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Extremum seeking control applied to operation of dividing wall column – DWC

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ABSTRACT

The dividing wall column (DWC) has significant energy saving potential compared to conventional column sequences. However, to reach these savings in practice, it is essential that the control structures can track the optimal operation point despite inevitable changes in feed properties, performance characteristics and other uncertainties. Otherwise, the energy consumption may rise significantly or, more commonly, the DWC becomes unable to produce pure products even at its maximum reboiler duty. Extremum seeking control (ESC) is a model-free optimisation technique that may mitigate off-optimal operation in this environment. By active perturbation of selected manipulative variables, the algorithm infers gradient properties of the measured cost function and, by that, enables tracking of a moving optimum. Extremum seeking control can be used also in combination with other approaches, e.g. self-optimising control. Applied to the DWC, the presented perturb-and-observe algorithm, which can be classified as ESC, appears to be able to track changes in the optimal values of the manipulated variables and setpoints.

Keywords: Distillation, Energy Efficiency, Process Control, Optimization, Machine Learning, Perturb and Observe, Dividing Wall Column

INTRODUCTION

The dividing wall column (DWC) is an attractive integrated distillation arrangement, since there are significant saving potentials in energy and capital cost compared to conventional column sequences [1, 2]. Despite that challenges in operation and design have been listed in the literature as a key obstacle, there are developments in this area that can enable more widespread industrial usage [3]. In a conventional column sequence, e.g., the direct split configuration, controlling the purities of each column to their specifications implicitly gives the lowest overall energy consumption [4]. In DWC operation, there are some pitfalls that may lead to either higher energy consumption and/or failure to produce the designed product purities [5]. Thus, it is of great interest to develop operating strategies that can handle real-world uncertainties and enable the DWC to operate such that it always maintains its designed capabilities in low energy consumption and simultaneous high product purity.

The characteristics of optimal operation have been described in detail for ideal mixtures and infinite number of stages [6] and also for finite stages [7], but the practical challenge is to use these insights to devise a control structure that can implement the optimal moves of the key manipulative variables as unmeasured disturbances affect the optimum process characteristics. This kind of on-line optimisation is needed in addition to the regulatory control that is required for maintaining the products inside the given purity specifications.

Herein, the focus is on perturbation-based optimisation of the available manipulators. Perturbation-based optimization methods such as extremum seeking control (ESC) are capable of optimizing many types of processes, even when these processes vary in time and model knowledge is limited [8–10]. In this paper, we consider the application of perturb-and-observe, a type of ESC, to approach optimal operation points for the dividing wall column. The main idea of this method is to iteratively perturb the available manipulative variables, observe the outcome and move in the direction of improvement. This leads to fast convergence to the optimal operation point, as well as fast tracking if the optimal operation point changes over time.

PERTURB AND OBSERVE

In this section, the perturb-and-observe method for model-free adaptive optimization is introduced. Consider the problem of minimizing an unknown, time-varying function $f(k, u(k))$ with input $u(k)$, where $k = 1, 2, \dots$ denotes the time index. We make the following assumptions:

1. The function f is unknown at any time k, but we can obtain noisy measurements of the form
2. $y(k) = f(k, u(k)) + \varepsilon(k)$. (1)
3. The magnitude of the measurement disturbance $\varepsilon(k)$ is small compared to the range of possible function values.
4. At any time k, function f has a unique minimum.
5. The rate of change of f is small compared to the sampling frequency.
6. The function $f(k, u(k))$ is static. If the underlying process contains dynamics, $f(k, u(k))$ represents the steady-state behavior.

Function $f(k, u(k))$ can represent many different processes. In the context of the dividing wall distillation column considered in this paper, a typical choice is the energy consumption V of the reboiler as a function of the liquid split R_l , i.e., $f(k, u(k)) = V(k, R_l(k))$ with $u(k) = R_l(k)$. The corresponding measurement $y(k)$ could be a direct, noisy measurement of the reboiler energy consumption, or an estimate based on, e.g., temperature measurements.

The aim is to find the input $u(k)$ that minimizes $f(k, u(k))$ and tracks this minimum while it changes over time. To this end, a perturb-and-observe method is employed. The main idea is to perturb the input variable u and determine the direction of improvement from the corresponding measurements y. Consider an initial input $u(1)$, for which output $y(1)$ is measured. The next input is chosen as

$$u(2) = u(1) + \Delta_u, \tag{2}$$

for some step size Δ_u. For $k = 2,3, \dots$, the corresponding measurement $y(k)$ is compared to the previous measurement $y(k-1)$ to obtain the direction of improvement $g(k+1) \in \{-1,1\}$ as follows:

$$g(k+1) = \begin{cases} g(k), & if \;\; y(k) \leq y(k-1), \\ -g(k), & if \;\; y(k) > y(k-1). \end{cases} \tag{3}$$

Next, the direction of improvement is used to determine the next input by taking

$$u(k+1) = u(k) + g(k+1)\Delta_u. \tag{4}$$

Thus, if input $u(k)$ leads to a smaller measured output $y(k)$ compared to the previous input $u(k-1)$, we are moving towards the minimum and the direction of movement remains unchanged. If $y(k) > y(k-1)$, we are moving away from the minimum and need to reverse the direction of movement. The iterative method is summarized in the flowchart in Figure 1.

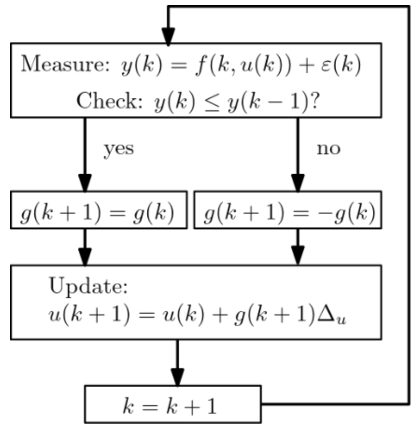

Figure 1. Flowchart for perturb and observe.

Implementation of perturb and observe

The implementation of the perturb-and-observe method described here is straightforward, as it consists only of updating an input parameter and observing the resulting outcome. Still, there are some implementation aspects that need to be considered. First, it is noted that the method can be applied to maximization instead of minimization of a function by changing the update rule for the direction of improvement to

$$g(k+1) = \begin{cases} g(k), & if \;\; y(k) \geq y(k-1), \\ -g(k), & if \;\; y(k) < y(k-1). \end{cases} \tag{5}$$

In this case, $y(k)$ being smaller than $y(k-1)$ leads to a change in direction.

A second consideration when implementing perturb and observe is that this method continues perturbing the input after the minimum has been found. In case of time-varying processes, this behavior is desired as it enables tracking of the time-varying optimal operating point. However, if the optimal operating point can be assumed to be time-invariant, it is in general preferred to reduce the step size Δ_u over time to limit the amount of unnecessary perturbation. Possible extensions also include resetting the automatic reduction of Δ_u once a change in the process is detected through additional sensors, see, e.g., [10] for ideas in this direction.

Third, the influence of the measurement disturbances $\varepsilon(k)$ needs to be considered. If $\varepsilon(k)$ is stochastic in nature and its magnitude is similar to the absolute difference between $y(k)$ and $y(k-1)$, one measurement might not be sufficient to determine the direction of improvement. In this case, it is recommended to keep the input $u(k)$ constant for several time steps and average the resulting measurements to obtain more accurate estimates of $f(k,u(k))$.

DWC CASE DESCRIPTION

We consider the “standard” ternary dividing wall column (DWC) as illustrated in Figure 2. It is operated at constant pressure. The feed (F) enters the prefractionator sections (1,2) at the left side of the wall and is described by its molar fractions (z) of pure components (A,B,C) and its liquid fraction (q). There are three product specifications, given by the main component purities in each product stream (D,S,B). There are five manipulated variables (MVs): 1) boilup vapor rate (V), 2) reflux rate (L), 3) side product draw rate (S), 4) the liquid split ratio (R_l) above the wall and 5) the vapor split ratio (R_v) below the dividing wall.

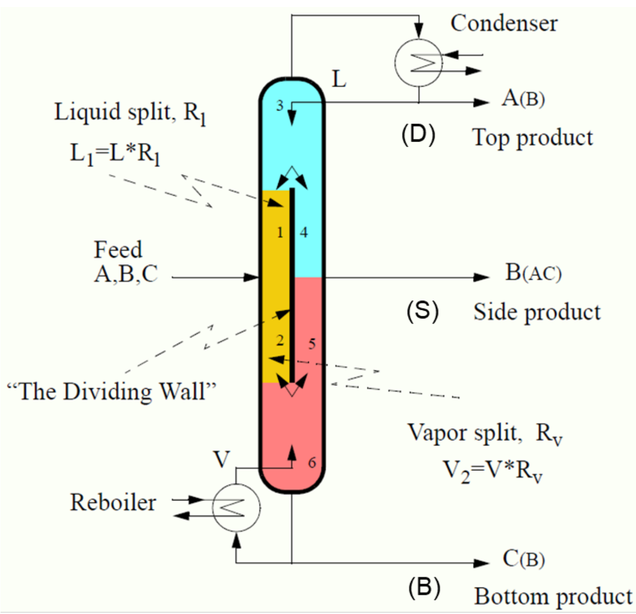

Figure 2. The ternary Dividing Wall Column (DWC). Section 1,2 is the prefractionator and 3,4,5,6 is denoted main column.

For the virtual infinite stage case and ideal feed mixture components, the minimum energy condition is precisely described by the Vmin-diagram [6]. Figure 3 shows this for the ideal case with equimolar feed, liquid fraction q=0.5 and relative volatilities $\alpha = [4,2,1]$ referred to the heavy component (C). The two peaks represent minimum vapor (in the top) for the sharp A/BC and AB/C splits. Point P is the so-called preferred split, which is minimum vapor for the AB/BC split. In a DWC, the highest peak sets the overall minimum reboiler vapor requirement. However, this minimum is only reachable when the prefractionator is operated at a point exactly on the (bold) line from the preferred split (P) on the boundary line to a balance point (B) (where all sections in the main column are at their minimum vapor simultaneously).

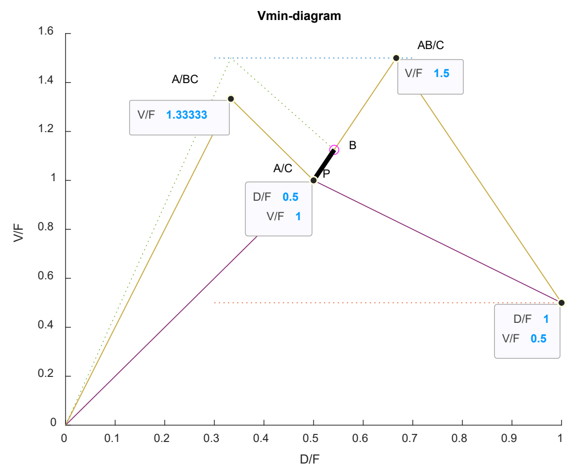

Figure 3. Vmin-diagram for the equimolar feed showing the optimality region for prefractinator operation at (bold) line from the so-called preferred split to the balance point (P to B).

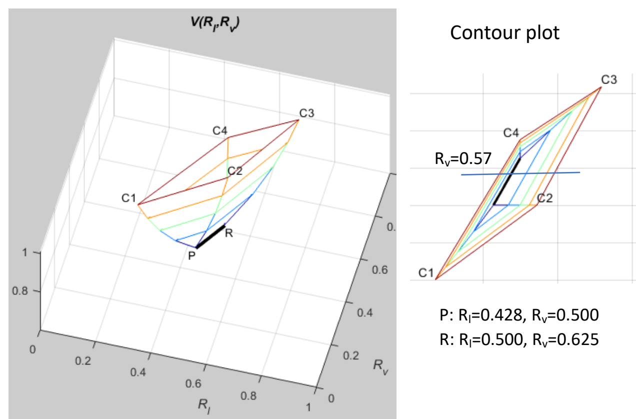

Figure 4. Perspective and contour view of the reboiler vapor solution surface V(R_l,R_v) for a sharp product split and an infinite number of stages. The optimality region PR maps directly from the Vmin-diagram to the R_l,R_v space. Each contour represents a 10% increase in V. The maximum reboiler saving for this case is as high as 39% (compared to conventional “direct split” and 30% in the condenser).

Most industrial DWC-implementations set the vapor split by design of the wall placement and tray/packing pressure drop characteristics. It is crucial that the obtained vapor split is properly checked to cover the expected feed property variation range, since the Vmin-diagram and solution surface will move whenever feed properties change. A constant, properly designed R_v=0.57 is illustrated in the contour plot in Figure 4 as a line that hits approximately the middle of the PR-line.

Then, it will usually be sufficient to adjust only the liquid split R_l to track the optimum on a moving solution surface.

In the simulation case considered here, the number of stages has been set to 16 in each of the 6 sub-sections. This leads to the real minimum boilup being about 6-7% above Vmin. In practice, this corresponds to about 2.3 times the minimum stage number for a product purity of 99%. A rigorous stage design should evaluate the pareto curve that balances capital and operational costs [3], but this level of detail is not required for this study.

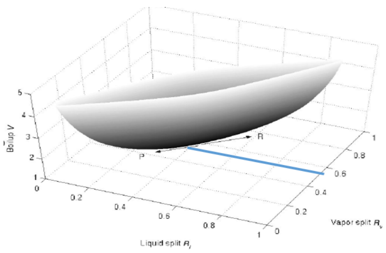

Figure 5. Finite stage solution surface, cf. Figure 4.

The impact to the finite stage solution surface, see Figure 5, is that it will be more rounded and not have the sharp edges shown in Figure 4. The optimality region will not be completely flat like the PR-line, but clearly much steeper perpendicular to the PR direction than along PR.

Temperature profiles

For each disturbance, a set of grid points $V(R_l)$ have been calculated as illustrated below in Figure 6.

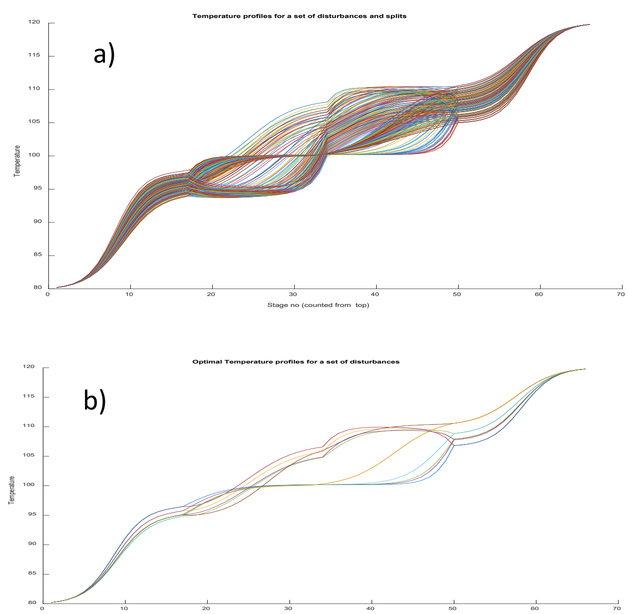

Figure 6. a) Temperature profiles for a set of disturbances and liquid split values, while the product composition is controlled, and b) The profiles where liquid split is optimised for each disturbance.

Figure 6 shows the all the executed runs in the upper plot but only the optimal profiles for each disturbance in the lower plot. The profiles may look a bit crowded, but a certain symmetry in the optimised plots can be observed, and that symmetry changes in different ways depending on which side of the optimum the DWC is operated. This gave rise to propose the so-called "Difference Temperature Symmetry" (DTS) described in more detail in [7], which is the sum of differences across the wall in the middle of sections 1,2,4,5 (see Figure 2): DTS=(T1-T4)-(T2-T5).

Self optimising control of the DWC

In self optimising control (SOC), the idea is to find a variable that contains some gradient-like information for a given cost function such that when this controlled variable (CV) is regulated to a setpoint by applying the available MV for optimisation as a regulatory controller MV, the deviation from the optimum upon disturbances is reduced compared to keeping the MV constant.

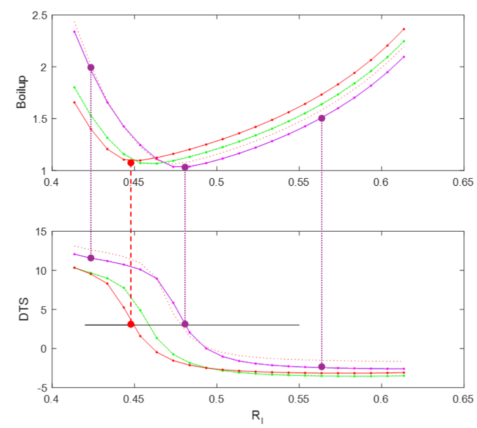

Figure 7. The relation between the SOC CV=DTS, the primary manipulated variable R_l and the boilup $V(R_l)$

Figure 7 indicates qualitatively that the DTS as a self optimising controlled variable has close to ideal properties; namely, that for low values the operation is at one side of the optimum and for high values on the other side. And by controlling to the setpoint, e.g. DTS=3, as for the nominal optimum, this value ensures close to optimum operation for the considered disturbances too.

But still, the optimal setpoint value for any given case cannot be assumed to be known, and this calls for a smart procedure for adjusting the setpoint. And here is where the perturb-and-observe algorithm is handy.

SIMULATION RESULTS

The relations between the MVs and the column state have been obtained by simulating the DWC with the products controlled to meet specifications by use of simple PI controllers for a range of grid-points for the optimisation MV around the calculated optimum for each disturbance scenario. The perturb-and-observe algorithm has no information about the active disturbance acting on a system and is just given a suitable (feasible) initial value for the MV. The perturb-and-observe algorithm considers basically the structure illustrated in Figure 8.

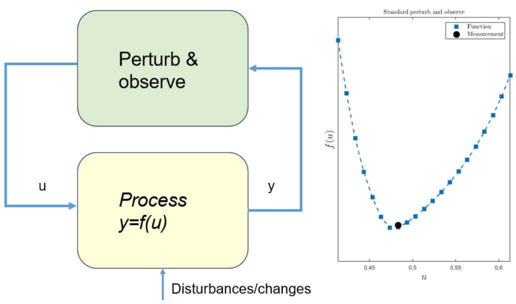

Figure 8. The general structure for the perturb-and-observe algorithm during extremum search.

Manipulating liquid split ratio

The first scenario is a case where some changes are applied to the feed composition and liquid fraction. The vapor split is set constant (as in most industrial cases).

Applied to the DWC, the structure of y=f(u) is illustrated in Figure 9 where u is the liquid split (R_l) and y is the reboiler vapor (V) which in this case is used as energy measure. It is an "output" in this context since its value is manipulated by the product purity controls to maintain the CVs at specification setpoints. The challenge is that the cost function is influenced by unknown disturbances and the actual optimum needs to be tracked by the perturb-and-observe algorithm.

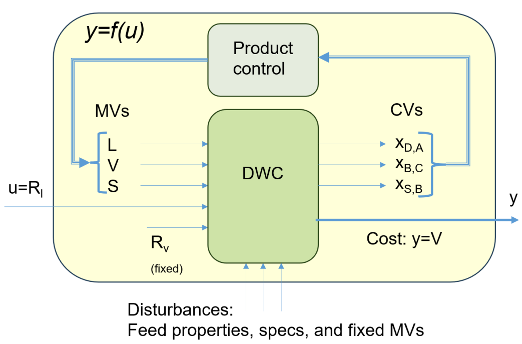

Figure 9. The cost function y=f(u) for the DWC is on top the basic regulatory product purity control. The optimisation MV is the liquid split u=R_l.

Figure 10 illustrates the minimisation search when the set of disturbances are applied. For each disturbance the cost function is moved as illustrated in the left graph. Noise is added to the measurement (y) and it can be observed in the upper right graph that the applied manipulated (perturbed) u-value (green) very soon after a change tracks the current optimal value (black) which is of course unknown to the algorithm. The feed composition and liquid fraction are changed at k=30,60,90,120. Note that white noise is added to the measurements to simulate measurement noise.

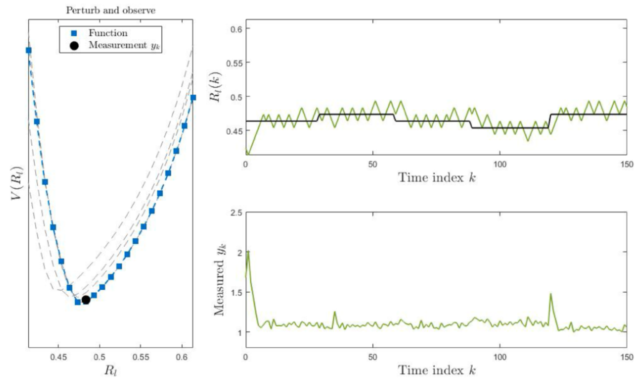

Figure 10. The resulting tracking of the optimal liquid split obtained by the perturb-and-observe algorithm.

Manipulating SOC setpoint

The structure of the basic regulatory control, including a self optimising control loop is shown in Figure 11. The perturb-and-observe algorithm will work with the SOC setpoint as its manipulative variable "u".

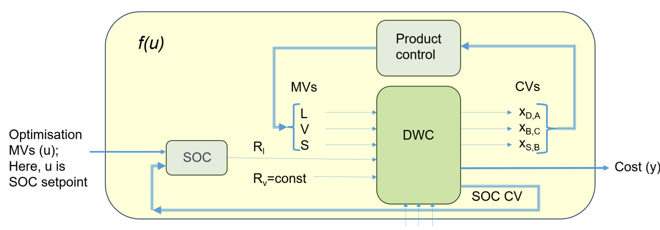

Figure 11. The cost y=f(u) when the perturb-and-observe is manipulating the SOC setpoint.

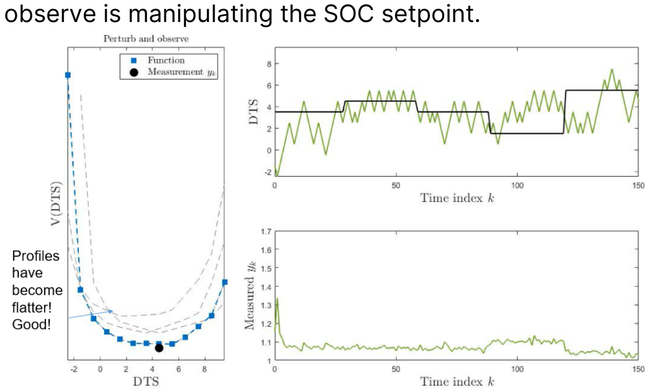

Figure 12. The tracking of y=f(u=DTS) for the same disturbance scenario as in Figure 7. Observe that the solution surface is "flattened" cf. Figure 7, thus being more robust for the exact setpoint.

A key advantage with use of the SOC is that it will reject fast disturbances in the regulatory layer. A disturbance may have some impact to the optimal setpoint, and then the perturb-and-observe algorithm can do the search for the best setpoint on the slower time scale. This configuration ensures less off-optimal operation time in the case with frequent variations in the disturbances.

DISCUSSION

The first question to be raised is how the perturb-and-observe method will perform in a true dynamic plant environment. The presented procedure is basically a steady-state method intended to be used on a process that is stabilised by its regulatory control system, and such that changes in the optimisation MVs do not interfere and degrade process transient performance and stability properties. This can typically be done by time-scale separation, e.g. by using a sample period in the ballpark of the typical closed-loop settling time for the regulatory control level. It is not required to have complete steady-state conditions (which in practice never occur perfectly in a plant) since the algorithm is robust against somewhat noisy measurements.

In distillation in general, including for the DWC, inferential product composition control is usually based on temperature measurements, and often on-line measurements of compositions are not available. Temperature based product purity control has been simulated too. The corresponding results are similar to the results for perturb-and-observe as shown in this work.

For products that are being quality controlled in a lab, it is possible to update the correlation between temperature setpoints and product composition. For temperature points in the profiles in the prefractionator, these kinds of correlations are not available, so some kind of tweaking of the setpoints will always be required. For this case, the perturb-and-observe algorithm fits perfectly since there is no alternative to pre-calculate the optimal setpoints exactly based on product data. Another desirable function for industrial practice is to gracefully handle a scenario where the optimal solution becomes constrained and to include some kind of safety supervision to prevent malfunction.

CONCLUSION

The simulation study indicates that extremum seeking control by the perturb-and-observe algorithm can track the moving optimum value of the available manipulated optimisation variable when the DWC is exposed to some unknown disturbances that alter the optimal operating point. The algorithm can also be used in combination with self optimising control on the regulatory layer by using that controller's setpoint as the manipulative optimisation variable.

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