A model predictive control interpretation of the generalized split-range control ⋆

José Diogo Forte de Oliveira Luna * **Rafael Oliveira Pedrisch** **
João Victor Bezerra da Silva **
José Souto Guimarães Ferreira Júnior **
Carlos Henrique Azevedo Guimarães ** **Kariston Dias Alves** **
Julio Elias Normey-Rico ***

* *Instituto Federal de Rondônia, RO, (e-mail: jose.luna@ifro.edu.br).*
** *Instituto Federal de Rondônia, RO*
*** *Universidade Federal de Santa Catarina*

Abstract: This study presents a Model Predictive Control (MPC) interpretation of the Generalized Split-Range Control (GSRC), focusing on a novel Mixed-Integer Quadratic Programming (MIQP) formulation that enforces sequential actuator usage. By replicating the input sequencing behavior of the GSRC, the proposed MPC controller allows for economically efficient decisions regarding input usage. In order to show this behavior, the same case study of the original paper on GSRC was simulated, comparing both approaches. The study suggests that while GSRC implementation may be more suitable for smaller processes, integrating the proposed MPC approach into larger industrial settings with existing MPC controllers could offer an equally economic efficient alternative to incorporate split-range economic properties.

Keywords: Model predictive control, split-range control, MISO systems, classical advanced control, optimization.

1. INTRODUCTION

Multiple-Input Single-Output systems emerge in industrial processes where two or more actuators operate together to drive a single process variable. This is the case when more than one input is required to cover the entire desired output steady-state range (Reyes-Lúa and Skogestad, 2020). They emerge in many applications such as pulp and paper industry (Allison and Ogawa, 2003), (Varshney et al., 2022), gas-liquid separators (Fatani et al., 2017), heat exchangers (Reyes-Lúa et al., 2018), chemical reactors (Alsop, 2016), HVAC (Reyes-Lúa et al., 2019) and even in medic drug delivery (Pawłowski et al., 2022).

The presence of multiple inputs allows for different control strategies to coordinate the usage of each actuator. This approaches can aim for improving dynamic behavior of the system and even minimizing the operational cost by prioritizing the usage of cheaper inputs (Pedrisch et al., 2023). In fact, several works in the last few years have proposed improvements on classic advanced control structures, which are able to deal with MISO systems, like mid-ranging control (Alsop, 2016), (Hägglund, 2021) or split range control.

In particular, split range control has been documented in the literature since the 40s (Eckman, 1945) but has received renewed attention in the recent years, as seem in the works of D. Machado (2022), F. García (2021) e Balbinot et al. (2021).

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As presented in Figure 1, for a TISO (two-input, single-output) system, this control technique employs a single primary controller C whose output is processed by a splitter block which, through affine functions, assigns a value to each control input u_i.

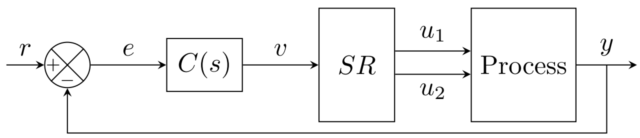

Figure 1. Split range control strategy for a Two-Input Single-Output system.

The inputs can be sequenced following an economic criteria, prioritizing cheaper inputs and only resorting to tapping into the more expensive resources once the cheaper inputs are saturated and can no longer drive the process output toward the set-point. Figure 2 shows an example for a two input system where u_2 is only used after u_1, assumed cheaper, saturates.

One of the drawbacks of the split-range control approach is that the primary controller must be tuned considering the dynamics of each input channel, and often lead to a compromise between what would have been chosen for each actuator individually (Reyes-Lúa et al., 2019). This issue has been addressed by Reyes-Lúa and Skogestad (2020) who proposed a generalized split range control (GSRC) based on a baton passing strategy. This approach provide each input with its own controller C_i, in order to overcome the tuning limitations present in the classic split-range control.

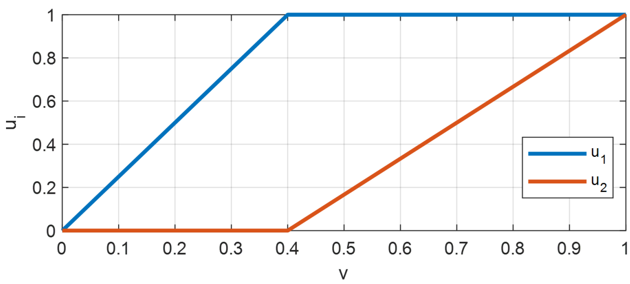

Figure 2. Split-range profile input usage regarding the controller output v.

Under a GSRC strategy, exemplified in Figure 3, at every instant, only one of the controllers has the baton and is allowed to actively control the process output by moving its own input. Once this actuator saturates it passes the baton to the next controller and remain saturated, thus emulating the input sequencing behavior seen in the classic split range control. This also enables the economic decision regarding which input should be prioritized considering their usage cost.

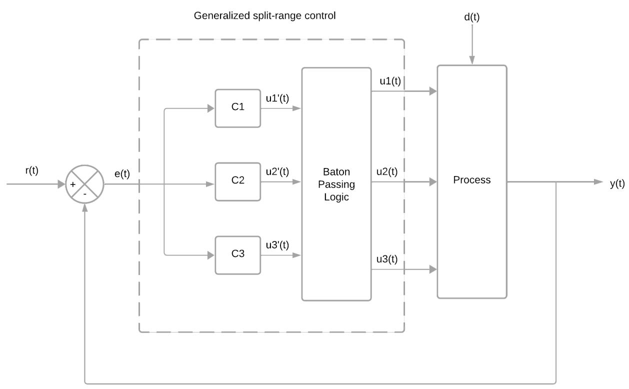

Figure 3. Generalized split-range control for a MISO system as proposed in Reyes-Lúa and Skogestad (2020).

For the sake of comparison Reyes-Lúa and Skogestad (2020) compared the performance of the GSRC with a model predictive control (MPC) approach. Their paper argues that while the dynamic behavior is quite similar, the MPC controller takes suboptimal decision regarding the usage of more expensive inputs during transient response, while the cheaper ones have not reached saturation yet.

While this could be improved by careful choice of the weights in the objective function to discourage the usage of the more expensive inputs or by the inclusion of input targets, provided by an economic-oriented optimization layer above, these solutions would not guarantee the input sequencing behavior seem in the split range control.

MPC strategies, however, are a very flexible framework that encompasses a large family of control methods that share common ideas founded upon the concept of model based prediction (Camacho et al., 2007). The general MPC controller is composed of a process model and a optimizer, as illustrated in Figure 4. The controller solves an optimization problem on each sampling instant to compute an optimal control sequence, but only the first value is actually applied to the process, as the MPC will compute a new optimal sequence on the next sampling instant. The constraints of the optimization problem can be used to enforce operational limitations and shape the behavior of the control inputs. In regard, the seminal work by Morari and Alberto (1999) proposed a predictive control scheme capable of dealing with mixed logic dynamic (MLD) systems. Said systems are described by physical laws, independent operating constraints and logical rules.

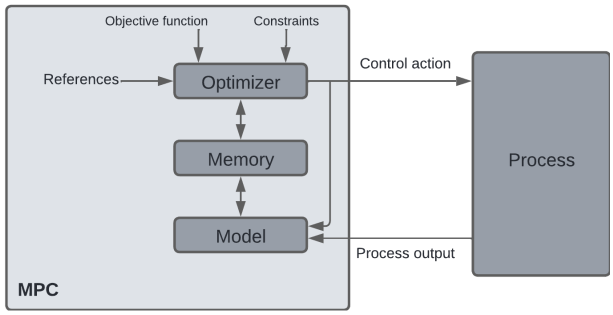

Figure 4. Basic MPC structure.

So, in order to emulate the desirable economic behavior of split range control in an MPC framework, the present study objective is to provide a novel formulation for enforcing input sequencing by introducing binary decision variables into the optimization problem.

2. GENERALIZED SPLIT RANGE CONTROL

The controller proposed by Reyes-Lúa and Skogestad (2020) comprises a controller switching scheme based on the so-called baton passing strategy. Each input is controlled by an independent controller C_i, whose output u_i' is considered a suggested control signal. Each suggested control signal is fed to the baton passing logic block that calculates the actual control signals u_i. This scheme is illustrated in Figure 3.

At any time during normal controller operation, only a single input u_i is actively controlling the process variable. Meanwhile, the other actuators are saturated, whether in the fully open or fully closed position. This is achieved with the baton passing logic, in which only the controller that owns the baton can move its control action, while the other remains saturated until the baton is passed to then eventually. On the other hand, the active input remains active until it becomes saturated, when this happens, it will pass the baton to the next input. And, because only one input is actively controlling the output at any given time, each C_i controller can be tuned independently as if it were in a SISO process, avoiding the compromise required in the conventional SRC primary controller.

As in the split-range control, the inputs in the GSRC are sequenced in relation to their cost of use: the cheapest input should be used first and, after saturation, the second, more expensive input is used, then the third, even more expensive, one and so on.

The baton passing strategy is implemented as follows. If the i-th controller has the baton at the instant t, it will calculate the suggested input $u_i'(t)$. Assuming that every input signal is scaled from 0 to 1, if $0 < u_i'(t) < 1$, C_i remains active and $u_i(t) = u_i'(t)$, while the other inputs remain saturated. Otherwise, if $u'_i(t) > 1$ or $u'_i(t) < 0$, then u_i saturates at the relevant limit and the baton is passed to the next or previous controller, depending on whether u_i is fully open or fully closed and if u_i has direct or reverse action.

3. INPUT SEQUENCING CONSTRAINTS

Analyzing the control behavior described in Reyes-Lúa and Skogestad (2020), it is possible to formulate a set of constraints that enforces the input sequencing behavior.

Initially, binary decision variables will be defined that will describe the current state of the control action of each actuator u_i. When the i-th manipulated variable is saturated at its maximum value, the binary variable σ_i must be set to 1. When it is saturated at its minimum value, the variable δ_i must be set as 1. As shown in (1) and (2), respectively.

$$u_i = u_i^{max} \rightarrow \sigma_i = 1 \tag{1}$$

$$u_i = u_i^{min} \rightarrow \delta_i = 1 \tag{2}$$

Firstly, the inputs should be grouped by their control action, either direct or reverse. Then, among each group the inputs must be ordered by their usage cost. For direct acting inputs, they should be ordered from cheapest to most expensive. Among the reverse acting inputs the ordering is reversed: from most expensive to cheapest.

For sake of clarity, consider a process with n inputs: $u_1, ..., u_m, ..., u_n$. Inputs 1 to m have reverse action (negative static gain) and inputs $m+1$ to n have direct action (positive static gain). When properly ordered, u_1 should be the most expensive reverse acting input and u_m the cheapest one. Conversely, u_{m+1} is the cheapest direct acting input and u_n the most expensive.

Provided that the inputs have been ordered as prescribed, the satisfaction of the following constraints for every step j of the control horizon will replicate the split range behavior:

$$\begin{bmatrix} 0 \\ 1-\delta_1(k+j) \\ \vdots \\ 1-\delta_{m-1}(k+j) \\ \sigma_{m+1}(k+j) \\ \vdots \\ \sigma_{n-1}(k+j) \\ 0 \end{bmatrix} \leq \begin{bmatrix} u_1(k+j) \\ u_2(k+j) \\ \vdots \\ u_m(k+j) \\ u_{m+1}(k+j) \\ \vdots \\ u_{n-1}(k+j) \\ u_n(k+j) \end{bmatrix} \leq \begin{bmatrix} 1-\delta_1(k+j) \\ 1-\delta_2(k+j) \\ \vdots \\ 1-\delta_m(k+j) \\ \delta_m(k+j) \\ \vdots \\ \sigma_{n-2}(k+j) \\ \sigma_{n-1}(k+j) \end{bmatrix} \tag{3}$$

Consider that, on a given moment, the process requires the reverse action to move towards the set-point. If the demand for reverse action is small enough and u_m is able do supply it alone, u_{m-1} is saturated at 0 and, therefore δ_{m-1} is set to 1, allowing u_m to move between 0 and 1, given that δ_m is set to 0. Having δ_m set to 0 locks the direct acting inputs turned off. If u_m saturates at 1 and u_{m-1} is needed δ_{m-1} is set to 0, so u_m is constrained to $1 \leq u_m \leq 1$, therefore saturating at its maximum value and freeing u_{m-1} to move up to 1. If u_{m-1} saturates at 1 then δ_{m-2} must be set to 0 so u_{m-1} is locked saturated in 1 and u_{m-2} can move freely between 0 and 1. This goes on until reaching u_1, the most expensive reverse acting input. Conversely, if direct action is needed to drive the process towards the set-point, in order to start using u_{m+1} the binary decision variable δ_m must be set to 1, thus forcing u_m to be locked at 0 and blocking all reverse action inputs. If u_{m+1} saturates at its maximum value σ_{m+1} is set to 1, forcing u_{m+1} to stay saturated and allowing u_{m+2} to increase up until 1. This repeats until reaching u_n, which is the most expensive direct acting input.

If the system has only direct or reverse acting actuators a sliced up version of the constraint (3) should be used. From u_1 up until u_m if only reverse action is available and from u_{m+1} up until u_n if only direct action is available. In the later case the upper bound for u_{m+1} is simply 1.

The addition of these constraints turn the quadratic programming (QP) optimization problem of a conventional MPC controller into a mixed-integer quadratic programming (MIQP) programming problem. MIQPs are still convex problems and, therefore, have a single global minimum which can be found with well known optimization techniques, so this should not be a hindrance to its usage as many modern commercial solvers are able to tackle MIQPs.

4. SIMULATION CASE STUDY

To compare the proposed GSRC structure with other techniques available in the literature, Reyes-Lúa and Skogestad (2020) carried out the simulation of a case study of temperature control in an indoor environment with four actuators present: two cooling sources and two heating sources.

- u_{AC}: air conditioning control signal;
- u_{CW}: cooling water control signal;
- u_{HW}: hot water control signal;
- u_{EH}: electric heater control signal;

The objective of the control is to maintain the temperature at T = T^{ref}, with the system constantly receiving the disturbance of external temperature T^{amb} being the nominal ambient temperature $T_0^{amb} = 17°C$. The same process will be used as a benchmark here for comparison purposes.

For simplicity the indoor space is described as a linear MISO system:

$$y(s) = G_p(s) \cdot u(s) + G_d(s) \cdot d(s) \tag{4}$$

where $y(s) = T(s)$, $u = [u_{AC}\ u_{CW}\ u_{HW}\ u_{EH}]^T$, $G_p(s) = [G_{AC}(s)\ G_{CW}(s)\ G_{HW}(s)\ G_{EH}(s)]$, which are the transfer functions between the temperature and the air conditioning, cooling water, hot water and electric heater control signals, respectively, and $d = T^{amb}$.

Table 1 shows the static gains (K_j), the time constants (τ_j) and dead times (θ_j) for G_p and G_d, modeled as first order transfer functions plus dead time.

Aiming to validate the novel formulation, a comparison between the GSRC and an MPC controller with the proposed set of constraints is presented below.

4.1 GSRC Results

The design of the GSRC starts with the definition of the sequence of inputs given their economic cost. In this

Table 1. Parameters for the first order plus dead time transfer function for $G_{p,i}(s)$ relating u_j to output $y = T$ and the disturbance $d = T^{amb}$ to $y = T$.

$G_{p,i}$	$K_{p,i}$	$\tau_i(min)$	$\theta_i(min)$
G_{AC}	-5	8	2
G_{AR}	-10	15	3
G_{AQ}	12	10	3
G_{AE}	8	5	1
G_d	1	15	6

case both cooling water and air conditioning are used to decrease the temperature, with the first being the cheaper input to use. Conversely, the hot water and the electric heater are the heating sources, with the hot water being the cheapest of the two. So, the control should favor using the cold and hot water first, and only tap into the most expensive actuators after the cheaper ones saturate.

Given that the dynamics for each channel are described by first order plus dead-time models, four PI controllers are used. For sake of comparison the same tuning of the original paper of Reyes-Lúa and Skogestad (2020) was used here:

Table 2. Tuning parameters for each PI controller.

C_j	$K_{c,j}$	$T_{i,j}$
C_1	- 0.4000	8
C_2	-0.2500	15
C_3	0.1389	10
C_4	0.3125	5

Each controller was discretized with a sample time of 0.1 minute and the GSRC was deployed in the simulation with the anti-windup scheme discussed in Reyes-Lúa and Skogestad (2020) where the integral action is turned off when a given controller does not have the baton.

For the simulated scenario the set-point temperature is held constant at 17°C and the external temperature, the disturbance, suffer six step changes. While the abrupt changes are unrealistic, they make easier to show the controller behavior than a smooth temperature profile would. The changes are akin to the scenario described in Reyes-Lúa and Skogestad (2020).This temperature profile and the behavior of the closed-loop system are shown in Figure 5.

While the external temperature is equal to the desired internal temperature there is no need for cooling or heating. However, at the first step change the controller uses only the cold water to drive the temperature back to the set-point. After the second step change the sequencing behavior is noticeable, as the PI controller increases u_{CW} until it saturates. At this point the baton passing logic passes the baton to the air conditioning, forcing the cold water to stay saturated while u_{AC} actively controls the temperature.

After the third step change, u_{AC} reaches zero, and the baton is passed back to the cold water keeping the air conditioning off. As the external temperature drops, cold water input is set to zero, passing the baton to the hot water, which then saturates and hands over to the electric heater, maximizing the use of the cheaper heating source.

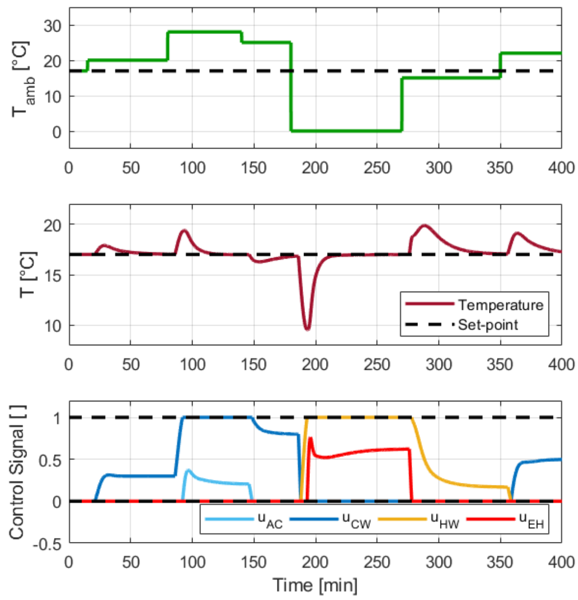

Figure 5. GSRC results. Top: Temperature profile for the scenario. Middle: Internal temperature. Bottom: manipulated variables for each actuator.

When the fifth step change arrives, the heating demand reduces, allowing for the electric heater controller to hand the baton back to the hot water controller. Finally, at the 350 minutes mark, the disturbance goes above the desired internal temperature, thus the hot water controller drives u_{HW} to 0 and hands the baton to the cold water, which works towards rejecting the disturbance.

4.2 MPC Approach

In order to control the internal temperature of the housing using an MPC controller with the addition of the input sequencing constraints, a Generalized Predictive Control (GPC) was adopted. The GPC is a well-established MPC controller that uses an input-output model known as CARIMA (Controller AutoRegressive Moving-Average) model (D.W. Clarke, 1987), which for the four input, single output case can be written as:

$$A(z^{-1})y(k) = \sum_{i=1}^{4} z^{-d_i} B_i(z^{-1})u_i(k-1) + \frac{C(z^{-1})}{\Delta}\varepsilon(k), \tag{5}$$

where A, B_i and C are polynomials in the time delay operator z^{-1}, $y(k)$ is the output of the system, $u_i(k)$ is the i-th manipulated variable, $\varepsilon(k)$ is gaussian white noise and $\Delta = 1 - z^{-1}$. Here it is assumed that $C(z^{-1}) = 1$.

The function to be minimized in the optimization problem is:

$$J = \sum_{j=N_1}^{N_2} \gamma(w(k+j)-\hat{y}(k+j))^2 + \sum_{i=1}^{4}\sum_{j=0}^{N_u-1} \lambda_i(\Delta u_i(k+j))^2 \tag{6}$$

where N_1 and N_2 are the limits of the prediction horizon, N_u is the size of the control horizon, $w(k+j)$ is the reference trajectory, $\hat{y}(k+j)$ is the predicted output, Δu_i is is the control increment for the i-th input and γ and λ_i

are the weights for the error and control effort terms of the cost function. This objective function can be understood as a trade-off between eliminating the error quickly and having a smooth control action.

For this four inputs case, the constraints shown in (3) become:

$$
\begin{gathered}
0 \leq \sum_{i=0}^{j} \Delta u_1(k+i) + u_1(k-1) \leq 1 - \delta_1(k+j) \\
1 - \delta_1(k+j) \leq \sum_{i=0}^{j} \Delta u_2(k+i) + u_2(k-1) \leq 1 - \delta_2(k+j) \\
\sigma_3(k+j) \leq \sum_{i=0}^{j} \Delta u_3(k+i) + u_3(k-1) \leq \delta_2(k+j) \\
0 \leq \sum_{i=0}^{j} \Delta u_4(k+i) + u_4(k-1) \leq \sigma_3(k+j)
\end{gathered} \tag{7}
$$

These four constraints should be enforced for every sample in the control horizon $j \in [0, N_u - 1]$ [1]. Additionally, since the GPC uses control increments Δu_i instead of the absolute control value u_i as decision variables for the optimization problem, each absolute control value, which should be bounded by (3) is found as:

$$
u_l(k+j) = \sum_{i=0}^{j} \Delta u_l(k+i) + u_l(k-1). \tag{8}
$$

The GPC runs at the same 0.1 min sampling time used for the GSRC. The prediction horizon was chosen as $N_1 = 10$, the smaller dead time among the four channels, and $N_2 = 65$. The control horizon is chosen as $N_u = 3$ for each input. The weights of the objective function are chosen as $\gamma = \dfrac{0.5}{N2 - N1}$ and $\lambda_i = \dfrac{1}{N_u K_{p,i}^2}$ for all the four inputs. The error term is divided by the length of the horizon and the control effort is divided by the length of the control horizon to normalize the terms and simplify the tuning. The 0.5 value was chosen experimentally aiming to encourage the control to take smoother control actions.

Finally, while it is simple to include the measurement of the disturbance to give the GPC some feedfoward capabilities, provided that the disturbance-to-output model is known, the simulation presented here does not include it, for the sake of a fair comparison.

The behavior of the system under the GPC with the proposed input sequencing mixed-integer constraints is show in Figure 6.

Similarly to the GSRC results, the MPC controller uses the inputs sequentially, always favoring the cheaper one for either cooling or heating demands. The controller only taps into the air conditioning or electric heater when the cheaper options are already saturated.

[1] In a standard GPC the lower and upper limits would simply be 0 and 1 for each manipulated variable. Further discussion is available in the supplementary material `https://github.com/jdiogoforte/split-range-MPC-2024/`

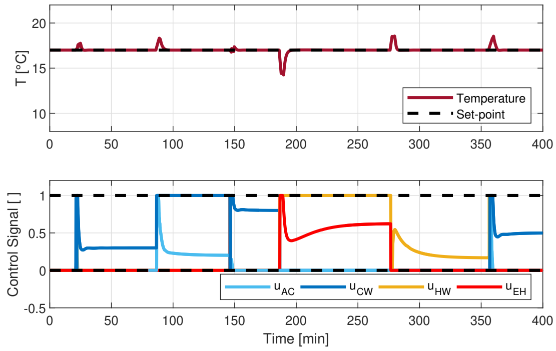

Figure 6. SRGPC results for the same disturbance sequence. Top: Internal temperature. Bottom: manipulated variables for each actuator.

4.3 Analysis of the results

To provide a quantitative comparison between the MPC approach and the GSRC the integral of the absolute error (IAE) and the cost of usage of the inputs was computed. While the original case study never stated actual values for the cost of each input, here, for simple illustration, the cost for hot water and cold water is considered to be the same and the cost of using the air conditioning and the electric heating is five times that much, reflecting the weights used in the MPC formulation used in (Reyes-Lúa and Skogestad, 2019). The comparison is presented in Table 3.

Table 3. Performance indexes for each approach.

Controller	IAE	Input Usage Cost
GSRC	269.1754	55.0938
GPC	34.1586	218.4820
SRGPC	31.5887	58.6588
SRGPC+FF	9.9120	50.1253

The comparison include both the GSRC and the GPC with the input sequencing constraints, dubbed SRGPC in the table, which were detailed in the previous section, along with a conventional GPC controller without binary decision variables, and a version of SRGPC with the inclusion of the measurement of the disturbance to provide feedforward capabilities for the controller (thus SRGPC+FF in the table) [2].

Firstly, when comparing the conventional GPC with the GSRC it is noticeable that the input usage cost for the MPC controller is significantly larger, as the controller has no incentive in the formulation to abstain from using the more expensive inputs. This could be tackled by increasing the λ_i terms associated with the electric heater and the air conditioner, but would not completely stop the GPC from using them simultaneously with the cheaper inputs.

The GPC with the proposed constraints, however, is able to improve the IAE in more than seven times with only a 6.7% increase in the operation cost when compared to the GSRC. It is arguable that with the inclusion of the constraints that replicate the input sequencing behavior of

[2] The GPC and SRGPC with FF response curves are available in the supplementary material.

the GSRC, the GPC controller will take an economically efficient decision regarding the usage of the available inputs, and the performance difference should be reported to the tuning of both controllers.

Moreover, given that the external temperature measurement could be easily implemented on a real system, the inclusion of the disturbance model into the GPC further improves the performance.

Finally, it is important to remark that, while the implementation of the GSRC is possibly more convenient for smaller processes, due to its lower computational power requirement, in an industrial setting with multiple control loops, specially where MPC controllers are already deployed, the usage of the proposed approach could prove to be a suitable alternative to integrate the split-range economic properties into the MPC framework. Additionally, the flexibility of the MPC approach is desirable when some of the sequenced inputs share limited resources with other inputs controlling different outputs.

5. CONCLUSIONS

This study has presented a MPC interpretation of the Generalized Split Range Control, based on a novel MIQP formulation that consists in the inclusion of a set of constraints that enforce the sequential usage of the actuators. This allow an MPC to emulate the behavior seen in Split Range Controllers where a cheaper input is favored and only when that input saturates the next, more expensive, input is used. Furthermore, the proposal is flexible enough to accommodate applications where the costs associated with using specific inputs can vary depending on production campaigns or market conditions. In such cases, the prioritization order of inputs could be adjusted by the control panel operator, which would simply require internal switching between multiple versions of the constraints sent to the MIQP solver.

The results from the simulation have shown that the proposed formulation works as intended, granting the MPC controller the same properties seen in the GSRC. It is important to remark that formulating the optimization problem as an MIQP instead of a QP due to the addition of binary decision variables does not hinder the convexity of the problem, so the global optimality remains ensured. Furthermore, the similar input usage cost is a convincing evidence that the proposed controller is suitable to address this class of MISO systems where different inputs have different usage costs.

The improvement in the IAE when compared to the GSRC can be reputed to the tuning of both controllers and the GPC capabilities of dealing with dead time. Therefore, future work could revisit the comparison using Smith Predictors as the primary controllers for the GSRC. Additionally, further investigation should be devoted to cases where overlap between inputs is used in conventional split-range control to mitigate non-linearities and improve controllability near valve travel extremes.

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