The theoretical basis of ratio control

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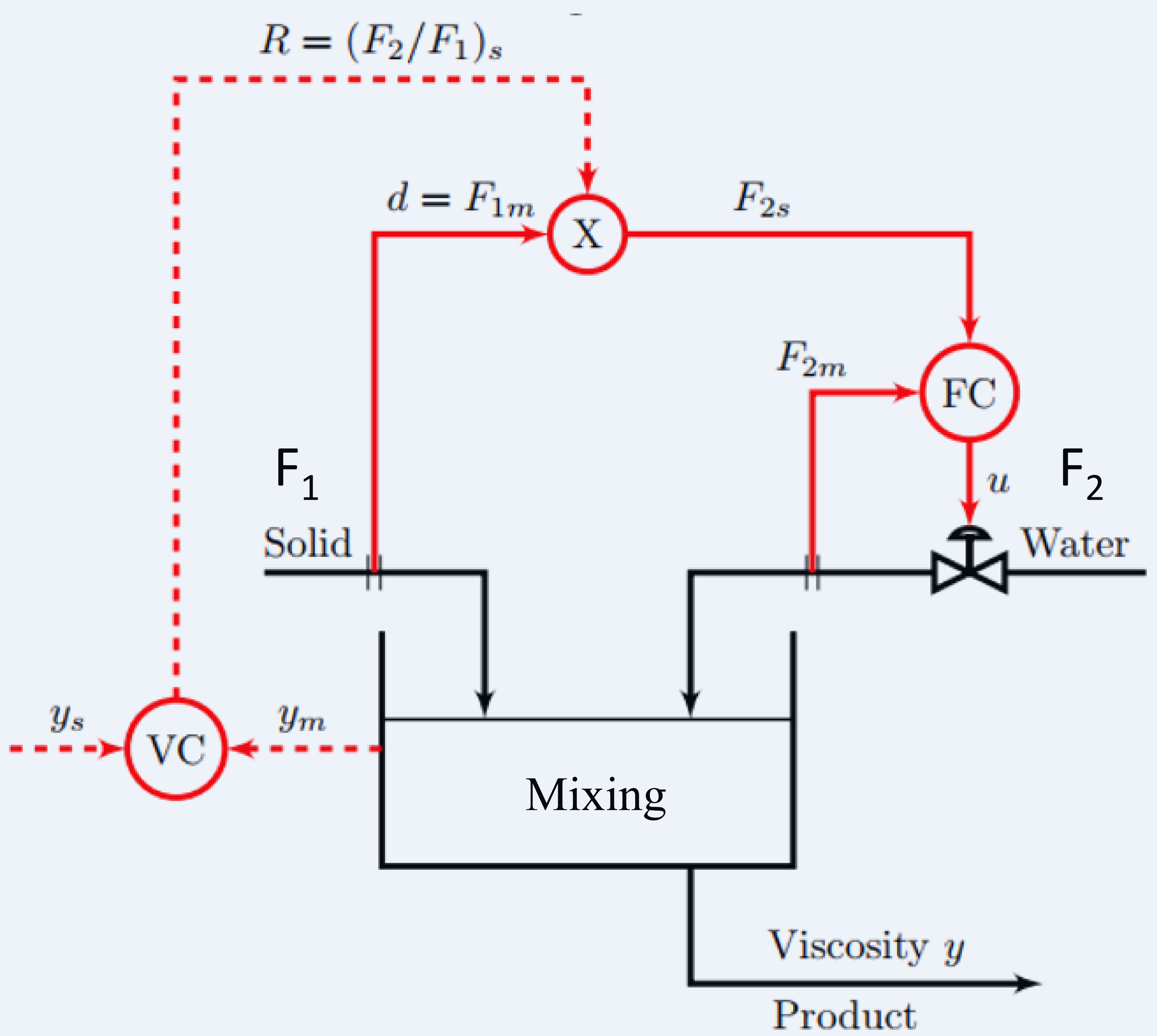
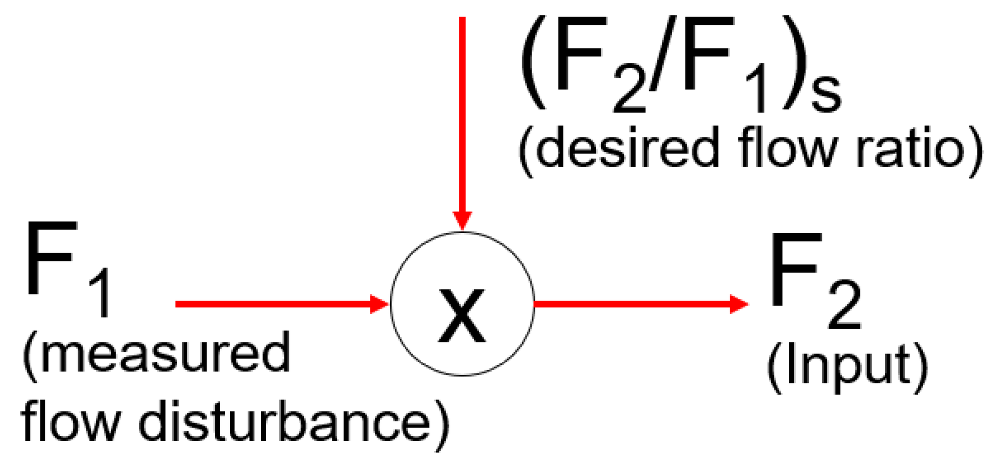
Background.

Ratio control is thousands of years old (think of recipes for food), but despite this, there exists no theoretical basis for its use. Because of this it is sometimes used incorrectly.



Typical ratio control scheme (solid red lines) with outer feedback loop (dashed red lines) to set the correct ratio.

- **No model is needed**, except for the **insight (Scaling property assumption)** that a constant ratio between the two mixed streams F_1 and F_2 (both extensive variables) gives a constant mixture property y (intensive variable).
- **The correct ratio setpoint is found by feedback control** (VC) based on keeping the measured controlled variable y at its desired setpoint y_s.
- In the figure, y is viscosity, but generally it can be any <u>intensive</u> variable.



Ratio control is sometimes viewed as a special case of feedforward control. It can also be used for decoupling.

- But ratio control differs from normal feedforward: Do <u>not</u> need a model for how F_1 and F_2 affect the controlled property y.
- The use of ratio control also linearizes the process gain (from the transformed input $R=(F_2/F_1)_s$ to the controlled property y).

When should we use ratio control?

- When fixing the ratio of two **extensive** variables (F_2/F_1) gives a constant **intensive** variable (y).
- The use of ratio control assumes the **scaling or scale-up property:** We get the same steady-state solution if we increase all extensive variables (flows and heat rates) by the same factor compared to a basis. For this to hold we must assume constant process efficiencies (Skogestad, 2009):
  - **The scaling property** holds if the process efficiency is independent of the throughput.

Similar to the use in thermodynamics, the scaling property holds for equilibrium systems. The scaling property (and thus the use of ratio control) applies to many process units, including

- **Mixing**
- Equilibrium reactors
- Equilibrium flash and equilibrium distillation

Cases where ratio control should <u>not</u> be used:

- Heat exchangers (because the area is fixed during operation)
  - This means that we should not fix the ratio between hot and cold flow.
  - For the scaling property to hold for a heat exchanger, we would need to increase the heat transfer area *A* proportionally to the flow rates. This is reasonable during design but not during operation (control).
- Non-equilibrium reactors (because the volume is fixed)
  - Need to include kinetics and reactor size
- Mixing processes where one feed stream is fixed
- Distillation with fixed heat input.

Scaling property, mathematical formulation.

Assume that all independent extensive variables (F_i) are increased by the same factor k, and that all independent intensive variables (x_i), are kept constant. Then all dependent intensive variables y remain constant, while all dependent extensive variables Y increase by the factor k:

y intensive: $y(x_1,x_2,kF_1,kF_2,kF_3) = y(x_1,x_2,F_1,F_2,F_3)$ **(1)**

Y extensive: $Y(x_1,x_2,kF_1,kF_2,kF_3) = k\,Y(x_1,x_2,F_1,F_2,F_3)$ **(2)**

Equivalently, we may say that y is homogeneous to the degree h, where h=0 if y is intensive and h=1 if y is extensive:

$$y(x_1, x_2, kF_1, kF_2, kF_3) = k^h\, y(x_1, x_2, F_1, F_2, F_3) \qquad (3)$$

The point of ratio control is that we want to keep the intensive variable y constant, that is, for control, (1) or (3) with h=0 is the important relationship.

Discussion

- In the literature there does not seem to be any clear statement of the **scaling property** and its assumptions. Reklaitis (1983,) proves the homogeneity of the material (p.40) and energy (p. 460) balance equations and uses this show that one may rescale the solution (p. 81). However, this is not enough to guarantee that we get the same steady-state solution, for example. for a non-equilibrium reactor or a distillation column where the stage efficiency depends on the load. Skogestad (1002) goes further and assumes constant efficiencies as a requirement for the scaling property.
- **In thermodynamics**, (3) with h=1 is used to derive Euler's theorem and from this we may derive the fundamental equation of thermodynamics, Legendre transformations and the Gibbs-Duhem equation.
- ***Ratio control is difficult to implement with model predictive control (MPC).*** With MPC, we need a nonlinear model for how *y* depends on the independent variables (*x* and *F)*, which may be a quite complex model, for example, if *y* is viscosity. We must also provide a value for the desired ratio y_s.

References

- M. Modell and R.C. Reid. *Thermodynamics and its applications*. 2nd edition, Prentice-Hall (1983). (See Appendix C on Euler's theorem)
- G.V. Reklaitis, *Introduction to material & energy balances*. Wiley (1983)
- S. Skogestad, "Consistency of Steady-State Models Using Insight about Extensive Variables", *Ind. Eng. Chem. Res*., 30, 4, 654-661 (1991)
- S. Skogestad. *Chemical end energy process engineering*, CRC press (2009)

Conclusion

Ratio control is very simple to use and gives nonlinear feedforward action without an explicit process model. It is almost always used to set the ratio of feed streams (mixing).

Demonstration with free drink

Ask the presenter for a demonstration of ratio control

F_1 = 10 parts water

F_2 = 8 parts alcohol

F_3 = 1 part whiskey concentrate or 0.5 parts brandy concentrate

Ratio: 10:8:1 (can be adjusted after feedback)

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