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Decentralized control for optimal operation under changing active constraints

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Abstract

Optimal economic operation of chemical plants requires control of active constraints, which may change because of disturbances. In addition, one should ensure optimality with respect to the unconstrained degrees of freedom by driving the reduced cost gradient to zero. One solution is to use on-line optimizing control, but the preferred approach in industry is to use decentralized control and selectors whenever possible. In this paper, we consider a new framework based on identifying the cost gradient projections that can be left uncontrolled when each specific constraint becomes active, leading to a decentralized logic. The proposed framework is applied to the optimal operation of the Williams-Otto reactor, which has two degrees of freedom and two constraints. The projection matrix, which depends on the gradient of the constraints, is assumed constant, resulting in a simple control structure. The approach works well in simulations and it switches between the four active constraint regions.

Keywords: optimal operation, feedback optimizing control, decentralized control, selectors.

1. Introduction

In spite of the potential economic benefits, real-time optimization (RTO) is less used in industry than one may expect. There are several reasons for this, and one is that standard RTO applications operate on a slow time scale, such that for cases with frequent disturbances, optimal operation is not satisfactorily achieved. An appealing solution is to move some of the optimization problem into the control layer design, which operates on a much faster time scale. This means that one should find controlled variables (CVs) that, when controlled to constant setpoints, result in minimal economic loss. This is the idea of self-optimizing control (Skogestad, 2000), which consists of obtaining these CVs as a combination of the available measurements. Another class of feedback optimizing strategies aim to estimate the plant cost gradients in order to control them to zero (Krishnamoorthy et al., 2019).

One of the main challenges that such approaches face is the presence of changing active constraints (Jäschke et al., 2017), which may drastically change the operation mode of the system. For example, if a constraint becomes active due to a disturbance, not taking its control into account leads to infeasible operation. Similarly, if a constraint becomes optimally inactive, the control of such constraint should be given up. For a given set of active constraints, if the cost gradient is measured, it is known that the control of the active constraints together with the control of a projection of the cost gradient over the nullspace of the active constraints' gradient leads to optimal operation (Jäschke and Skogestad, 2012). However, the existence of several active constraint regions may deem necessary the use of several independent control structures, each of them being able to provide near-optimal operation for their respective region of design. The switching between such control structures would also become an issue since the lack of a feedback-based switching strategy could lead to improper operation.

In practical applications of process systems, logic elements have been extensively used for reconfiguring control loops, which is often needed for attaining optimal operation (Reyes-Lúa et al., 2018). In particular, selectors have been successfully applied as a tool for automatic detection of active constraint switching for single input systems (Krishnamoorthy and Skogestad, 2020). In this work, we extend this analysis for multivariable systems, proposing a simple framework for decentralized optimal operation under changing active constraints, with the use of PID controllers and selectors.

2. Methodology

The main idea of this framework consists in identifying the optimal CVs for each region, and consequently proposing control loops for dealing with such CVs, in a way that reconfiguring is minimized. Consider the following convex optimization problem:

$$\begin{aligned} &\min_u J(u,d) \\ s.t. \quad & g_i = g_{u,i}^T u + g_{d,i}(d) \leq 0, \qquad i = 1, \dots, n_g \end{aligned} \tag{1}$$

For the purposes of this work, we consider the steady-state cost gradient J_u to be known. Based on that, and assuming that $n_u \geq n_g$ and all vectors $g_{u,i}$ are linearly independent (LI), we can devise a simple control strategy for optimal operation. Firstly, the unconstrained degrees of freedom related to the nullspace of $G_u = [g_{u,1} \quad \cdots \quad g_{u,n_g}]^T$ will always be optimally controlled regardless of how many constraints are active, and therefore $N_0^T J_u$, with N_0 being a basis for the nullspace of G_u, should always be controlled to zero. In addition to that, if all but one constraint is active, an extra unconstrained degree of freedom needs to be considered. The degree of freedom freed by the constraint g_i becoming inactive can be determined by the nullspace of the matrix $G_{u_{-i}}$ comprised of all the remaining rows $g_{u,j}^T$, $j \neq i$. By definition, this nullspace will include the space generated by N_0, and it is sufficient to pick any projection $N_{g,i}$ that is LI from N_0. A unique solution can be obtained by picking $N_{g,i}$ orthogonal to N_0.

With these definitions, we propose the following control strategy for a given active constraint set $\mathcal{A}$:

- if $n_u > n_g$, find N_0 such that $G_u N_0 = 0$ and control $N_0^T J_u = 0$;
- for $i = 1, \dots, n_g$:
    - find $N_{g,i}$ such that $\begin{bmatrix} G_{u_{-i}} \\ N_0^T \end{bmatrix} N_{g,i} = 0$;
    - control $g_i = 0$ if $i \in \mathcal{A}$; else, control $N_{g,i}^T J_u = 0$.

It can be verified that the operating point defined by the forementioned CVs leads to the solution of (1). Furthermore, if decentralized PID control is used for every CV, the decision of controlling $g_i = 0$ or $N_{g,i}^T J_u = 0$ can be performed locally by comparing the corresponding control loops. This problem may be solved with selectors, such that constraint control becomes active when necessary, and the unconstrained degree of freedom is controlled whenever the constraint is not violated. If the system is such that

decentralized PID control can be used for every set of CVs, the framework will lead to optimal operation.
The proposed framework considers linear constraints with respect to the plant inputs, which is a strong assumption that is not accurate for most real systems. We will now demonstrate the effectiveness of the framework in a case study with nonlinear constraints, evaluating the loss that is obtained by the application of the proposed method.

3. Results

3.1. Case study description

We now consider the optimal operation of the Williams-Otto reactor (Williams and Otto, 1960). The optimal operation of the reactor is described by:

$$\begin{aligned} \min_u J^{ec} &= p_A F_A + p_B F_B - (F_A + F_B)(p_E x_E + p_P(1 + \Delta p_P) x_P) \\ s.t. \quad & g_1 = x_A - 0.12 \leq 0 \\ & g_2 = x_E - 0.3 \leq 0 \end{aligned} \tag{2}$$

The degrees of freedom for operation are $u = [F_B, T_R]$, with T_R being the reactor temperature, and the considered disturbances are $d = [F_A, \Delta p_P]$. The solution of the above optimization problem as a function of the disturbances leads to the pattern shown in Figure 1, where each region correspond to the set of active constraints at the solution.

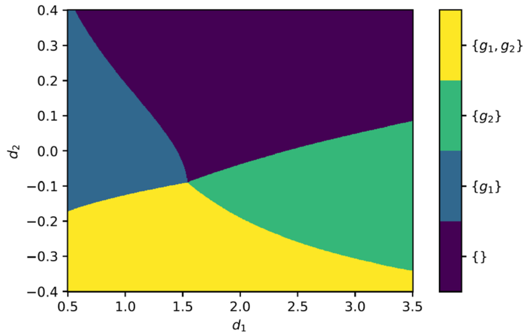

Figure 1: Optimally active constraints as function of the disturbances

3.2. Control structure design

In order to apply the proposed framework, we linearize the constraints at a nominal point. With this, we assume a constant constraint gradient with respect to the inputs, and this ultimately results in constant cost gradient projections to be controlled inside each region. The nominal point was chosen to be the optimal point for $d^* = [1.0 \quad 0.0]$, and the linearization of the constraints at this point results in:

$$\Delta g = \begin{bmatrix} g_{u,1}^T \\ g_{u,2}^T \end{bmatrix} \Delta u + g_d \Delta d = \begin{bmatrix} -0.0492 & 0.0032 \\ -0.0328 & -0.0026 \end{bmatrix} \Delta u + g_d \Delta d \tag{3}$$

Since $n_g = n_u = 2$, there are no unconstrained degrees of freedom that remain active at the fully constrained case, when $\mathcal{A} = \{g_1, g_2\}$. Therefore, no cost gradient projection $N_0^T J_u$ is needed, and all operational degrees of freedom are filled by active constraint control. For $\mathcal{A} = \{g_1\}$, in addition to controlling $g_1 = 0$, we must control one cost

gradient projection $N_{g,2}^T J_u = 0$ in order to fill the remaining degree of freedom, where $N_{g,2}$ is chosen such that $G_{u_{-2}} N_{g,2} = g_{u,1}^T N_{g,2} = 0$. Similarly, in the region where $\mathcal{A} = \{g_2\}$, the optimal CVs will be $g_2 = 0$ and $N_{g,1}^T J_u = 0$, where $N_{g,1}$ is chosen such that $G_{u_{-1}} N_{g,1} = g_{u,2}^T N_{g,1} = 0$. In the unconstrained region $\mathcal{A} = \{\,\}$, the optimal CVs are all the components of the cost gradient $J_u = 0$. However, controlling $N_{g,1}^T J_u = 0$ and $N_{g,2}^T J_u = 0$ simultaneously implies in $J_u = 0$, as the constraints are independent and therefore $[N_{g,1} \quad N_{g,2}]$ is full rank. This means that the same gradient projections used for the partly constrained regions can be used for the fully unconstrained region.
The control structure that results from the application of the proposed methodology is presented in Figure 2, where K_{g_i} and K_{c_i} denote PID controllers related to control of constraints and gradient projections, respectively. It can be seen that control of $N_{g,1}^T J_u$ can be optimally given up when the control of g_1 becomes active, and the same happens with the pair $N_{g,2}^T J_u$ and g_2. This pairing results in optimal operation for all possible active constraint regions, with the switching being performed by max selectors on the controller outputs for this case study.

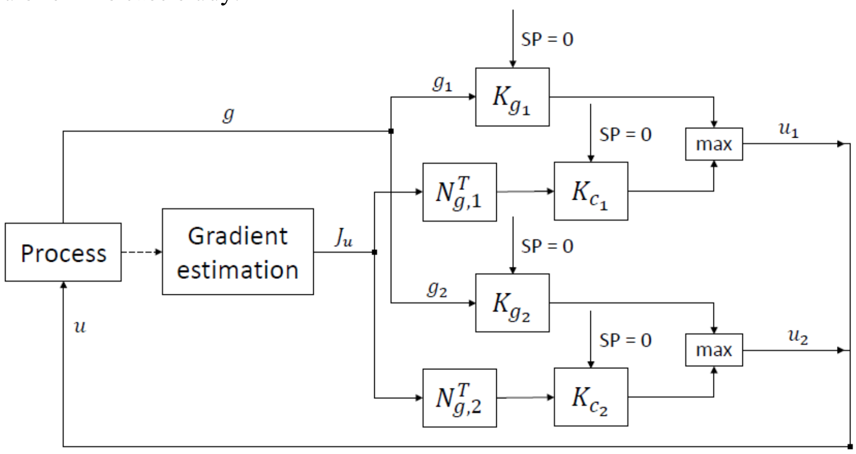

Figure 2: Control structure for optimal operation of case study

3.3. Simulation results

The behavior of the proposed control structure is illustrated by the simulations presented in Figure 3. The disturbance realizations were chosen such that the system operates at all active constraint regions. In the simulations, J_u was obtained using the automatic differentiation tools from CasADi (Andersson et al., 2019) with known disturbances. In practice, an estimator would be needed if the disturbances are not measured.
The system starts at the fully unconstrained region, where perfect optimal operation is possible because the cost gradient, J_u, is known. Once the system moves to the region $\mathcal{A} = \{g_1\}$ at $t = 4$ h, constraint violation is avoided as $g_1 = 0$ becomes the CV chosen by the selector. Even though the inputs are not driven to their optimal value, the choice of CVs is such that low economic loss is achieved. At $t = 6$ h, the disturbance value is equal to the nominal point, where the linearization of the constraints was performed. For this reason, the controlled cost gradient projection $N_{g,2}^T J_u$ corresponds exactly to the optimality conditions, and perfect optimal operation is attained. At $t = 8$ h, the system starts operating at the region $\mathcal{A} = \{g_1, g_2\}$. As all degrees of freedom are associated to active constraint control, perfect optimal operation is achieved. From $t = 12$ h, the system operates at $\mathcal{A} = \{g_2\}$, and near-optimal operation is achieved with low economic loss.

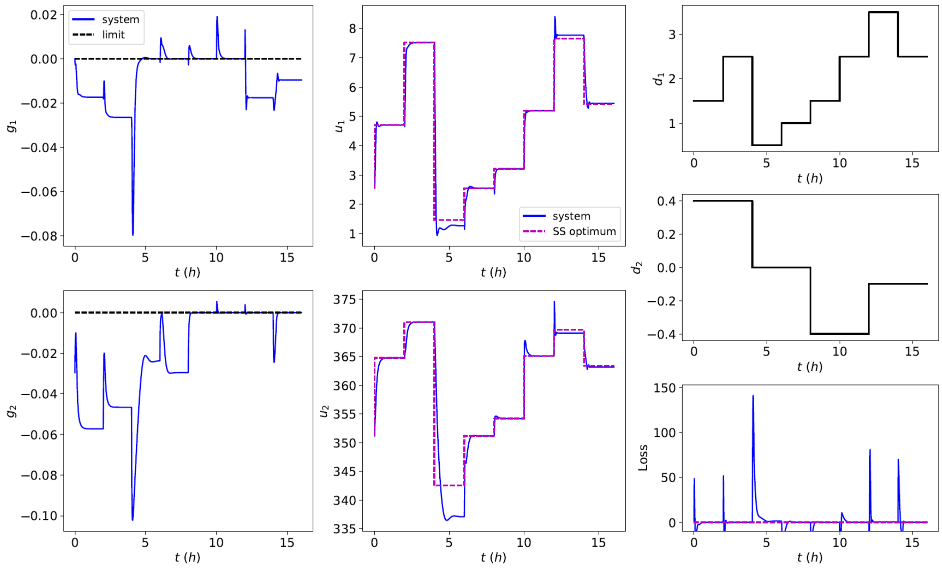

Figure 3: Dynamic simulation of proposed control structure

The steady-state economic loss for the proposed control structure is presented in Figure 4 as a function of the disturbances. Similar to what was observed in the dynamic simulations (Figure 3), the regions with nonzero loss are concentrated at the partly constrained regions. This is because the optimal CVs corresponding to the cost gradient projections change due to the nonlinearity of constraints. In contrast, the fully constrained and fully unconstrained regions mostly present zero operational loss, because the optimal CVs remain constant inside these regions, and the information about J_u is accurate. The linearization of the constraints was performed inside the region $\mathcal{A} = \{g_1\}$, where the operational loss is effectively zero at this reference point, and it increases as the system moves away from it. It can also be seen that optimal switching between regions is subject to errors related to the linearization of the constraints. The largest operational loss is obtained in the switching between regions $\mathcal{A} = \{g_1, g_2\}$ and $\mathcal{A} = \{g_2\}$, where an inaccurate cost gradient projection is tracked, that is, controlling $N_{g,1}^T J_u = 0$ does not lead to exact optimal operation. Therefore, the switching is not performed at the exact optimal boundary, as can be seen in the results, but it nonetheless leads to a reasonable switching policy between regions, and good resulting economic performance.

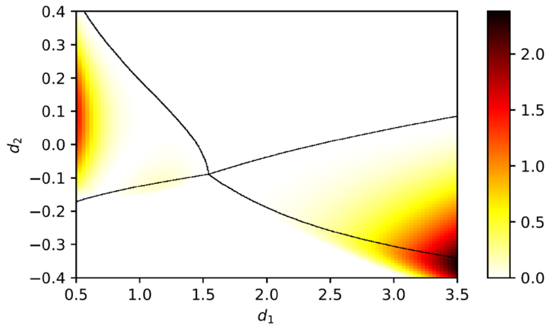

Figure 4: Steady-state economic loss of the proposed control structure as a function of disturbances (black lines represent optimal region switching)

4. Discussion

The control structure resulting from the framework proposed in this work (Figure 2) makes use of selectors as simple elements for switching between operating regions. These elements are frequently used in practice for coordinating conflicting control objectives. In this work, we show how these elements can be used for the optimal operation of systems under changing active constraints in a systematic manner. The min/max nature of the selectors is ultimately related to the nature of the constraint with respect to the input (Krishnamoorthy and Skogestad, 2020).

The approach proposed in this work is based on the analysis of the partly constrained regions, where we find gradient projections that can be optimally controlled. These projected gradients are also the optimal CVs of other regions, which minimizes the number of control loops that are necessary to account for all regions. Moreover, the proposed switching operates independently for each plant input, which means that the detection of each constraint is done independently, and feasible operation is safely achieved. However, the reconfiguring of CVs done by the selectors may significantly change the interactions between loops, and therefore careful tuning of the controllers is necessary, such that a good performance is achieved regardless of which loops are active.

5. Conclusion

In this work, we propose a framework for decentralized optimal operation under changing active constraints, applicable to a class of multivariable problems. Even though the approach is based on the linearization of the constraints, and therefore the quality of the linearization plays a relevant role in the economic performance, the strategy proved to be successful in a nonlinear case study, which encourages its use in other relevant problems of process systems engineering, especially when the gains from the inputs to the constraints do not change greatly in the operating range. The use of adaptive cost gradient projections would also be beneficial for improving economic performance. More theoretical aspects regarding the proposed framework will be expanded in future work.

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