

Systematic Pairing Selection for Economic-oriented Constraint Control

Risvan Dirza, and Sigurd Skogestad*

*Dept. of Chemical Engineering, Norwegian Univ. of Science & Technology (NTNU),
NO-7491 Trondheim, Norway
sigurd.skogestad@ntnu.no*

Abstract

This work considers the problem of minimizing economic losses due to system-wide production systems, where different subsystems share hard coupling constraints. The hard coupling constraints need to be tightly controlled, and it is important that it is done in a way that the overall system remains close to optimal in the time it takes for the much slower optimization layer to implement the required input changes. The particular application that we study is a large-scale subsea gas-lifted oil production network, where different subsystems have a local objective and the shared constraint can be a common compressor, but the method has general applicability to any system with time-scale separation between control and optimization layers.

Keywords: Production Optimization, Self-optimizing Control, Active Constraint Control.

1. Introduction

Determining optimal operation of a large and complex process and production system, such as an oil and gas production system, is a challenging task. Decomposing the process into several subprocesses/subsystems is usually recommended since optimizing a small system is practically less complex. Thus, decomposition strategy requires each local process system/cluster/subsystem to have a local optimizer to ensure local optimal process operation. This decomposition strategy is also responsible for coordinating these subsystems to achieve system-wide optimal process operation. The optimal process operation involves making decisions in real-time to meet production goals. This is typically done in the context of real-time optimization (RTO) using process models and real-time measurements. RTO is developed based on mathematical concepts, and with it, production performance improved.

In the 80s, there was an increasing interest in replacing model-based numerical solvers with a simple feedback loop, named feedback-optimizing control. The idea is to translate the economic objective into process control objective by finding a function of the controlled variables (CVs), and when it is held constant, it leads to the optimal adjustment of the manipulated variables (MVs) (Morari et al., 1980). Twenty years later, Skogestad (2000) introduced the concept of self-optimizing control (SOC). In SOC, when the optimum lies at some constraints, we use active constraint control where the available MVs tightly control the constrained variables. The idea of tight active constraint control is one of the primary motivations of this work to ensure the feasibility and obtain a (near-) optimal process operation.

2. Problem Statement

Consider the following steady-state optimization problem of N different subsystems.

$$\min_{\mathbf{u}} J(\mathbf{u}, \mathbf{d}) = \sum_{i=1}^N J_i(\mathbf{u}_i, \mathbf{d}_i) \quad (1a)$$

$$s. t. \quad g(\mathbf{u}, \mathbf{d}) \leq 0 \quad (1b)$$

where $\mathbf{u}_i \in \mathbb{R}^{n_{u_i}}$ denotes the MVs for subsystem i , n_{u_i} is the number of MVs in subsystem i , and $\mathbf{u} = [\mathbf{u}_1 \ \dots \ \mathbf{u}_N]^T$, $\mathbf{d}_i \in \mathbb{R}^{n_{d_i}}$ denotes the disturbances in subsystem i , n_{d_i} is the number of disturbances in subsystem i , and $\mathbf{d} = [\mathbf{d}_1 \ \dots \ \mathbf{d}_N]^T$, $J_i: \mathbb{R}^{n_{u_i}} \times \mathbb{R}^{n_{d_i}} \rightarrow \mathbb{R}$ is a function denoting the local objective of subsystem i , $g: \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \rightarrow \mathbb{R}^{n_g}$ is a function denoting the inequality constraints. n_g is the number of constraints.

The Lagrangian function of problem (1) is

$$\mathcal{L}(\mathbf{u}, \mathbf{d}, \lambda) = \sum_{i=1}^N J_i(\mathbf{u}_i, \mathbf{d}_i) + \lambda^T g(\mathbf{u}, \mathbf{d}) \quad (2)$$

where $\lambda \in \mathbb{R}^{n_g}$ is the shadow price/ Lagrange multiplier of active constraints $g(\mathbf{u}, \mathbf{d})$.

The goal of problem (1) is to determine optimal MVs to achieve system-wide steady-state optimal operation. Our motivation is to solve problem (1) using a feedback control structure that handles changing active constraints.

One possible approach is primal-dual feedback-optimizing control that can eliminate the need for a numerical solver (Krishnamoorthy, 2020; Dirza et al., 2021). Moreover, this approach is flexible in handling active constraint changes. This method has a central coordinator acting as a central constraint controller in a slow timescale in the upper layer. However, this approach has no near-optimal performance strategy due to the non-performing upper layer. There are many practical reasons why the central constraint controller may fail to update the Lagrange multipliers. For example, when the disturbance occurs much faster than the sampling time of the central constraint controller. Another example is when constrained variables from a local system are not updated on time since the optimizer of the other local system may need a numerical solver. This solver requires time to solve the optimization problem. Having different types of local optimizers is normal since every subprocess is unique. In addition, having a central constraint controller in a very slow timescale leads to a longer transient. Constraint violation may occur during this transient. These imply that it is essential to have a good pairing of a (primal) MV to a constrained variable. Thus, “*a systematic pairing procedure is necessary to determine which MV should be paired with a constrained variable such that the pairing minimizes the loss most (near-optimal performance)*”. Finally, this procedure is necessary for selecting the primal MV in primal-dual with direct constraint control proposed by Dirza et al., 2022.

3. Systematic Pairing Formulation

To pair the constrained variables with the right MV, we propose a pairing procedure based on MV's sensitivities to its local disturbances, assuming no saturation issues in the possible MVs, no back-off problem, and equal value of constraints - MVs gain.

Meanwhile, the remaining MVs control their self-optimizing control variables. To describe this proposal, we consider an Indirect control problem formulation.

Without losing the generality, we consider a case where we have two MVs (i.e., $u_1 \in \mathbb{R}^1$ and $u_2 \in \mathbb{R}^1$), and we want to control the gradient of the Lagrange function w.r.t its input, denoted by $\mathcal{L}_{\mathbf{u}}(\lambda, \mathbf{u}) \in \mathbb{R}^{2 \times 1}$, and the active constrained variable, denoted by $g(\mathbf{u}) \in \mathbb{R}^1$, where λ is Lagrange multiplier for constraint function $g(\mathbf{u})$, and $\mathbf{u} = [u_1 \ u_2]^T$. $\mathcal{L}_{\mathbf{u}}(\lambda, \mathbf{u})$ consists of $\mathcal{L}_{u_1}(\lambda, u_1) \in \mathbb{R}^1$ and $\mathcal{L}_{u_2}(\lambda, u_2) \in \mathbb{R}^1$.

Assume that we want to control the constrained variables tightly with u_2 , and we consider the disturbance, $\mathbf{d} \in \mathbb{R}^{2 \times 1}$, influences input u_2 . It could be the local disturbance of subsystem 2 or a change of u_2 caused by setpoint changing. This setpoint change may occur due to the changes in subsystem 1. Since λ is constant (due to the non-performing upper layer), and $g_{u_1}(u_1)$ is also in many cases (i.e., resource allocation), we only need to control $J_{u_1}(u_1)$. This formulation can be written as an indirect control problem as follows,

$$J_{u_1}(u_1) = G_{11}u_1 + G_{12}u_2 \quad (3a)$$

$$g(\mathbf{u}) = G_{21}u_1 + G_{22}u_2 \quad (3b)$$

$$u_2 = G_d d + \hat{u}_2 \quad (3c)$$

where G_{11} is the gain from u_1 to $J_{u_1}(u_1)$, G_{12} is the gain from u_2 to $J_{u_1}(u_1)$, G_{21} is the gain from u_1 to $g(\mathbf{u})$, G_{22} is the gain from u_2 to $g(\mathbf{u})$ and G_d is the disturbance gain that influences u_2 .

Fig. 1 illustrates this formulation, where we want to ‘tightly’ control $g(\mathbf{u})$ to reference r_2 directly using a direct constraint controller (DCC). In addition, we also want to find the right u_2 such that u_2 can also contribute to controlling $J_{u_1}(u_1)$ to reference r_1 indirectly or by pairing u_2 with $g(\mathbf{u})$. This control structure has a better ability to control $J_{u_1}(u_1)$ than the other possible structure.

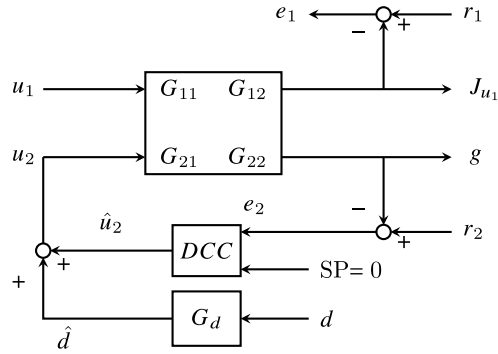


Figure 1: Indirect control problem formulation for systematic pairing

We assume that G_{22} is square and invertible. Otherwise, we can replace the solution with the pseudoinverse. By rearranging Eq. 1 and assuming a ‘perfect’ control $g(\mathbf{u}) \approx r_2$, we obtain $J_{u_1}(u_1) \approx G_{12}G_{22}^{-1}r_2$. Thus, we must choose r_2 such that $r_2 \approx G_{22}G_{12}^{-1}r_1$. According to Skogestad and Postlethwaite (2005), $G_{12}G_{22}^{-1}$ should be small. Usually, it implies that we need to select the pairing that gives the largest G_{22} , where $G_{22} = \nabla_{u_2}g(\mathbf{u})$. However, based on this formulation, selecting the pairing based on G_{22} is insufficient. This formulation shows that we should also consider small G_{12} in addition to large G_{22} in the framework of primal-dual approach with direct constraint control, especially when we have faster disturbances (or non-performing upper layer). Selecting based on G_{12} is then essential and complementary to the common rule.

We consider these rules as a near-optimal performance strategy for the primal-dual with direct constraint controls framework (Dirza et. al., 2022).

Defining $\hat{d} = G_d d$, then $G_{12} \approx \frac{\Delta J_{u_1}}{\Delta u_2} \approx \frac{\Delta J_{u_1}}{\Delta(\hat{d} + \hat{u}_2)}$. If one keeps $\hat{u}_2$ at the same value to control $g(\mathbf{u})$, then a change in $\hat{d}$ can represent any change. Considering Eq. 1c, then $G_{12} \approx \frac{\Delta J_{u_1}}{\Delta \hat{d}}$. Furthermore, assuming that the stationary point is at the local optimum and knowing that J_{u_1} is controlled by u_1 , and J_{u_2} is uncontrolled, then any disturbance on J_{u_1} leads to ΔJ_{u_2} ($\mathcal{L}_{u_2}$ being drifted away from 0). It implies that any disturbance on J_{u_1} leads to the total profit loss ΔJ . Therefore, we can estimate G_{12} as $\frac{\Delta J}{\Delta \hat{d}}$ ($G_{12} \approx \frac{\Delta J}{\Delta \hat{d}}$).

4. Numerical Results

We demonstrate the presented rules in a subsea gas-lifted oil production optimization problem with a fixed gas lift compressor described in Dirza et al. (2022). Moreover, we consider a subsea gas-lifted oil production well network that consists of two wells to provide a better demonstration. Fig. 2 illustrates the case study.

The objective function is to maximize the oil production income while minimizing the cost of the gas lift. The optimization problem is as follows,

$$\min_{\mathbf{w}_{gl}} J = \sum_{i=1}^N (-p_{o,i} w_{po,i} + p_{gl,i} w_{gl,i}) \quad (4a)$$

$$s. t. \quad \mathbf{f}(\mathbf{x}, \mathbf{w}_{gl}, \mathbf{d}) = \mathbf{0} \quad (4b)$$

$$\mathbf{g}(\mathbf{x}, \mathbf{w}_{gl}, \mathbf{d}) \leq \mathbf{0} \quad (4c)$$

$$\mathbf{g}_s(\mathbf{x}, \mathbf{w}_{gl}, \mathbf{d}) = Pow_{gl} - Pow_{gl}^{max} \leq 0 \quad (4d)$$

where $p_{o,i}$, $p_{gl,i}$, and $w_{po,i}$ are the price of produced oil, the cost of gas-lift, and the oil production rate of well i , respectively. Pow_{gl} is the total power consumed by a fixed compressor to inject the sum of gas-lift rate i , and Pow_{gl}^{max} is the maximum available power. The vector $\mathbf{x} \in \mathbb{R}^{n_x}$, and $\mathbf{d} \in \mathbb{R}^{n_d}$ are the vectors of states, and disturbance (i.e., gas-oil ratio) for the entire system. n_x is the number of states. $\mathbf{w}_{gl} \in \mathbb{R}^{n_{wgl}}$ is the vector of inputs for the entire system, where $\mathbf{w}_{gl} = [w_{gl,1} \ \dots \ w_{gl,N}]^T$. Constraint (4b) and (4c) represent model and physical constraints, respectively. We assume that Constraint (4c) is locally managed to maintain the focus of the discussion. Eq. (4a) is additively separable, and eq. (4d) is a linear and hard constraint. The total gas lift rate is supplied by a fixed-efficiency gas lift compressor.

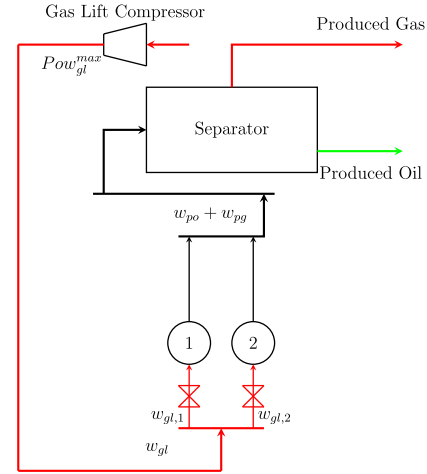


Figure 2: Field illustration

The simulation considers a case where we have a non-performing upper layer (λ is not updated). We show the numerical results of the presented near-optimal performance strategy (Structure 1). As a benchmark, we also show the results of the asynchronous protocol (Structure 0), that the local controllers keep controlling the gradient of the Lagrange function w.r.t input to 0, given any value of the

Lagrange multipliers from the central constraint controllers. In addition, we also show the results of another possible structure (Structure 2).

We solve the steady-state optimization problem (4) to obtain the ‘true’ optimal cost for any considered disturbance cases. We assume that based on historical data, the largest possible error of the disturbance is $\pm 5\%$. The (profit-) loss is the difference between the steady-state cost of structure j to the optimal cost, which can be expressed mathematically as $\Delta J_j = J_j - J^*$, where j is the index of the structure (*i.e.*, $j \in \{0,1,2\}$).

First, we simulate for any possible largest error for Structure 0. The simulation shows that the above case study experiences the largest possible disturbance that happens sequentially starting from $GOR_1 + 5\%$, $GOR_1 - 5\%$, $GOR_2 + 5\%$, $GOR_2 - 5\%$, $Pow_{gl}^{max} + 5\%$, and finally $Pow_{gl}^{max} - 5\%$.

As it can be seen in Fig. 3, Structure 0 fails to satisfy steady-state constraint when either GOR_1 , GOR_2 , or Pow_{gl}^{max} decreases 5% (see time window 18-32 hr, 48-62 hr, and 78-90 hr), which validates the necessity to have a near-optimal strategy in the primal-dual approach.

As mentioned in Section 3, the first general rule is pairing input and active constraint with the largest $G_{22,j} = \nabla_{w_{gl,j}} \mathbf{g}_s(\mathbf{x}, \mathbf{w}_{gl}, \mathbf{d})$. Based on this definition, $G_{22,1} = 3.6740$, and $G_{22,2} = 3.6740$, which corresponds to the assumption of equal value of constraints - MVs gain. This also validates the necessity to have an additional rule to select the pairing that gives the most economic-oriented result.

The additional rule is pairing input and active constraint with the smallest $G_{12,j}$, which one can estimate by calculating $\frac{\Delta J_j}{\Delta GOR_j}$ using the finite difference method. The obtained result is that the smallest $G_{12,1}$ is 1.4441, and the smallest $G_{12,2}$ is 1.4642. According to

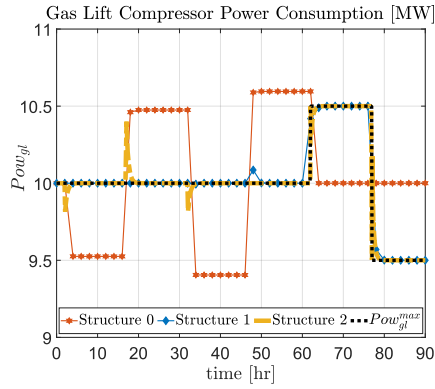


Figure 3: Steady-state constraint satisfaction

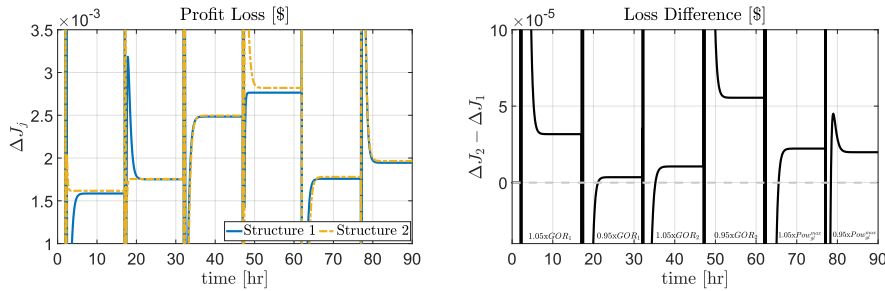


Figure 4: Left figure: Profit loss comparison. Right figure: Loss difference between Structure 1 and Structure 2.

the presented method in Section 3, this result indicates that the most economic-oriented pairing is Structure 1, where we pair the active constraint with $w_{gl,2}$.

Fig. 4 shows the profit loss comparison between Structure 1 and Structure 2, and the right figure shows that, at any possible extreme disturbance, Structure 1 can minimize more the steady-state loss than Structure 2. Additionally, Tab. 1 shows the steady-state profit loss for 24 hours with different extreme disturbance cases.

Table 1: Steady-state profit loss for 24 hours

Structure	GOR_1		GOR_2		Pow_{gl}^{max}	
	+5%	−5%	+5%	−5%	+5%	−5%
0	630.7200	sscv*	751.6800	sscv*	656.6400	sscv*
1	136.3423	151.6720	214.7048	238.8445	151.4331	168.2676
2	139.7161	151.6730	215.0000	243.8771	153.3618	169.9576

sscv*: steady-state constraint violation.

5. Conclusion

In this paper, we have shown that the proposed rule (smallest G_{12}) is complementary to the existing pairing rule (largest G_{22}), especially in the framework of the primal-dual approach. This systematic pairing selection procedure can assist the designer in pairing for economic-oriented constraint control in the primal-dual with direct constraint controls. In addition, this procedure can minimize steady-state loss in the primal-dual framework when we have a non-performing upper layer.

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