($F_2 = L$) and vapor boilup ($F_3 = V$), and we fix the ratios $F_2/F_1 = L/F$ and $F_3/F_1 = V/F$. However, we do not need to use $F_1 = F$ as the basis, that is, we do not need to measure the feed disturbance F to apply ratio control. We get the same result at steady steady if we use two other ratios, for example, $F_2/F_4 = L/D$ and $F_3/F_2 = V/L$ where $F_4 = D$ is the measured distillate product flowrate. Furthermore, in distillation, the boilup $F_3 = V$ is not directly manipulated by its own valve. Rather, it is indirectly manipulated by a valve which changes the heat to the other side of the reboiler.

Ratio control is often viewed as a special case of feedforward and decoupling control, because the “basis” in ratio control may be viewed as a disturbance d (as in feedforward control) or as another process input u_i (as in decoupling). For example, Shinskey (1979) (page 173) writes that ”ratio control systems are feedforward systems” and Liptak ((Liptak, 1970, 2006) writes that “ratio control actually portray the most elementary form of feedforward control”. However, this is misleading. First, as just explained, ratio control does not require that we measure the disturbance. Another fundamental difference is that feedforward control is model-based whereas ratio control is data-based. Feedforward control requires explicit process models for how the disturbance d and the input u affect the output y (e.g., $\Delta y = G\Delta u + G_d\Delta d$ for the linear case). The feedforward controller then inverts the input model to compute the input u based on the measured d (e.g. $\Delta u = -G^{-1}G_d\Delta d$). On the other hand, ratio control it based on data (measurements of u and d) and the physical insight that a constant ratio u/d will keep the variable y constant (and no explicit model is needed for this). We do not even need a model to set the ratio setpoint, because this, as shown in Figure 1, may be set by an outer feedback loop.

The published literature on ratio control is rather scarce. The text books of Young (1955) and Hengstenberg et al. (1964) (p. 1351) show ratio control systems similar to the proposed one in Figure 3a. Buckley (1964) (Fig. 17.7) shows an example of a cascade system (similar to Figure 1) where a composition controller updates the desired ratio of the two feeds to a reactor. Shinskey (1967) considers the two schemes in Figure 3 and to avoid nonlinearity in the control loop (involving FC or RC) he recommends the scheme in Figure 3a with an FC and a multiplication element [1] A good and simple treatment of ratio control is Riggs (1999). He uses the multiplication block for implementing ratio control and also shows how to include feedback correction, similar to Figure 1, for a neutralization process. Riggs (1999) is the only one who, to my knowledge, links ratio control to the scaling property. More recently, Hägglund (2001, 2017) has suggested flexible and robust ways of implementing ratio control.

The main theoretical assumption behind the use of ratio control is that the system satisfies the scaling property, which is discussed in Section 2. In Section 3 we discuss implementation of ratio control. In Section 4, we discuss more complex implementations, including the use of “dual” ratio control for the case when the manipulated variable u may saturate, and cross-limiting control for ratio control of combustion processes.

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[1] The “multiplication element” was in earlier literature given some rather non-descriptive names, such as “ratio relay” (Buckley, 1964, page 157) and “ratio station” (Shinskey, 1967, page 161), presumably because performing multiplication was not straightforward with analog (usually pneumatic) control equipment. Unfortunately, the non-descriptive term “ratio station” is still (2025) used in some publications and text books.

2. The scaling assumption and the theoretical basis of ratio control

Despite its long history, it seems that no one has provided a theoretical basis for ratio control. Therefore, its use is usually based on intuition. However, intuition and physical insight have imitations, and because of the lacking theoretical basis for ratio control, it is sometimes used wrong. As an example, it has been common to recommend using a fixed reflux-to-feed ratio L/F in a distillation column (e.g., Young (1955) (p. 321), Luyben (2022b)). However, this is not a good solution if the heat input (V) is constant (Bang & Skogestad, 2025), for example, due to saturation. This is because, as in discussed in detail below, *all independent extensive variables* (including the boilup V) must be increased proportionally to apply ratio control.

2.1. The scaling assumption

Ratio control is based on the scaling assumption. To state the scaling assumption, we need to understand the difference between intensive and extensive variables (e.g., Modell & Reid (1983):

- *Intensive variables* are properties that do not depend on the size of the system. Common examples are composition, density, viscosity, pressure and temperature. Note that a ratio is an intensive variable.
- *Extensive variables* scale with the size of the system. Examples include flowrate, heatrate, volume, mass, energy and area.

The scaling assumption may for a steady-state process be formulated as follows:

> *For a process that satisfies the* **scaling assumption***, we have that scaling (changing)* <u>*all*</u> *independent extensive variables (X_i) by the* <u>*same*</u> *factor k, with all independent intensive variables (x_i) constant, scales (changes) all dependent extensive variables (y) by the same factor k and keeps all the dependent intensive variables (Y) constant.*

A simple example is a food recipe, where we know that when the amount of all ingredients (extensive variables, X_i) are increased proportionally (with fixed ratios, X_i/X_j), then all properties (intensive variables y) of the product will remain constant, including the taste.

To state mathematically the *scaling assumption*, we consider, for simplicity, a case with five independent variables (but it could by any number). The independent variables are divided into two classes:

- Intensive variables: x_1 and x_2.
- Extensive variables: X_1, X_2 and X_3 (typically flow rates, F_i)

The dependent variables, which are either intenstive (y) or extensive (Y), are functions of the independent variables, and the steady-state model may be written as

$$y \text{ intensive}: \quad y = f_y(x_1, x_2, X_1, X_2, X_3) \tag{1a}$$

$$Y \text{ extensive}: \quad Y = f_Y(x_1, x_2, X_1, X_2, X_3) \tag{1b}$$

where f_y and f_Y are nonlinear functions. Mathematically, if a system satisfies the scaling assumption, the following relationships hold:

$$y \text{ intensive}: \quad \underbrace{f_y(x_1, x_2, kX_1, kX_2, kX_3)}_{y(k)} = \underbrace{f_y(x_1, x_2, X_1, X_2, X_3)}_{y} \tag{2a}$$

$$Y \text{ extensive}: \quad \underbrace{f_Y(x_1, x_2, kX_1, kX_2, kX_3)}_{Y(k)} = k\,\underbrace{f_Y(x_1, x_2, X_1, X_2, X_3)}_{Y} \tag{2b}$$

Here, y and Y are the original values of the dependent variables (with $k = 1$), and $y(k)$ and $Y(k)$ are the values after scaling all the independent extensive variables X_i by a factor k. Thus, the intensive variables y remain constant, $y(k) = y$, whereas the extensive variables Y scale with the factor k, $Y(k) = kY$. Mathematically, we say that the intensive variables are homogeneous to the degree $h = 0$ (since $k^h = 1$ for $h = 0$) and the extensive variables are homogeneous to degree $h = 1$ (since $k^h = k$ for $h = 1$) (e.g., Appendix C in Modell & Reid (1983)). (2) holds generally for thermodynamic systems in equilibrium (Callen, 1960).

2.2. Implications of the scaling assumption for ratio control

The objective of control is to keep constant dependent variables. It then follows that for ratio control, (2a) in terms of y (intensive variable) is the key relationship. However, (2b) is also important since it emphasizes that all the extensive variables X_i and Y need to increase by the same factor k. If only one extensive variable fails to do this, for example, because it is kept constant (X_j = constant for some j), then the scaling assumption does not hold. This important point is also implicit in the underlined words <u>same</u> and <u>all</u> in the scaling assumption.

In general, with n independent extensive variables X_i, all these n variables need to increase by the same factor k for the scaling assumption to hold. To understand what this implies in terms of ratios, assume that one independent extensive variable, say X_1, is changed by a factor k. Then for the scaling assumption to hold, the remaining $n-1$ independent variables X_i need to change by the same factor k, which will be satisfied if $n-1$ independent ratios are kept constant (for example, X_i/X_1 = constant for $i = 2$ to n). However, since from (2a), all dependent intensive variables y remain constant when we scale the system, me may more generally, instead of a ratio (e.g., X_i/X_1), keep any selected intensive variable y_i constant (assuming y_i is selected so that satisfying (2a) results in unique values for X_i).

In conclusion, from the scaling assumption, as formulated mathematically in (2), we arrive at the following important rules for the use of ratio control:

- (R1) The controlled variable y is implicitly assumed to be an intensive variable, for example, composition, density, viscosity, taste or temperature.

- (R2) The system must satisfy the scaling assumption (2).

- (R3) Since all extensive variables must be scaled by the same factor k, there can only be one independent extensive disturbance variable. This variable is sometimes called the "basis", "wild variable", "master variable", "flow disturbance" or "throughput manipulator" (TPM).

- (R4) If the system has n independent extensive variables X_i, then from (2) we need to manipulate $n-1$ of these variables to keep $n-1$ ratios (or more generally, $n-1$ dependent intensive variables y_i) constant. For a change (disturbance) in the throughput (basis, wild flow) this will result in keeping *all* dependent intensive variables constant, including the controlled variable(s) y (at steady state).

The terms "independent" and "dependent" variables here have the meaning of "inputs/disturbances" and "outputs" from a control point of view. The independent intensive variables (x_i) are typically feed property disturbances, and the dependent intensive variables are typically product properties (y).

For example, the process in Figure 4 has $n = 3$ independent extensive variables at steady state (F_1, F_2, F_3). If the process satisfies the scaling assumption (say, it's a mixing process or a distillation column), then using $n - 1 = 2$ of these variables (F_2, F_3) to keep $n-1 = 2$ ratios (or more generally, 2 dependent intensive variables) constant, will keep *all* dependent intensive product variables constant when there are disturbances in F_1.

The scaling assumption, which is the basis for ratio control, only holds if the independent intensive variables x_i are constant, that is, if there are no intensive variable disturbances (like feed composition). This is rarely satisfied, but in practice this may not be a serious limitation if they change relatively slowly, because such disturbances can be handled by an outer feedback controller which adjusts the ratio setpoint(s) (see CC in Figure 1).

2.3. When does the scaling assumption hold?

The scaling assumption holds for all thermodynamic equilibrium systems. Thus, the scaling assumption (and thus the use of ratio control) applies to many process units, including

- Mixers

- Equilibrium reactors

- Equilibrium distillation with constant stage efficiency

If in doubt, one can check if the system satisfies (2a) and (2b). More generally, for the scaling assumption to hold for a process, we must assume constant "efficiencies". Skogestad (2009) (page 66) writes: "An initial basis for a stream is often chosen. We can later rescale (up or down) all the streams to the desired quantity. Mass, energy, volumes, etc. (all extensive variables) will scale with the same factor *provided that the efficiencies of the units remain constant.*"

2.4. When does the scaling assumption not hold?

However, there are also many process units where the scaling assumption does not hold and therefore ratio control should not be used, or at least used with care. This includes, for example, non-equilibrium reactors (where kinetics are important and the reactor volume matters) and heat exchangers. For the scaling assumption to hold for a heat exchanger, we would need to increase the heat transfer area A proportionally to the flow rates. This is reasonable during design, but not during operation (control) when the equipment is fixed. Nevertheless, ratio control of a heat exchanger is suggested by Smith (2010) (page 204) and simulations show that it works quite well. However, this is probably by luck, because heat exchangers do not satisfy the scaling assumption.

From rule R3, ratio control requires that all extensive variables are scaled by the same factor. This means that we need to be careful when applying ratio control to processes with many independent extensive variables. The problem is that if we keep one extensive variable constant (except for the basis disturbance), then the scaling assumption does not hold. For example, a distillation column with a fixed heat input V (and with the feedrate F as a disturbance) does not satisfy the scaling assumption, which means that L/F ratio control should not be used unless also V/F is constant (see the Discussion section for details).

3. Implementation of ratio control

The recommended implementation of ratio control for F_2/F_1, with a multiplication element and a flow controller, was shown in the introduction, see Figures 1, 2 and 3a.

The implementation with a division element and ratio controller (RC) in Figure 3b is *not* recommended. Flower & Parr (2003) writes that it is "an intuitive, but incorrect, method of ratio control" where "the loop gain varies with throughput" (see (7) below). Love (2007) writes that it is "commonly used throughout industry, although in two different accounts, the indirect method [using the multiplication element] is superior". The two different accounts are the potential of zero division and the nonlinearity (see (7) below).

3.1. Flow controller

In Figures 1 and 3a, F_1 denotes the "master flow" (wild variable, basis or flow disturbance) and the flow controller for the manipulated flow F_2 has the setpoint

$$F_{2s} = (F_2/F_1)_s F_1 \tag{3}$$

The F_2 flow controller usually uses the corresponding valve position z_2 as the manipulated variable (MV). The response from z_2 (MV) and F_2 (CV) is usually very fast and often the dynamic process model is assumed to be static, $\Delta F_2 = k_v \Delta z_2$. In such cases, the best controller is a pure I-controller with integral gain $K_I = \frac{1}{k_v \tau_c}$, where τ_c is the closed-loop time constant (including any process time delay) (Skogestad, 2003). A typical value for τ_c for a flow controller is between 5s and 15s. It is also possible to use a PI-controller with integral time τ_I equal to the process time constant τ for the valve, and controller gain K_c selected such that the integral gain $K_I = K_c/\tau_I$ remains unchanged, that is, $K_c = \frac{1}{k_v} \frac{\tau}{\tau_c}$.

It is possible to implement ratio control without a flow controller, for example, using feedforward control from F_{2s} to the valve position z_2. However, this is usually not sufficiently accurate because it involves inverting the valve equation relating the flowrate F_2 [kg/s, m^3/s, mol/s] to the valve position z_2. This equation is uncertain and depends on the pressure drop over the valve which may vary.

Flower & Parr (2003) refer to the flow controller for F_2 as the slave loop and to F_1 as the master flow. If there is also a flow controller for the master flow F_1, then we may use the setpoint $F_{1,s}$ instead of F_1 as the basis for the multiplication element when computing the setpoint for F_2 in (3) (Buckley, 1964, page 158). It is also possible to use a "blend" of the two variables as the basis, for example, $\gamma F_{1,s} + (1-\gamma)F_1$ where γ is an adjustable parameter between 0 and 1 (Hägglund, 2001). Using F_{1_s} as a replacement for F_1 may be advantageous to speed up the response for F_2, but the disadvantage is that logic must be added for the case when the controller is not active or if F_1 does not reach its setpoint, for example, because of valve saturation for z_1 (Hägglund, 2017). To speed up the response for F_2, a better solution may be to tune the flow controller for F_2 to be fast (with a small value of τ_c) or to add some "feedforward action" from $F_{1,s}$, for example, from $F_{1,s}$ to z_2.

3.2. Nonlinearity

We here show that the implementation with the multiplication element in Figures 1 and 3a, results in a linear response with a constant gain, both for the inner flow controller (FC) as well as for the outer property controller that sets the ratio setpoint (CC in Figure 1).

3.2.1. Outer loop nonlinearity (for controller CC in Figure 1)

One advantage with ratio control is that it makes the response from the ratio setpoint $R_s = (F_2/F_1)_s$ to y for the outer controller (CC in Figure 1) independent of the flow disturbance (F_1). In some sense this is obvious, because the scaling assumption, which is the basis for ratio control, says that y will be constant when the ratio R is constant, independent of the values of the extensive variables.

Nevertheless, to understand it better, consider the simple mixing process in Figure 1 where F_1 is the disturbance and F_2 is the manipulated variable (the true

MV is usually the valve position z_2, but assume we have a flow controller for F_2 as shown in Figure 1). Assume that y is the fraction of a given component in the product and that x_1 and x_2 are the feed fractions. Then, from the component material balance, we have that y is the average of the feed fractions,

$$y = \frac{x_1 F_1 + x_2 F_2}{F_1 + F_2} \tag{4}$$

Next, we linearize this model for two cases:

1. *No ratio control.* In this case, the MV for the outer loop (CC) is F_2 (rather than F_2/F_1). The process as seen from the outer loop is then from F_2 to y, and the linearized model becomes $\Delta y = K \Delta F_2$ with gain

   $$K = \frac{F_1(x_2 - x_1)}{(F_1 + F_2)^2} = \frac{x_2 - x_1}{F_1(1 + R)^2} \tag{5}$$

   Note that the gain K depends on the disturbance F_1 and becomes infinite as the throughput $F_1 + F_2$ approaches zero.

2. *With ratio control.* Here, the process as seen from the outer loop is from $R = F_2/F_1$ to y, and the linearized model becomes $\Delta y = K_R \Delta R$ with gain

   $$K_R = \frac{x_2 - x_1}{(1 + R)^2} \tag{6}$$

   With constant ratio $R = F_2/F_1$, the gain K_R is independent of the disturbance F_1 (as expected).

In summary, the use of ratio control makes the process gain for the outer loop (CC in Figure 1) independent of the throughput (e.g., F_1). This makes it possible to use linear controllers over a wider range of throughputs.

3.2.2. Inner loop nonlinearity (for controllers FC and RC in Figure 3))

We here compare the two ratio control schemes in Figure 3 using the analysis of Shinskey (1967) (page 160) and show that the implementation with a multiplication element is better than with a division element.

For the scheme with the multiplication element in Figure 3a, we have linearity for the inner flow controller FC (provided the valve is linear) This follows since for a linear valve, the response for the FC, from the manipulated variable z_2 (valve position) to the controlled variable F_2, is

$$\Delta F_2 = k_v \Delta z_2$$

where k_v is a constant.

On the other hand, for the scheme with the division element in Figure 3b, we have a strong nonlinearity for the inner ratio controller RC. This follows since the response for the RC, from the manipulated variable z_2 (valve position) to the controlled variable $R = F_2/F_1$, is (again assuming a linear valve)

$$\Delta R = \frac{k_v}{F_1}\Delta z_2 = \frac{1}{F_1}\Delta F_2 \quad (7)$$

The gain becomes infinite if F_1 goes to zero. It does not help to control the inverse ratio, $R' = F_1/F_2$, because here we get $\Delta R' = -\frac{F_1}{F_2^2}\Delta F_2$ and the gain becomes zero if F_1 goes to zero and infinite if F_2 goes to zero. Another option is to control the normalized ratio, $R_N = F_2/(F_1 + F_2)$, which gives $\Delta R_N = \frac{F_1}{(F_1+F_2)^2}\Delta F_2$, but here the gain becomes zero if F_1 goes to zero.

3.3. Dynamic elements

The scaling assumption, which is the basis of ratio control, applies to the steady-state behavior. To achieve better dynamic behavior for the recommended scheme in Figure 3a, one may introduce dynamic compensation as shown in Figure 6. Typically, we use a lead-lag element with delay on the form

$$\text{Dynamic element} = \frac{Ts + 1}{(\tau_1 s + 1)(\tau_2 s + 1)}e^{-\theta s}$$

where any of the parameters $(T, \tau_1, \tau_2, \theta)$ may be zero. Note that the steady-state gain is 1. The dynamic element is usually on the "basis" signal F_1 (see Figure 6), to avoid that it becomes part of the outer control loop which adjusts the ratio setpoint $(F_2/F_1)_s$.

For example, if we in a distillation column use the flow ratio $F_2/F_1 = V/F$ (where $F_2 = V$ is the boilup and $F_1 = F$ the feedrate), we may use the dynamic element to delay the measurement of the feedrate F (disturbance) because it takes some time for a change in F to reach the bottom of the column (which is where we want to control the composition y and where the boilup V enters). On the other hand, if the flow control loop (FC) for $F_2 = V$ is slow, maybe due to the slow dynamics in the reboiler in the distillation column, then we can speed up the response for F_2 by choosing $T > \tau_1$ in the dynamic element. In this case, T is the closed-loop time constant of the FC-loop.

Luyben (2022a) Luyben (2022b) compares linear dynamic feedforward control (which he calls "additive" feedforward control) with static ratio control (which he calls "multiplicative" feedforward control), and concludes that the "additive" structure is often better. However, this is not a fair comparison, because dynamic compensation can easily be added to ratio control, as shown in Figure 6. This would make ratio control the preferred choice (assuming that the scaling assumption holds), because ratio control gives the correct action at steady state. That is, it corrects for the nonlinearity in the process, for example, as given by the nonlinear relationship from F_1 and F_2 to y in (4).

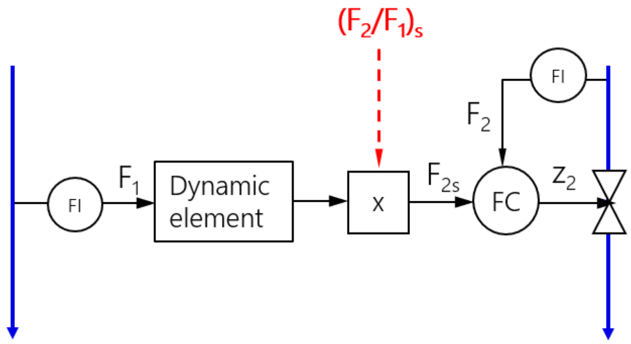

Figure 6: Ratio control with dynamic compensation. The dynamic element has no effect at steady state.

4. More complex ratio control implementations

4.1. Dual ratio control

Conventional ratio control, as shown Figure 1, fails if the manipulated variable (MV) used to control the ratio (z_2 in Figure 1) saturates (e.g., at a fully open valve position) or if its value is set by some other more important control task (override). If keeping the desired ratio has high priority, then the solution is to use *"dual" ratio control, where we let a second manipulated variable (MV) take over the task of ratio control when the original MV cannot be manipulated.* This is an example of MV-MV switching for which there generally are three alternatives (Skogestad et al., 2023):

1. Split-range control (SRC; one controller with two MVs)
2. Split-parallel control (SPC; two controllers, one for each MV, with setpoint separation)
3. Valve position control (VPC; the second MV is used, when necessary, to avoid saturation for the original MV)

The last alternative (VPC) has the advantage that the original MV (say z_2) is always used to control the ratio. However, this means that this MV (z_2) is not allowed to saturate (at 100%), which may result in an economic loss. Furthermore, the outer VPC loop (e.g., using z_1 to control z_2 at 80% position) may be too slow to avoid saturation in z_1, which may lead to temporary loss of ratio control. The VPC alternative may be attractive for cases where the second MV (z_1) is an on/off variable, for example, an extra pump which may be turned on under certain conditions, because then we cannot use z_1 (alone) to control the ratio. If tight ratio control is required, then the second alternative (SPC) is not desirable as it requires a quite large setpoint separation to work well (Forsman et al., 2025). This has also been confirmed by simulations (not included) for the dual ratio control case. It therefore seems that the first alternative (SRC) is usually the best for dual ratio control, and this is in line with industrial practice (K. Forsman, personal communication, 2025). In many cases, the second MV (z_1) is already paired with some other controlled variable(CV), like control of flowrate, so we need to change the CV for the second MV. We then need to combine the MV-MV switching with CV-CV switching (Skogestad, 2023). The latter normally requires a MIN- or MAX-selector, although we will see below that sometimes a multiplication element may be used instead.

An example of dual ratio control using split-range control is shown in Figure 7. This is a process with two feeds (F_1 and F_2) and two controlled variables (flowrate F_1 and ratio F_2/F_1, where ratio control has priority). Valve position z_2 is the "original" MV for controlling the ratio F_2/F_1, and valve position z_1 is the second MV that takes over if z_2 saturates at 100% (fully open valve). However, z_1 is normally used for controlling the flowrate F_1 at a given setpoint, so we use a MIN-selector to perform the override. In the implementation Figure 7, the ratio block (multiplication element) multiplies the measured value of F_1 with the ratio setpoint $(F_2/F_!)_s$ to compute the setpoint F_{2s}. This is send to the the split-range flow controller FC-SR for F_2 which computes the internal variable u and sends it to the logic SR-block. The SR-block manipulates the original MV z_2 when u is less than 50% and the second MV z_1 when u is above 50%. The table for the SR-block in Figure 7 gives the values where we change the linear relationship between the input signal u and the output signals z_2 and z_{12}. The valve positions z_1 and z_2 are here assumed to be in the range 0-100%. Note that the split value $u = 50\%$ may be adjusted to make the effective gains for the two control loops different (but the integral and derivative times are the same). For example, changing 50% in the first column to 25% gives a 3 (=75%/25%) times higher effective gain for z_2 than for z_1.

The proposed scheme scheme works well as shown by the dynamic simulations in Figure 8 where there are flowrate setpoint changes at times 1, 2 and 3 [min] and ratio setpoint changes at times 5, 6 and 8 [min]. The increase in ratio setpoint at t=6, results in F_2 reaching its maximum value (F2MAX=1 kg/s at $z_2 = 100\%$) and the split range block switches to using z_1 for ratio control. To reach the new ratio setpoint of 1.4, F_1 is then reduced to 1/1.4=0.71 kg/s which is below its setpoint (F1SP=0.8 kg/s), that is, we have to give up control of F_1 and we get an "unavoidable offset". Importantly, the SRC scheme in Figure 8 is able to maintain ratio control under all conditions, as desired. However, the dynamic response for reaching the new ratio setpoints at t=6 and t=8 is a bit delayed. The reason is a "limbo" effect because the switch to using z_1 for ratio control does not occur immediately when z_{12} in Figure 7 drops below 100%, because z_{11} (the output from the flow controller for F_1) starts at about 80%. This means that the integral action in the split-range flow controller (FC-SR) needs to increase u to a value above 50% before z_{12} drops to 80% and switching with the MIN-selector occurs.

The "limbo" effect is discussed in Appendix E in Zotică et al. (2022) who propose to speed up the switching by using bias updates. However, a much simpler solution is proposed by the modified SRC scheme in Figure 9. Here, the signal z_{12} from the split-range block (which must be in the range 1 to 0) is

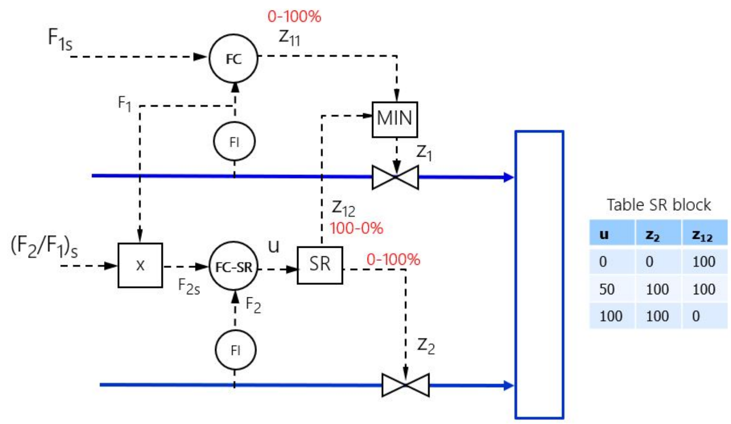

Figure 7: "Dual" ratio control using split-range control (SRC)

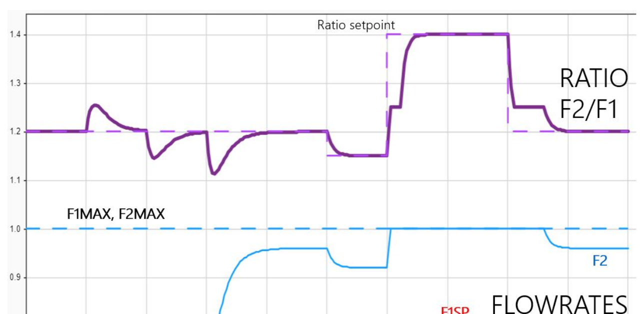

Figure 8: Simulation of dual ratio SRC scheme in Figure 7 The limbo-effect, caused by a delay from the MIN-selector, is seen for the ratio at times 6 and 8.

multiplied by the valve position z_{11} desired by the flow controller and the switch occurs immediately when z_{12} drops below 1. The MIN-selector may be omitted because $z_{12} \leq 1$. Initially, when z_{12} drops below 1, there will be some "fighting" between controllers FC and FC-SR, because both controllers are connected to z_1, but the fighting will stop quite soon due to the anti-windup action in controller FC which manipulates z_{11}. The split-range controller FC-SR does not have anti windup. The "multiplication trick" in Figure 9 results in much better ratio control at times 6 and 8 as shown by the simulation in Figure 10. In conclusion, the modified split-range scheme in Figure 9 is recommended for dual ratio control applications. In the discussion section, it is argued that the "multiplication trick" may be used more generally to avoid the "limbo effect" for split-range control (see Figures 15 and 16).

In the simulations in Figures 8 and 10, the only dynamics come from the two flow controllers. These are pure I-controllers which give a loop transfer function $5/s$ (corresponding to a closed-loop time constant of 0.2 [min]). Controller FC for F_1 has anti-windup with tracking coefficient K_t equal to 5 (using the PID controller in Matlab Simulink) (corresponding to a tracking time $\tau_T = \frac{1}{K_t} = 0.2$ [min]. In summary, the PI-controller is (Skogestad, 2023)

$$u(t) = u_0 + K_c e(t) + \int_{t_0}^{t} (K_I e(t') + K_t e_T(t'))dt' \tag{8}$$

with $e(t) = y - y_s$ and $e_T = \tilde{u} - u$, where for controller FC for $y = F_1$, we have $u = z_{11}$ and $\tilde{u} = z_1$. In the simulations, $K_c = 0, K_I = 5$ and $K_t = 5$. Controller FC-SR for $y = F_2$ has $K_c = 0, K_I = 5$ and $K_t = 0$, that is, this controller does not have anti windup. The valves (process) are linear and static with gain 1, that is, $F_1 = k_1 z_1$ and $F_2 = k_2 z_2$ with $k_1 = k_2 = 1$ [%/%]. The flowrates in the simulations have been divided by 100% for better visualization.

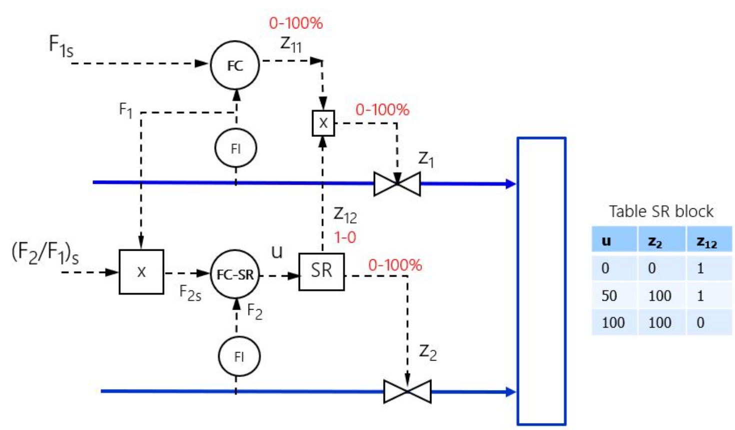

Figure 9: Modified split-range control (SRC) scheme for "dual" ratio control where the MIN-selector in Figure 7 is replaced by multiplication to avoid the limbo effect.

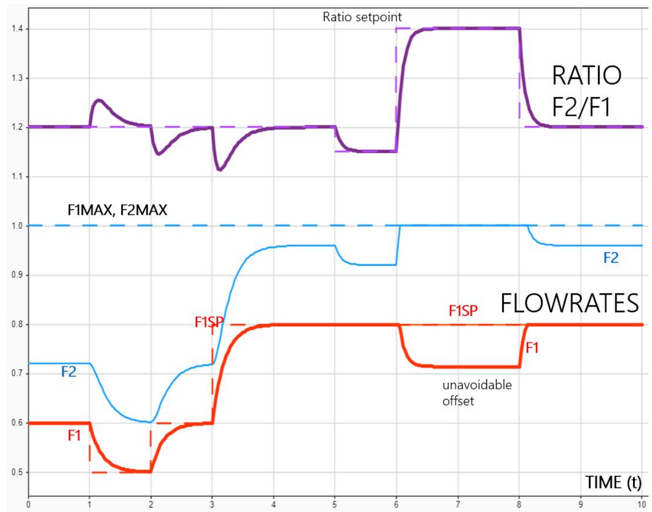

Figure 10: Simulation of modified dual ratio SRC scheme in Figure 9.

An alternative to the dual ratio schemes in Figures 7 and 9 is the "tracking ratio station" scheme of Hägglund (2017). However, this scheme is rather complicated to implement, and, more seriously, it results in undesirable frequent switching.

4.2. Cross-limiting ratio control

Another common, but fairly complex, application of ratio control is cross-limiting control in combustion power plants (Liptak, 1973; Wade, 2004). Here, the objective is to mix air (2) and fuel (1) in a given ratio, but during dynamic transients, when there will be deviations from the given ratio, one should make sure that there is always an excess of air, that is, we should always have $F_1/F_2 < (F_1/F_2)_s$.

A conventional ratio control scheme for combustion processes is shown in Figure 11. The setpoint for the ratio, $(F_1/F_2)_s$, could be set by a feedback controller (not shown) which controls, for example, the remaining oxygen after combustion. The setpoint for the fuel, $F_{1,s}$ could be set a feedback controller (not shown) which controls, for example, the power or the steam pressure. The conventional scheme in Figure 11 is not able to main excess of air under all dynamic transients, because normally (and always in the linear case) if $F_1/F_2 < (F_1/F_2)_s$ during one transient then $F_1/F_2 > (F_1/F_2)_s$ for the opposite transient (e.g., see the ratio for changes down and up in the flow setpoint in Figure 8 at times 1 and 2).

Interestingly, the scheme in Figure 12 with crossing min- and max- selectors achieves $F_1/F_2 < (F_1/F_2)_s$ during all transients. The scheme is widely used in industry and is mentioned in many industrial books (e.g., Liptak (1973); Wade (2004).

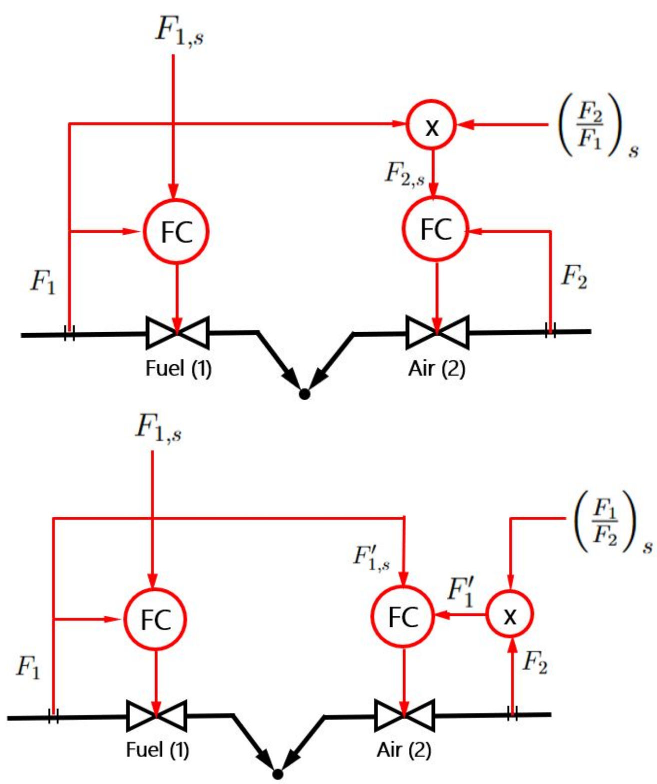

Figure 11: Conventional ratio control implementations for combustion. The two schemes are equivalent for practical purposes. The second scheme with the inverse ratio is shown as an intermediate step to the cross-limiting scheme in Figure 12.

A dynamic simulation is shown in Figure 13 for an increase in flow setpoint ($F_{1,s}$) at times 1 an 3, decrease at time 5, and finally a change down and up in the ratio setpoint at times 7 and 9. How does it work?

Let us first consider the change in fuel rate (F_1), which is the most important. When we increase the fuel setpoint ($F_{1,s}$), the air flow (F_2) will increase first, while the MIN-selector holds back the fuel increase. On the other hand, when we decrease the fuel setpoint ($F_{1,s}$), the fuel flow (F_1) decreases first while the MAX-selector holds back the air flow (so it remains high for a longer time). In summary, we always have excess of air during the dynamic transients in fuel rate.

However, when we decrease the ratio setpoint $(F_1/F_2)_s$ (at time 7), the ratio F_1/F_2 is initally above its setpoint, although the control system reduces the fuel (F_1) to help decrease the ratio during the transient. In any case, this situation is unavoidable, since it is not possible to react immediately to a setpoint change using feedback control. Furthermore, setpoint changes in the fuel ratio are not expected to be large and frequent.

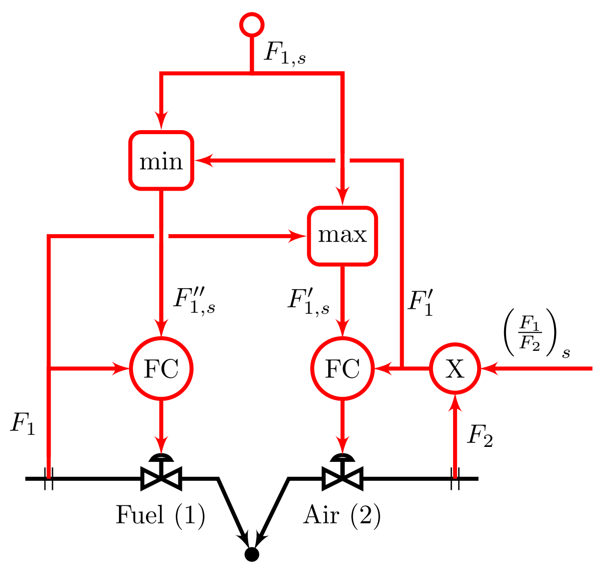

Figure 12: Cross-limiting ratio control for combustion where air (2) should always be in excess of fuel (1) (Smith & Corripio, 1997; Wade, 2004).

In the simulations, the valves are assumed to be linear ($\Delta F = k_v \Delta z$) and the only dynamics come from the two flow controllers. These are pure I-controllers which give a loop transfer function $5/s$ for F_1 (corresponding to a closed-loop time constant of 0.2 [min]) and $10/s$ for F_2 (corresponding to a closed-loop time constant of 0.1 [min]). Note that the cross-limiting solution works well in all cases, independent of the tuning of the flow controllers.

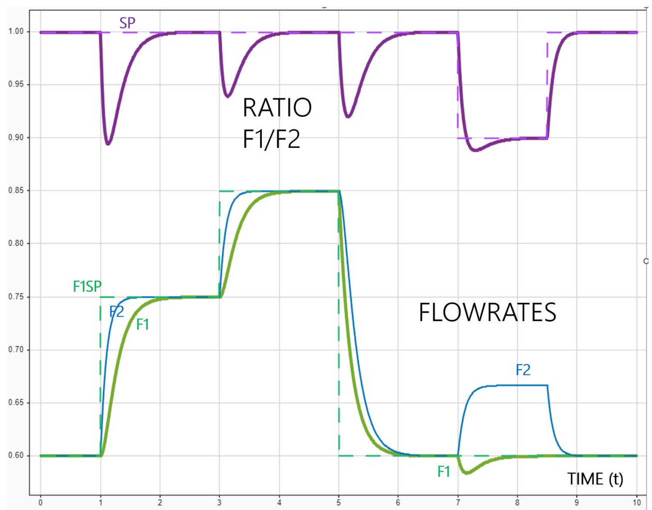

Figure 13: Simulation of the cross-limiting ratio control scheme in Figure 12.

5. Discussion

5.1. The scaling assumption

In the literature, there appears to be no clear statement of the scaling assumption and its relationship to ratio control. The one who comes closest is Riggs (1999) who writes the following in the first paragraph in his section on ratio control: "Many processes scale directly with the feedrate to the process, e.g. distillation columns and wastewater neutralization. For distillation columns, all the liquid and vapor flow rates within the column are directly proportional to the column feed rate if the product purities are maintained and the tray efficiency is constant".

The idea of scaling all flowrates (and more generally all extensive variables) with respect to a "basis" is extensively used for mass and energy balance calculations in process engineering. However, its theoretical basis, namely the scaling assumption, is not mentioned in most standard text books. One exception is Reklaitis (1983) who proves the homogeneity (2b) (with $h = 1$) of the material balance equations (page 40) and shows that this leads to the scaling assumption: "If any set of flows F_i satisfies the balance equations and if α is any number, the flow rates αF_i also satisfy the balance equations". He notes that the same applies to the energy flows in the energy balance (page 460).

However, this is not enough to guarantee that we get the same steady-state solution with constant intensive variables (y). For example, it will not be the case for a non-equilibrium reactor or a distillation column where the stage efficiency depends on the load. Riggs (1999) and Skogestad (2009) state that also constant efficiencies are required for the scaling assumption and (2) to hold. Skogestad (2009) writes (page 66): "An initial basis for a selected stream is often chosen, for example 1 kg or 100 mol/s. If necessary, we can later rescale (up or down) all the streams to the desired quantity. Mass, energy, volumes, etc. (all extensive variables) will scale with the same factor provided that the efficiencies of the units remain constant."

In thermodynamics, (2b) (with $h = 1$) is used to derive Euler's theorem and from this we may derive the fundamental equation of thermodynamics, Legendre transformations and the Gibbs-Duhem equation (Modell & Reid, 1983). Skogestad (1991) states the scaling assumption and stresses the difference between extensive and intensive variables. Following Modell & Reid (1983), he derives Euler's theorem in thermodynamics and uses this to derive consistency relationships (eq. 27 in his paper) for linear steady-state models for systems that satisfy the scaling assumption, and shows that many published models do not satisfy this and therefore are incorrect. The counterpart of these consistency equations in thermodynamics is the integrated Gibbs-Duhem equation.

5.2. Distillation

An example where it is important to keep *all* ratios constant is distillation. Typically, for distillation we need keep two ratios constant (and not only one), see Figure 5. More generally, Rule R4 says that for an application with $n = 2$ independent extensive variables at steady state (like distillation with fixed pressure), we need to keep $n = 2$ ratios constant. In addition, from rule R3, we can only have a single extensive disturbance. This means that ratio control should *not* be applied a distillation column with a fixed heat input V (and with the feedrate F as a disturbance). Nevertheless, reflux-to-feed (L/F) ratio control is often used in practice, even for cases where the heat input (which is an extensive variable) is constant. However, Bang & Skogestad (2025) show with simulations that with constant heat input (that is, boilup V is constant), a constant reflux ratio L/F gives the wrong (opposite) response in reflux L to a change in feedrate F. This is easy to explain: For an increase in the feedrate F, we need to increase the distillate product flowrate D to maintain approximately constant product composition (y). With constant boilup V and a liquid feed, the vapor rate V_T up the column is approximately constant (see Figure 5). At steady state $V_T = L + D$, so to increase D we need to reduce the reflux L (and certainly not to increase L as would result from keeping the ratio L/F constant).

5.3. Generalized ratio control using transformed inputs

One limitation with ratio control is that it assumes that there are no changes (disturbances) in the independent intensive variables (x_i in (1) and (2); x_1 in Figure 1). These disturbances may be handled by updating the ratio setpoint using an outer feedback controller (controller CC in Figure 1), but if this is not sufficient, then we need to make use of model-based feedforward control. For the static case, a simple and powerful nonlinear feedforward approach is provided with the use of ideal transformed inputs (Skogestad et al., 2023; Skogestad, 2023).

Consider the steady-state model (the subscript 0 is used to emphasize that we use a static model)

$$y = f_0(u, d) \tag{9}$$

where y is the controlled variable (CV), u is the manipulated variable (MV) and d is the disturbance variable (including both extensive and intensive disturbances). All variables are vectors in the general case. The ideal transformed input v_0 (controller output) is selected as the right-hand-side of the steady-state model,

$$v_0 = f_0(u, d) \tag{10}$$

For implementation, one needs to invert the model by solving (10) with respect to u for given values of v_0 and d. We can formally write the solution as

$$u = f_0^{-1}(v_0, d) \tag{11}$$

At steady state, the resulting transformed system then trivially becomes

$$y = v_0 \tag{12}$$

That is, we have $y = Iv_0$, so we have perfect feedforward control, decoupling and linearization at steady state. It looks like magic, but it works in practice. To have perfect control, we must assume that all disturbances d are measured (or at least estimated), but if this is not the case then one may use a simpler variant of f_0 as the transformed input v, where we fix the value of unmeasured disturbances to get partial feedforward or decoupling. To correct for model error and unmeasured disturbances, the value (setpoint) for v_0 may be adjusted by an outer controller C (usually a decentralized PID controller).

As an example, consider the simple mixing process in Figure 14 where $u = F_2$ is the manipulated variable (the true MV is usually the valve position z_2, but assume we have a flow controller for F_2 as shown in Figure 14) and y (product mass fraction) is the controlled variable. The disturbance (d) variables are F_1 and the feed compositions x_1 and x_2 (mass fractions).

From the steady-state component material balance, we have that y is the weighted average of the feed fractions (recall (4))

$$y = f_0(u, d) = \frac{x_1 F_1 + x_2 F_2}{F_1 + F_2} \tag{13}$$

Note that $u = F_2$. The transformed input is defined as the right-hand side of this equation, $v_0 = f_0(u, d)$. Note that v_0 is the output from the feedback controller. Inverting (13), we find how the input $u = F_2$ depends on the transformed input v_0:

$$u = F_2 = f_0^{-1}(v_0, d) = \frac{x_1 - v_0}{v_0 - x_2} F_1 \tag{14}$$

This may realized using the improved ratio control scheme in Figure 14 where, compared to Figure 1 we have added a computation block which uses (14) to compute the setpoint for F_2/F_1. This will result in constant composition y at steady state for all disturbances, even without a feedback controller (CC) to update v_0. Of course, this assumes that the model is correct and that we measure or estimate the feed disturbances x_1 and x_2.

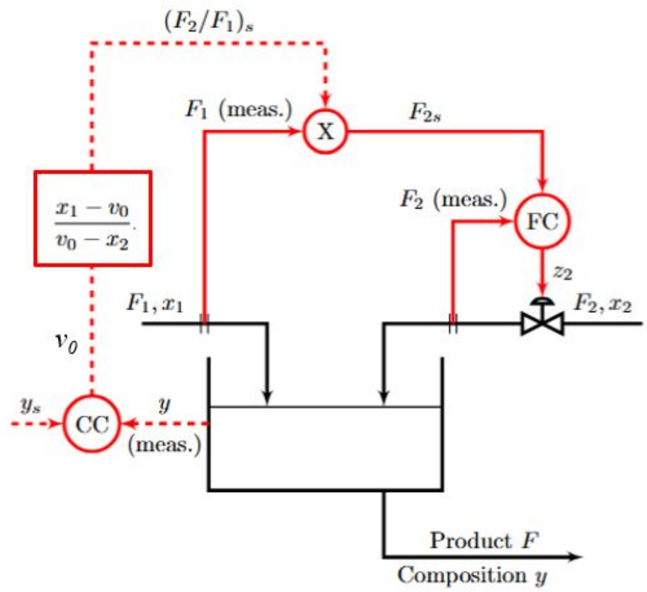

Figure 14: Improved ratio control scheme for mixing process using transformed input v_0. The feed mass fractions x_1 and x_2 that enter the computation block, need to be measured or estimated.

5.4. Normalized ratio

In the paper, we have considered the ratio

$$R = F_2/F_1$$

(or sometimes the inverse F_1/F_2) which is simple to implement using a multiplication element. An somewhat more complex alternative is the normalized ratio (King, 2011; Skogestad, 2023)

$$R_N = \frac{F_2}{F_1 + F_2} \tag{15}$$

The two ratios are equivalent in the sense that fixing one keeps the other constant (since $R_N = (R^{-1} + 1)^{-1}$). However, the normalized ratio has some properties that may makes it better for implementation (Skogestad et al., 2023). First, R_N is always in the range 0 to 1, whereas R may vary between 0 and ∞.

Second, for many applications, the ratio R_N is a special case of the ideal static transformed input (v_0) and provides linearization (e.g. see (13)) (Skogestad et al., 2023). To understand this better, consider again the mixing process in Figure 1. The steady-state model equation (4) becomes (King, 2011)

$$y = x_1(1 - R_N) + x_2 R_N$$

which is linear in R_N, so the linearized model as seen from the outer controller (CC in Figure 1) becomes $\Delta y = K_N \Delta R_N$ with (King, 2011)

$$K_N = x_2 - x_1 \tag{16}$$

Notice that the K_N for the normalized ratio R_N is independent of both R and the throughput F_1, whereas the gain K_R for R in (6) depends quite strongly on R when R is large. One way to reduce this nonlinearity for $R = F_2/F_1$, would be to use the inverse ratio $R' = F_1/F_2$ for cases when $R = F_2/F_1$ is large. However, more generally, if large variations in R are expected (which is not so common), an even better solution is to use the normalized ratio $R_N = F_2/(F_1 + F_2)$ as the output of the outer controller.

However, the process gain K_N will still vary if there are changes in $x_2 - x_1$, and in particular this may be a problem if the quality difference $x_2 - x_1$ is small, because then a relatively small change in either x_1 or x_2 will cause a proportionately large change in the difference. This problem is noted by King (2011) who considers a very similar mixing example, but with temperature rather than composition. King (2011) proposes to add some feedforward action to compensate for changes in the temperature difference $T_2 - T_1$. Probably, the best way to add steady-state feedforward action is to use the transformed input $v_0 = f_0(u, d)$ in (13) as the outer controller output, which results in the implementation in Figure 14. Here, the steady-state model $\Delta y = K_0 v_0$ has a constant process gain $K_0 = 1$.

5.5. MPC and ratio control

Unlike conventional feedforward control, ratio control does not need an explicit model of how the controlled property variable y depends on the extensive variables (X_i, F_i). This makes ratio control more powerful and simpler to apply than many people think. However, it also implies that conventional model-based control approaches, like model predictive control (MPC), are not ideally suited for implementing ratio control.

First, MPC may be a good solution if maintaining a given ratio setpoint is a primary control objective and may be defined as a controlled variable (CV) for MPC. One application is MPC for air-to-fuel ratio (AFR) control in engines (Trimboli et al., 2009; Honek et al., 2015). In this case, we may use a conventional industrial linear MPC implementation (with MV constraints). MPC is also a good alternative to achieve "dual" ratio control (as in Figures 7 and 9). However, the use of MPC with ratio as a CV, requires a ratio element to compute the measured ratio (e.g., F_2/F_1). This is similar to the ratio controller

(RC) implementation in Figure 3b and it shares some of its problems, including the danger of dividing by zero and the nonlinearity from the MV (e.g., F_2) to the CV (e.g., F_2/F_1), see (7). The nonlinearity may be handled by using nonlinear MPC.

Next, consider the mixing process in Figure 1, where the main measured disturbance is the flowrate $d = F_1$ and the primary objective is to use the input $u = F_2$ (MV) to control the composition y (CV). Assuming that there is some delay associated with the measurement of y, we know intuitively that it may be an advantage to use ratio control, where $u/d = F_2/F_1$ is kept constant on a fast time scale. How are we going to make MPC do this? The obvious solution is to supply MPC with a model, such as (4), which tells MPC that y will be constant when the ratio $u/d = F_2/F_1$ is constant. This should work well if we also tell MPC that there is some delay associated with y, so that it knows that feedforward action from $d = F_1$ will improve control of y. However, note that the use of ratio control gives nonlinear feedforward action, so this require a nonlinear MPC implementation (which is not commonly used in industry today). Also, what should we do if we do not have a good model for how y depends on u and d, for example, if y is a more complex property variable, such as viscosity or color? In this case, we may have to create an artificial model which tells MPC that y will be constant if we keep the ratio u/d constant.

Alternatively, for the mixing process in Figure 1, we may use a cascade implementation where MPC is combined with another controller (e.g., PID control), and the ratio is defined as either a CV or MV for MPC. This avoids some of the modelling issues and allows for using a linear MPC implementation. The simplest, as was discussed initially, is to let MPC be a slave controller and define the ratio as a controlled variable (CV) for MPC. The ratio setpoint is set by a separate slower master controller which controls y (like CC in Figure 1). An alternative cascade implementation is to use MPC as the master controller. Here, the ratio setpoint, like $(F_2/F_1)_s$, is defined as a degree of freedom (MV) for MPC, and the ratio setpoint is implemented by the faster regulatory layer, below MPC, for example using the simple implementation in Figure 3a or a "dual" ratio implementation (e.g., Figure 9).

5.6. Generality of multiplication trick for split range control with override

The trick of replacing the MIN-selector with a multiplication element to avoid the limbo-effect with split-range control (SRC) for dual ratio control (see Figures 7 and 9) may be generalized and shown in Figures 15 and 16. It is assumed that control of y_2 has higher priority than control of y_1. Controller C1 has anti-windup but not the split-range controller C2. (Note that if the controlled variables y_1 and/or y_2 are not flows, then one may add slave flow controllers (not shown in Figures 15 and 16) to compensate for valve nonlinearity and pressure disturbances, and in such cases, z_1 would be the setpoint $F_{1,s}$ and z_2 the setpoint $F_{2,s}$.)

Consider first the standard implementation in Figure 15. In the unsaturated case, the two MVs (valve positions z_1 and z_2) are used to control their associated CVs (y_1 and y_2) using single-loop controllers (C1 and C2) (usually PID

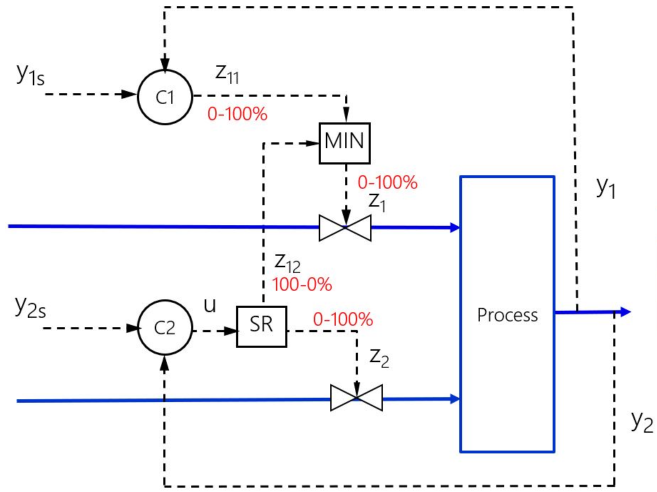

Table SR block

u	z_2	z_{12}
0	0	100
50*	100	100
100	100	0

Figure 15: General split-range control for MV-MV switching combined with override MIN-selector for CV-CV switching (for case where control of y_2 has higher priority than control of y_1). Controller C1 has anti-windup but not the split-range controller C2. (*The split value of 50% in the table may be changed to adjust the effective controller gain).

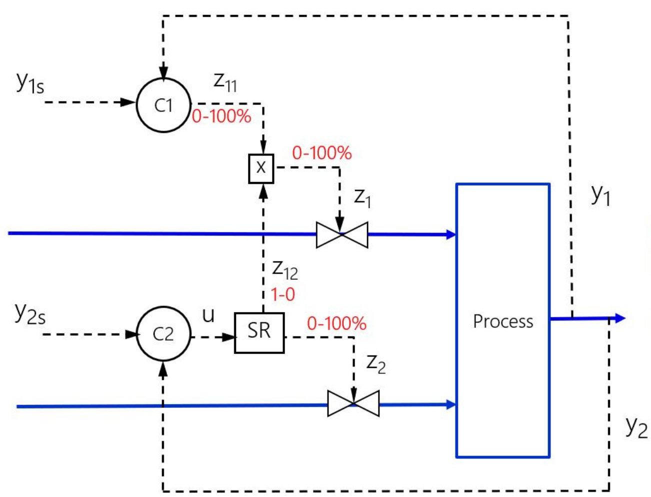

Table SR block

u	z_2	z_{12}
0	0	1
50*	100	1
100	100	0

Figure 16: Avoiding limbo-effect with split-range control by replacing the MIN-selector in Figure 15 by a multiplication element. Controller C1 has anti-windup but not the split-range controller C2. (*The split value of 50% in the table may be changed to adjust the effective controller gain for C2).

controllers). Control of y_2 is has higher priority than control of y_1, so if z_2 saturates fully open (100%), the SR block and the MIN-selector make the system switch to using z_1 for control of y_2. However, as noted in the simulation in Figure 7 (at times 6 and 8), the switching does not occur immediately because z_{12} has to "wind down" from 100% to the present value of z_{11}.

This limbo effect may be avoided if we replace the MIN-selector with a multiplication element, as shown in Figure 16. Here, the switching occurs immediately when z_{12} drops below 1. There will be some "fighting" initially between controllers C1 and C2, because both controllers are connected to z_1, but the fighting will stop quite soon because of the anti-windup action in C1.

A minor problem with the solution in Figure 16 is that it introduces a nonlinearity through the multiplication with z_{11}. That is, a value of z_{11} less than 100% will reduce the effective gain from z_{12} to y_2 and slow down the control of y_2 when z_2 is saturated.

The trick of avoiding the limbo-effect by replacing the selector by multiplication with a signal between 1 and 0 only works to replace a MIN-selector. However, note that a MAX-selector may be replaced by a MIN-selector by simply redefining the sign of the variables, for example, by defining a fully open valve to be 0% rather than 100% and a closed valve to be 100% rather than 0%. Thus, the multiplication trick can be used also to replace a MAX-selector.

6. Conclusion

Ratio control is very simple to use and it gives nonlinear feedforward action without needing an explicit process model. It is almost always used for chemical processes to set the ratio of the reactant feed streams. Ratio control is sometimes viewed as a special case of feedforward control, but note that we do not need a model for the controlled property y for ratio control, whereas such a model is needed for feedforward control.

The theoretical basis for ratio control is the scaling assumption which says that we get the same steady-state solution if we increase all extensive variables (flows and heat rates) by the same factor compared to a basis. Similar to the use in thermodynamics, the scaling assumption holds for equilibrium systems with constant efficiencies.

The scaling assumption is formulated mathematically in (2). From this we derived the following rules for the use of ratio control:

- (R1) The controlled variable y is implicitly assumed to be an intensive variable, for example, composition, density, viscosity, taste or temperature.
- (R2) The system must satisfy the scaling assumption (2).
- (R3) Since all extensive variables must be scaled by the same factor k, there can only be one independent extensive disturbance variable. This variable is sometimes called the "basis", "wild variable", "master variable", "flow disturbance" or "throughput manipulator" (TPM).

- (R4) If the system has n independent extensive variables X_i, then from (2) we need to manipulate $n-1$ of these variables to keep the $n-1$ ratios constant (or more generally, $n-1$ dependent intensive variables y_i). For a change (disturbance) in the throughput (basis, wild flow), this will result in keeping *all* dependent intensive variables constant, including the controlled variable(s) y (at steady state).

The paper has also discussed the practical implementation of ratio control using a multiplication element. Figure 1 shows a typical cascade implementation where an outer loop controls y and sets the ratio setpoint. Note that no model is needed to implement this solution. More advanced implementations are dual ratio control for the case with saturation (Figure 10) and cross-limiting control to keep one component (typically oxygen) in excess during dynamic transients (Figure 12). Ratio control can be generalized to include model information using the idea of transformed inputs (Figure 14). Finally, Figure 16 shows how the multiplication trick (used in Figure 12) applies more generally to avoid the limbo effect in split-range control.

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Supplementary material

The MATLAB/Simulink files for the dynamic simulations are available at the home page of Sigurd Skogestad.

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