Origins of

DYNAMIC MATRIX CONTROL

The Early Writings of Charles R. Cutler

PSE PRESS

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Origins of Dynamic Matrix Control

The Early Writings of Charles R. Cutler

by Sigurd Skogestad and Thomas A. Adams II

Norwegian University of Science and Technology

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I had this idea of using an LP to control a process at multiple constraints. While taking an electrical engineering course on Z transforms, I realized the process dynamics could be represented numerically by truncating the infinite series and evaluating the terms of the series at each time interval.

...

I was locked in the refinery for 3 or 4 months so I wrote the program.

-Charles R. Cutler, December 2010

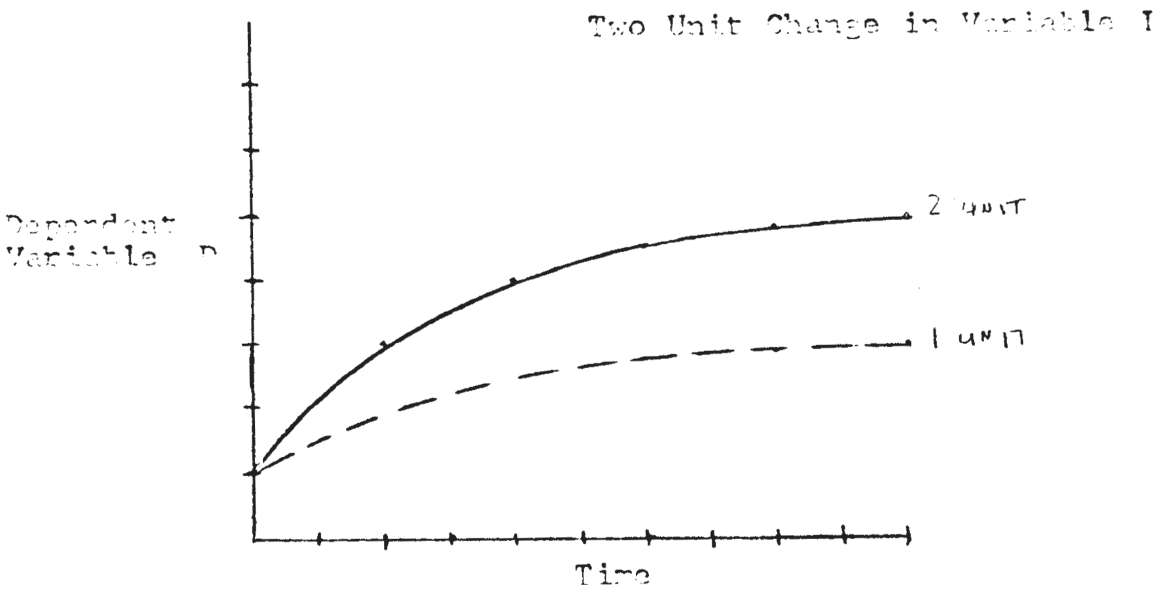
Introduction

At a National AIChE meeting in Houston in April 1979, engineers from Shell presented a paper (published also at the JACC in 1980[1]) that attracted the attention of many attendees. According to Prof. Manfred Morari at the University of Pennsylvania, it was essentially impossible to understand the described techniques, which were referred to as "dynamic matrix control (DMC)," but they were intriguing because they seemed very different from the industrial standards at that time, and they "worked." The authors claimed that they had solved the notoriously difficult cat cracker optimizing control problem[2].

This pioneering paper on DMC turned out to be the genesis of Model Predictive Control (MPC), where the central idea is to formulate the control problem as a repeated open-loop optimization problem with a moving time horizon. MPC is today the most widely used multivariable control approach, in both academia and industry. Even the original DMC formulation of MPC, which was invented for the early age of digital computing, is still extensively used in most of the larger refineries in the world.

However, it is a common misunderstanding that DMC was developed in the 1980s, or, that the credit of its development should go to the Shell Oil Company[3]. In fact, as documented in the following, the only one who should be credited for the development of DMC is **Charles R. Cutler** (1936-2020). The history goes back to his 1969 PhD proposal to the Chemical Engineering Department at the University of Houston[4]. There, Cutler laid out his first ideas, at the time completely untested in practice. He had the fortunate chance to experiment on a real refinery when, while working in the as the process manager of the catalytic cracking process at Shell's New Orleans refinery in 1973, a plant strike forced him to be locked into the refinery for three to four months. Having access, authority, and perhaps boredom, he used this extremely rare opportunity to prove out his ideas. The experiments were wildly successful and DMC was adopted at Shell after that. However, it would take him nearly 14 years to finish the thesis, due no doubt to the part-time nature of his PhD studies and the delay in getting permission to have the industrial experimental data published.

We have come into possession of Cutler's original 1969 thesis proposal and some interesting communications from Cutler, before his death, describing the history behind it. Because the timeline and credit of the development of DMC are often incorrectly attributed, we think it is important to science historians that his 1969 thesis proposal be published along with these communications. It proves that Cutler developed the main points of the theory behind DMC all the way back to 1969, before any testing, and thus should be fully attributed to him. For example, he used essentially the same diagram in his development of his concept for the PhD proposal as was published in the 1980 work:



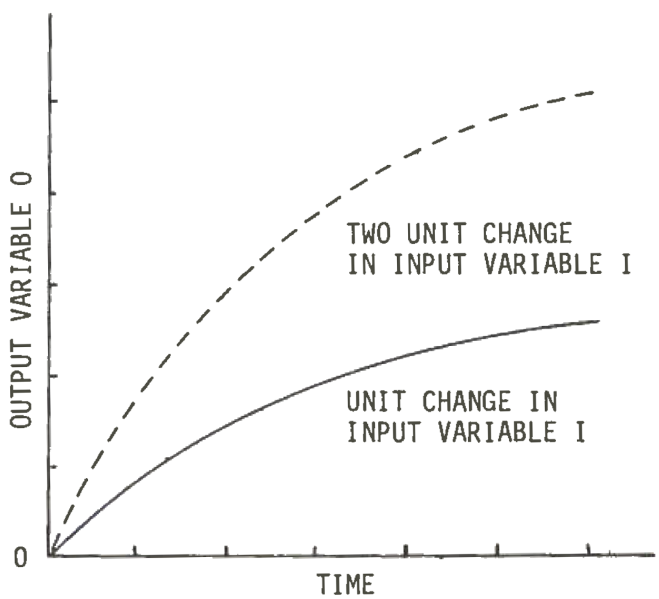


FIG. 2 PRESERVATION OF SCALE FACTOR

Top: Figure 2 from Cutler's 1969 thesis proposal (the original manuscript quality is poor).
Bottom: Figure 2 from the first resulting publication in 1980.[2]

This book contains a copy of his original thesis proposal from 1969, sent by Cutler to his colleague Prof. Manfred Morari, then at ETH Zurich in Switzerland. The original page images are quite deteriorated, so some digital enhancements were made for clarity. Prof. Sigurd Skogestad from the Norwegian University of Science and Technology, has "translated" the document into plain text for easy searchability and reading. He has also provided some commentary.

Cutler (*center*) and colleagues at a workshop in Åbo, Finland in August 2001 where he received the 5th Nordic Process Control Award. *Left-to-right*: Sigurd Skogestad, Leif Hammarström, Charlie Cutler, Elling Jacobsen, and Kurt Erik Häggblom.

References

1. Cutler CR, Ramaker BL. Dynamic matrix control — A computer control algorithm. *Proceedings of the 1980 Joint Automatic Control Conference*, Vol 1. Paper WP5-B. Aug 13-15, San Francisco, CA, USA (1980).

2. Morari M. Model predictive control: The genesis of an idea. *IEEE Control Systems*. August (2025). https://doi.org/10.1109/mcs.2025.3573842

3. Prett DM, Ramaker BL, Cutler CR. Dynamic matrix control method. *United States Patent* 4,349,869 (1982)

4. Cutler CR. Dynamic matrix control: An optimal multivariable control algorithm with constraints. *University of Houston Thesis* (1983).

Letters from Cutler

Cutler sent the below email to his colleague Prof. Morari with an explanation of the history behind the design of DMC. Morari had written to request more about the story. The email contained several attachments, including his thesis, his 1980 paper, and various letters to and from Shell Oil Company employees about the work between 1973 and 1976. It is not necessary to publish all the letters, but they are requests for permission to publish his work in various forms, and their subsequent approvals. Two letters from 1975 and 1976 included a draft of the 1980 paper, indicating how early even that public work was written. One particular note for historians is that C. C. Williams III, Manager of Process Engineering-Refining and Engineering-Products at Shell, wrote in his letter on January 28, 1976 that Shell could "not patent the control technique since it has been in commercial use for more than one calendar year." This means DMC was fully commercialized sometime between 1973 and 1974.

The emails have been reformatted for clarity. Addresses have been redacted in italics. Original spellings are left intact.

From: Charles Cutler <*redacted*@cutler-tech.com>
Subject: Cutler's Matrix Control
Date: December 16, 2010 at 9:50:03 PM GMT+1
To: *Manfred Morari*
Cc: 'Matthew Hetzel' <*redacted*@cutler-tech.com>, 'June Cutler' <*redacted*@cutler-tech.com>

Hello Man fred

It was good to make contact again with you. I have attached a file that contains communications I had when I was working for Shell Oil Company. It provides some historical perspective on the evolution of the dynamic matrix control algorithm. I was working for Dr. Huang on an experimental thesis at the University of Houston, when Shell offered me a group leader position in a newly created process control group in their New Orleans refinery. I had this idea of using an LP to control a process at multiple constraints. While taking an electrical engineering course on Z transforms, I realized the process dynamics could be represented numerically by truncating the infinite series and evaluating the terms of the series at each time interval. I ask Huang to let me develop this idea remotely. My proposal to him in 1969 is included in the attachment. He agreed to let me return to Houston once a month to review my progress and he would let me switch my dissertation topic.

In 1973, the refinery was struck and staff personal took over the operation of the refinery. My control group in 1970 and 1971 had build a RTO and computer control system for the catalytic cracking unit. Since at the time of the strike I was the process manager of the catalytic cracking department, I did not have to ask anyone for permission to test my matrix control concept. I was locked in the refinery for 3 or 4 months so I wrote the program. Ramaker's contribution was a program that he had help develop at LSU while working on this PHD that fit a first order model to data. The inlet temperature to the furnace that went through a surge drum before entering the furnace. Ramaker's program permitted us to get the disturbance model for the effect of inlet temperature to the surge drum to the outlet temperature of the furnace. The effect of the fuel gas and draft damper were taken from plots of the data versus time. As you can see from my proposal to C.J. Huang, the concepts for algorithm were develop years before Ramaker's involment.

I kept the file on the communications in Shell in the event someone challenged me as to the ownership of the algorithm. I was never challenged, but the word on the street was the algorithm belonged to Shell. It did prevent me from getting project work for my new control company. Stone and Webster Engineering Company gave me my first opportunity to use the algorithm outside of Shell, but only after their lawyers kept the attached file for two months.

I still work 50 plus hours per week for my company. Our new controller is a significantly more powerful than the version Aspen still sells. We build a controller this last year that has 41 manipulated variables and 160 control variables on the largest single train ammonia plant in the world in Saudi Arabia. The controller runs at a 10 second frequency, has 420 coefficients, and executes in less than a half a second.

If you get to San Antonio for some reason, I would love to have lunch or dinner with you.

Have a great day !!!

Charlie

Selections from the attachments follow.

SHELL OIL COMPANY

DATE MARCH 4, 1973

TO PROCESS SUPERINTENDENT - EAST OPERATIONS

FROM PROCESS MANAGER
CATALYTIC CRACKING
NORCO REFINERY

cc - Circulate: Refinery Manager
Refinery Superintedent
Chief Technologist

SUBJECT PUBLISHING OF CO FURNACE CONTROL DATA

As you know, I have been working on my PHD dissertation in the area of Process Control. Recently I implemented one of the techniques from my dissertation research on the Cat Cracker CO Furnace. The technique is working exceeding well and substantiates the theory on which my dissertation is based. I would like to obtain permission from the company to publish this data in my dissertation. The use of actual process data will give the dissertation credibility and statue.

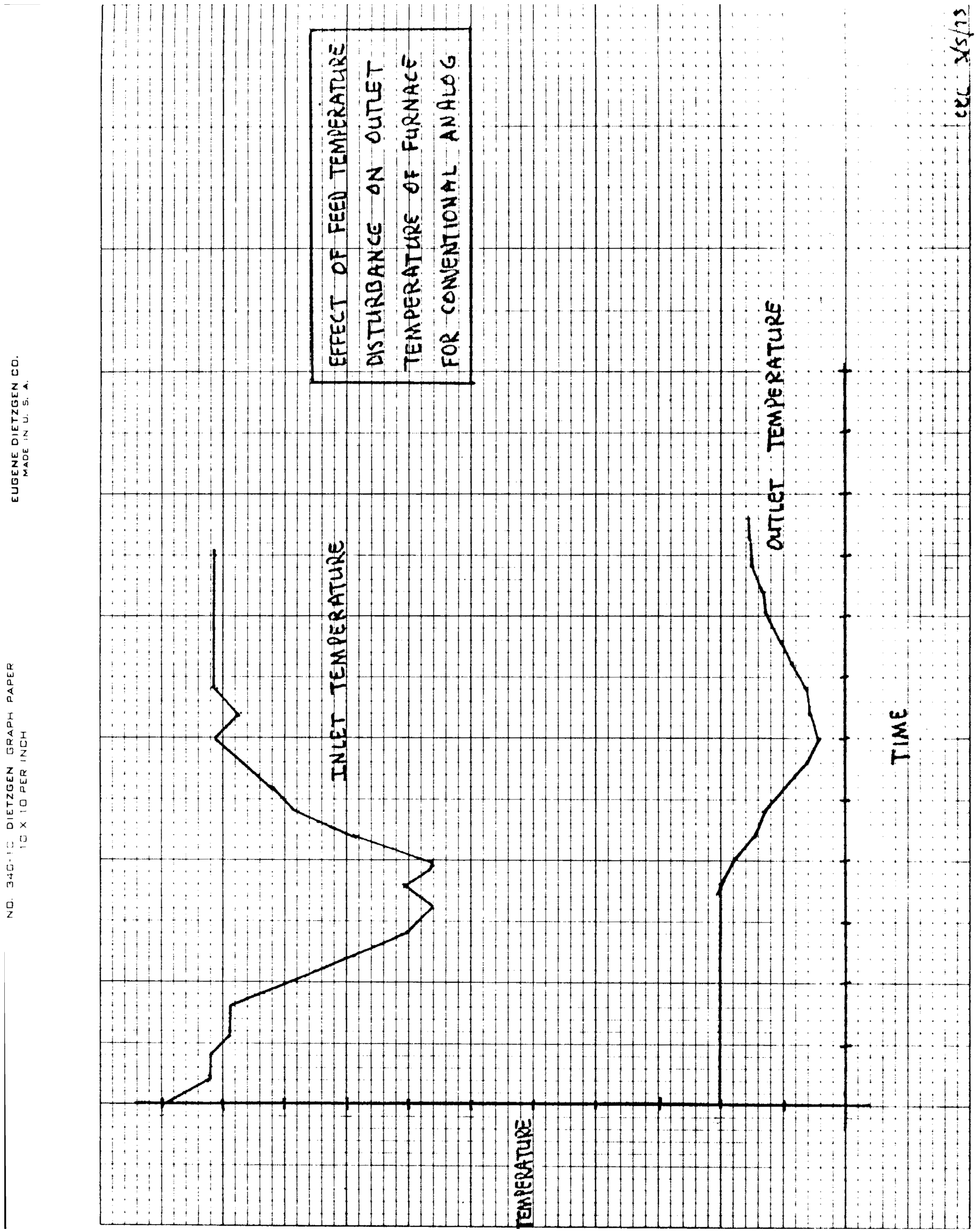
The data may be presented in dimensionless terms to avoided any association with the actual process or levels of operation. Attached are the data presented in graphical terms, that I would use.

The theory to which I have referred is familiar to you, Egon Doering, Charles Gillard, and Stan Marple in Head Office. However, for others who may read this letter, I have attached a copy of the original research proposal I gave my professor. The technique used on the CO Furnace is the least square approach to reducing the error described in the proposal.

Your consideration of this matter will be appreciated.

C. R. Cutler

Attachment



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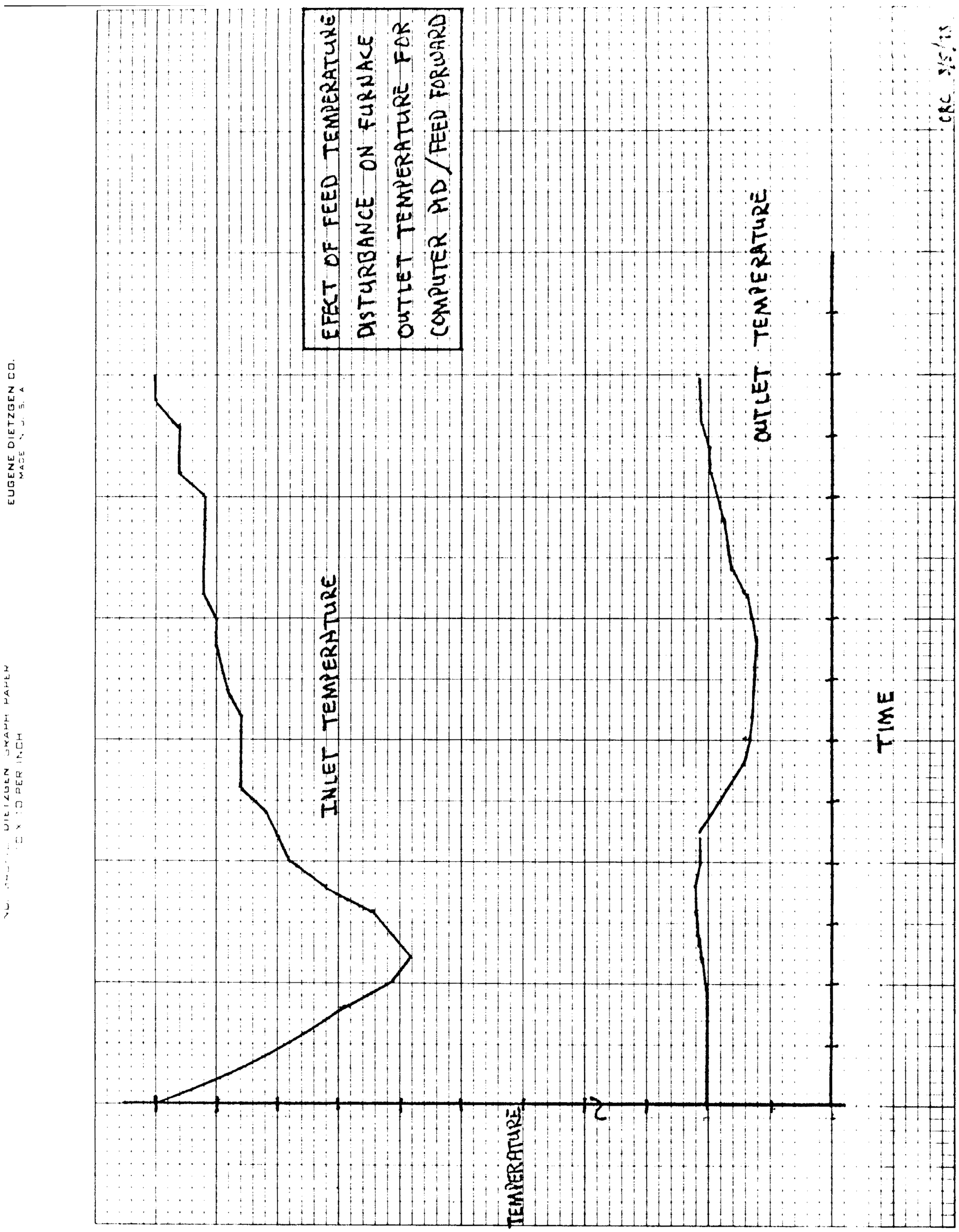
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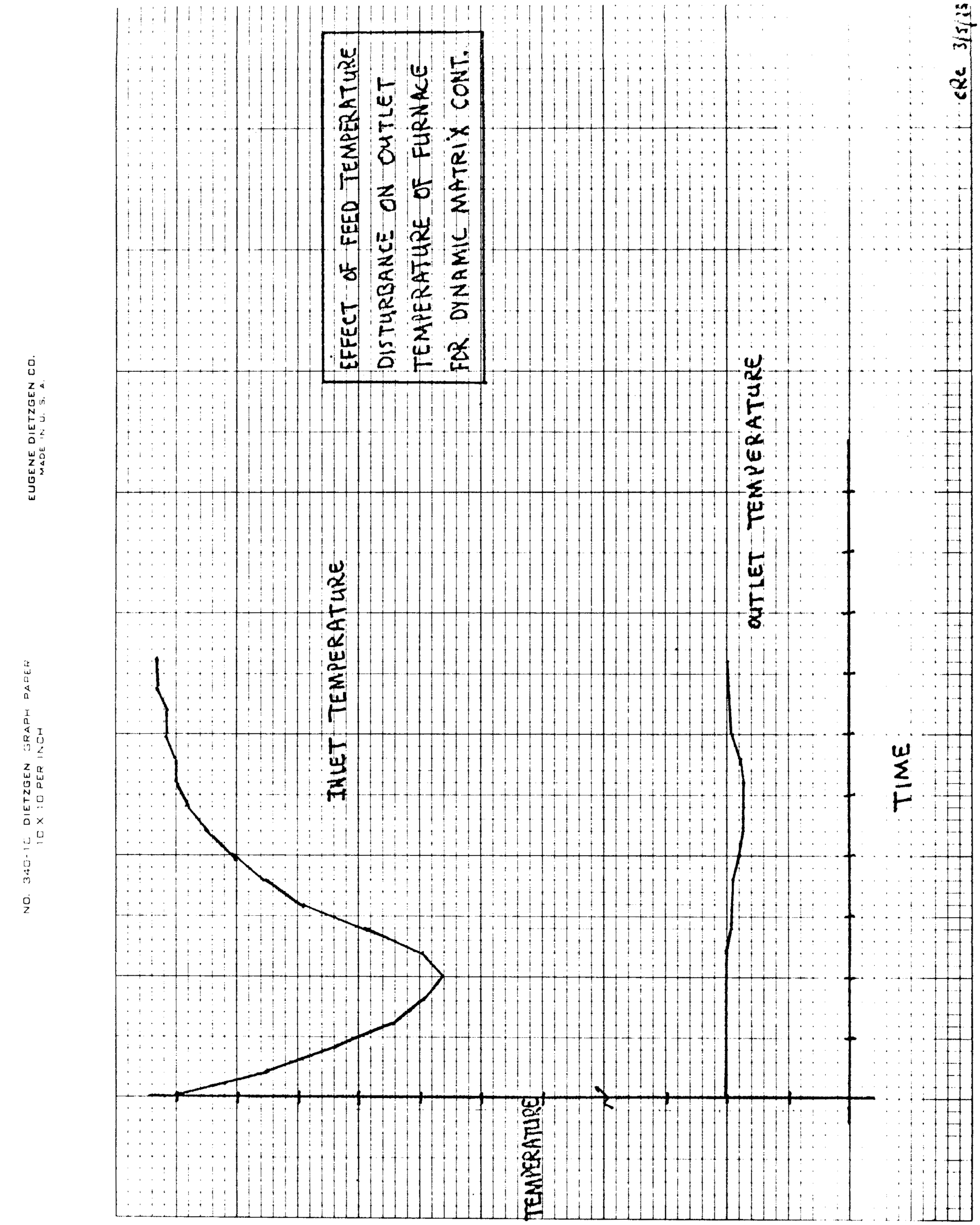
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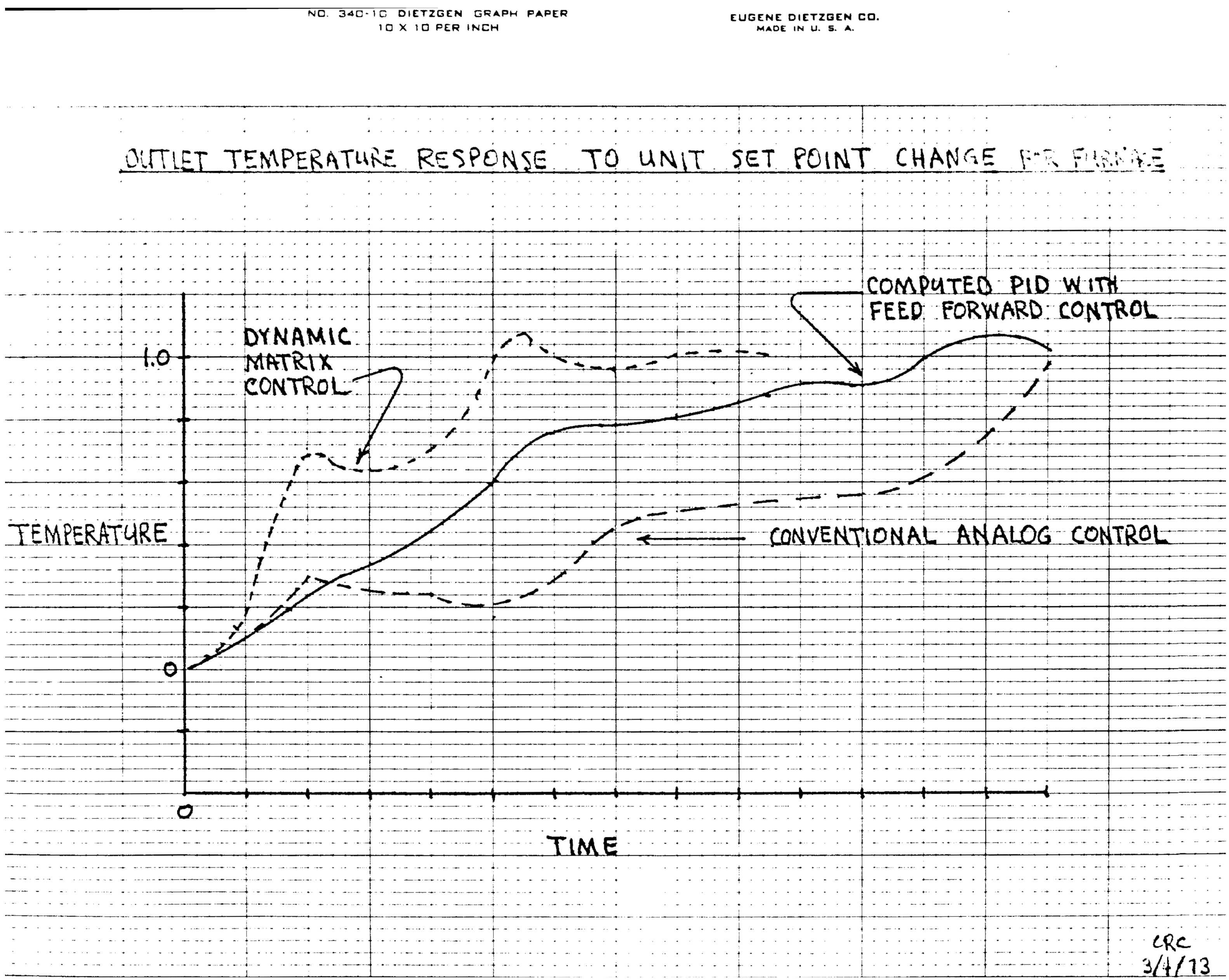
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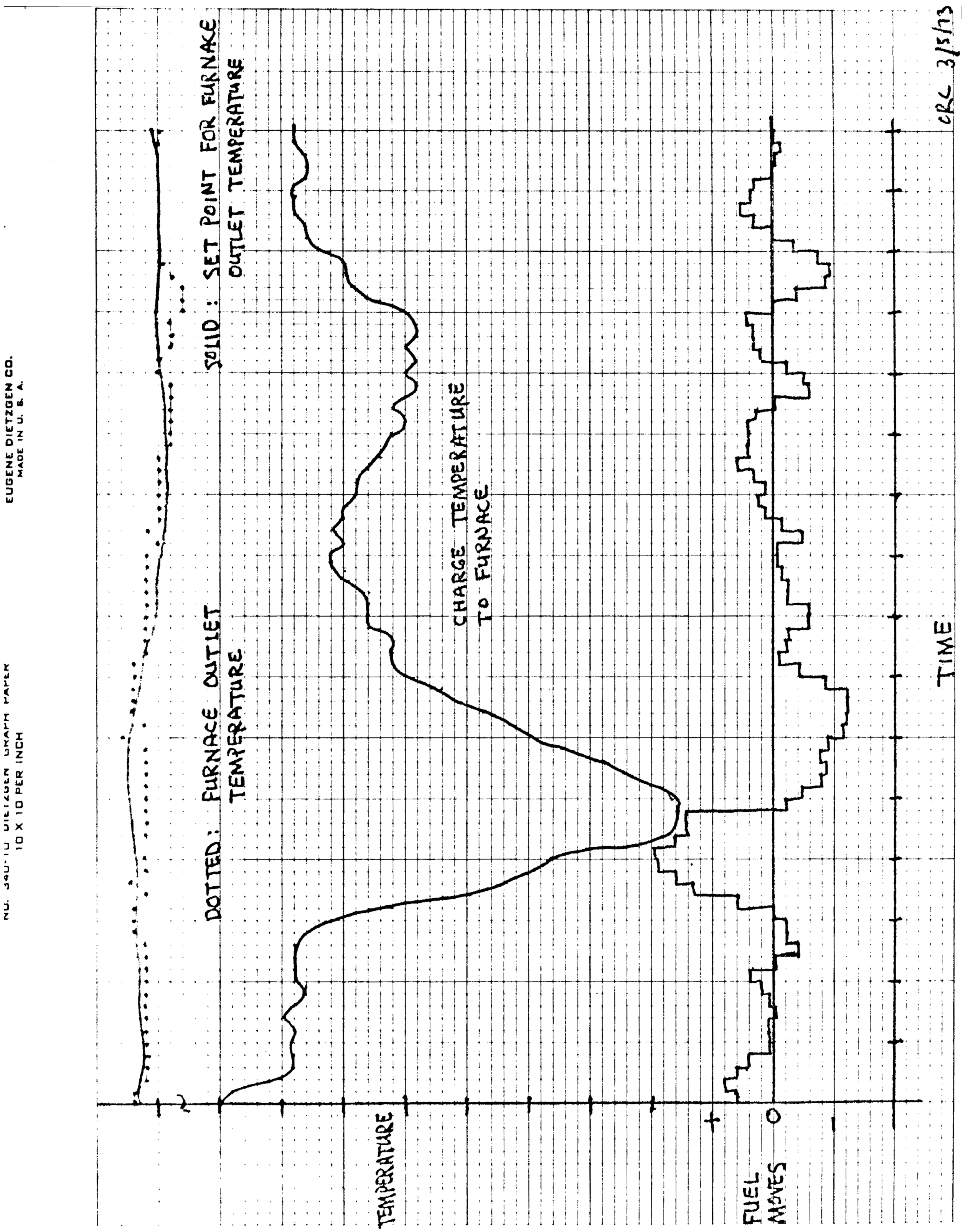
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The 1969 thesis draft

I have “cleaned up” the original document which was of rather poor quality. There are a couple of places where I’m still unsure and here I put in italics. It is interesting to note that final resulting DMC modeling approach (using a discrete step response) and control algorithm using linear programming is well developed. He also puts a lot of emphasis on manipulated variable constraints, and he goes a long way in describing the steady-state LP solution which still to this day is used in commercial software to find a feasible steady-state. The approach (which is not clearly mentioned in the PhD proposal) is to order the setpoints in priority order and give up less important setpoints until a feasible steady-state solution is found. Interestingly, this powerful two-step approach (steady-state feasibility followed by dynamic optimization) is still today (2025) largely unknown in the academic community.

—Sigurd Skogestad
Professor, Norwegian University of Science and Technology

August 24, 1969

Dr. S.J. Huang
Chemical Engineering Department
University of Houston
Houston, Texas

Dear Dr. Huang:

I have outlined in the following pages my proposal for a multivariable control algorithm to fulfil my dissertation requirement. I believe this is an original approach in a number of ways that will become evident as the algorithm is developed. I would like to apply it to the control of complex distillation column that has several reflux systems and draw streams. Preferably this should be an actual column, but if I cannot get Shell Oil to let me try it on an actual column, then I would use a computer simulation of a column to test the control algorithm. The algorithm was formulated to satisfy the following criterion:

1. The manipulated variables should be moved on the basis of a profit criterion, rather than

on some type of minimum error criterion that most optimal control systems are based.

2. The algorithm should recognize the ability to use the manipulated variables at future times. Prior knowledge of the overall system dynamics should permit the calculation of the trajectory of the manipulated variables as well as the controlled variables.

3. The algorithm should include feedback to compensate for prediction errors in the dynamics and for disturbances that enter the system.

4. The control algorithm should recognize that control variables are many times set at their maximum or minimum values by an overall process optimization. This means the set point for the control variable must be approached asymptotically and must not be exceeded.

5. The algorithm should recognize that the manipulated variable has a limited range over which to operate, before the valve, compressor, etc., reaches saturation.

6. The algorithm should be capable of controlling D^n variables with I^k manipulated variables where $n \leq k$.

The explanation of the proposed algorithm is proposed in the following manner. First, I will demonstrate how a set of linear algebraic equations can be used to describe the dynamic response of more than one dependent variables I_k. Then I will show how the system can be expanded to describe the dynamic response of more than one dependent variable D_n with the same independent variable I_k.

By changing from absolute values to differences for the independent and dependent variables, the dependent variables may be described in terms of an error relative a set *point*.

At this point in the development of the algorithm, I will have demonstrated how to calculate the change in error in time for the n dependent control variables with a change in the k independent manipulated variables. The complete response will be calculated by solving a system of linear algebraic equations. The next degree of sophistication will demonstrate how previous calculated moves in the independent variables and the resulting transient condition may be incorporated into current calculations. Also the feedback corrections for the errors predicted by dynamic response predictions and disturbances will be added at this point in the algorithm development. I shall discuss and show how a least square solution of the equations developed would lead to a workable multivariable control system. This will be helpful in visualizing the subsequent steps in the algorithm development. The nest step is to show how the linear programming algorithm may be used to solve the dynamic equations and converge. This last development will be to show how steady state calculations of the process economics can be incorporated in the dynamic control equations to obtain the most profitable set of *control moves*.

Description of System Dynamics By A Set of *Linear Mathematic* Equations

The following analysis is based on the assumption that the process dynamics van be described by a set of linear differential equations. This is not a serious limitation since the nonlinear systems may be linearized in the region of interest. For a linear system the magnitude of the scale *factor* is preserved. This is demonstrated in Figure 1 where the response of the dependent variable D is shown for a unit change in independent variable I.

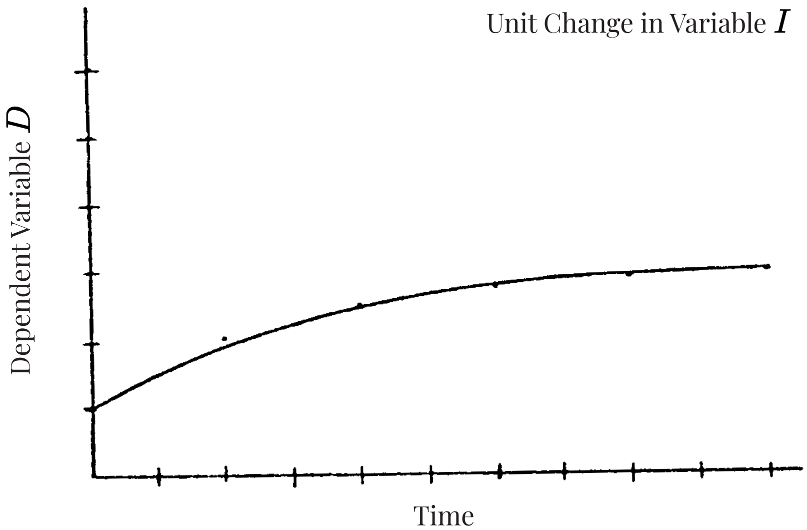

Figure 1

If a two unit change is made in independent variable I, then the same response is obtained from D except that the change in amplitude is multiplied by 2. This is demonstrated in Figure 2.

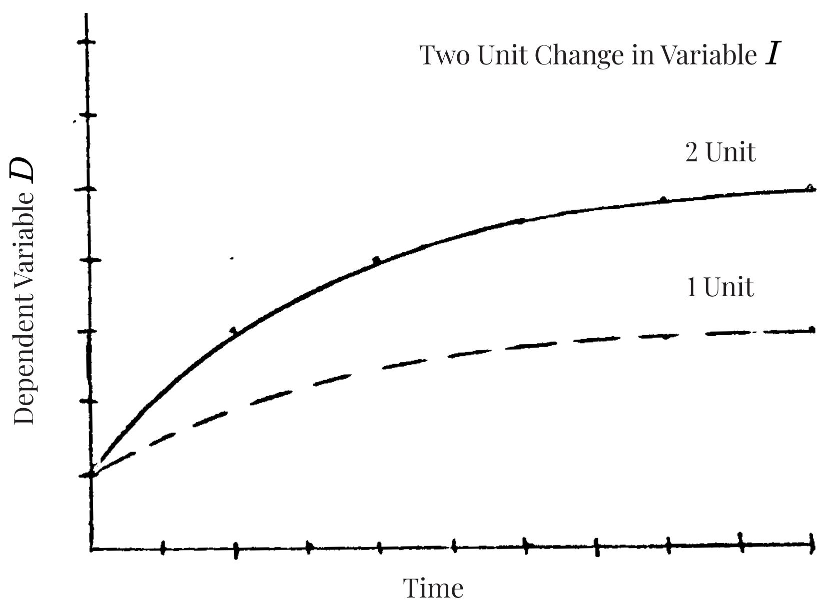

Figure 2

For a linear system the principle of superposition applies and this is demonstrated in Figure 3.

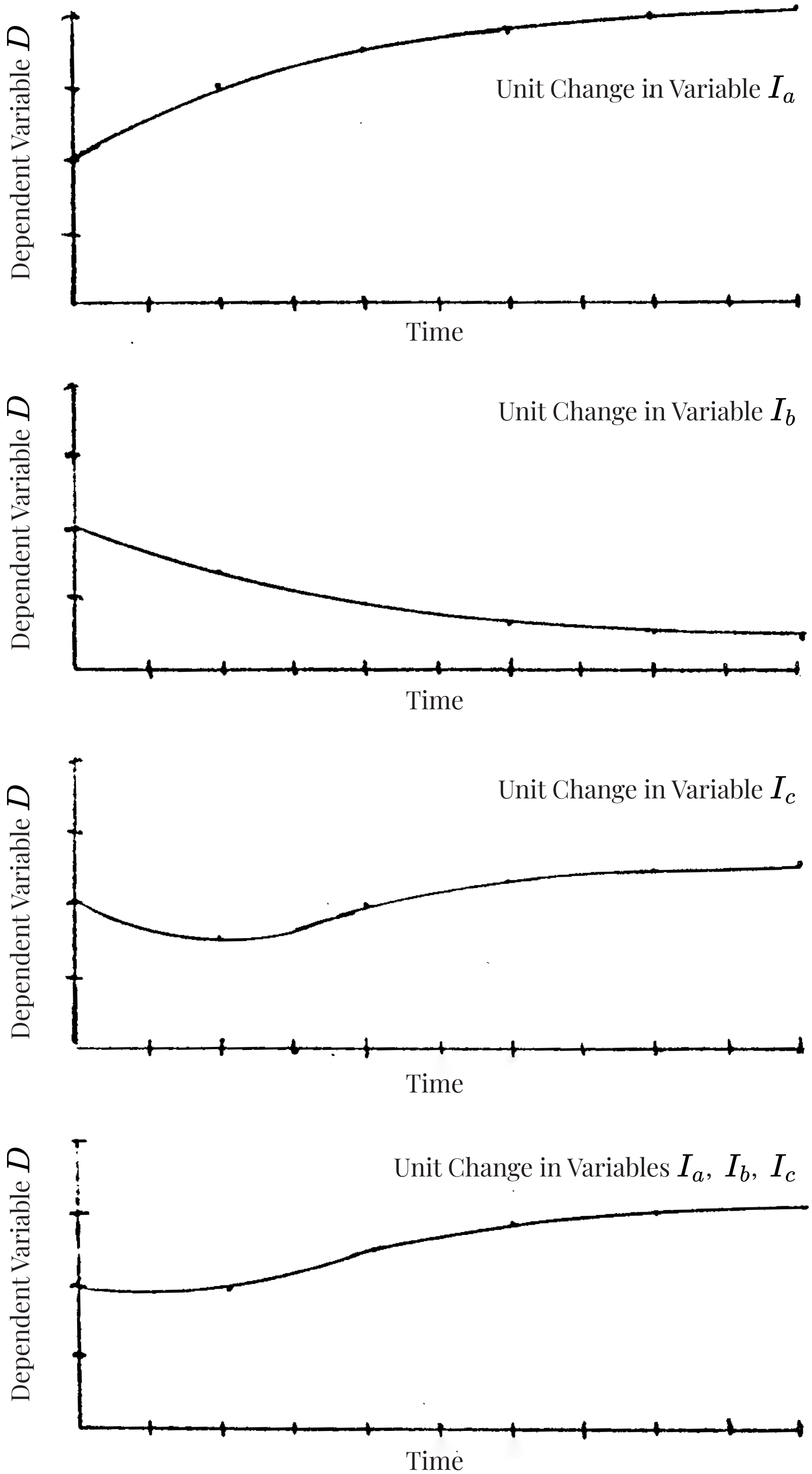

Figure 3

Let the numerical subscript on the dependent variable D represent the value of D at some discrete time interval after the change in the independent variables. The change in value of D at any time interval may be determined by a set of linear algebraic equations if the change in the independent variables are known.

$$D_1 = A_{11}I_a + A_{12}I_b + A_{13}I_c$$

$$D_2 = A_{21}I_a + A_{22}I_b + A_{23}I_c$$

$$D_3 = A_{31}I_a + A_{32}I_b + A_{33}I_c$$

$$D_4 = A_{41}I_a + A_{42}I_b + A_{43}I_c$$

$$D_5 = A_{51}I_a + A_{52}I_b + A_{53}I_c$$

$$D_6 = A_{61}I_a + A_{62}I_b + A_{63}I_c$$

$$\ldots = \ldots + \ldots + \ldots$$

Equation Set 1

where D_1, D_2, D_3, etc. are for the time intervals 1, 2, 3 etc. , respectively and represent the change in D that results from the cgange in I_a, I_b, and I_c. The coefficients A_{ij} describe the system dynamics and may be determined experimentally by a number of published techniques. The relationship between this approach to describing system dynamics and finite difference differential equations is obvious. The coefficients A_{ij} in the equation are nothing more than the solution of the differential difference equations. It is a simple matter to extend this calculation procedure to more than one dependent variable. For example two dependent variables D_a and D_b.

$$D_{a1} = A_{11}I_a + A_{12}I_b + A_{13}I_c$$

$$D_{a2} = A_{21}I_a + A_{22}I_b + A_{23}I_c$$

$$D_{a3} = A_{31}I_a + A_{32}I_b + A_{33}I_c$$

$$\ldots = \ldots + \ldots + \ldots$$

$$D_{b1} = B_{11}I_a + B_{12}I_b + B_{13}I_c$$

$$D_{b2} = B_{21}I_a + B_{22}I_b + B_{23}I_c$$

$$D_{b3} = B_{31}I_a + B_{32}I_b + B_{33}I_c$$

$$\ldots = \ldots + \ldots + \ldots$$

Equation Set 2

The Change In A Dependent Variable Can Be Expressed As A Change In The Error And A Control Scheme Developed Using The Exact Sample Method To Solve For The Current Value.

The change in the dependent variable D_a and D_b can be expressed as a change in the error when the set point for each variable is known:

$$D_{a1} = D_a(\text{At Time} = 1) - D_A(\text{At Time} = 0)$$

$$D_{a1} = (D_a \text{ Set Point} - D_a \text{ at time} = 0) - (D_a \text{ Set Point} - D_a \text{ at time} = 1)$$

$$D_{a1} = (\text{Error at time} = 0) - (\text{Error at time} = 1) = D_{a1}$$

Equation Set 3

For illustration, assume that a steady state error exists in one of the dependent variables D_a. It is also assumed that the most desirable path for the variable to return to the setpoint is known. This is illustrated in Figure 4.

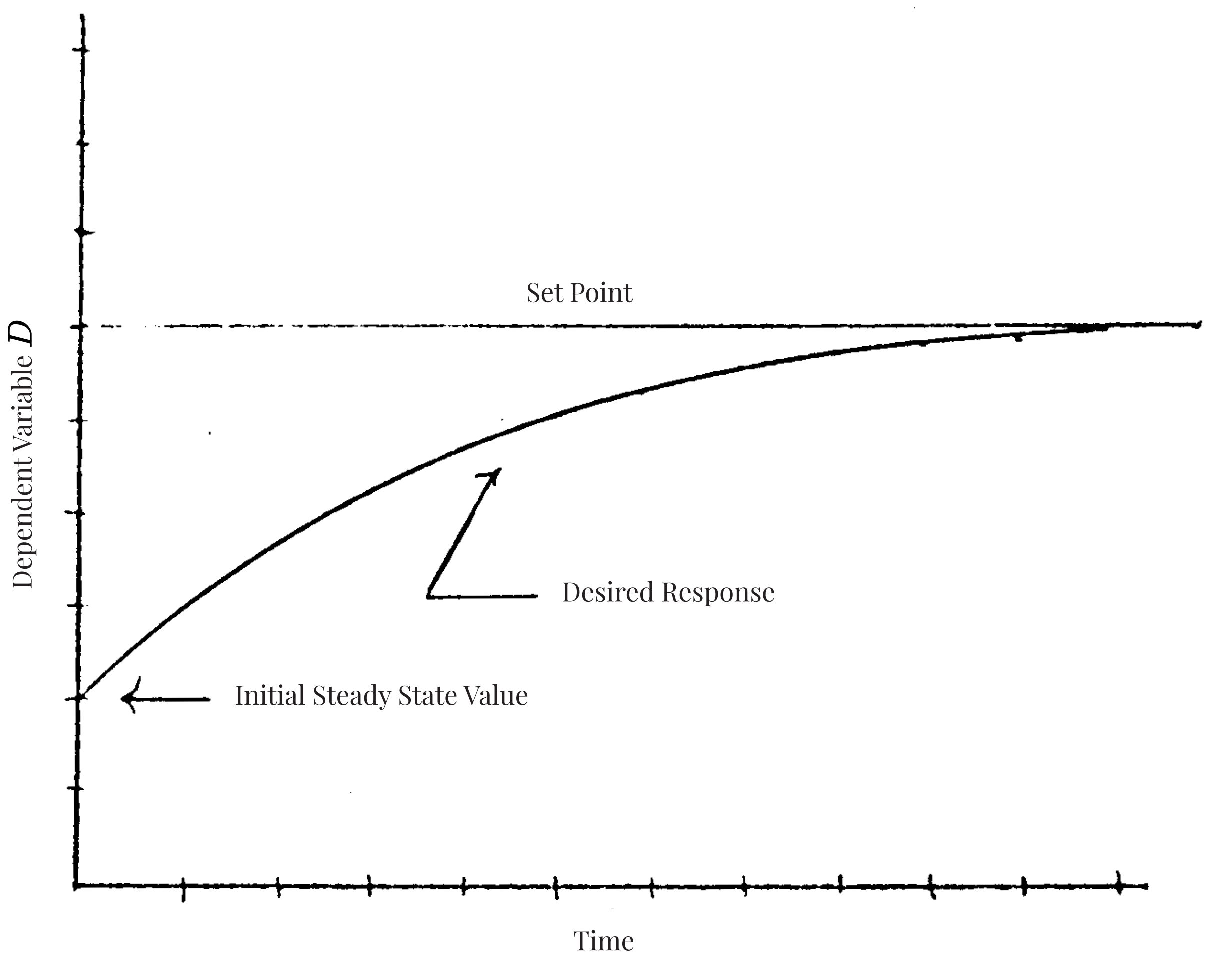

Figure 4

Since the desired change in D_a is now known and the coefficients A_{ij} in the dynamic response are known, the best set of moves in the independent variables to accomplish this control response can be determined by the method of least squares. The concept is easily expanded to multivariable control since the change in independent variables determined may influence a number of dependent control variables.

$$D_{a1} = A_{11}I_a + A_{12}I_b + \cdots\ A_{1j}I_k$$

$$D_{a2} = A_{21}I_a + A_{22}I_b + \cdots\ A_{2j}I_k$$

$$D_{a3} = A_{31}I_a + A_{32}I_b + \cdots\ A_{3j}I_k$$

$$D_{a4} = A_{41}I_a + A_{42}I_b + \cdots\ A_{4j}I_k$$

$$\ldots \qquad \ldots \qquad \ldots \qquad \ldots.$$

$$D_{ai} = A_{j1}I_a + A_{j2}I_b + \cdots\ A_{ij}I_k$$

$$D_{b1} = B_{11}I_a + B_{12}I_b + \cdots\ B_{1j}I_k$$

$$D_{b2} = B_{21}I_a + B_{22}I_b + \cdots\ B_{2j}I_k$$

$$D_{b3} = B_{31}I_a + B_{32}I_b + \cdots\ B_{3j}I_k$$

$$D_{b4} = B_{41}I_a + B_{42}I_b + \cdots\ B_{4j}I_k$$

$$\ldots \qquad \ldots \qquad \ldots \qquad \ldots.$$

$$D_{bi} = B_{j1}I_a + B_{j2}I_b + \cdots\ B_{ij}I_k$$

$$D_{c1} = C_{11}I_a + C_{12}I_b + \cdots\ C_{1j}I_k$$

$$D_{c2} = C_{21}I_a + C_{22}I_b + \cdots\ C_{2j}I_k$$

$$D_{c3} = C_{31}I_a + C_{32}I_b + \cdots\ C_{3j}I_k$$

$$D_{c4} = C_{41}I_a + C_{42}I_b + \cdots\ C_{4j}I_k$$

$$\ldots \qquad \ldots \qquad \ldots \qquad \ldots.$$

$$D_{ci} = C_{j1}I_a + C_{j2}I_b + \cdots\ C_{ij}I_k$$

$$where\ j \geq i$$

Equation Set 4

The number of independent variables fix the size of the matrix to be inverted, therefore any degree of accuracy can be achieved in describing the dynamic response by reducing the length of the time interval.

Time Dependent Moves in the Independent Variables May Be Incorporated Into the Control Algorithm

An additional improvement can be achieved by permitting the independent variables to move in each time interval in the future. For example $I_{\mathrm{a}0}$, $I_{\mathrm{a}1}$, $I_{\mathrm{a}2}$, etc. are moves in the independent variable I_a at times 0, 1, 2 respectively.

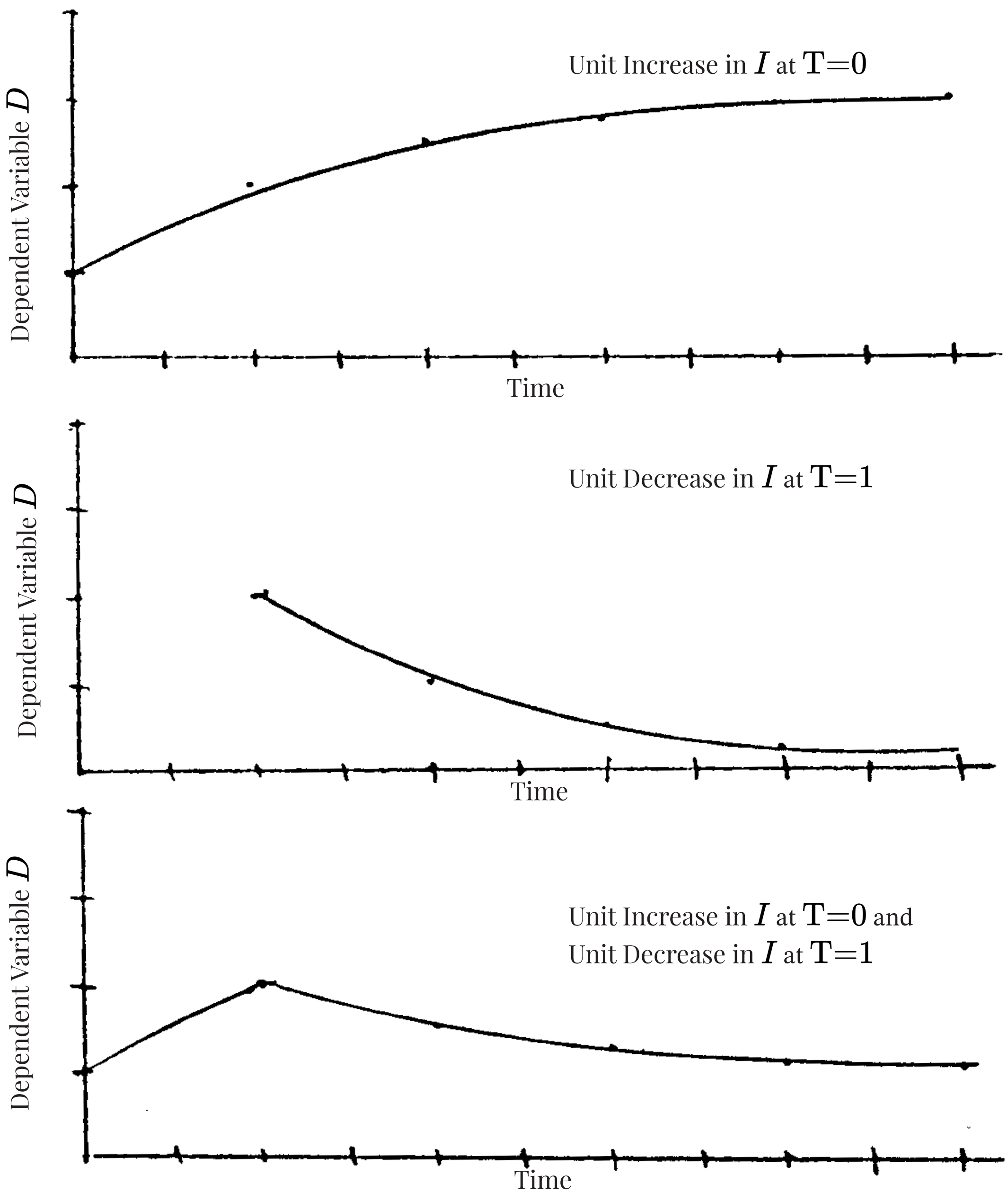

Figure 5

The response of the dependent variable D_a to changes in one variable I_a is shown in Figure 5. The changes in I_a are made at time 0 and time 1 and represent a unit increase and decrease, respectively, The addition of this concept to the algorithm adds another dimension to the control since it allows the "planning" of future moves of the manipulated variables. There isn't any additional dynamic information to extend the algorithm to time dependent moves in the independent variable since the dynamic matrix repeats itself. This is illustrated below for 2 independent variables.

$$D_{a1} = A_{11}I_{a0} + A_{12}I_{b0} + \cancel{A_{11}I_{a1}}^{0} + \cancel{A_{12}I_{b1}}^{0} + \cdots$$

$$D_{a2} = A_{21}I_{a0} + A_{22}I_{b0} + A_{\mathbf{11}}I_{a1} + A_{\mathbf{12}}I_{b1} + \cdots$$

$$D_{a3} = A_{31}I_{a0} + A_{32}I_{b0} + A_{\mathbf{21}}I_{a1} + A_{\mathbf{22}}I_{b1} + \cdots$$

$$D_{a4} = A_{41}I_{a0} + A_{42}I_{b0} + A_{\mathbf{31}}I_{a1} + A_{\mathbf{32}}I_{b1} + \cdots$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots.$$

Equation Set 5

The concept of time varying independent variables is easily extended to more than one dependent variable by analogy to equation set 2 where only one move in each of the independent variable at time zero was shown. For clarification this situation is illustrated below:

$$D_{a1} = A_{11}I_{a0} + A_{12}I_{b0} + \cancel{A_{11}I_{a1}}^{0} + \cancel{A_{12}I_{b1}}^{0} + \cdots$$

$$D_{a2} = A_{21}I_{a0} + A_{22}I_{b0} + A_{\mathbf{11}}I_{a1} + A_{\mathbf{12}}I_{b1} + \cdots$$

$$D_{a3} = A_{31}I_{a0} + A_{32}I_{b0} + A_{\mathbf{21}}I_{a1} + A_{\mathbf{22}}I_{b1} + \cdots$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots.$$

$$D_{b1} = B_{11}I_{a0} + B_{12}I_{b0} + \cancel{B_{11}I_{a1}}^{0} + \cancel{B_{12}I_{b1}}^{0} + \cdots$$

$$D_{b2} = B_{21}I_{a0} + B_{22}I_{b0} + B_{\mathbf{11}}I_{a1} + B_{\mathbf{12}}I_{b1} + \cdots$$

$$D_{b3} = B_{31}I_{a0} + B_{32}I_{b0} + B_{\mathbf{21}}I_{a1} + B_{\mathbf{22}}I_{b1} + \cdots$$

Equation Set 6

Correction For Errors in the Dynamic Model and Disturbances To the System

The expression in the algorithm for errors in the dynamic predictions and disturbances would be made comparing the predicted error against the actual error. If the actual error is less than the predicted error then there would not be any corrections made in the independent variables. If on the other hand the actual error is greater that the predicted error the discrepancy between the errors would be corrected by moving the independent variables. The use of the least square method of solution in conjunction with the specification of a desired system response represents a practical control algorithm for a multivariable control system. The algorithm would satisfy four of the six desired criterion listed on the first two pages of this letter. The profit criterion is not covered directly with this type of approach; however if the set points were the result of an optimization, this would not be a serious limitation.

The method does not provide a technique for recognizing the physical limitations of the independent (manipulated) variables. Although this problem can probably be circumvented by an iterative process, there appears to be a simpler alternative that will be considered next.

Use Of the Linear Programming Algorithm To Solve the Dynamic Equations

The sum of the moves in each of the independent variable determines the final steady state position of the process. The most desirable steady state is the one which is optimum from the standpoint of the profit functions. On this basis the best set of control moves is the set that culminates in an optimum steady state operation. The linear programming algorithm provides the basis for finding the steady state optimum and also provides a means of constraining the response of the process being optimized when the dynamics are described in the manner presented in the previous paragraph. The beauty of this approach is that by adjusting the allowable error at such interval of time, the convergence of system can be guaranteed. In addition, the controlled variable can be allowed to converge in a cyclic manner or in an over-damped manner depending on how the limits are set at each time interval. This characteristic permits the control of a system adjacent to the constraints of the process, which is essential if the maximum benefits are to be derived from the control. This principle is illustrated in the next figure.

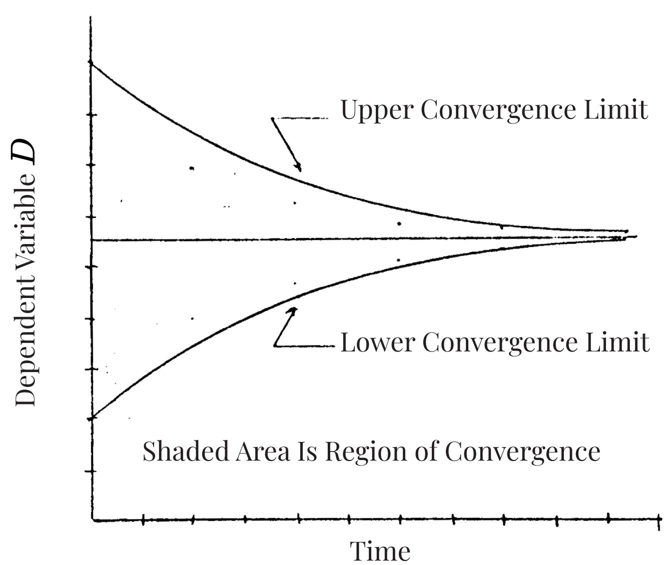

Figure 6

An optimization by definition forces a process to some set of constraints, therefore an algorithm that has an economic motivation must be capable of approaching while not exceeding a constraint on control variable.

The elements in the profit row of the linear programming would be derived from a steady state model of the process as reported by a number of writers in the literature.

The control algorithm proposed in this letter is the result of a number of years of studying and searching for a practical method of controlling multivariable systems. The criterion listed for the control algorithm are not just desirable but necessary if a computer is to successfully control a chemical process **at an optimum condition**. The conventional control theory in text books fails miserable short of these criterion, therefore I believe this proposal of mine is worth pursuing. To the best of my knowledge the approach has not been proposed or attempted before and although my experience with process control at Shell cannot be disassociated from my technical background, I can say without hesitation that it does not resemble any technique now being used by Shell or under development by Shell.

Your comments, even if you do not agree with my points would be appreciated.

Respectfully yours,

Charles R. Cutler

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The original documents follow.

August 24, 1969

Dr. C. J. Huang
Chemical Engineering Department
University of Houston
Houston, Texas

Dear Dr. Huang:

I have outlined in the following pages my proposal for a multivariable control algorithm to fulfil my dissertation requirement. I believe this is an original approach in a number of ways that will become evident as the algorithm is developed. I would like to apply it to the control of a complex distillation column that has several reflux systems and draw streams. Preferably this would be an actual column, but if I cannot get Shell Oil to let me try it on an actual column, then I would use a computer simulation of a column to test the control algorithm.

The algorithm was formulated to satisfy the following criterion:

1. The manipulated variables should be moved on the basis of a profit criterion, rather than some type of minimum error criterion that most optimal control systems are based.

2. The algorithm should recognize its ability to move the manipulated variables at future times. Prior knowledge of the overall system dynamics should

permit the calculation of the trajectory of the manipulated variable as well as the controlled variable.

3. The algorithm should include feedback to compensate for prediction errors in the dynamics and for disturbances that enter the system.
4. The control algorithm should recognize that control variables are many times set at their maximum or minimum values by an overall process optimization. This means the set point for the control variable must be approached asymptotically and must not be exceeded.
5. The algorithm should recognize that the manipulated variable has a limited range over which to operate before the valve, compressor, etc., reaches saturation.
6. The algorithm should be capable of controlling D^n variables with I^k manipulated variables where $n \neq k$.

The explanation of the proposed algorithm is presented in the following manner. First, I will demonstrate how a set of linear algerbraic equations can be used to describe the dynamic response of a dependent variable D to the change in a number of independent variables I_k. Then I will show how the system can be expanded to describe the dynamic response of more than one dependent variable D_n with the same independent variables I_k.

By changing from absolute values to differences for the independent and the dependent variables, the dependent variable may be described in terms of an error relative to a set point.

At this point in the development of the algorithm, I will have demonstrated how to calculate the change in error in time for n dependent control variables with a change in K independent manipulated variables. The complete response will be calculated by solving a system of linear algerbraic equations. The next degree of sophistication will demonstrate how previous calculated moves in the independent variables and the resulting transiant condition may be incorporated into current calculations. Also the feedback corrections for the errors generated by dynamic responses predictions and disturbances will be added at this point in the algorithm development. I shall discuss and show how a least square solution of the equations developed would lead to a workable multivariable control system. This will be helpful in visualizing the subsequent steps in the algorithm development. The next step is to show how the linear programming algorithm may be used to solve the dynamic equations and guarantee stability and convergence. The last step in development will be to show how steady state calculations on the process economics can be incorporated with dynamic control equations to obtain the most profitable set of control [illegible].

<u>Description of System Dynamics By A Set Of Linear Algebraic Equations</u>

The following analysis is based on the assumption that the process dynamics can be described by a set of linear differential equations. This is not a serious limitation since the non-linear systems may be linearized in the region of interest. For a linear system the magnitude of the scale factor is proserved. This principle is demonstrated in Figure 1 where the response of the dependent variable D is shown for a unit change in independent variable I.

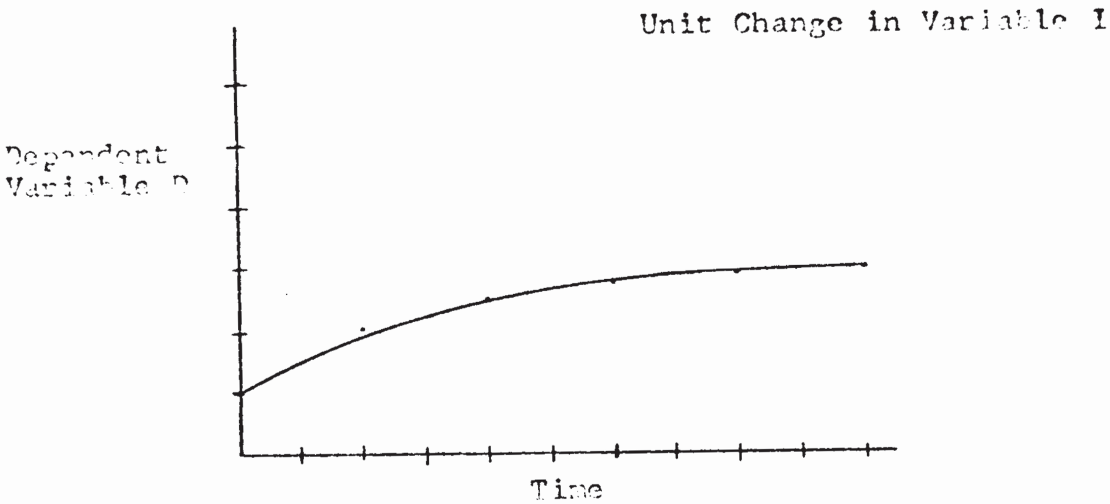
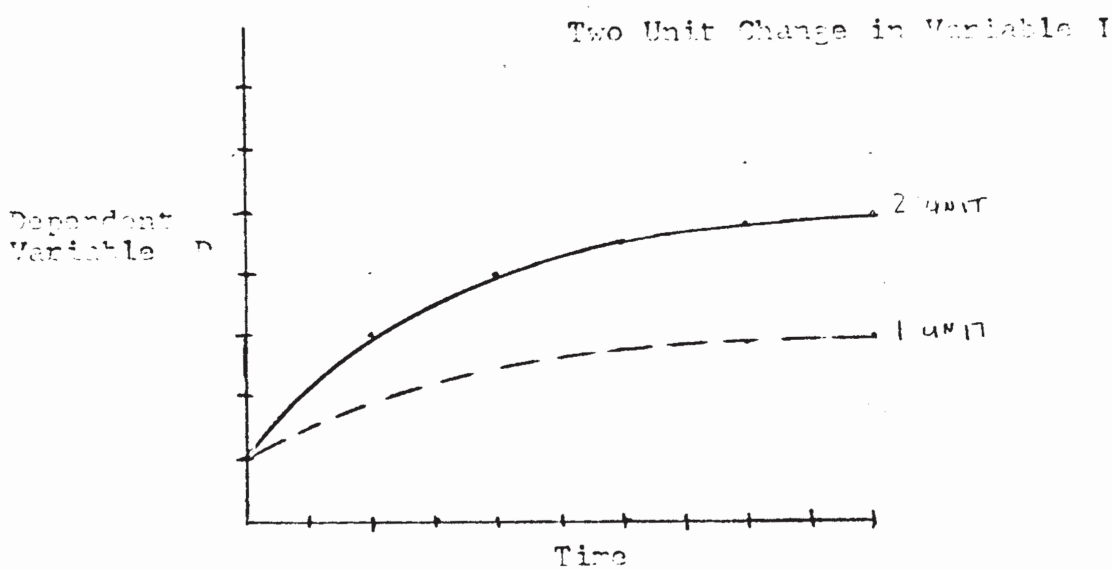


Figure 1

If a two unit change is made in independent variable I, then the same response is obtained from D except that the change in amplitude is multiplied by 2. This is demonstrated in figure 2.



For a linear system the principle of superposition applies and this is demonstrated in Figure 3.

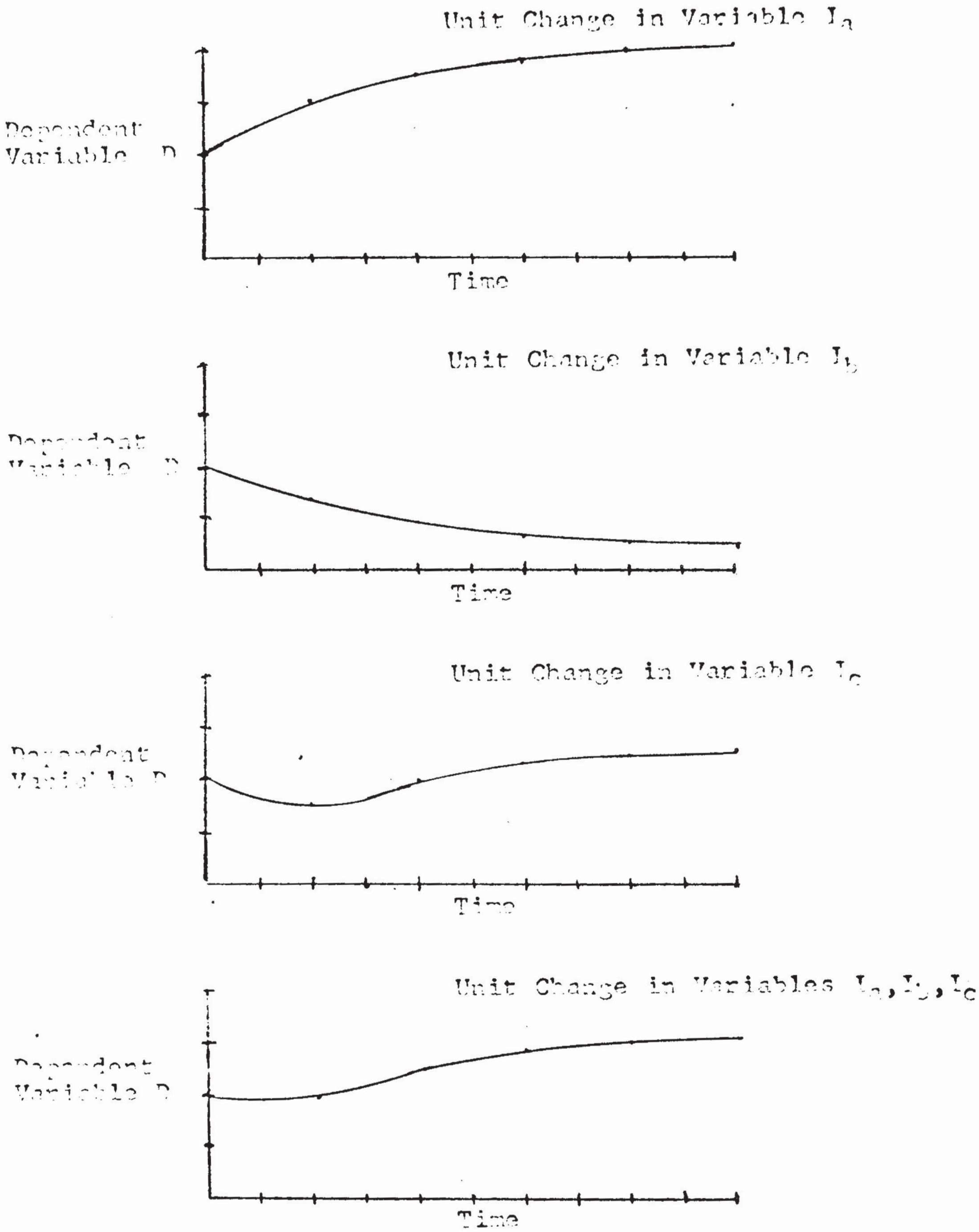


Figure 3

Let the numerical subscription on the dependent variable D represent the value of D at some discrete time interval after the change in the independent variables. The change in value of D at any time interval may be determined by a set of linear algerbraic equations if the change in the independent variables are known.

$$
\begin{aligned}
D_1 &= A_{11} I_a + A_{12} I_b + A_{13} I_c \\
D_2 &= A_{21} I_a + A_{22} I_b + A_{23} I_c \\
D_3 &= A_{31} I_a + A_{32} I_b + A_{33} I_c \\
D_4 &= A_{41} I_a + A_{42} I_b + A_{43} I_c \\
D_5 &= A_{51} I_a + A_{52} I_b + A_{53} I_c \\
D_6 &= A_{61} I_a + A_{62} I_b + A_{63} I_c \\
\vdots & \quad \vdots \qquad\quad \vdots \qquad\quad \vdots
\end{aligned}
$$

Equation Set 1

where D_1, D_2, D_3, etc. are for time intervals 1,2,3, etc. respectively and represent the change in D that results from the change in I_a, I_b, and I_c. The coefficients Aij describe the system dynamics and may be determined experimentally by a number of published techniques. The relationship between this approach to describing systems dynamics and finite difference differential equations is obvious. The coefficients Aij in the matrix are nothing more than the solution of the differential difference equations. It is a simple matter to extend this calculational procedure to more than one dependent variable. For example two dependent variables D_a and D_b.

$$
\begin{array}{lcl}
D_{a1} & = & A_{11}I_a + A_{12}I_b + A_{13}I_c \\
D_{a2} & = & A_{21}I_a + A_{22}I_b + A_{23}I_c \\
D_{a3} & = & A_{31}I_a + A_{32}I_b + A_{33}I_c \\
\vdots & & \vdots \qquad\quad \vdots \qquad\quad \vdots \\
D_{b1} & = & B_{11}I_a + B_{12}I_b + B_{13}I_c \\
D_{b2} & = & B_{21}I_a + B_{22}I_b + B_{23}I_c \\
D_{b3} & = & B_{31}I_a + B_{32}I_b + B_{33}I_c \\
\vdots & & \vdots \qquad\quad \vdots \qquad\quad \vdots
\end{array}
$$

Equation Set 2

The Change In A Dependent Variable Can Be Expressed As A Change In The Error And A Control Scheme Developed Using The Least Square Method To Solve For The Control Moves

The change in the dependent variable D_a and D_b can be expressed as a change in the error when the set point for each variable is known.

$$D_{a1} = D_a(\text{At Time} = 1) - D_a(\text{At Time} = 0)$$

$$D_{a1} = (D_a \text{ Set Point} - D_a \text{ at time} = 0) - (D_a \text{ Set Point} - D_a \text{ At Time} = 1)$$

$$D_{a1} = \text{Error At Time} = 0 - \text{Error At Time} = 1 = E_{a1}$$

Equation Set 3

For illustration, assume that a steady state error exists in one of the dependent variables D_a. It is also assumed that the most desirable path for the variable to return to the set point is known. This is illustrated in Figure 4.

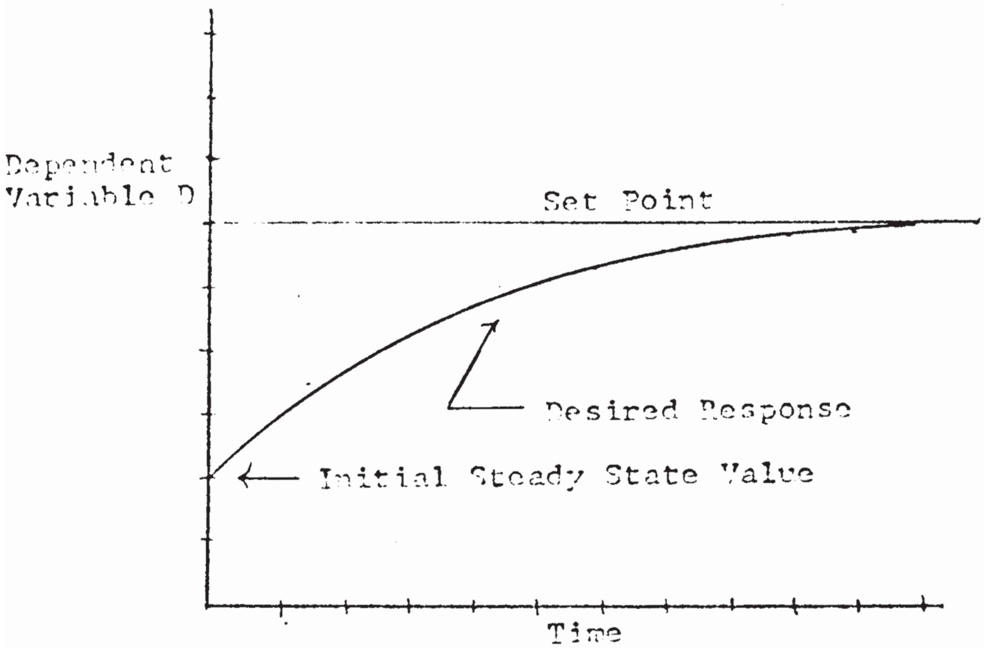


Figure 4

Since the desired change in D_a is now known and the coefficients Aij in the dynamic response matrix are known, the best set of moves in the independent variables to accomplish this control reponse can be determined by the method of least squares. The concept is easily extended to multivariable control since the change in independent variables determined may influence a number of dependent control variables.

$$
\begin{array}{lllllll}
D_{a1} & = & A_{11} I_a & + & A_{12} I_b & + & --- & A_{ij} I_k \\
D_{a2} & = & A_{21} I_a & + & A_{22} I_b & + & --- & A_{2j} I_k \\
D_{a3} & = & A_{31} I_a & + & A_{32} I_b & + & --- & A_{3j} I_k \\
D_{a4} & = & A_{41} I_a & + & A_{42} I_b & + & --- & A_{4j} I_k \\
\vdots & & \vdots & & \vdots & & & \vdots \\
D_{ai} & = & A_{i1} I_a & + & A_{i2} I_b & + & --- & A_{ij} I_k \\
D_{b1} & = & B_{11} I_a & + & B_{12} I_b & + & --- & B_{1j} I_k \\
D_{b2} & = & B_{21} I_a & + & B_{22} I_b & + & --- & B_{2j} I_k \\
D_{b3} & = & B_{31} I_a & + & B_{32} I_b & + & --- & B_{3j} I_k \\
D_{b4} & = & B_{41} I_a & + & B_{42} I_b & + & --- & B_{4j} I_k \\
\vdots & & \vdots & & \vdots & & & \vdots \\
D_{bi} & = & B_{i1} I_a & + & B_{i2} I_b & + & --- & B_{ij} I_k \\
D_{c1} & = & C_{11} I_a & + & C_{12} I_b & + & --- & C_{1j} I_k \\
D_{c2} & = & C_{21} I_a & + & C_{22} I_b & + & --- & C_{2j} I_k \\
D_{c3} & = & C_{31} I_a & + & C_{32} I_b & + & --- & C_{3j} I_k \\
D_{c4} & = & C_{41} I_a & + & C_{42} I_b & + & --- & C_{4j} I_k \\
\vdots & & \vdots & & \vdots & & & \vdots \\
D_{ci} & = & C_{i1} I_a & + & C_{i2} I_b & + & --- & C_{ij} I_k
\end{array}
$$

where $j \geq i$

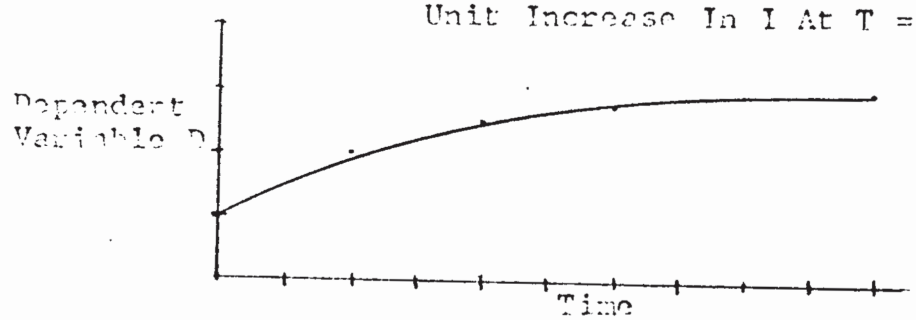
Equation Set 4

The number of of independent variables fix the size of the matrix to be inverted, therefore any degree of accuracy can be achieved in discribing the dynamic response by reducing the length of the time interval.

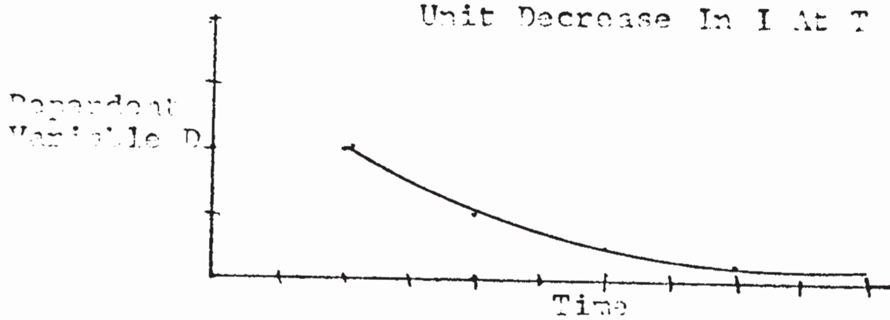
__Time Dependent Moves In The Independent Variables May Be Incorporated Into The Control Algorithm__

An additional improvement can be achieved by permitting the independent variable to move in each time interval in the future. For example I_{ao}, I_{a1}, I_{a2}, etc. are moves in the independable variable I_a at times 0,1,2 respectively.

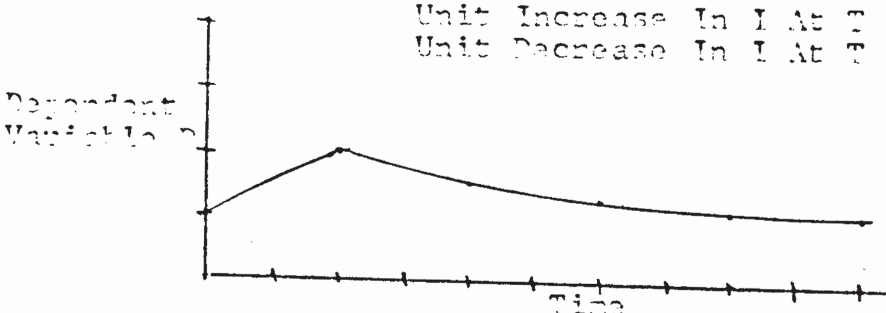
Unit Increase In I At T = 0



Unit Decrease In I At T = 1



Unit Increase In I At T = 0 And
Unit Decrease In I At T = 1



The response of the dependent variable D_a to changes in one variable I_a is shown in Figure 5. The changes in I_a are made at time 0 and time 1 and represent a unit increase and decrease respectively. The addition of this concept to the algorithm adds another dimension to the control since it allows the "planning" of future moves of the manipulated variable. There isn't any additional dynamic information required to extend the algorithm to time dependent moves in the independent variable since the dynamic matrix repeats itself. This is illustrated below for 2 independent variables.

$$
\begin{array}{lllllllllll}
D_{a1} & = & A_{11} I_{ao} & + & A_{12} I_{bo} & + & A_{01} I_{a1} & + & A_{02} I_{b1} & + & --- \\
D_{a2} & = & A_{21} I_{ao} & + & A_{22} I_{bo} & + & A_{11} I_{a1} & + & A_{12} I_{b2} & + & --- \\
D_{a3} & = & A_{31} I_{ao} & + & A_{32} I_{bo} & + & A_{21} I_{a1} & + & A_{22} I_{b1} & + & --- \\
D_{a4} & = & A_{41} I_{ao} & + & A_{42} I_{bo} & + & A_{31} I_{a1} & + & A_{32} I_{b1} & & --- \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots & &
\end{array}
$$

<u>Equation Set 5</u>

The concept of time varying independent variables is easily extended to more than one dependent variable by analogy to equations Set 2 where only one move in each of the independent variables at time zero was shown. For clarafication this situation is illustrated below:

(Illustration is listed on next page (page 12)

$$
\begin{array}{lllllllll}
D_{a1} & = & A_{11} I_{ao} & + & A_{12} I_{bo} & + & \overset{0}{\cancel{A_{11}}} I_{a1} & + & \overset{0}{\cancel{A_{12}}} I_{b1} \\
D_{a2} & = & A_{21} I_{ao} & + & A_{22} I_{bo} & + & A_{11} I_{a1} & + & A_{12} I_{b1} \\
D_{a2} & = & A_{31} I_{ao} & + & A_{31} I_{bo} & + & A_{21} I_{a1} & + & A_{22} I_{b1} \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots \\
D_{b1} & = & B_{11} I_{ao} & + & B_{12} I_{bo} & + & \overset{0}{\cancel{B_{11}}} I_{a1} & + & \overset{0}{\cancel{B_{12}}} I_{b1} \\
D_{b2} & = & B_{21} I_{ao} & + & B_{22} I_{bo} & + & B_{11} I_{a1} & + & B_{12} I_{b1} \\
D_{b3} & = & B_{31} I_{ao} & + & B_{32} I_{bo} & + & B_{21} I_{a1} & + & B_{22} I_{b1} \\
\vdots & & \vdots & & \vdots & & \vdots & & \vdots
\end{array}
$$

Equation Set 6

Correction For Errors In The Dynamic Model And Disturbances To The System

The correction in the algorithm for errors in the dynamic predictions and disturbances would be made by comparing the predicted error against the actual error. If the actual error is less than the predicted error then there would not be any corrections made in the independent variables. If on the other hand the actual error is greater than the predicted error, then the descrepancy between the errors would be corrected by moving the independent variables. The use of the least square method of solution in conjuction with the specification of a desired system response represents a practical control algorithm for a multivariable control system. The algorithm would satisfy four of the six desired criterion listed on the first two pages of this letter. The profit criterion is not covered directly with this type of appproach; however if the set points were the result of an optimization, this would not be a serious limitation.

The method does not provide a technique for recognizing the physical limitations on the independent (manipulated) variables. Although this problem can probably be circumvented by an iteration process, there appears to be a simplier alternative that will be considered next.

Use Of The Linear Programming Algorithm To Solve The Dynamic Equations

The sum of the moves in each of the independent variable determines the final steady state position of the process. The most desirable steady state is the one which is optimum from the standpoint of a profit function. On this basis the best set of control moves is the set that culminates in an optimum steady state operation. The linear programming algorithm provides the basis for finding the steady state optimum and also provides a means of constraining the response of the process being optimized when the dynamics are described in the manner presented in the preceding paragraphs. The beauty of this approach is that by adjusting the allowable error at each interval of time the convergence of the system can be guaranteed. In addition the controlled variable can be allowed to converge in a cyclic manner or in an over damped manner depending on how the limits are set at each time interval. This characteristic permits the control of a system adjacent to the constraints of the process, which is essential if the maximum benefits are to be derived from the control. This principal is illustrated in the next figure.

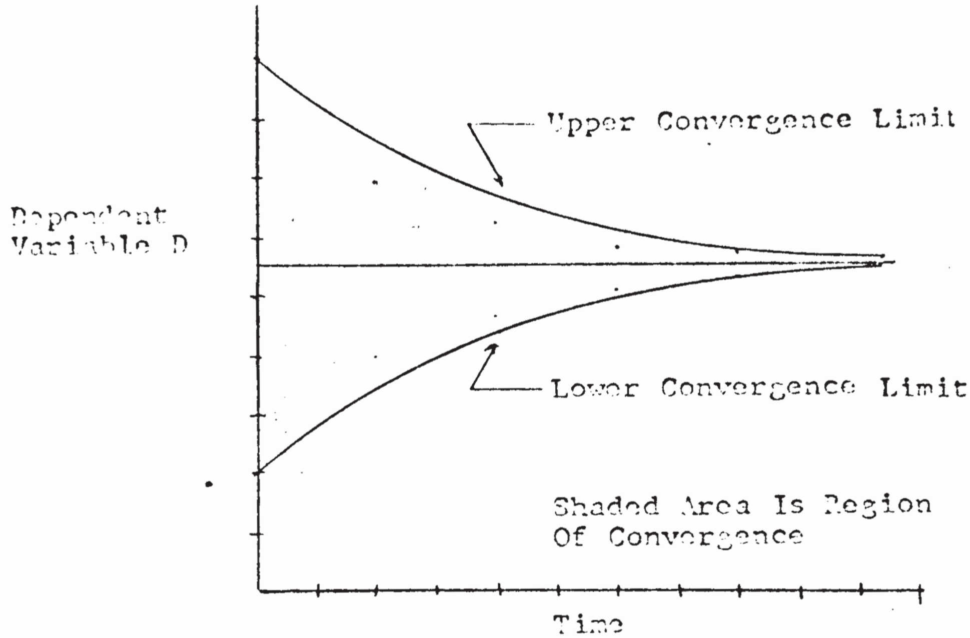


Figure 6

An optimization by its definition forces a process to some set of constraints, therefore an algorithm that has an economic motivation must be capable of approaching while not exceeding a constraint or control variable.

The elements in the profit row of the linear program matrix would be derived from a steady state model of the process as reported by a number of writers in the literature.

The control algorithm proposed in this letter is the result of a number of years of studying and searching for a practical method of controlling a multivariable system. The criterion listed for the control algorithm are not just desirable but necessary if a computer is to successfully control a chemical process at an optimum condition. The conventional control theory presented in text books falls miserable short of these criterion, therefore I believe this proposal of mine is worth pursuing. To the best of my knowledge the approach has not been proposed or attempted before and although my experience with process control at Shell cannot be disassociated from my technical background, I can say without hesitation that it does not resemble any techniques now being used by Shell or under development by Shell.

Your comments, even if you do not agree with my view points would be appreciated.

Respectfully yours,

Charles R. Cutler

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1504 Mason Smith
Metairie, La. 70003

CRC/jc

The unpublished 1975 paper

The following "1975 paper" is an early draft paper that was never published. In early 1976, Cutler got permission from Shell to publish it, but it seems it never happened. This draft is fairly different from the conference paper published in 1980[1], and the authors are different also. Huang, who was the original PhD advisor (but not a control expert) is the first author on the 1975 paper, but he is not mentioned in the final 1980 paper which was published at JACC (where the authors are Cutler and Ramaker, both from Shell). Some of the figures, are dated March of 1973 and are from his earlier correspondence within Shell (see previous).

This "final" DMC paper (which currently has more than 3000 citations on Google Scholar) was published in the proceedings of the Joint American Control Conference (JACC) is San Francisco in August 1980[1]. However, it seems a version of the paper was discussed with colleagues during or following the AIChE Annual meeting in Houston in April 1979.

References

1. Cutler CR, Ramaker BL. Dynamic matrix control — A computer control algorithm. Proceedings of the 1980 Joint Automatic Control Conference, Vol 1. Paper WP5-B. Aug 13-15, San Francisco, CA, USA (1980).

The original documents follow.

A COMPUTER CONTROL ALGORITHM FOR COMPLEX CONTROL PROBLEMS

BY

DR. C. J. HUANG & C. R. CUTLER

A Control Algorithm for digital computers has been developed which permits the solution of control problems that are not handled adequately by conventional feed back control. The principles were developed at the University of Houston and the Control Algorithm was tested at Shell Oil Companies Norco Refinery. The Algorithm evolved from a technique of representing process dynamics with a set of numerical coefficients. The numerical technique, in conjunction with a least square formulation to minimize the integral of the error/time curve, make it possible to solve complex control problems in a unique manner. The control problem on which the Algorithm was tested was a preheat furnace. The furnace is located downstream of a number of other process units that at times introduce significant distrubances in the inlet temperature. The control problem is further enhanced by a dead time and a large time constant for the response of the outlet temperature to a change in the fuel. These response curves are shown in Figure 1.

The Algorithm which was named the ''Dynamic Matrix Control'' is compared against the conventional Analog Control of the furnace in Figure 2. As can be observed, the Dynamic Matrix Control of the outlet temperature of the furnace for a unit change in the set point is substantially better than the Analog Control. Another comparison is made in Figures 3A and 3B between the Dynamic Matrix and a computer implemented PID Algorithm for a distrubance in the feed inlet temperature. The Computer PID control has a feed forward feature that is a significant improvement over the feedback Analog Control which is shown in Figure 3C. However the Dynamic Matrix Control is substantially better than either of these modes of control.

Any system which can be described or approximated by a system of linear differential equations can utilize the Dynamic Matrix Control technique. The technique is based upon the numerical representation of the system dynamics. Two properties of linear systems makes the numerical representation possible. The first of these principles is the preservation of the scale factor. It is illustrated in Figure 4 where the response of the dependent variable D is shown for a change in the independent variable I. The solid line represents the response of the dependent variable to a unit change in the independent variable and the dashed line illustrates the response for a two-unit change in the independent variable. Note the response of the two-unit change has twice the amplitude of the one-unit change. For a linear system, the response of the dependent variable for any size change in the independent variable may be obtained by multiplying the scalar value of the independent variable times the unit response curve for the dependent variable. Further, note on Figure 4 that the unit response curve can be approximated by a set of numbers if the curve is broken into discrete intervals of time. The two-unit response curve in Figure 4 can be obtained by multiplying the set of numbers for the unit response by 2. The second charateristic of a linear system is the principle of superposition. This principle is illustrated in Figure 5 where the response of a dependent variable is shown for a unit change in two independent variables. Also, the response of the dependent variable to a simultaneous unit change in both independent variables is shown. The response for this curve was obtained by summing the responses for the unit response curves for the independent variables.

Mathematically the change in the dependent variable with time for the change in N independent variables is given by:

$$\Delta D_1 = \sum_{j=1}^{N} A_{1j} \quad \Delta I_j$$

$$\Delta D_2 = \sum_{j=1}^{N} A_{2j} \quad \Delta I_j \qquad (1)$$

'' '' '' ''
'' '' '' ''

$$\Delta D_i = \sum_{j=1}^{N} A_{ij} \quad \Delta I_j$$

Where the ΔD_i are the changes in the dependent variable from its initial value to its value at time interval i and the ΔI_j are the changes in the independent variables from their initial value at time equal to zero. The A_{ij} are the numerical coefficients referred to in the proceeding paragraphs.

The concept of describing process dynamics with numerical coefficients is easily expanded to more than one dependent variable by providing a set of equations similiar to Equation Set (1) for each dependent variable. Inasmuch as the independent variables affect the response of all dependent variables for an interacting system, the complete system response is determined when the changes in the independent variables are known. The following set of equations demonstrate this relationship:

$$\Delta D_{i^1} \Big|_{i=1}^{M} = \sum_{j=1}^{N} A^1_{ij} \quad \Delta I_j \Big|_{i=1}^{M}$$

$$\Delta D_{i^2} \Big|_{i=1}^{M} = \sum_{j=1}^{N} A^2_{ij} \quad \Delta I_j \Big|_{i=1}^{M} \qquad (2)$$

'' '' '' '' '' ''
'' '' '' '' '' ''

$$\Delta D_{ik} \Bigg|_{i=1}^{M} = \sum_{j=1}^{N} A_{ij}^{k} \quad \Delta I_j \Bigg|_{i=1}^{M}$$

Where M is the number of time intervals, N is the number of independent variables, k is the number of dependent variables, i is the index on the time intervals, j is the index on the independent variables, and the superscript carried on the A_{ij} indicates a different set of numerical coefficients for each dependent variable. The notation $\Big|_{i=1}^{M}$ implies the vector ΔD_{ik} takes on values form i equal 1 to M for each value of k. Further, note that the ΔI_j are common to all equation sets.

The idea of introducing the movement of the independent variables in future intervals is accomplished by shifting the vector of dynamic coefficients down one time interval, filling the vacated element with a zero, and adding a movement column for each independent variable for that interval of time. Expanding Equation Set (2) to include this concept yields the following:

$$
\begin{aligned}
\Delta D_{i1} \Bigg|_{i=1}^{M} &= \sum_{j=1}^{N} A_{ij}^{1} \quad \Delta I_{1j} + \sum_{j=1}^{N} B_{ij}^{1} \quad \Delta I_{2j} + \ldots \Bigg|_{i=1}^{M} \\
\Delta D_{i2} \Bigg|_{i=1}^{M} &= \sum_{j=1}^{N} A_{ij}^{2} \quad \Delta I_{1j} + \sum_{j=1}^{N} B_{ij}^{2} \quad \Delta I_{2j} + \ldots \Bigg|_{i=1}^{M} \\
\vdots \quad \vdots \quad & \quad\quad\quad \vdots \quad \vdots \quad\quad\quad\quad \vdots \quad \vdots \quad\quad\quad \vdots \qquad (3) \\
\Delta D_{ik} \Bigg|_{i=1}^{M} &= \sum_{j=1}^{N} A_{ij}^{k} \quad \Delta I_{1j} + \sum_{j=1}^{N} B_{ij}^{k} \quad \Delta I_{2j} + \ldots \Bigg|_{i=1}^{M}
\end{aligned}
$$

Where M is the number of time intervals, N is the number of independent variables, k is the number of dependent variables, i is the index on the time intervals, j is the index on the independent variables, and the superscript on the A_{ij} and B_{ij} indicate a different set of numerical coefficients for each dependent variable. The B_{ij} matrix has the same set of numerical coefficients as appear in the A_{ij} matrix except the first row is all zeros in the B matrix and the i+1 row of the B matrix is equal to the i row of the A matrix.

The representation of the response of a dynamic system by a set of linear equations as outlined in the preceeding paragraphs permits the prediction of a systems response if the changes in the independent variables are known. The Control Algorithm substracts the predicted system response from the set point for each dependent variable. The vector of projected errors are set equal to the matrix of dynamic coefficients. A least square fit of the data yields the best set of moves in the independent variables to minimize the projected error. The obvious difficulty with the use of the least square method to calculate the movement of the manipulated variable is the unconstrained nature of the solution. The method without constraint will yield very large changes in the manipulated variables that would not be physically realizable.

One technique for suppressing the change in the manipulate variable is to multiply by a number greater than one, the main diagonal elements of the square matrix that evolves from the least square formulation. The effectiveness of such a multiplier is illustrated in Figure 6 where a square wave change in set point was made. The unconstrained least square reduction in the error resulted in a total change in the absolute value of the manipulated variable moves of 37.57 taken over 10 intervals of time. With a multiplier of 1.005 the total moves where 4.91 and with 1.010 the moves were 3.42. The multiplier effectively adds another row to the original data for each independent variable for each interval in which it is allowed to move. All elements in the row are zero except for the specific independent variable which has a coefficient related to the size of the multiplier. As can be seen from Figure 6, the suppression of the independent variable moves by an order of magnitude did not significantly impair the reduction in the projected error.

The matrix of coefficients which describe the dynamics of the system are the basis for the Dynamic Matrix Control Algorithm. For the furnace control problem the response of the dependent variable was considered for 30 intervals of time and the movement of the fuel gas was considered for 10 intervals. Thirty intervals of time represents about 4 1/2 time constants for the response of the outlet temperature to a change in the fuel. At the tenth time interval, the outlet temperature has 3 times constants to settle from the last change in the fuel. This choice of time intervals results in a matrix with 10 columns and 30 rows. To initialize the Algorithm, the measured outlet temparature is stored into the 30 element vector that represents predicted values of the dependent variable. This assumes the system is at steady state, but is not a necessary criterion. An error is then calculated from the expected value of the variable and the set point for the 30 intervals of time. This vector of errors become the right hand side for the 10 by 30 matrix. The least square fit of these data yields the best set of fuel moves to eliminate the projected errors for 30 time intervals. The projected set of fuel moves are used to calculate the outlet temperature change for the forthcoming 30 intervals of time, and the temperature changes are then added into the 30 element vector for the predicted value of the dependent variable. The first fuel move is implemented and the entire vector of predicted dependent variable values is shifted forwared one interval of time. At the start of the next interval of time the predicted value of the dependent variable is compared with the measured value. The error in the projection is used to adjust all 30 values in the predicted dependent variable vector. This adjustment in the prediction provides the feedback to compensate for distrubances and errors in the dynamic prediction. At the next interval the set of errors between the set point and the predicted values of the dependent variable is used to solve for another set of 10 fuel moves. The 9 remaining fuel moves from the previous calculations are summed with newly calculated fuel moves and the current fuel move is implemented. The pattern is repeated at each successive interval of time.

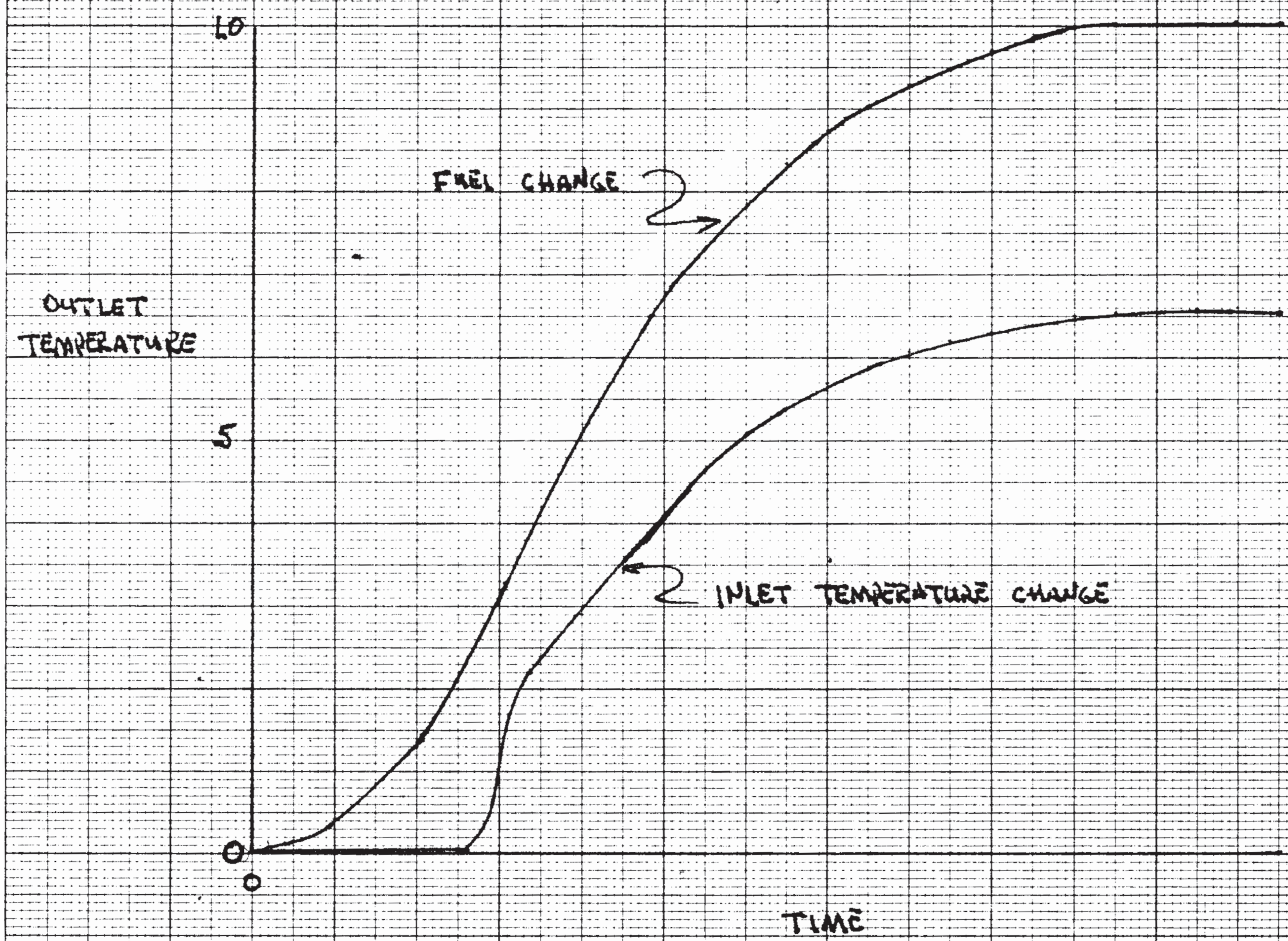
Feed forward control is implemented by measuring the change in the feed inlet temperature between time intervals, multiplying by the numerical coefficient for the effect of the inlet temperature on the outlet, and summing this response of the outlet temperature into the vector of predicted values for the dependent variables.

The description of the technique for the Dynamic Matrix was given in some detail to foster understanding. The actual calculations involved in the technique are at least two orders of magnitude less than would be required by the outlined procedure. The first simplification is to recognize that the matrix of coefficients representing the dynamics are fixed and only the right hand side changes from one time interval to the next. Furthermore, the square matrix that results from the formulation of the least square procedure does not change, which also means the inverse matrix does not change. The errors between the projected vector of dependent variable and the set point, appear in the calculation of the right hand side for the least square formulation.

Consequently the inverse matrix multiplied times the right hand sides yields the set of projected moves in the independent variable. Further it can be shown that only the first row of the inverse matrix and the right hand sides are needed to solve the control program. This calculation yields the control move to make at the present interval of time. The other control moves in the independent variables are imbeded in the first row of the inverse matrix, in the successive updating of the right hand sides and by the coupling of time intervals from adding in the projected response from the fuel moves in each interval of time. In the case of the furnace control discussed earlier the calculations for the Dynamic Matrix Algortithm reduced to the multiplication of the 10 elements in the first row of the inverse matrix times the right hand sides calculated for the least square formulation.

In summary, the Dynamic Matrix Control Algoithm has proven to be a valuable technique for computer control of difficult problems. The opportunity for solution of multivariable problems was not emphasized, but is one of the real attributes of the Algorithm. The least square procedure was used to solve the Dynamic Matrix, however, the Linear Programming Technique is an effective tool for solving the same type problem and has many interesting features.

FURNACE OUTLET TEMPERATURE RESPONSE TO FUEL AND INLET TEMPERATURE



FUEL COEFFICIENTS	INLET TEMPERATURE COEFFICIENTS
.014	.0
.043	.107
.086	.240
.143	.290
.214	.340
.307	.410
.414	.465
.507	.508
.600	.540
.676	.570
.736	.590
.786	.609
.836	.622
.874	.632
.904	.640
.929	.651
.949	.653
.972	.656
.986	.658
1.000	.659

FIGURE 1

OUTLET TEMPERATURE RESPONSE TO UNIT SET POINT CHANGE FOR FURNACE

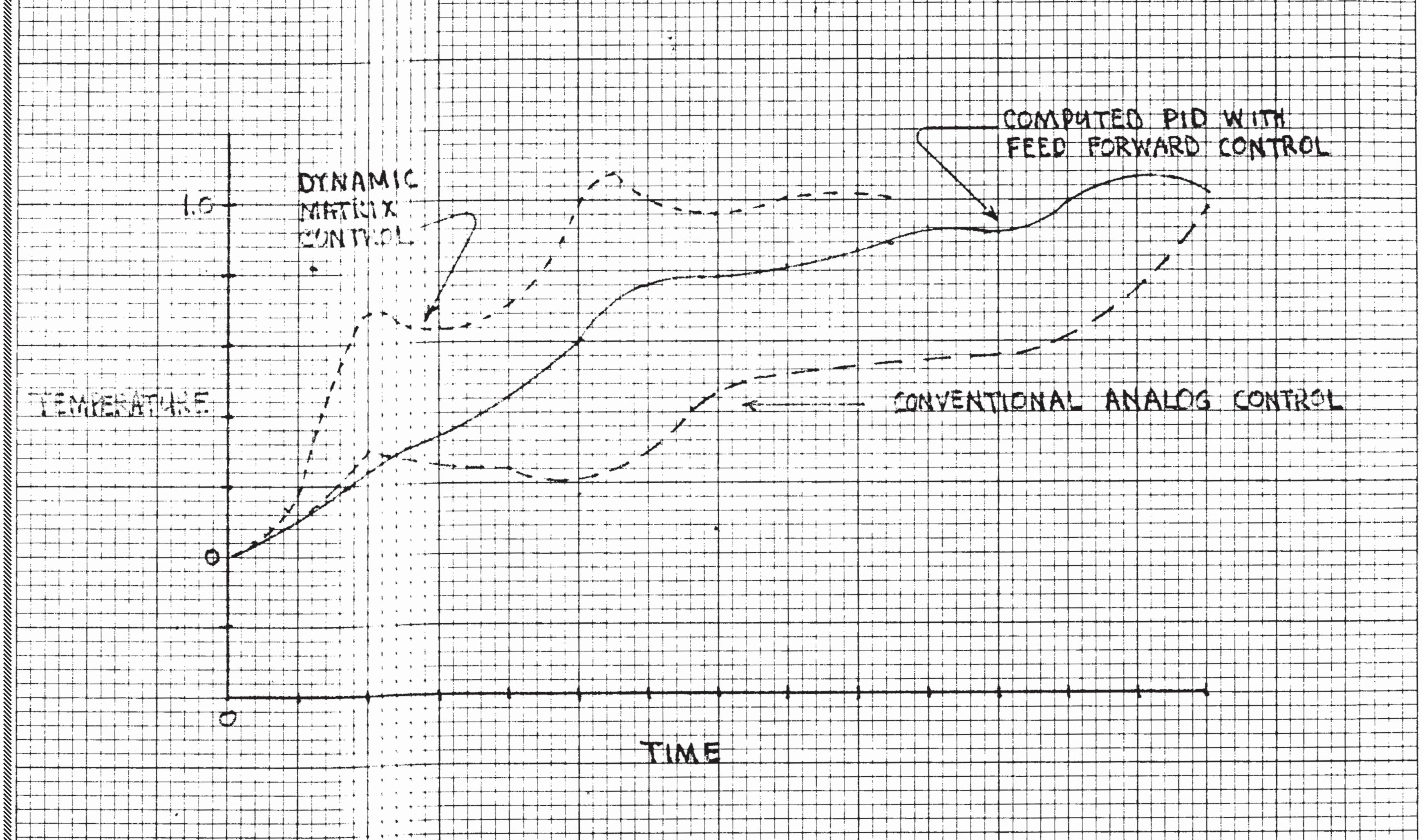
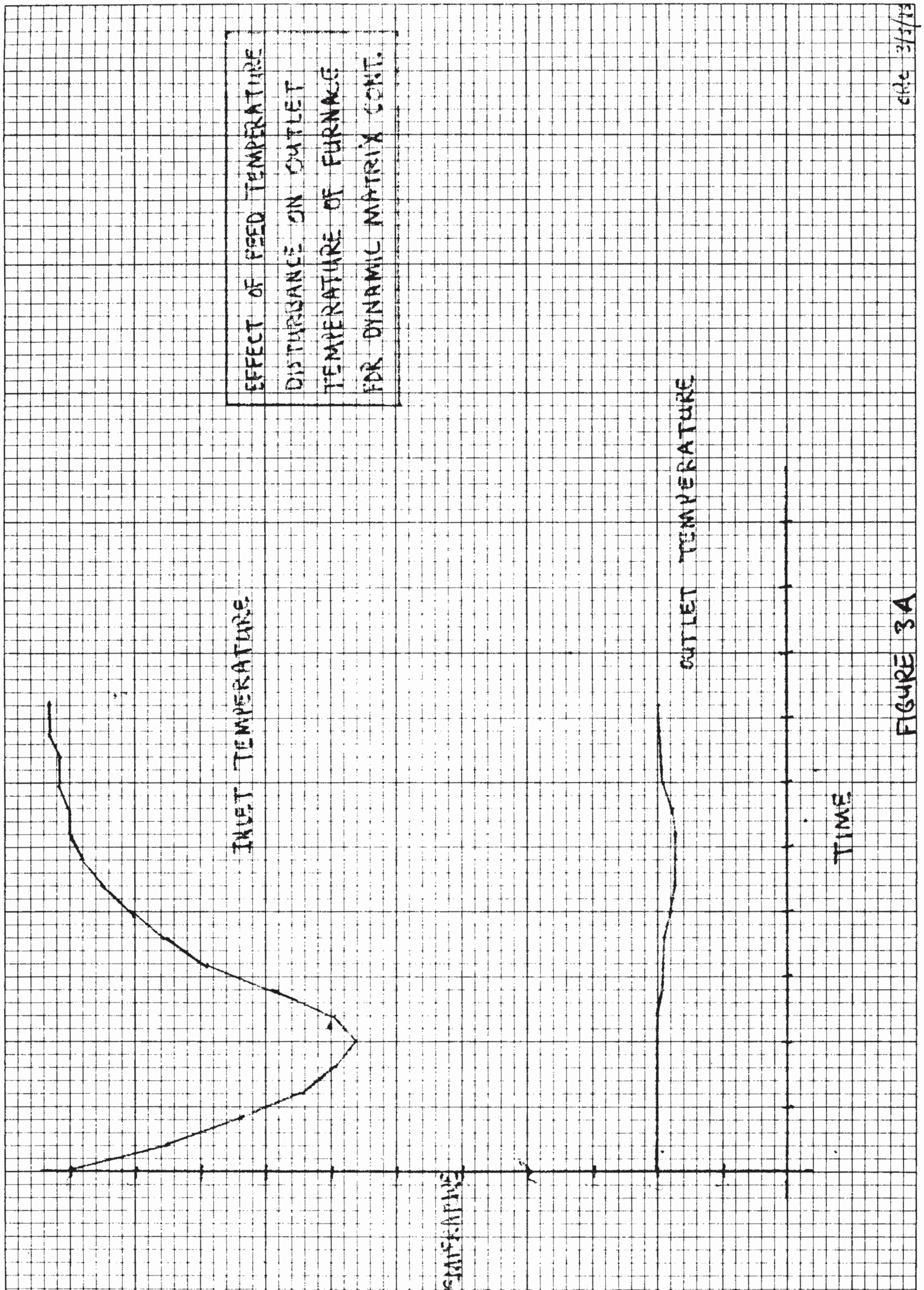


FIGURE 2

CRC
3/4/13


EFFECT OF FEED TEMPERATURE
DISTURBANCE ON OUTLET
TEMPERATURE OF FURNACE
FOR DYNAMIC MATRIX CONT.
INLET TEMPERATURE
OUTLET TEMPERATURE
TEMPERATURE
TIME
FIGURE 3A

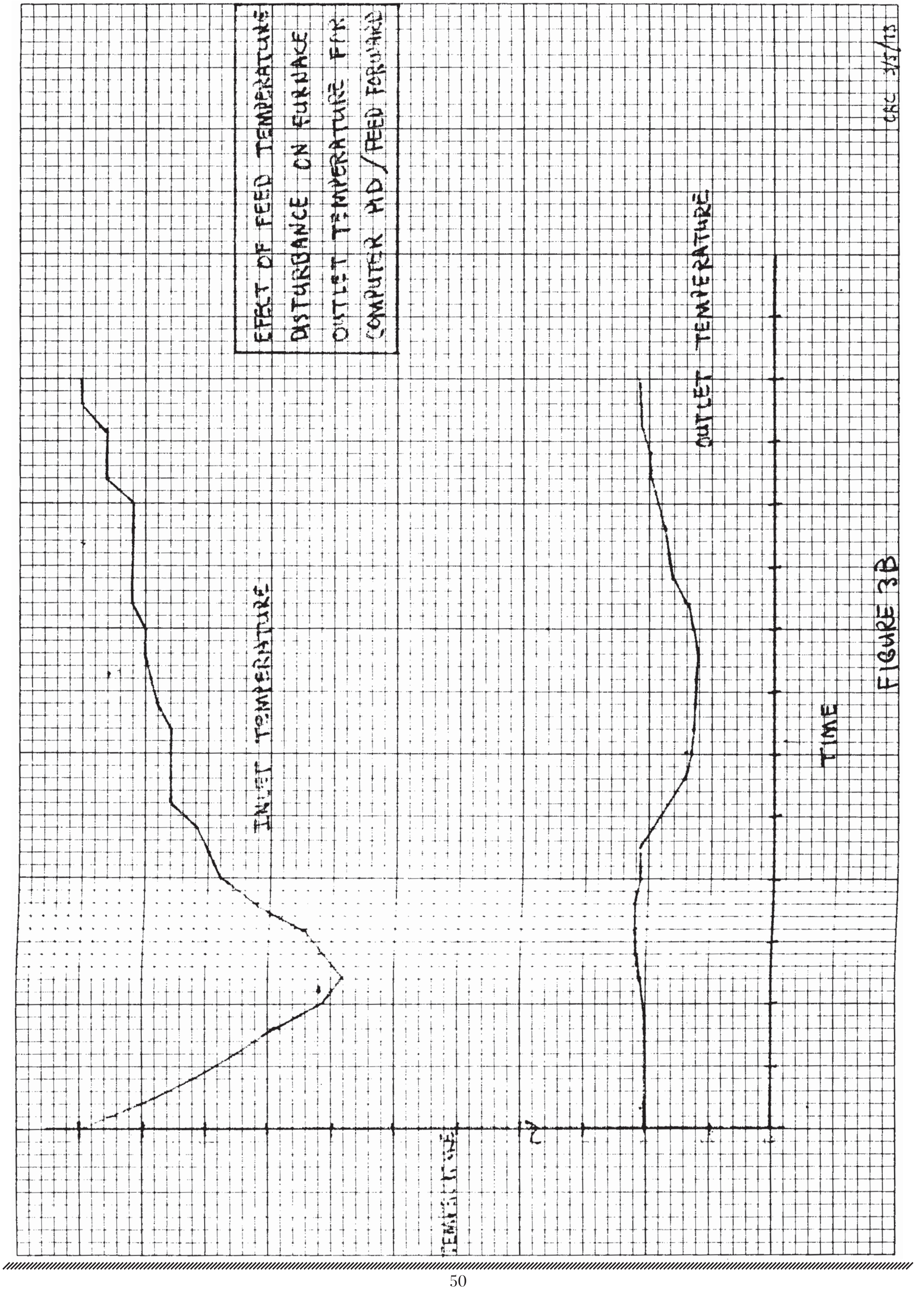
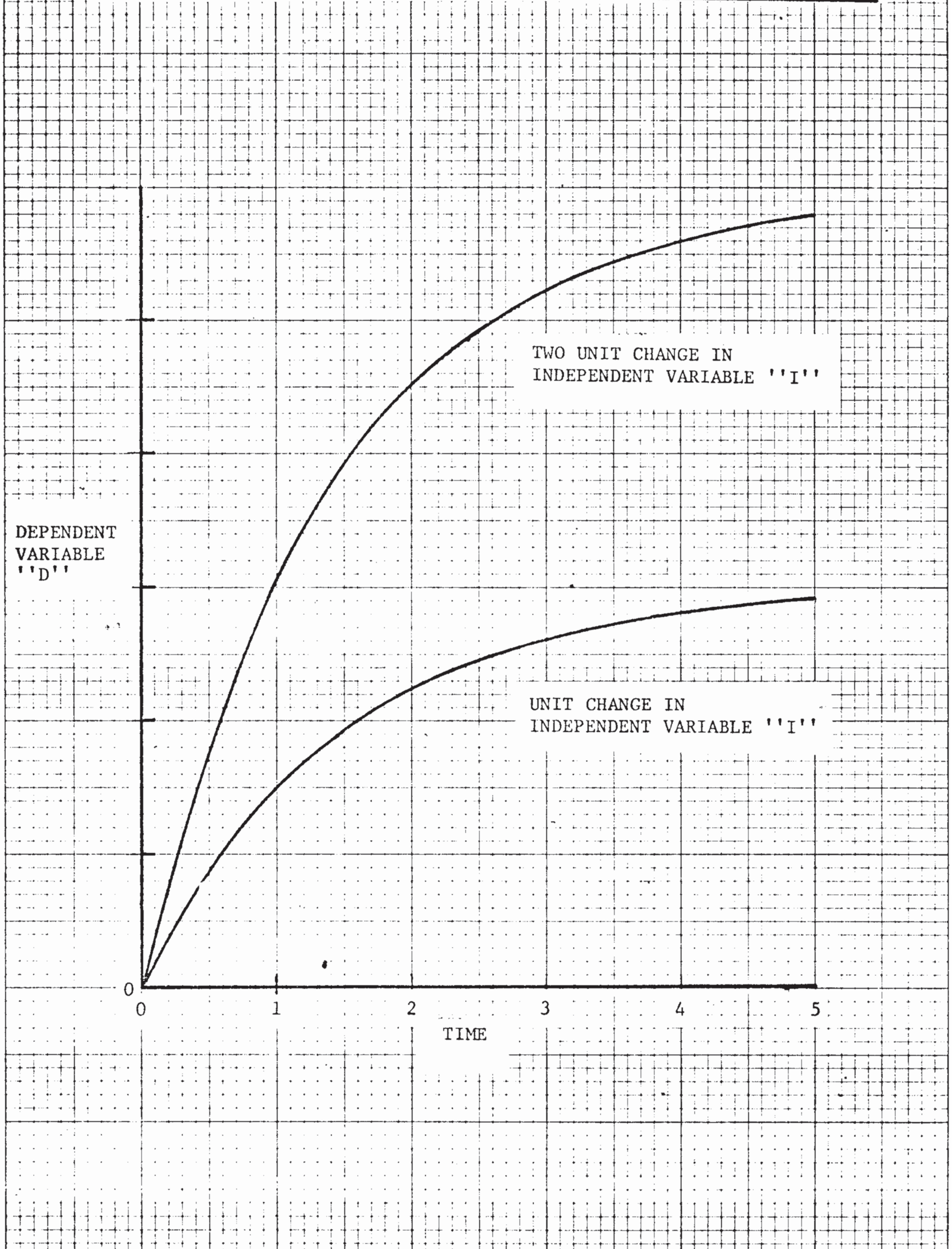

EFFECT OF FEED TEMPERATURE
DISTURBANCE ON FURNACE
OUTLET TEMPERATURE FOR
COMPUTER PID / FEED FORWARD
INLET TEMPERATURE
OUTLET TEMPERATURE
TEMPERATURE
TIME
FIGURE 3B

FIGURE 4

ILLUSTRATION OF THE PRESERVATION OF SCALE FACTOR FOR A LINEAR SYSTEM



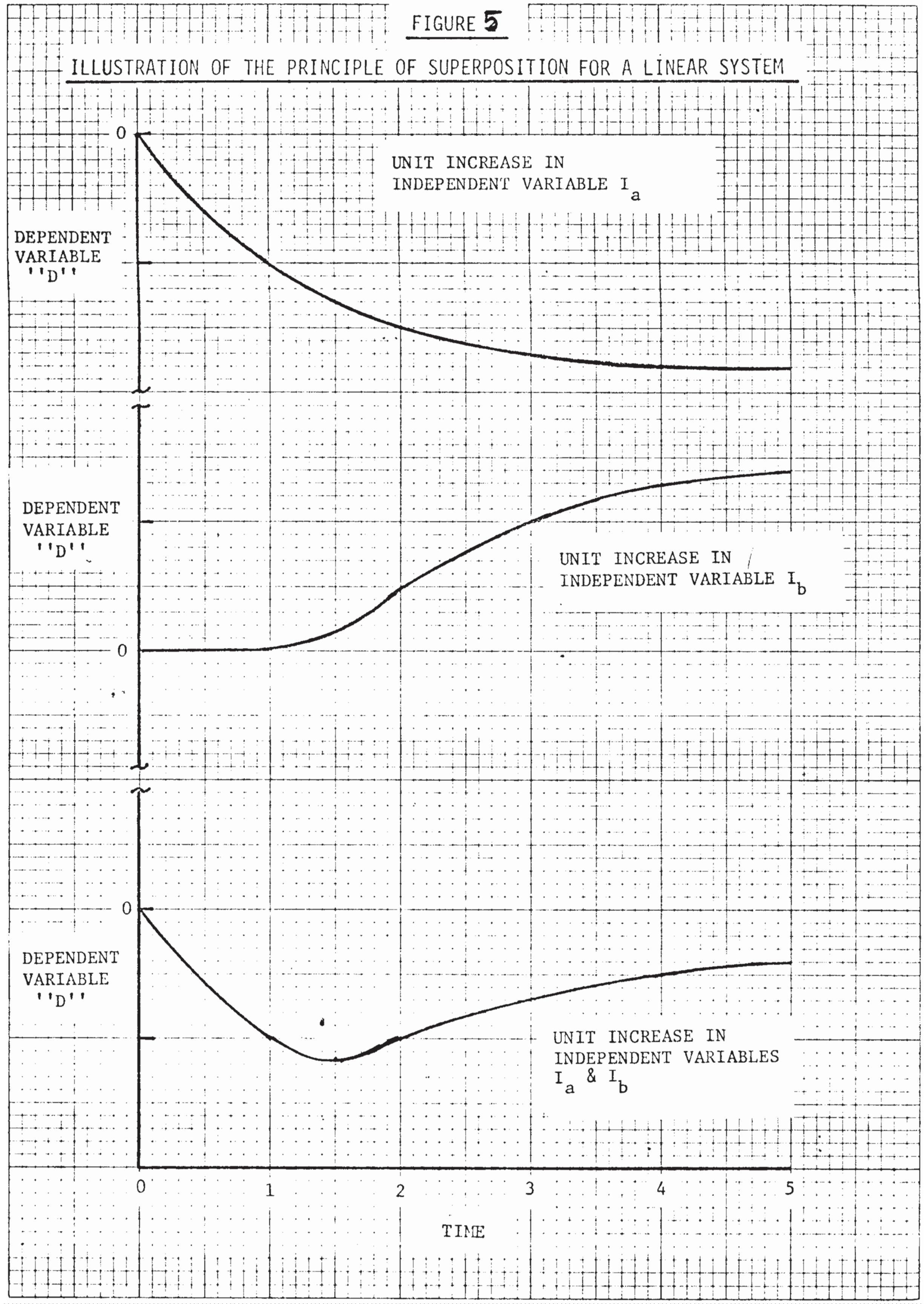
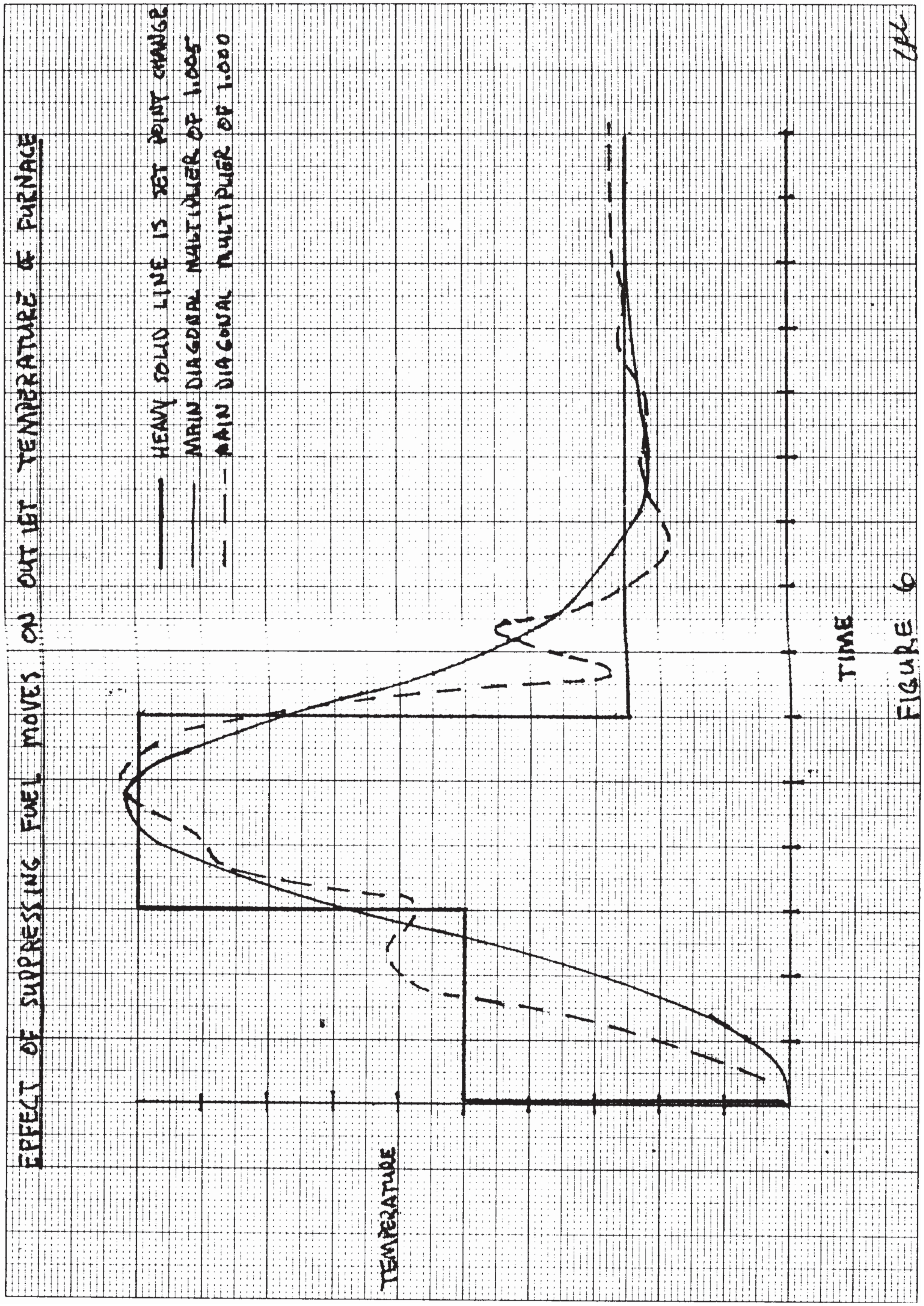


FIGURE 5
ILLUSTRATION OF THE PRINCIPLE OF SUPERPOSITION FOR A LINEAR SYSTEM
0
UNIT INCREASE IN INDEPENDENT VARIABLE I_a
DEPENDENT VARIABLE ''D''
DEPENDENT VARIABLE ''D''
UNIT INCREASE IN INDEPENDENT VARIABLE I_b
0
0
DEPENDENT VARIABLE ''D''
UNIT INCREASE IN INDEPENDENT VARIABLES I_a & I_b
0
1
2
3
4
5
TIME


EFFECT OF SUPPRESSING FUEL MOVES ON OUTLET TEMPERATURE OF FURNACE
HEAVY SOLID LINE IS SET POINT CHANGE
MAIN DIAGONAL MULTIPLIER OF 1.005
MAIN DIAGONAL MULTIPLIER OF 1.000
TEMPERATURE
TIME
FIGURE 6

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