

## Optimal Operation of Heat Exchanger Networks with Changing Active Constraint Regions

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### Abstract

In this paper, we study the optimal operation of heat exchanger networks with stream splits. In particular, we extend previous approaches on the unconstrained optimization of the system to the constrained case, with temperature constraints on each flow branch, and with changing disturbances so that the set of optimally active constraints changes during operation. The simplest way to achieve optimal operation when some of the constraints are active, is to control the constraints to their limiting value, known as active constraint control. For the remaining unconstrained degrees of freedom, we propose to control linear combinations of the gradient as self-optimizing controlled variables. To automatically switch between the different active constraint regions, we use classical advanced control elements such as selectors, thereby achieving optimal operation using only the temperature measurements as feedback in different active constraint regions. The performance of the proposed feedback optimizing control structure for the heat exchange network is compared with the traditional model-based real-time optimization using simulations. In the presence of structural plant-model mismatch, we show that our proposed approach performs optimally for all disturbances, while traditional real-time optimization fails to converge for some cases, as the optimization problem becomes infeasible depending on the estimated disturbances.

**Keywords:** process control, optimal operation, self-optimizing control, applications

### 1. Introduction

In the context of optimal operation of process systems, the choice of controlled variables plays a vital role, as it will dictate how efficiently a process can operate without interference of higher layers (Skogestad, 2000). The ideal design of a supervisory control layer would result in a structure that is able to operate optimally under constant setpoints. This concept is known as self-optimizing control, and recent developments aim for systematic choice of control objectives (Krishnamoorthy and Skogestad, 2019). A known challenge in supervisory layer design is the change in optimally active constraints during operation, which can be caused by changes in disturbances that affect process objectives. When that happens, reconfiguration of the controlled structure is usually desired to minimize the operational losses. If that does not happen, interactions with the higher optimization layer become stronger, as the sensitivity of the optimal setpoint values with relation to the changing disturbances is high when there are no changes in the control structure. Krishnamoorthy and Skogestad (2019) discusses the handling of changes in active constraints through

feedback control, without the solution of online optimization problems, by selector-based control structures. This approach is to be evaluated in this work, compared to the solution of real-time optimization (RTO) problems, which can be problematic in the presence of model-plant mismatch.

## 2. Case study modeling

The case study considered in this work consists of three heat exchangers in parallel, see Figure 1. Each exchanger has its own source of hot fluid, such that the cold fluid is split and sent to the exchangers, and the operational goal is to maximize the outlet temperature of the cold fluid, subject to constraints related to the maximum temperature in the individual exchangers.

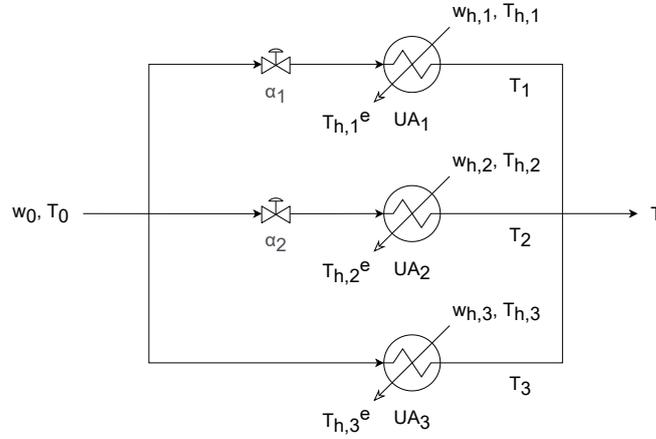


Figure 1: Heat exchanger network scheme

In addition to the mass and energy balances, an additional relation is necessary for calculating the total exchanged heat in each equipment,  $Q_i$ . The analytic solution, assuming constant heat capacities and countercurrent flow, is given by Eq.(1).

$$Q_i = UA_i \Delta T_{LM,i} \quad (1)$$

In this equation,  $\Delta T_{LM,i}$  represents the logarithmic mean of temperature differences inside the heat exchanger. Although exact, this model presents some numerical challenges, especially when the heat capacities are too close, or when the temperature differences assume opposite signs during iteration. A simplified linear version of this model makes use of the arithmetic mean of temperature differences,  $\Delta T_{AM,i}$ , and for this model, simple analytic expressions for the gradient can be derived (Jäschke and Skogestad, 2014).

The steady-state optimization problem considered for the optimal operation of this system can therefore be written as:

$$\begin{aligned}
\min_{\alpha} \quad & J = -T \\
\text{s.t.} \quad & g_i = T_i - T_{max} \leq 0, \quad i = 1, 2, 3
\end{aligned} \tag{2}$$

### 3. Proposed control structure

The optimal operation of heat exchanger networks has been extensively studied by Jäschke and Skogestad (2014) for the unconstrained case. In this case, the gradient  $J_u$  to be driven to zero can be approximately written in terms of the Jäschke temperatures. For the constrained case, however, the set of controlled variables need to change so that optimal operation is achieved. Given that the active constraints  $g_A$  are effectively controlled, there are still unconstrained degrees of freedom that need to be used for optimal operation. As proven by Krishnamoorthy and Skogestad (2019), we can find the additional controlled variables as a linear combination of the gradient such that the necessary conditions of optimality are satisfied. These correspond to  $c = N^T J_u$ , where  $N$  is the nullspace of the gradient of the active constraints with relation to the inputs,  $\nabla_u g_A$ , at the optimal point. This procedure results in a set of controlled variables per region, defined by the respective set of active constraints.

For this case study, there are 7 feasible operating regions, one of which is fully unconstrained, 3 being partially constrained (one active constraint per region), and the remaining being fully constrained (two active constraints per region). The case with all 3 constraints being active is infeasible with the available degrees of freedom, and will therefore not be considered. The fully unconstrained region can be optimally operated by controlling the plant gradient to zero, and the fully constrained regions are optimally operated through active constraint control. For the optimal operation in the partially constrained regions, the combinations of the gradient to be controlled in addition to the active constraints are given in Table 1.

Active constraint	$N^T$
$g_1$	$\begin{bmatrix} 0 & 1 \end{bmatrix}$
$g_2$	$\begin{bmatrix} 1 & 0 \end{bmatrix}$
$g_3$	$\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

Table 1: Linear combinations of gradient per active constraint

The next step for the design of a simple control structure is defining the pairing between manipulated and controlled variables, and the switching between active controllers. In the current case study, there are 2 manipulated variables and 3 constraints, which means that the constraints cannot be assigned to one specific input if optimal operation over all regions is desired. Therefore, at least one of the constraints needs to be controlled by multiple inputs.

Based on this reasoning, this work proposes an adaptive control structure to deal with all possible active constraint regions. The full control structure, showing the logic blocks and controllers, is presented in Figure 2. and the pairing between manipulated and controlled

variables is summarized in Table 2. All presented controllers have integral action, so that steady-state offset is eliminated.

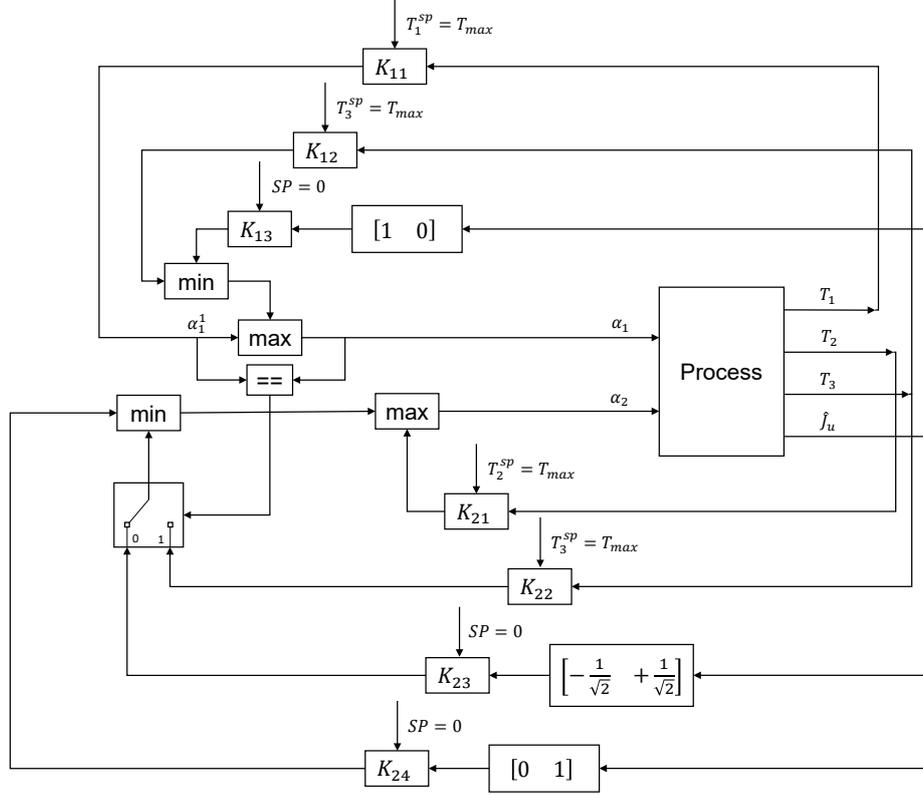


Figure 2: Proposed adaptive control structure

$\alpha_1$	$\alpha_2$ ( $T_1$ inactive)	$\alpha_2$ ( $T_1$ active)
$T_1$	$T_2$	$T_2$
$[1 \ 0] J_u$	$[0 \ 1] J_u$	$[0 \ 1] J_u$
$T_3$	$\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} J_u$	$T_3$

Table 2: Proposed adaptive pairing for all operating regions

#### 4. Simulation results and discussion

The control structure previously presented is now evaluated in closed-loop simulation face to changing disturbances. Figure 3 shows the simulation results, where all 7 possible regions are explored. As the process itself is considered to be at steady state at all times, the dynamics of the system is fully attributed to the tuning of the controllers. Operation

in the fully constrained regions is optimal at steady state, whereas there is some deviation from the optimal conditions in the partially constrained and unconstrained regions. This is due to the estimation of gradients by Jäschke temperatures, which does not fully represent the plant model, but gives a reasonable estimate for control, so that low operational loss is achieved.

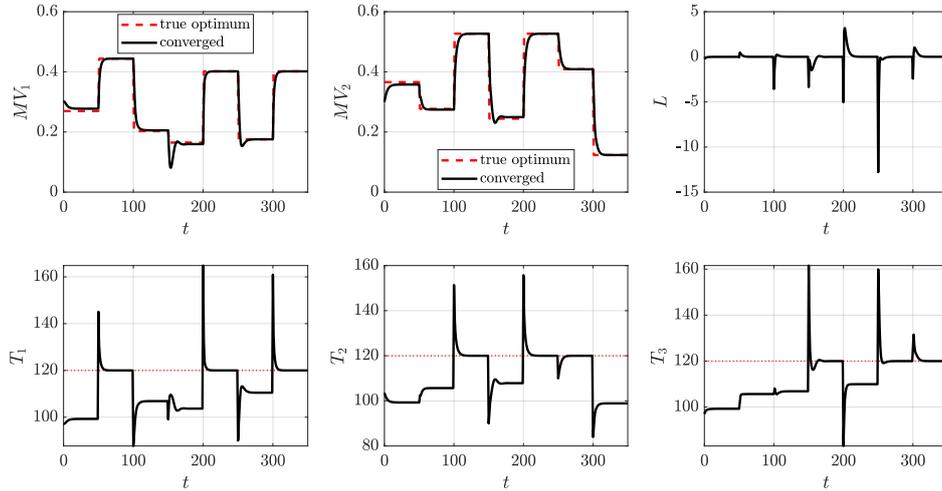


Figure 3: Simulation of region-based control structure using Jäschke temperatures

These results are compared with a traditional RTO implementation, see Figure 4. This implementation consists of a two-step approach, with disturbance estimation followed by model-based constrained optimization. The system converges in few iterations, with similar steady-state behavior to the region-based control structure. The unconstrained and partially constrained regions suffer from deviations from the true optima, due to model-plant mismatch, and the converged state is quite similar to that of the region-based control structure. This is to be expected, as Jäschke temperatures represent the gradient information extracted from the model used in the RTO framework.

In the RTO simulation, a curious undesired behavior is observed. From  $t = 40$ , in the fifth simulated region, the system converges to an infeasible point. This happens because the disturbance estimation step returns parameter values that make the optimization problem infeasible, meaning that there are no inputs that satisfy all constraints on the model with the given parameters, even if the estimation step returns parameters that agree with the plant measurements. Some workarounds are therefore deemed necessary for the effective implementation of the RTO strategy, such as the adaptation of the optimization problem itself, based on the estimation of gradients from the true plant (Marchetti et al., 2009).

## 5. Conclusion

In this work, we extended previous work on the optimal operation of heat exchanger networks to the constrained case, where the ideal self-optimizing variables known as Jäschke

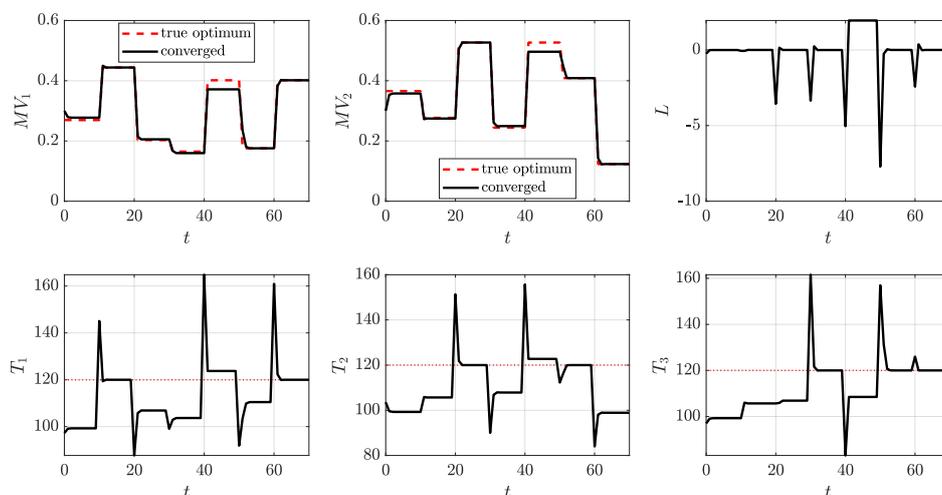


Figure 4: Simulation of steady-state RTO with model-plant mismatch

temperatures cannot be applied to every operating condition. Instead, control of the active constraints becomes necessary for optimal operation, and the challenge lies in deciding automatically what are the best controlled variables during operation. This has been achieved with the use of selectors, with steady-state performance comparable to a traditional model-based RTO implementation. With the proposed control implementation, one avoids the solution of online optimization problems, which can be problematic, as highlighted by the presented results. However, the simultaneous use of the presented tools is encouraged, so that near-optimal operation is achieved in the faster timescales, and optimization tools can correct for mismatches under more careful evaluation of the results.

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