

# Optimal Resource Allocation using Distributed Feedback-based Real-time Optimization<sup>\*</sup>

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## Abstract:

This paper considers the problem of steady-state optimal resource allocation in an industrial symbiosis, where different companies share common resources. Such optimal resource allocation problems are commonly studied in the context of distributed optimization to limit information sharing. One such framework is the Lagrangian decomposition approach, where the different subproblems are locally optimized for a given shadow price of the shared resource, which is updated by a master coordinator. In the traditional distributed RTO approach, this involves solving a numerical optimization problem online for each subproblem, which can be computationally intensive. In order to avoid the need for solving numerical optimization problems, this paper proposes a distributed feedback-based real-time optimization framework, where each subproblem is locally optimized for a given shadow price using simple feedback controllers. The proposed feedback-based distributed RTO scheme is applied to an industrial symbiotic subsea oil production system, where the different wells are operated by different companies. The simulation results show that the proposed feedback-based distributed RTO scheme can optimally allocate the shared resources.

*Keywords:* Distributed optimization, Dual decomposition, Feedback control

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## 1. INTRODUCTION

With increasing energy demand and tightening environmental regulations, the necessity for sustainable production increases, and there is an explicit need to focus on resource efficiency. In the process and manufacturing industries, there is an increasing trend of industrial symbiosis, where different organizations share resources and equipment in a mutually beneficial manner, and is seen as a key driver for sustainable production.

Optimal process operation involves taking decisions in real-time to meet production goals and emission targets. This is typically done in the context of real-time optimization (RTO) using process models and real-time measurements. As the process industry is embracing industrial symbiosis with common resources, this creates new challenges, as the companies may not wish to share such information across the different companies due to various reasons such as market competitiveness and intellectual property. Thus, there is a clear need to optimally allocate the shared resources with limited information sharing across the different organizations.

Distributed real-time optimization using the dual (Lagrangian) decomposition approach is a potential solution that facilitates industrial symbiosis (Wenzel et al., 2016). In this framework, the different subsystems, that represent different companies, are locally optimized, and a master coordinator updates the shadow price for the shared resource in order to match the supply and demand of the shared resource in this micro market setting. This only requires sharing limited information, such as the shadow prices and the total resource consumption. The different subsystems only report the total resource consumption/production to the master coordinator. If the supply of a shared resource exceeds the demand, the coordinator will decrease the shadow price to encourage consumption of the shared resource by the subsystems. Similarly, if the supply is lower than the demand, the coordinator will increase the price to reduce consumption. This price-based coordination scheme imitates the tatonnement process, where the coordinator has the responsibility to find the equilibrium price iteratively by dynamic pricing based on responses of the subsystems (Walker, 1987; Wenzel et al., 2016). To this end, the different subproblems and the master coordinator are solved iteratively until the problem converges to a feasible and optimal solution.

However, repeatedly solving numerical optimization problems can be computationally intensive. Although different approaches have been recently proposed to speed up the convergence of the distributed optimization problem (see

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for example Wenzel et al. (2020) for a recent overview), solving the numerical optimization problem online in itself may be a fundamental limiting factor in many applications. Furthermore, many traditional process industries may also prefer to use simple feedback controllers as opposed to model-based RTO tools due to lack of technical expertise or corporate culture.

Recently, there is an increasing interest in a class of methods for RTO, known as “feedback-optimizing control” (Morari et al., 1980), which eliminates the need to solve numerical optimization problems by indirectly moving the optimization problem into the control layer. Feedback optimizing control has predominantly been studied in the context of a single optimization problems and in particular, the focus has been on what to control for the unconstrained degrees of freedom. See for example Skogestad (2000), Engell (2007), Chachuat et al. (2009), and Srinivasan and Bonvin (2019) and the references therein. However, the problem of distributed RTO with a master coordinator in the context of feedback optimizing control has received very little attention.

This paper aims to address this gap, where we propose an “optimizing” controlled variable for each subproblem, which is given as a function of the shadow price. By controlling the proposed controlled variable in each subproblem independently, the different subproblems can locally optimize their processes for a given shadow price. As the master coordinator updates the shadow prices, this would lead to optimal operation of the overall system. The main contribution of this paper is a distributed feedback-based real time optimization (DFRTO) framework based on Lagrangian decomposition, that achieves optimal steady-state operation in a distributed manner, without the need to solve numerical optimization problems online.

The remainder of the paper is organized as follows. Section 2 describes the problem formulation. Section 3 presents the proposed method where the Lagrangian dual decomposition framework is translated into a distributed feedback control method. Section 4 compares the performance of the proposed DFRTO approach with the centralized optimization approach for a large-scale subsea oil production system before concluding the paper in Section 5.

## 2. PROBLEM FORMULATION

Consider a generic optimal resource sharing problem in  $N$  different subsystems with additively separable cost

$$\min_{x_1, \dots, x_N} \sum_{i=1}^N f_i(x_i) \quad (1a)$$

$$\text{s.t.} \quad \sum_{i=1}^N A_i x_i - \bar{x} \leq 0 \quad (1b)$$

where  $x_i \in \mathbb{R}^{n_{x_i}}$  denotes the decision variables for subsystem  $i$  and  $n_{x_i}$  is the number of decision variables in subsystem  $i$ ,  $A_i \in \mathbb{R}^{m_x \times n_{x_i}}$  is a matrix that couples different subsystems,  $\bar{x} \in \mathbb{R}^{m_x}$  is the shared resource constraints and  $m_x$  is the number of shared resource constraints, and  $f_i : \mathbb{R}^{n_{x_i}} \rightarrow \mathbb{R}$  is a function that denotes the local objective of subsystem  $i$ . Note that  $x_i > 0$  implies that the shared resources is consumed by subsystem  $i$ . Each subsystem  $i$

may also have local constraints that are assumed to be locally managed by each company, and are not explicitly shown in the problem formulation (1).

The objective of the aforementioned problem is to determine optimal shared resource allocation in order to achieve system-wide steady-state optimal operation in a distributed fashion with limited information sharing.

The Lagrangian of problem (1a) reads as

$$\mathcal{L}(x_1, \dots, x_N, \lambda) = \sum_{i=1}^N f_i(x_i) + \lambda^\top \left( \sum_{i=1}^N A_i x_i - \bar{x} \right) \quad (2)$$

where  $\lambda \in \mathbb{R}^{m_x}$  is the Lagrange multiplier of the shared resource constraints (1b). Defining some  $h_i \in \mathbb{R}^{m_x}$  with  $0 \leq h_i \leq \bar{x}$  and  $\sum_{i=1}^N h_i = \bar{x}$  (e.g.  $h_i = \frac{\bar{x}}{N}$ , for  $i = 1, \dots, N$ ), we can see that problem (2) becomes additively separable, where each subproblem is given as a function of the shadow price  $\lambda$

$$\mathcal{P}_i(\lambda) : \quad \min_{x_i} \mathcal{L}_i(x_i, \lambda) \quad (3)$$

where

$$\mathcal{L}_i(x_i, \lambda) = f_i(x_i) + \lambda^\top (A_i x_i - h_i)$$

This is known as dual or Lagrangian decomposition (Ladson, 2002; Boyd et al., 2007).

Starting from an initial guess  $\lambda_0$ , the master coordinator then updates the Lagrange multipliers using the subgradient method

$$\lambda^{k+1} = \max \left[ 0, \lambda^k + \alpha \left( \sum_{i=1}^N A_i x_i^k - \bar{x} \right) \right] \quad (4)$$

where  $\alpha$  is step length, and  $x_i^k$  is optimal resources allocation at iteration  $k$ . The Lagrange multipliers  $\lambda$  denotes the shadow price of the shared resource, which has an economic interpretation of matching the supply and demand of the shared resource as mentioned earlier. Note that, according to the KKT conditions,  $\lambda \geq 0$  must hold for the inequality (coupling) constraints in problem (1b). This requirement is ensured by using a max operator in Eq. (4).

## 3. PROPOSED METHOD

In this paper, we aim to solve the steady-state optimization problem (1) using feedback control in a distributed manner. To this end, we want to find an “optimizing” controlled variable for each subproblem (3) as a function of the shadow prices such that the master problem (4) can be used to coordinate the local feedback controllers, thus achieving system-wide optimal operation. This framework was recently introduced as *Distributed Feedback-based RTO* (DFRTO) by Krishnamoorthy (2020).

From the Lagrangian decomposition framework, it can be seen that for the stationary condition of subproblem (3), we need  $\nabla \mathcal{L}_i(x_i, \lambda) = 0$ . Based on this idea, the optimizing controlled variable of subsystem  $i$  can be expressed as a function of the Lagrange multiplier  $\lambda$

$$c_i(\lambda) = \nabla \mathcal{L}_i(x_i, \lambda) = \nabla_{x_i} f_i(x_i) + A_i^\top \lambda \quad (5)$$

which must be controlled to a constant set-point of  $c_i^{sp} = 0$  in order to satisfy the necessary condition of optimality. Assuming that the stationary point of (3) is also a local minimum, then this leads to optimal operation of the local

subsystem for a given shadow price  $\lambda$ . As  $\lambda$  is updated in (4) to reach market equilibrium, then this leads to optimal operation of the overall optimization problem (1).

In order to evaluate (5), each subsystem  $i$  is required to estimate its local steady-state cost gradient  $\nabla_{x_i} f_i(x_i)$  which can be achieved locally using any model-based or model-free gradient estimation. See for example Srinivasan et al. (2011), and François et al. (2012) for a list of gradient estimation techniques for RTO that can be used here. This is also shown in Fig. (1).

Eq. (5) shows that by driving  $\nabla_{x_i} \mathcal{L}_i(x_i, \lambda)$  to 0 in each subsystem  $i$ , and updating the Lagrangian multipliers iteratively using Eq. (4), we can avoid solving the optimization problem online. Moreover, the master coordinator has only limited knowledge about the different subsystems, and can only influence the Lagrangian multipliers (i.e. shadow prices). Thus, the different subsystems avoid the need to share local information such as the detailed models, measurements, local constraints, and the objective function. The only information that needs to be shared is the total resource consumption,  $\sum_{i=1}^N A_i x_i(t)$ .

Since the proposed DFRTO framework does not need to solve numerical optimization problems online, the sampling rate is not limited by the computation time. Moreover, the proposed framework also enables the master coordinator and the different subproblems to be implemented at different sampling rates. Depending on the choice of the gradient estimation scheme used, the proposed DFRTO scheme may avoid the steady-state wait-time issue, or may avoid the need for detailed process models (François et al., 2012).

In traditional distributed optimization framework, the master coordinator problem and the subproblems require several iterations to converge to the optimal solution at each time step (Lasdon, 2002; Boyd et al., 2007). In the proposed DFRTO scheme, we use real-time measurements  $x_i(t)$  to update the Lagrange multiplier

$$\lambda(t+1) = \max \left[ 0, \lambda(t) + \alpha \left( \sum_{i=1}^N A_i x_i(t) - \bar{x} \right) \right]$$

This can be seen as iterating between the master coordinator and the subproblems in real-time. A distributed feedback RTO scheme using augmented Lagrangian was recently proposed by Krishnamoorthy (2020), where it was shown that, under certain conditions as  $t \rightarrow \infty$ , the different subproblems converge to a stationary point of problem (1).

#### 4. ILLUSTRATIVE EXAMPLE

In this section, we apply the proposed DFRTO approach on a large-scale subsea gas-lifted oil production well network (see Fig. 2) with  $N = 4$  subsea manifolds (clusters), each operated by different companies (cf. Table 1).

The objective is to maximize the total revenue from each subsystem: maximize oil production and minimize the costs associated with the gas lift compression. The four companies share a common topside process facility that has the gas compression station as shown in Fig. 2. The lift gas  $w_{gl}$  is a shared resource that must be optimally

allocated amongst the four companies. In addition, the topside processing facility also has gas processing capacity limitations. This limits the total gas flow rate that can be produced by the different wells. To this end, the total lift gas and the total gas processing constraint couples the different subsystems. Thus, the optimization problem is given as

$$\min_{w_{gl,1}, \dots, w_{gl,N}} -\$_o \sum_{i=1}^N w_{po,i} + \$_{gl} \sum_{i=1}^N w_{gl,i} \quad (6a)$$

$$\text{s.t.} \quad \sum_{i=1}^N w_{gl,i} - \bar{w}_{gl} \leq 0, \quad (6b)$$

$$\sum_{i=1}^N w_{pg,i} - \bar{w}_{pg} \leq 0 \quad (6c)$$

where  $\$_o$  is the oil price, and  $\$_{gl}$  is the cost of gas compression. In this case, gas-lift injection rate,  $w_{gl,i}$ , are the decision variables,  $w_{po,i}$  and  $w_{pg,i}$  are the oil and gas production rates, respectively, which depend on the gas lift injection. The local objective function is given by  $f_i(x_i) = -\$_o w_{po,i} + \$_{gl} w_{gl,i}$ .

The gas-to-oil ratio (GOR), which is a reservoir property, is a time varying disturbance for the different wells (feed disturbance). In this simulation study, the GOR for the different wells are assumed to vary as shown in Fig. 3. High GOR indicates that the well has a lighter fluid requires less amount of gas-lift injection rate compared to the wells with lower GOR to produce the same amount of oil.

In order to achieve optimal operation in a distributed manner, we control the proposed self-optimizing variable (5). In this example, (5) can be represented as,

$$\nabla_{w_{gl,i}} \mathcal{L}_i(w_{gl,i}, \lambda) = \hat{\mathbf{J}}_{w_{gl,i}} + \lambda_{gl} + (\nabla_{w_{gl,i}} w_{pg,i})^\top \lambda_{pg} \quad (7)$$

where  $\hat{\mathbf{J}}_{w_{gl,i}} := -\$_o \nabla_{w_{gl,i}} w_{po,i} + \$_{gl}$ . To estimate the steady-state gradients in (7), in this paper we use the model-based gradient estimation framework proposed by Krishnamoorthy et al. (2019). Note that the proposed DFRTO framework is not just restricted to this gradient estimation approach, and one may instead use any other model-based or model-free gradient estimation scheme (Srinivasan et al., 2011).

Once the gradients are estimated using local measurements, each subsystem uses feedback controllers or a combination of them to drive the proposed self-optimizing controlled variable (7) to a constant setpoint of zero. In this paper, we use PI controllers for each well, which are tuned using the SIMC tuning rules (Skogestad, 2003). The controller is designed with a sampling time of 1 second.

The overall plant is modelled as an Index-1 DAE with a total of 30 differential states, 120 algebraic states, and 10 inputs. The model equations can be found in

Table 1. Companies and Oil Production Wells

Subsystem	Company	Oil Production Well ID
1	1	1,2,3
2	2	4,5,6
3	3	7,8
4	4	9,10

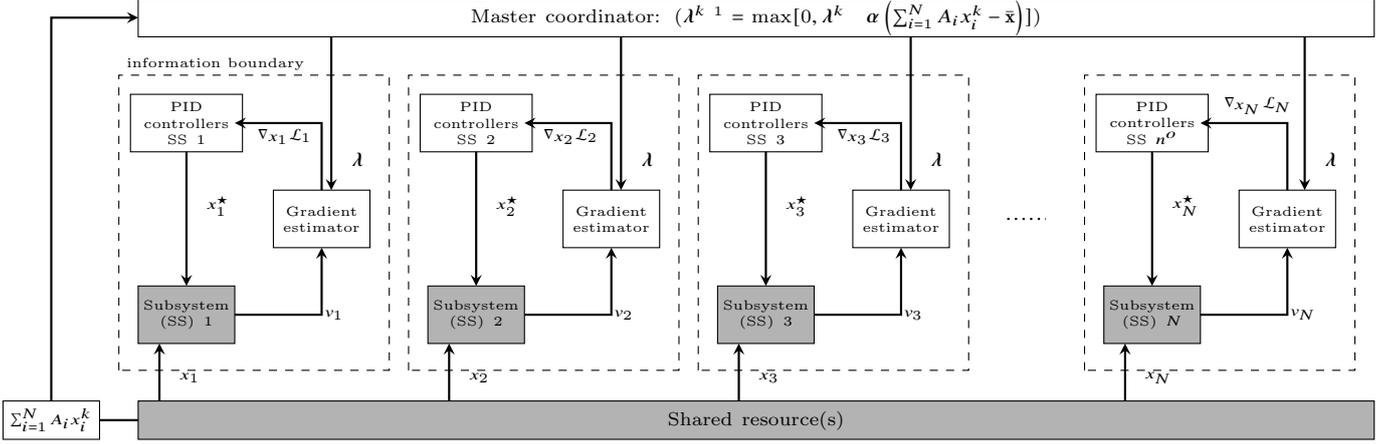


Fig. 1. The proposed DFRTO structure for optimal resource allocation using simple PID controllers. The grey box represents the physical system and the white box represents the computation block.  $v_i$  denotes real time measurements at subsystem  $i$ .

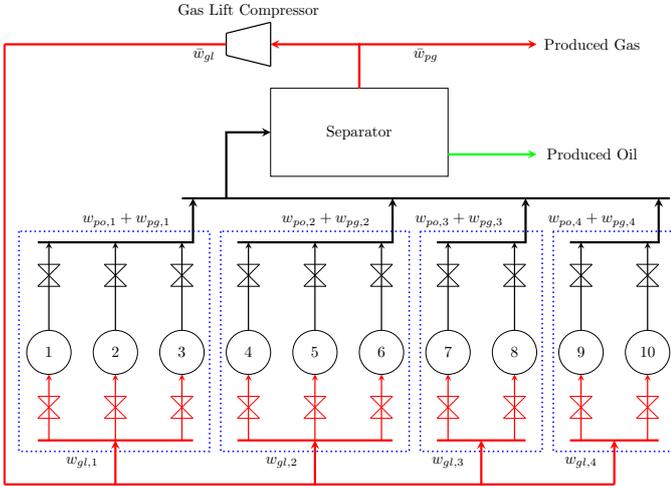


Fig. 2. A simplified process diagram of a large-scale offshore field with shared gas-lift injection resource, equipped with constraints in both gas-lift and produced gas compressor.

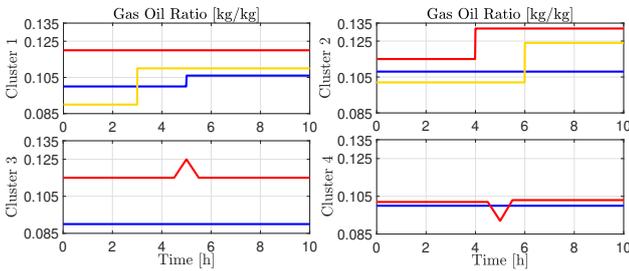


Fig. 3. GOR variations in the ten wells in the associated clusters

(Krishnamoorthy et al., 2018, Appendix B). The model parameters used in this paper are shown in Table A.1-A.3.

The plant simulator is developed using CasADi v3.5.1 (Andersson et al., 2019) with MATLAB R2019b and is simulated using the IDAS integrator (Hindmarsh et al., 2005). The simulations are performed on a 2.11 GHz processor with 16 GB memory. The simulations are performed for a

total simulation time of 10 hours. The GOR for all wells varies as shown in Fig. 3, where it can be seen that the system is frequently subject to disturbances. The total available gas-lift ( $\bar{w}_{gl}$ ) and gas processing capacity ( $\bar{w}_{pg}$ ) also varies, which affects the optimal allocation of the gas-lift. The disturbances in GOR may also lead to an unconstrained case, where both the coupling constraints (6b) and (6c) are inactive.

First, we solve the centralized production optimization problem (7) to obtain the ideal optimal setpoint as the baseline. Then, we implement the proposed strategy described in Section 3. Fig. 4 shows the simulation results comparing the ideal optimum, and the DFRTO with limited information sharing. The absolute error between the ideal optimum and DFRTO is also shown in the same figure for the total oil, total produced gas, and total gas-lift injection rate, which indicates that the proposed method is able to converge to the ideal optimum at steady-state.

Fig. 4 - 5 show that during times  $t = 2 - 4$  h, no constraint is active (i.e.,  $\lambda = 0$ ), where the optimal allocation utilizes less than the total available gas lift, and the total gas processing capacity. The total available gas-lift constraint is active during times  $t = 4 - 8$  h. Subsequently, the gas processing capacity constraint is active during times  $t = 9 - 10$  h.

It is important to note that during the transition, due to changes in GOR and/or constraints, the associated Lagrange multipliers converge in real-time (see Fig. 5(d)). Consequently, the total gas-lift injection rate and/or the total produced gas rate may violate the constraints dynamically for some time (dynamic violation) but the constraints are satisfied at steady-state (no steady-state violation). As can be seen right after  $t = 8$  h, due to the sudden increase of total gas lift constraint at time  $t = 8$  h, the gas processing capacity constraint should be ideally active immediately. In fact, the total available gas-lift constraint is still active for some time since  $\lambda_{gl}$  is still positive even though decreasing. Meanwhile,  $\lambda_{pg}$  is still increasing gradually, that violates the gas processing capacity constraint dynamically. It can be clearly seen

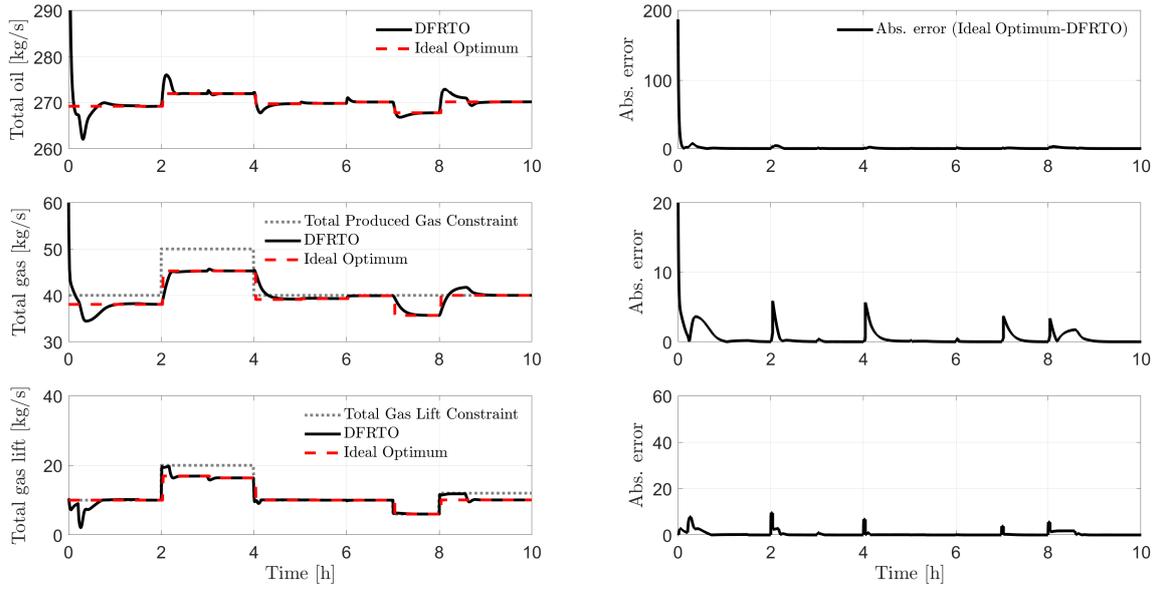


Fig. 4. Simulation results showing the performance of the DFRTO framework compared to the ideal optimum provided by centralized approach, along with the absolute error between them (in the right-hand plots)

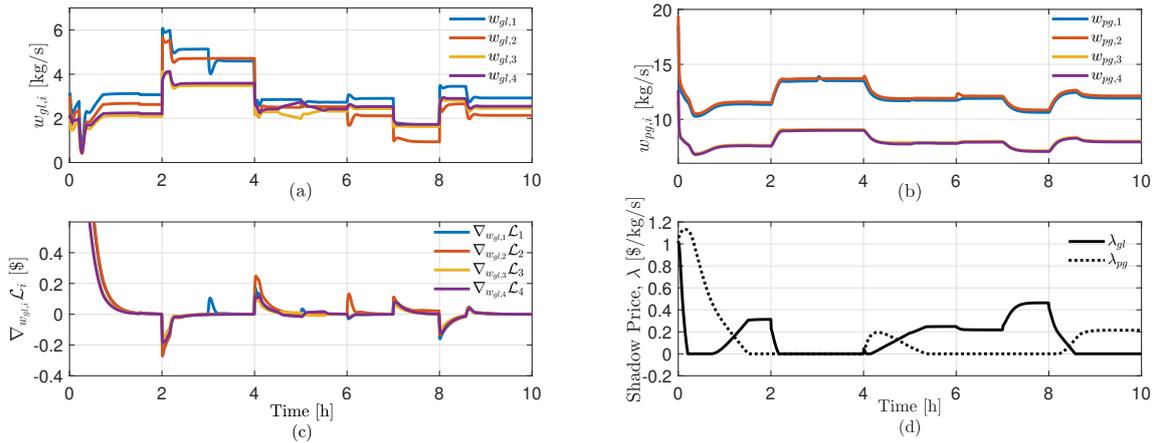


Fig. 5. (a) The allocation of the shared gas-lift resources among the clusters. (b) The exported produced gas among the clusters. (c) The controlled estimates gradients of the Lagrangian function among the clusters. (d) The shadow prices of the coupling constraints

that, as the Lagrange multipliers converge, the steady-state operation corresponds to the ideal optimum.

## 5. CONCLUSION

In this paper, we presented a distributed feedback-based RTO framework, where we showed that the Dual decomposition framework can be converted into a feedback control problem by controlling (5) to a constant setpoint of zero. Since (5) is a function of the Lagrange multipliers, we showed that by using a standard master coordinator (4) that updates the Lagrange multipliers using the subgradient method, the proposed approach leads to system-wide optimal operation. This avoids the need for solving numerical optimization problems online and enables system-wide optimal operation with limited information sharing. Any model-based or model-free gradient estima-

tion scheme may be used with the proposed framework, making it broadly applicable.

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## Appendix A. GAS LIFT NETWORK PARAMETERS

Table A.1. List of well parameters and their corresponding values of Subsystem 1 used in the simulation results.

Parameter [units]	Well 1	Well 2	Well 3
$L_w$ [m]	1500	1500	1500
$H_w$ [m]	1000	1000	1000
$D_w$ [m]	0.121	0.121	0.121
$L_{bh}$ [m]	500	500	500
$H_{bh}$ [m]	500	500	500
$D_{bh}$ [m]	0.121	0.121	0.121
$L_a$ [m]	1500	1500	1500
$H_a$ [m]	1000	1000	1000
$D_a$ [m]	0.189	0.189	0.189
$\rho_o$ [kg m <sup>-3</sup> ]	8	8	7.9
$C_{iv}$ [m <sup>2</sup> ]	1E-4	1E-4	1E-4
$C_{pc}$ [m <sup>2</sup> ]	2E-3	2E-3	2E-3
$p_r$ [bar]	150	155	155
$PI$ [kg s <sup>-1</sup> bar <sup>-1</sup> ]	7	7	7
$T_a$ [°C]	28	28	28
$T_w$ [°C]	32	32	32

Table A.2. List of well parameters and their corresponding values of Subsystem 2 used in the simulation results.

Parameter [units]	Well 4	Well 5	Well 6
$L_w$ [m]	1500	1500	1500
$H_w$ [m]	1000	1000	1000
$D_w$ [m]	0.121	0.121	0.121
$L_{bh}$ [m]	500	500	500
$H_{bh}$ [m]	500	500	500
$D_{bh}$ [m]	0.121	0.121	0.121
$L_a$ [m]	1500	1500	1500
$H_a$ [m]	1000	1000	1000
$D_a$ [m]	0.189	0.189	0.189
$\rho_o$ [kg m <sup>-3</sup> ]	8	8.2	8.05
$C_{iv}$ [m <sup>2</sup> ]	1E-4	1E-4	1E-4
$C_{pc}$ [m <sup>2</sup> ]	2E-3	2E-3	2E-3
$p_r$ [bar]	160	155	155
$PI$ [kg s <sup>-1</sup> bar <sup>-1</sup> ]	7	7	7
$T_a$ [°C]	28	28	28
$T_w$ [°C]	32	32	32

Table A.3. List of well parameters and their corresponding values of Subsystem 3 and 4 used in the simulation results.

Parameter [units]	Well 7	Well 8	Well 9	Well 10
$L_w$ [m]	1500	1500	1500	1500
$H_w$ [m]	1000	1000	1000	1000
$D_w$ [m]	0.121	0.121	0.121	0.121
$L_{bh}$ [m]	500	500	500	500
$H_{bh}$ [m]	500	500	500	500
$D_{bh}$ [m]	0.121	0.121	0.121	0.121
$L_a$ [m]	1500	1500	1500	1500
$H_a$ [m]	1000	1000	1000	1000
$D_a$ [m]	0.189	0.189	0.189	0.189
$\rho_o$ [kg m <sup>-3</sup> ]	7.9	8.2	8	8.05
$C_{iv}$ [m <sup>2</sup> ]	1E-4	1E-4	1E-4	1E-4
$C_{pc}$ [m <sup>2</sup> ]	2E-3	2E-3	2E-3	2E-3
$p_r$ [bar]	155	165	150	155
$PI$ [kg s <sup>-1</sup> bar <sup>-1</sup> ]	7	7	7	7
$T_a$ [°C]	28	28	28	28
$T_w$ [°C]	32	32	32	32