Bidirectional Inventory Control with Optimal Use of Intermediate Storage and Minimum Flow Constraints ⋆

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Abstract: Bidirectional inventory control has been shown to solve the problem of maximizing the production of units arranged in series, through automatic reconfiguring of the inventory control loops, when temporary or permanent bottlenecks occur in any section of the process. This control system deals with constraints related to maximum flow, but minimum flow constraints are also typical in process systems to avoid improper operation. This work proposes an extension to the bidirectional inventory control structure that incorporates minimum flow constraints, through the use of additional level controllers with intermediate setpoints, and additional selector blocks. The order in which the selectors are implemented indicates the priority for giving up on the controlled variables, and the intermediate setpoint values affect how long the process can run in feasible operation. The proposed control structure successfully prevents constraint violation when the problem is feasible, retaining the reconfiguring properties of bidirectional inventory control.

Keywords: Decentralized control, inventory control, selectors, throughput manipulator.

1. INTRODUCTION

When operating a chemical plant, the periodic shutdown of units becomes necessary, either due to planned maintenance, or due to failures that must be corrected. To decouple the effect of these temporary shutdowns, buffer tanks are employed, such that production is continued using the accumulated inventory, or accumulated until unit reactivation, without compromising the overall processing rate. The same principle is applied to maintaining a constant production rate in periodic batch/semi-continuous processes (Karimi and Reklaitis, 1983). Managing the inventory of these units during operation becomes then paramount for maximizing the processing rate of the plant during these events. The problem of inventory management face to disturbances is also relevant for supply chain management (Schwartz and Rivera, 2010).

The installation of buffer tanks is especially important around critical units that operate normally at full capacity, since unnecessary shutdown of these units leads to irrecoverable losses that could otherwise be avoided with proper planning. The unit that limits the overall processing capacity of a plant is known as the process bottleneck. In terms of maximizing production, a good idea is therefore to set the production rate of the plant close to this bottleneck (Downs and Skogestad, 2011). The valve that sets the overall production rate of a plant is known as the throughput manipulator (TPM), and the control of the inventories must be defined as a function of this TPM. For a consistent inventory control layer, the input-output pairs should radiate from the chosen TPM, being in the direction of the flow for downstream units, and opposite to the flow direction for upstream units (Price et al., 1994). This rule is sufficient for processes with units arranged in series, which are the focus of this work.

The management of buffer levels when shutdowns occur is often performed by the plant operators, which switch the affected inventory controllers to manual mode, until normal operation is restored. If the planned stop is long, this may be accompanied with some accumulation prior to the unit shutdown, such that production may be continued without problems. This strategy, although often optimal, needs human intervention, and therefore an automatic control framework that deals with this issue is desired. On the other hand, the control strategies often employed in the literature rely on dynamic models and optimization (Chong and Swartz, 2013; Boucheikhchoukh et al., 2022), which is costly to implement, and does not reflect the simplicity of the policy that is implemented by experienced operators. The bidirectional control structure, presented by Shinskey (1981) and further discussed in Zotică et al. (2022), is able to solve this problem in a simple automatic control framework, comprised of PI controllers and selectors.

The main idea of the bidirectional control structure, shown in black in Figure 1, is to use the inventory of each unit for maximizing the time in which the process can run with maximum throughput. At steady state, the control structure treats the process bottleneck as the TPM, as it is saturated and therefore cannot be used for inventory control. As a consequence, due to the reconfiguring logic of

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the control loops, all downstream inventories will operate at the minimum level $M_i = L$ controlled by the respective outlets z_i, and all upstream inventories will operate at the maximum level $M_i = H$ controlled by the respective inlets z_{i-1}. If a new bottleneck is introduced anywhere before the current TPM, reducing the flow at that point, the downstream unit inventory is depleted until the minimum, becoming then controlled by its outlet, generating a cascade effect that ends with the previous TPM being used for inventory control of the unit before it. This results in an effective change in the TPM position in the process, since the introduced bottleneck sets the production rate. A similar analysis can be made for new bottlenecks after the original TPM. The variation in the inventories affected by this chain of events serves as a buffer time in which the system can operate with the same overall production, and if the introduced bottleneck is active only during a small period, the system can revert to its original behavior without affecting production.

Although the solution given by bidirectional level control is valid for processes with varying bottlenecks, care must be taken when implementing this control structure in processes where a minimum flow must be guaranteed in certain sections of the process, a limitation that naturally appears for some types of equipment. In these cases, if the inventory before the constrained section is critically low, or if the inventory after it is critically high, there is no margin for satisfying this constraint dynamically. A reasonable strategy in these cases would be then to use an intermediary value for the inventory of the neighboring units, so that there is enough dynamic margin for satisfying these constraints, as well as temporary bottlenecks. Together with this, additional control logic must be implemented, in order to automatically reconfigure the control structure when these minimum flow constraints become relevant. The additional control logic for dealing with minimum flow constraints is the novelty presented in this paper.

Mathematically, the goal can be defined as the maximization of the overall production $\bar{F}$ over a sufficiently long time horizon T of a series of N buffer inventories subject to constraints in the manipulated variables, which are the valve positions z_i, $i = 0, \ldots, N$, and constraints in the inventory levels M_i, $i = 1, \ldots, N$, according to:

$$
\begin{aligned}
\max_{z(t)} \; \bar{F} = \frac{1}{T}\frac{1}{N+1}\sum_{i=0}^{N}\int_0^T F_i(t)\; dt & \\
\text{s.t. } M_i^{min} \le M_i(t) \le M_i^{max}, \qquad & i = 1, \ldots, N \\
z_i^{min} \le z_i(t) \le z_i^{max}, \qquad & i = 0, \ldots, N \\
\frac{dM_i}{dt} = \frac{1}{V_i^t}\left(F_{i-1} - F_i\right), \qquad & i = 1, \ldots, N \\
F_i = C_i z_i, \qquad & i = 0, \ldots, N
\end{aligned}
\tag{1}
$$

Here, V_i^t represents the total capacity of the i-th inventory, and C_i represents the flow coefficient of the i-th valve. While bidirectional level control can be used to solve this problem when only the constraints $z_i \le z_i^{max}$ and $M_i^{min} \le M_i \le M_i^{max}$ are relevant, through the previously described logic, this work considers the case where the constraints $z_i \ge z_i^{min}$ may also be activated during operation. To solve this problem, we propose an extension to the bidirectional inventory control framework, shown in red in Figure 1. In the proposal, we account for the minimum flow constraints by using controllers with intermediary setpoints and additional selectors. This portion of the control logic will be active as long as it is feasible to satisfy the minimum flow constraints. The control framework will be now presented and exemplified in a case study.

2. PROPOSED CONTROL STRUCTURE

In this work, we consider a system of three tanks in series, described in Zotică et al. (2022). The physical parameters necessary to simulate the system are reproduced in Table 1. For simplicity, we consider that only z_2 is subject to a minimum flow constraint, but the approach can naturally be extended to include minimum flow constraints on other sections of the process. For designing the inventory control layer, the desired closed-loop time constant is chosen as $\tau_c = 0.5$ min, and is used as the tuning parameter for tuning the PI controllers following the SIMC rules (Skogestad, 2003). All controllers are implemented with antiwindup action, using the backcalculation strategy (Åström and Rundqwist, 1989) with tracking time constant $\tau_T = \tau_I/4$, where τ_I is the controller integral time.

The proposed control structure is presented in Figure 1. In this structure, normal inventory control done by z_2

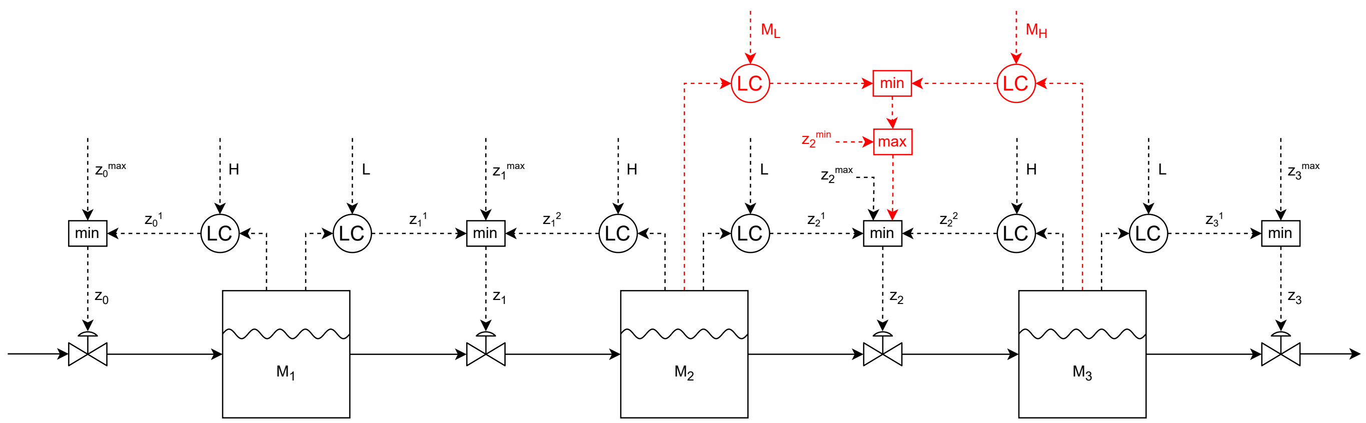

Fig. 1. Proposed bidirectional inventory control structure with minimum flow constraint handling (black denotes original bidirectional structure, red denotes the addition proposed by this work)

i	C_i [m^3/min]	V_i^t [m^3]
0	1	-
1	1.25	2.3
2	1.428	4.2
3	1.667	6.4

Table 1. Physical parameters for the three tank system

aims for intermediary setpoints M_L and M_H. This normal mode may be overriden by the minimum flow constraint $z_2 \geq z_2^{min}$, and in this case inventory control is given up until normal operation is reestablished, or until the inventory control that was given up reaches the associated critical value, L or H. In this moment, inventory control must be resumed, to avoid complete inventory depletion or overflow.

In terms of satisfying the minimum flow constraint on z_2, it is desired that a high value for M_L and a low value for M_H are selected, as this allows for continued transfer from M_2 to M_3 when temporary bottlenecks appear. However, this conflicts with the usage of the buffer inventories for maximizing production, as the solution provided by the bidirectional inventory control dictates. This shall be evidenced by the simulations presented next.

3. SIMULATION RESULTS

We present simulation results for two control structures:

- Proposed bidirectional structure with minimum flow constrain on F_2 (Figure 1).
- Simple bidirectional structure (without the red parts in Figure 1).

The simulations represent some common disturbance scenarios, allowing us to highlight when the current proposal succeeds or fails. Under all simulations, a minimum flow at z_2, $z_2^{min} = 60\%$, is desired, and the high and low level setpoints are chosen as $H = 90\%$ and $L = 10\%$. In addition, as a compromise for maximizing immediate production and satisfying minimum flow constraints, the intermediate setpoints are initially chosen as $M_H = M_L = 50\%$. This choice will be further analyzed.

First, we show that the structures have similar behavior when the disturbances do not affect the minimum flow constraint, and when these disturbances are only present for a short period. In Figure 2 we present a simulation where the steady-state bottleneck is at the process outlet with $z_3 = 100\%$, and therefore all nominal inventories are at a high state. At $t = 10$ min, a temporary bottleneck is introduced at the process inlet, with $z_0 = 50\%$, and due to that, all levels are depleted, in order to keep production at the maximum. When the temporary bottleneck is removed at $t = 40$ min, all tanks are filled back, and operation is back to steady state, without affecting the production at the steady-state bottleneck. When the production rate is changed at the TPM at $t = 80$ min and $t = 100$ min, both control structures quickly respond accordingly, due to all inventory controllers being active. Instead, if the temporary bottleneck on z_0 is instead removed at $t = 90$ min, see Figure 3, both control structures must reduce the flow at the original bottleneck. It can be seen that the proposed control structure must reduce the flow at z_3 earlier than the simple bidirectional structure, which is expected since there is less inventory to be used for rejecting the disturbance, as $M_3 = M_H$ initially. Additionally, both approaches tend to satisfy steady-state mass balances, working as consistent inventory control structures.

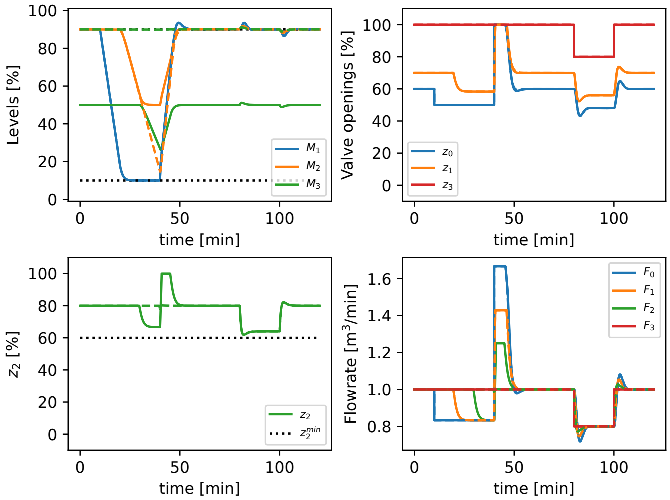

Fig. 2. Both control structures are able to maximize production at the bottleneck under temporary disturbances (continuous lines represent the proposed structure, dashed lines represent simple bidirectional control) — simulation with TPM at z_3 with short flow reductions at z_0 and z_3

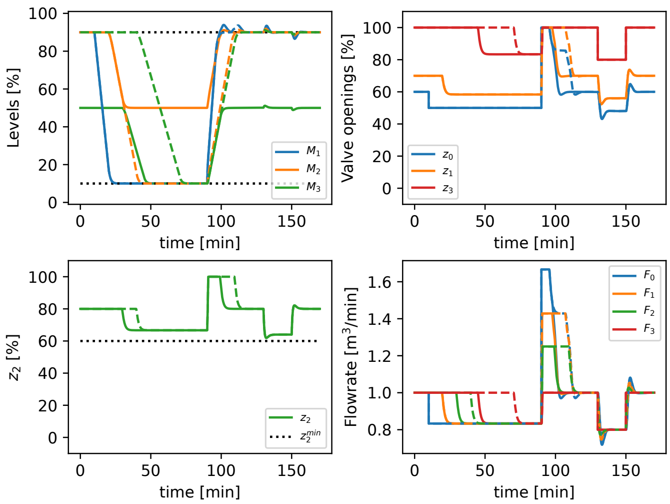

Fig. 3. Long disturbances force reduction on production at steady-state bottleneck, with the proposed structure (continuous lines) being affected before simple bidirectional control (dashed) — simulation with TPM at z_3 with flow reductions at z_0 and z_3

We now consider cases where the minimum flow constraint may be violated, and the differences between the control structures are highlighted. In the simulation presented in Figure 4, the steady-state bottleneck is still at the process outlet, but greater temporary bottlenecks are introduced. As the first disturbance, z_1 is lowered to 40% at $t = 7$ min, which forces M_2 to be emptied. While the inventory is uncontrolled until $M_2 = L$ for the simple bidirectional structure, the proposed framework activates the minimum flow constraint when M_2 reaches M_L, since it cannot keep normal inventory control at $M_2 = M_L$ without violating z_2^{min}. That behavior can be kept until $M_2 = L$, but as the temporary bottleneck on z_1 is removed before the inventory reaches its critical value (at $t = 20$ min), feasibility is maintained. This behavior comes at the expense of slightly affecting the steady-state bottleneck, as M_3, which started from M_H, was depleted during the event. Afterwards, at $t = 40$ min, the original TPM (z_3) is further constrained to 50%, forcing all the flows in the bidirectional control structure to drop almost immediately to attain inventory control, which leads to infeasible operation. The proposed control framework is able to use the margin between $M_3 = M_H$ and $M_3 = H$ to satisfy the minimum flow constraint, and after M_3 reaches the upper limit, operation becomes infeasible until the bottleneck is removed at $t = 60$ min. After that, M_3 must be emptied out until M_H while satisfying $z_2 \geq z_2^{min}$ in the proposed framework, while all the flows go immediately up for the simple bidirectional scheme. In terms of mass balances, the proposed framework is forced to operate in imbalance for the longest possible time, due to the minimum flow constraint.

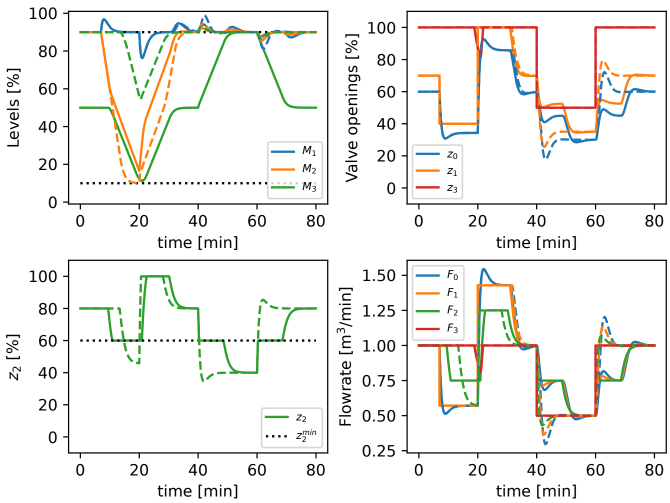

Fig. 4. The proposed structure (continuous line) allows for feasible operation during longer periods than simple bidirectional control (dashed) — simulation with TPM at z_3 with larger flow reductions at z_1 and z_3

For the first disturbance in Figure 4, it is interesting to note that there is an inversion of behavior during operation. In a very short timescale, while M_2 is between H and M_L, the control structures behave equally. If the disturbance continues to be active, the original bidirectional control becomes best performing, as it still maximizes the flow through z_2, while the proposed control structure is more conservative. In the longer run, however, the original bidirectional structure loses feasibility first, and in that case the proposed control structure performs best.

Figure 5 illustrates the case where the bottleneck is at the process inlet, with $z_0 = 50\%$, and therefore all inventories are initially at the lower state. The introduced disturbances in the simulation are $z_0 = 40\%$, from $t = 5$ min to $t = 20$ min, and $z_3 = 60\%$, from $t = 45$ min to $t = 70$ min. Analogously to the previous simulation, the reduction on z_0, which was the original TPM, immediately makes traditional bidirectional control infeasible, whereas the proposed framework uses the buffer from $M_2 = M_L$ until $M_2 = L$ to keep feasibility. The temporary bottleneck on z_3 leaves M_3 uncontrolled in the simple bidirectional control until it reaches the limit H, whereas the minimum flow constraint becomes active in the proposed framework when M_3 passes through M_H. The proposed control structure is able to keep feasibility for both disturbances, while simple bidirectional control violates the constraint on both cases.

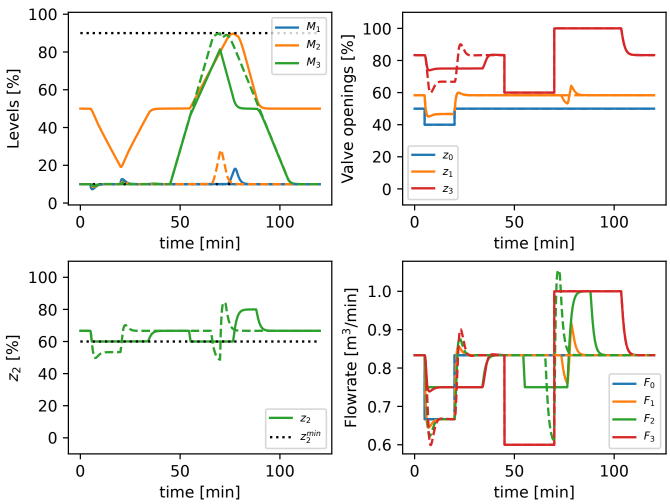

Fig. 5. The proposed structure (continuous line) completely avoids violating the minimum flow constraint, as opposed to simple bidirectional control (dashed) — simulation with TPM at z_0 with flow reductions at z_0 and z_3

It must also be noted that changes in the intermediary setpoint values may improve response face to some disturbances. Figure 6 illustrates the effect of raising all intermediary inventory setpoints when the process bottleneck is originally at its outflow, z_3. In this case, the disturbances are $z_1 = 40\%$ from $t = 5$ min to $t = 20$ min, and $z_3 = 65\%$ from $t = 50$ min $t = 65$ min. A larger gap between M_L and L allows for improving operation when bottlenecks appear before z_2, in the sense that the use of a higher intermediate setpoint lets the system operate with feasibility for longer. In addition, the higher M_H is, the slower the inventory M_3 is consumed, which allows for keeping z_3 unaltered for longer. Conversely, when z_3 is forced to be lowered, the gap between M_H and H dictates how long the system can be kept feasible, and a low M_H would be desired.

Figure 7 illustrates the case where feasibility is prioritized in operation, with high value of M_L and low value of M_H. The original bottleneck is at $z_0 = 50\%$, and the tested disturbances are $z_0 = 40\%$ from $t = 5$ min to $t = 35$ min, and $z_3 = 65\%$ from $t = 75$ min to $t = 120$ min. While the use of normal intermediary setpoints fail to keep the process feasible face to these disturbances, the adjust of setpoints allow for that end, at the expense of reducing the flow at the steady-state bottleneck.

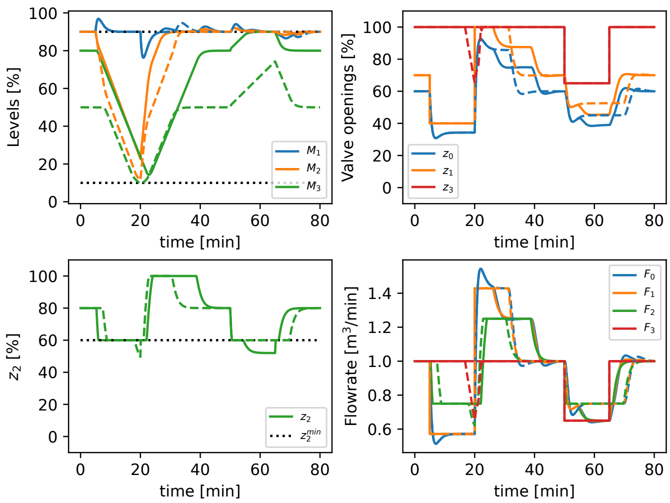

Fig. 6. With TPM at z_3, higher intermediary setpoints (continuous line, $M_H = M_L = 80\%$) improve operation when inlet is disturbed, but worsen performance when outlet is disturbed (dashed lines represent $M_H = M_L = 50\%$)

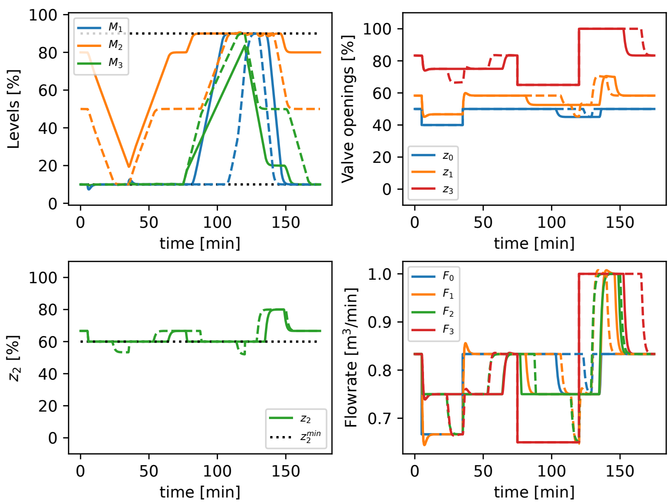

Fig. 7. With intermediary setpoints farther from critical values (continuous line, $M_H = 20\%$, $M_L = 80\%$), the period of feasible operation is maximized for disturbances on z_0 and z_3 (dashed lines represent $M_H = M_L = 50\%$)

4. DISCUSSION

The case study presented in this paper aims to reproduce a simplified version of unit operations in series. These unit operations are for simplicity represented as valves, and the buffer tanks represent the holdups between units. Therefore, for the simulations presented in this work, temporary constraints on maximum allowed valve opening represent temporary reductions on the operating capacity of the units. For example, the case presented in Figures 2 and 3 refers to a hypothetical process with a steady-state capacity bottleneck on the last unit ($z_3 = 100\%$), and all other units operate below their nominal capacity. The temporary bottleneck introduced by setting $z_0 = 50\%$ represents a temporary limit on the processing capacity of that unit. Finally, the unit represented by z_2 must always operate above a certain throughput ($z_2^{min} = 60\%$), so that abnormal operation is avoided.

As evidenced by the presented results, the margin between intermediary and extreme inventory levels is used for satisfying minimum flow constraints during transients. However, satisfying this constraint is not always feasible, since the mass balances are forcibly not satisfied to attain feasibility. This contrasts with the principle of consistent inventory control, which states that mass balances should be satisfied at steady state with the proposed control structure. Due to this, satisfaction of the minimum flow constraint must be given up, being overriden by consistent inventory control loops at critical inventory levels. It must be noted that the problem of maximizing the flow through a series of tanks is always feasible, and it is solved automatically by the bidirectional inventory control structure.

As can be noted from the bidirectional control structure, the use of min selectors automatically yields the maximum feasible inputs, since the inputs that were not selected would violate the objective corresponding to the input that was selected. Although this maximizes production, a max selector must be used to check for violation of the minimum flow constraint. Since satisfying the minimum flow constraint may not always be feasible, such constraint must be placed at a lower priority than level control at extreme conditions. Similarly to Krishnamoorthy and Skogestad (2020), where min and max selectors are combined for optimal switching between constraints at steady state, the order in the implementation of the selectors is related to the order in which the objectives must be given up. Therefore, if the minimum flow constraint is to be given up face to critical inventory levels, the max selector must be implemented before the min selector related to the extreme inventory control loops in Figure 1.

While the extreme inventory levels are defined by conditions such as drying out or overflow, there are several conflicting objectives when selecting the intermediary inventory setpoints, as shown by the simulations. In some cases, the override with the minimum flow constraint can be regarded as too conservative, since for short enough disturbances this override may prove unnecessary. This requires some knowledge on the nature of the expected disturbances, so that a reasonable value for those setpoints is selected. For instance, if the minimum flow constraint was to be fully prioritized, the choice of intermediary setpoints as $M_H = L$ and $M_L = H$ would maximize the time for which the system can run feasibly, at the expense of bypassing the buffering ability of the inventories for dealing with temporary bottlenecks. On the other hand, choosing $M_H = H$ and $M_L = L$, which is the same as removing the red portion of Figure 1, maximizes production under bottlenecks, ignoring the minimum flow constraints. The selection of $M_L = M_H = 50\%$ is the more conservative approach, when information about the possible scenarios is not available. If the buffer tanks are large enough, this choice will be sufficient to reject all types of disturbances, whether they affect the minimum flow constraint or not.

It was shown in Figure 3 that implementing intermediate level setpoints leads to some loss in terms of rejecting temporary bottlenecks, since the period for which the system can run with maximum production is proportional to the gap between low and high inventory setpoints. The bidirectional inventory control structure implements the optimal policy of maximizing production at the bottleneck, constrained to the inventory bounds, and the control logic added on top of it makes a compromise between this objective and minimum flow constraints.

In industrial applications with minimum flow constraints, if such constraint is to be violated, the system must be shut down to prevent equipment damage. This is done until inventories are restored to operational levels, and operation can be then restarted. Instead, if a shutdown is not desired, the minimum flow constraints can be often dealt with by anti-surge systems, which recycle part of the outflow of the unit so as to guarantee minimum flow. However, this solution may be too expensive, since it generates a recycle flow, which is in turn tied to more pumping costs in the operation. The control structure proposed in this work may reduce these costs while normal operation is feasible, and can also be overriden by anti-surge control when the system reaches critical levels, which is a simple and effective way of solving the feasibility issues of the current proposal.

If the optimization problem from Equation (1) was to be solved through dynamic optimization, assumptions about the nature of the disturbances should be clearly made. For example, if economic MPC was to be implemented in the present case study, the optimal levels for operation would not matter, unless some disturbance is expected. Therefore, in order to determine the optimal operating inventory levels, and to make a compromise between the conflicting cases we presented in the simulations of this work, robust approaches such as multi-stage NMPC (Lucia et al., 2014) should be employed. This would come at the expense of implementation complexity, in terms of system and disturbances modeling, and the high computational cost inherent to the tool. The control structure presented in the current work aims to solve the operational problem using simple control structures, and the compromise between objectives is done by setting reasonable values to the setpoints of the inventory control loops. This can be done with offline analysis through simulation of the different disturbance scenarios, but it can also be easily done manually after implementation. Such flexibility is hardly obtained when using centralized optimization-based strategies.

5. CONCLUSION

The control structure proposed in this work was able to account for constraints related to minimum allowed flow, when satisfying these constraints is feasible dynamically. When compared to the original bidirectional control structure, similar behavior is observed when the system is far from the constraint, and a more conservative behavior is observed when the constraint becomes active, where feasible operation is favored over maximizing immediate production. The implementation of this control structure is therefore recommended when constraints related to maximum and minimum allowed flow must be considered simultaneously, together with other strategies that ensure feasibility, such as anti-surge loops.

The main limitation of the proposed control strategy is that, being built as an extension of the bidirectional inventory control, it only considers a linear arrangement of the inventories. As splits and recycles are very common in process systems, an interesting topic of research would be to propose extensions to this framework to more complex arranges of inventories, such that production maximization is achieved, respecting operational constraints, using simple feedback control elements.

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