# 149h - The Theoretical Basis of Ratio Control

Tuesday, November 7, 2023

3:30 PM - 5:00 PM

*Regency Ballroom R/S (Convention Level, Hyatt Regency Orlando)*

## Abstract


### Background

Ratio control is extensively used in the process industries, in particular for mixing processes and for feed flows to chemical rectors. Ratio control is most likely the oldest control approach (think of recipes for making food and chemical compounds), but despite this, there exists no theoretical basis for its use.

### What is ratio control?

The figure shows a typical ratio control scheme for mixing with an outer feedback loop to set the ratio. Note that no model is needed, except for the insight that the a constant ratio is desired. The correct ratio setpoint is found by feedback based on keeping the measured controlled variable y at its desired setpoint ys. In the figure, y is the viscocity, but generally it can be any intensive variable,

Ratio control is more powerful than most people think, because its application only depends on a "*scaling assumption*" and does require an explicit model for the controlled variable (y).

### Scaling assumption

For a mixing process, the "*scaling assumption"* says when all feed flows are increased proportionally (with a fixed ratio), then at steady state any intensive variable y will remain constant. Note that all extensive variables (flows, heat rates, sizes of certain equipment) need to be scaled by the same factor for the scaling property to hold, and that also the independent intensive variables (typical feed composition and temperature) need to be kept constant.

The *scaling assumption* may be formulated as follows (e.g., Skogestad, 1991):

- *In words:* Scaling all the independent extensive variables by a factor k (with the independent intensive variables constant) scales all the dependent extensive variables by the same factor k and keeps all the dependent intensive variables constant,

- *Mathematically:* Consider a steady-state relationship $y=y(x_1, x_2, F_1, F_2, F_3)$ where the independent variables are divided in two classes:
  - Intenstive variables, $x_1$ and $x_2$
  - Extensive variables, $F_1$, $F_2$ and $F_3$ (typically flow and heat rates)

Assume that all extensive variables are increased by the same factor k compared to their nominal values ($^0$), that is, $F_1=kF^0{}_1$, $F_2=kF^0{}_2$, $F_3=k\,F^0{}_3$. Then

y intensive: $y(x_1,x_2,F_1, F_2, F_3) = y(x_1, x_2, F^0{}_1, F^0{}_2, F^0{}_3)$ (1)

y extensive : $y(x_1,x_2,F_1, F_2, F_3) = k\, y(x_1, x_2, F^0{}_1, F^0{}_2, F^0{}_3)$ (2)

The point of control is that we want to keep the controlled variable y constant, that is, for ratio control (1) is the important relationship.

**Main result**

From (1) we arrive at the following important conclusions for the use of ratio control:

(A) The systems must satisfy the scaling assumption,

(B) The controlled variable y is implicitly assumed to be an intensive variable, for example, a property variable like composition, density or viscosity, but it could also be a temperature or pressure.

(C) If the system has n independent extensive variables, then we need to keep n-1 ratios constant in order to keep the (any) independent intensive variable y constant.

(D) Since all extensive variables must be scaled by the same factor, there must be at most one extensive variable disturbance, typically the throughput.

Note that for achieving perfect control with only ratio control, we must assume that there are no intensive variable disturbances (like feed composition), but in practice this is not a serious limitation because these disturbances can be handled by an outer feedback controller which adjusts the ratio setpoint(s).

When does the scaling assumption hold? Similar to the use in thermodynamics, it holds for all equilibrium systems.

The scaling property (and thus the use of ratio control) applies to many process units, including

- Mixers,
- equilibrium reactors,
- equilibrium flash and equilibrium distillation

*Example distillation*. For distillation we must assume that the pressure and efficiency (the number of theoretical stages) is constant. For a typical distillation column, there are three independent extensive variables at steady state, so at steady state we need to keep two ratios constant, for example, L/F and V/F where L is the reflux, V is the boilup and F is the federate (often a disturbance).

*Example mixing:* Ratio control is very common for mixing processes because here the assumption of constant efficiency holds, at least at steady state when we assume perfect mixing. As an example, consider a mixing process with three feeds ($F_1$, $F_2$, $F_3$) and one mixed product outflow (F). Because of the steady-state mass balance ($F=F_1+F_2+F_3$), there are three independent variables at steady state. Assume now that one flow is given and is therefore the disturbance d from a control point of view (e.g., $d=F_3$). With the level loop closed (to satisfy the steady-state mass balance) there are then two independent (manipulated) variables left (e.g. $u_1= F_1$ and $u_2 = F_2$) and the system will be fully specified by fixing two flow ratios, for example, $u_1/d$ and $u_2/d$, and this will result in constant mixture properties. Importantly, note that $u_1$, $u_2$ and d can be any of the four flows $F_1$, $F_2$, $F_3$ and F. The important thing is that we keep two ratios constant.

However, there are also many process units where ratio control should not be used, because the scaling property does <u>not</u> hold. This includes, for example, non-equilibrium reactors (where kinetics are important) and heat exchangers.

For the scaling property to hold for a heat exchanger, we would need to increase the heat transfer area A proportionally to the flow rates. This is reasonable during design but not during operation (control) when the equipment is fixed.

**Discussion**

*MPC and ratio control.* Note that ratio control is difficult to implement with MPC. With MPC, we need a nonlinear model for how y depends on u and d, which may be a quite complex model, for example, if y is viscosity. On the other hand, a simple ratio control implementation (see Figure) does not require a model for how y depends on u and d, we just need the physical insight that y will be constant if we keep the ratio u/d constant. We also do not need to have a model to find the correct setpoint for the ratio u/d to keep y constant, as this can be found indirectly through the action of feedback (see Figure).

*Warning.* Even for units where the scaling property should hold, ratio control only applies when *all* extensive variables are increased by the same factor k.

*Example mixing (for case where ratio control should not be used***).** Assume we want to make a drink by mixing three ingredients (feeds), namely

$F_1$= lemon juice,

$F_2$ = lime juice (both of which are quite sour),

$F_3$ = sugar.

There are three independent extensive variables, and according to the scaling property, we will achieve constant product composition if we keep any two flow ratios constant, for example, $F_2/F_1$ and $F_3/F_1$. However, assume that we are limited on sugar, so $F_3 = F_{3,max}$ = constant. Thus, it will be impossible to increase $F_3$ further and ratio control should *not* be used, in spite of the fact the scaling property holds for this system.

In fact, for this particular example, if we increase $F_1$ and keep $F_3$ (sugar feed) constant (at its maximum), then keeping the ratio $F_2/F_1$ constant (and thus *increasing* $F_2$) is most likely the *opposite* of what we should do, because it makes the drink even more sour. Instead, if we increase $F_1$ (lemon) with $F_3$ (sugar) constant, then the drink will taste best if we *decrease* $F_2$ (lime).

*Relationship to thermodynamics*, The scaling property may be used to derive steady-state gain consistency relationships and is also important in thermodynamics (Skogestad, 1991). We here assume that y is homogeneous to the degree h in the extensive variables, where h=0 if y is intensive and h=1 if y is extensive:

$$y(x_1, x_2, kF^0_1, kF^0_2, kF^0_3) = k\, y(x_1, x_2, F^0_1, F^0_2, F^0_3) \quad (3)$$

In thermodynamics, (3) is used to derive Euler's theorem and from this derive the fundamental equation of thermodynamics, Legendre transformations and the Gibbs-Duhem equation.

**Conclusion**

Ratio control is very simple to use and it gives nonlinear feedforward action without needing an explicit process model. It is almost always used for chemical processes to set the ratio of the reactant feed streams. Ratio control may be viewed as a special case of feedforward control, but note that we do not need a model for the property y for ratio control, whereas such a model is needed for feedforward control.

**Reference**


1. Skogestad, "Consistency of Steady-State Models Using Insight about Extensive Variables", Ind. Eng. Chem. Res., 30, 4, 654-661 (1991)


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